GRPS1024

Library of the groups of order 1024.

0.0.5

20 June 2022

David Burrell

David Burrell
Email: davidburrell@ufl.edu
Homepage: https://davidburrell.github.io/

Contents

Chapter 1

Groups of Order 1024

1.1 Overview

This library gives explicit access to the following groups of order 1024:

- The rank 1 group
- All rank 2 groups
- All rank 3 groups
- All rank 4 groups
- Rank 5 groups with p-class at least 3
- Rank 6 groups with p-class at least 3
- Rank 7 groups with p-class at least 3
- Rank 8 groups with p-class at least 3
- Rank 9 groups with p-class at least 3
- The rank 10 group

This library gives partial information on the remaining groups of order 1024:

- Rank 5 groups with p-class 2
- Rank 6 groups with p-class 2
- Rank 7 groups with p-class 2
- Rank 8 groups with p-class 2
- Rank 9 groups with p-class 2

For the groups that are not explicitly available the following information is available:

- Parent Group ID

- Parent Group Order
- p-class
- Rank
- Age

The groups are sorted first by their parent group ids and then by the pc codes of the standard presentations for the groups. The data contained in this library was used in the 2021 enumeration of the groups of order 1024 [Bur21]. The available groups were generated using the p-group generation algorithm [O'B90] as implemented in the ANUPQ package [GNOH19]. The information on the remaining groups was calculated using the cohomological methods for enumerating p-groups of p-class 2 as introduced in [EO99].

Chapter 2

Functionality

2.1 Methods

Once the package is loaded the user may call `SmallGroup(1024,i)` and receive either a group if available or a *partially constructed group* which has the following attributes set

- p-class
- Rank
- Heritage
- Order

Example

```
gap> G:=SmallGroup(1024,1);
<pc group of size 1024 with 10 generators>
gap> Rank(G);
1
gap> PClassPGroup(G);
10
gap> GRPS1024_Heritage(G);
[ 512, 1, 1 ]
gap> H:=SmallGroup(1024,3568); #this is a partially constructed group
<pc group with 0 generators>
gap> PClassPGroup(H);
2
gap> RankPGroup(H);
5
gap> GRPS1024_Heritage(H);
[ 32, 51, 1 ]
gap> K:=SmallGroup(1024,3569); #this is a partially constructed group
<pc group with 0 generators>
gap> PClassPGroup(K);
2
gap> RankPGroup(K);
5
gap> GRPS1024_Heritage(K);
[ 32, 51, 2 ]
#notice that H,K have the same parent group but their age differs
```

2.1.1 Groups1024Information

▷ `Groups1024Information(arg)` (function)

prints information on the groups of the specified order.

Example

```
gap> Groups1024Information();
##################  Groups Information  ########################
There are 49487367289 groups of order 1024
They are sorted by rank, p-class, parent group and then age

Group  1                                 has rank 1 and pclass 10
Group  2                                 has rank 2 and pclass 3
Groups 3-1912                                 have rank 2 and pclass 4
Groups 1913-6569                         have rank 2 and pclass 5
Groups 6570-8638                         have rank 2 and pclass 6
Groups 8639-9077                         have rank 2 and pclass 7
Groups 9078-9117                         have rank 2 and pclass 8
Groups 9118-9122                         have rank 2 and pclass 9
Groups 9123-319435                         have rank 3 and pclass 3
Groups 319436-708057                         have rank 3 and pclass 4
Groups 708058-781241                         have rank 3 and pclass 5
Groups 781242-789631                         have rank 3 and pclass 6
Groups 789632-789820                         have rank 3 and pclass 7
Groups 789821-789829                         have rank 3 and pclass 8
Groups 789830-793395                         have rank 4 and pclass 2
Groups 793396-7180625                         have rank 4 and pclass 3
Groups 7180626-8792073                         have rank 4 and pclass 4
Groups 8792074-8843732                         have rank 4 and pclass 5
Groups 8843733-8844822                         have rank 4 and pclass 6
Groups 8844823-8844836                         have rank 4 and pclass 7
Groups 8844837-387473667                 have rank 5 and pclass 2 ## Not Available ##
Groups 387473668-752623856                 have rank 5 and pclass 3
Groups 752623857-754063194                 have rank 5 and pclass 4
Groups 754063195-754066166                 have rank 5 and pclass 5
Groups 754066167-754066184                 have rank 5 and pclass 6
Groups 754066185-48452082590                 have rank 6 and pclass 2 ## Not Available ##
Groups 48452082591-48760455837                 have rank 6 and pclass 3
Groups 48760455838-48760467931                 have rank 6 and pclass 4
Groups 48760467932-48760467954                 have rank 6 and pclass 5
Groups 48760467955-49487311927                 have rank 7 and pclass 2 ## Not Available ##
Groups 49487311928-49487364283                 have rank 7 and pclass 3
Groups 49487364284-49487364310                 have rank 7 and pclass 4
Groups 49487364311-49487367243                 have rank 8 and pclass 2
Groups 49487367244-49487367275                 have rank 8 and pclass 3
Groups 49487367276-49487367288                 have rank 9 and pclass 2 ## Not Available ##
Group  49487367289                       has rank 10 and pclass 1
This library was created by David Burrell (2022).
```

References

[Bur21] D. Burrell. On The Number of Groups of Order 1024. *Communications in Algebra*, 0(0):1–3, 2021. 4

[EO99] B. Eick and E. A. O'Brien. Enumerating p -Groups. *Journal of the Australian Mathematical Society. Series A. Pure Mathematics and Statistics*, 67(2):191–205, dec 1999. 4

[GNOH19] G. Gamble, W. Nickel, E. O'Brien, and M. Horn. ANU p-Quotient, 2019. 4

[O'B90] E. A. O'Brien. The p-group generation algorithm. *Journal of Symbolic Computation*, 9(5):677–698, oct 1990. 4

Index