The Grover search algorithm

Quantum Computing Minicourse

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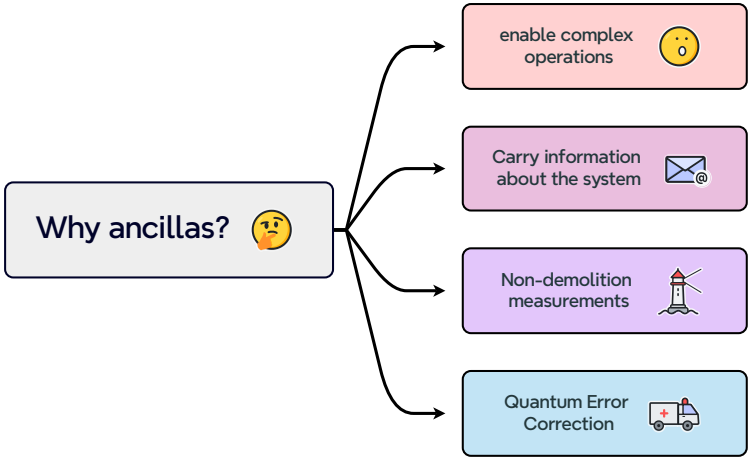
Ancillary qubits

Ancilla qubits are extra qubits, which help a qubit system in some computations. For example:



The phase kickback

One clever trick we can implement using ancillas is the **phase kickback**.

1. Take into account two qubits: a system qubit q and an ancilla a.
2. Suppose we prepare q into a superposed state and a into the excited state:

$$|q\rangle = H|0\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}, \qquad |a\rangle = |1\rangle .$$

3. Let's consider now a controlled Z operation.
4. What happen if we now apply the CZ using q as control and a as target?
5. we could expect something happen on the target qubit! Not really:

$$CZ\left(\frac{|0\rangle + |1\rangle}{\sqrt{2}} \otimes |1\rangle\right) = CZ\left(\frac{|01\rangle + |11\rangle}{\sqrt{2}}\right) = \frac{|01\rangle + Z|11\rangle}{\sqrt{2}} = \frac{|01\rangle - |11\rangle}{\sqrt{2}} = \frac{|0\rangle - |1\rangle}{\sqrt{2}} \otimes |1\rangle .$$

6. acting on a we got a phase kickback on the system!

Important

This happens if the ancilla is prepared into a state which is eigenvector of the controlled operation.