

# GRPS39

## Library of the groups of order 19683

0.0.1

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# Chapter 1

## Groups of Order 19683

### 1.1 Overview

This library gives complete access to the following groups of order 19683:

- The rank 1 group
- All rank 2 groups
- All rank 3 groups with p-class not equal to 3
- All rank 4 groups with p-class at least 4
- All rank 5 groups with p-class at least 4
- All rank 6 groups with p-class at least 4
- All rank 7 groups with p-class at least 4
- All rank 8 groups with p-class at least 3
- The rank 9 group

This library gives partial information on the remaining groups of order 19683:

- Rank 3 groups with p-class 3
- Rank 4 groups with p-class 2
- Rank 4 groups with p-class 3
- Rank 5 groups with p-class 2
- Rank 5 groups with p-class 3
- Rank 6 groups with p-class 2
- Rank 6 groups with p-class 3
- Rank 7 groups with p-class 2

- Rank 7 groups with p-class 3

For the groups that are not explicitly available the following information is available:

- Parent Group ID
- Parent Group Order
- p-class
- Rank
- Age

The groups are sorted first by their rank, then by p-class, then by parent group ids and finally by age. The data contained in this library was used in the 2022 enumeration of the groups of order 19683 [Bur22]. The computational tools used were developed in the 2021 enumeration of the groups of order 1024 [Bur21]. The available groups were generated using the p-group generation algorithm [O'B90] as implemented in the ANUPQ package [GNOH19]. The information on the remaining groups was calculated using the cohomological methods for enumerating p-groups as introduced in [EO99].

## Chapter 2

# Functionality

### 2.1 Methods

Once the package is loaded the user may call `SmallGroup(19683,i)` and receive either a group if available or a *partially constructed group* which has the following attributes set

- `p-class`
- `Rank`
- `Heritage`
- `Order`

Additional information can be obtained via `SmallGroupsInformation(19683)`.

Example

```
gap> G:=SmallGroup(19683,1);
<pc group of size 19683 with 9 generators>
gap> RankPGroup(G);
1
gap> PClassPGroup(G);
9
gap> GRPS39_Heritage(G);
[ 6561, 1, 1 ]
gap> H:=SmallGroup(19683,546913);
<group> #this is a partially constructed group
gap> PClassPGroup(H);
3
gap> RankPGroup(H);
3
gap> GRPS39_Heritage(H);
[ 729, 122, 1 ]
gap> K:=SmallGroup(19683,546913+1);
<group> #this is a partially constructed group
gap> PClassPGroup(K);
3
gap> RankPGroup(K);
3
gap> GRPS39_Heritage(K);
```

```
[ 729, 122, 2 ]
```

```
#notice that H,K have the same parent group but their age differs
```

### 2.1.1 Groups19683Information

▷ Groups19683Information(*arg*)

(function)

prints information on the groups of order 19683 (SmallGroupsInformation(19683) also works).

Example

```
gap> Groups19683Information();
```

```
##### Groups Information #####
```

```
There are 5,937,876,645 groups of order 19,683
```

```
They are sorted by rank, p-class, parent group and then age
```

```
Group          1                has rank 1 and pclass 9
```

```
Groups         2-12             have rank 2 and pclass 3
```

```
Groups         13-1431          have rank 2 and pclass 4
```

```
Groups         1432-12482       have rank 2 and pclass 5
```

```
Groups         12483-16608      have rank 2 and pclass 6
```

```
Groups         16609-17592      have rank 2 and pclass 7
```

```
Groups         17593-17600      have rank 2 and pclass 8
```

```
Group          17601            has rank 3 and pclass 2
```

```
Groups         17602-88975117   have rank 3 and pclass 3
```

```
Groups with IDs 546913-88541296 are not available and are immediate descendants of 729#122
```

```
Groups         88975118-91756404 have rank 3 and pclass 4
```

```
Groups         91756405-91883170 have rank 3 and pclass 5
```

```
Groups         91883171-91887160 have rank 3 and pclass 6
```

```
Groups         91887161-91887185 have rank 3 and pclass 7
```

```
Groups         91887186-91953852 have rank 4 and pclass 2
```

```
Groups with IDs 91887186-91953852 are not available and are immediate descendants of 81#15
```

```
Groups         91953853-340943248 have rank 4 and pclass 3
```

```
Groups with IDs 92241644-209726506 are not available and are immediate descendants of 729#425
```

```
Groups with IDs 209726507-227782267 are not available and are immediate descendants of 729#440
```

```
Groups with IDs 227782268-258162927 are not available and are immediate descendants of 729#453
```

```
Groups with IDs 274401608-281443634 are not available and are immediate descendants of 2187#6044
```

```
Groups with IDs 304813802-317529080 are not available and are immediate descendants of 2187#6576
```

```
Groups         340943249-355792512 have rank 4 and pclass 4
```

```
Groups         355792513-355820119 have rank 4 and pclass 5
```

```
Groups         355820120-355820169 have rank 4 and pclass 6
```

```
Groups         355820170-4032242074 have rank 5 and pclass 2
```

```
Groups with IDs 355820170-4032242074 are not available and are immediate descendants of 243#67
```

```
Groups         4032242075-4717133004 have rank 5 and pclass 3
```

```
Groups with IDs 4032242075-4032242205 are not available and are immediate descendants of 2187#909
```

```
Groups with IDs 4045509432-4062606128 are not available and are immediate descendants of 2187#911
```

```
Groups with IDs 4062606129-4103654980 are not available and are immediate descendants of 2187#912
```

```
Groups with IDs 4110319952-4123052906 are not available and are immediate descendants of 2187#912
```

```

Groups with IDs 4123052907-4451185904 are not available and are immediate descendants of 2187#913
Groups with IDs 4451185905-4500755636 are not available and are immediate descendants of 2187#913
Groups with IDs 4500755637-4599685634 are not available and are immediate descendants of 2187#913
Groups with IDs 4599685635-4612426328 are not available and are immediate descendants of 2187#913
Groups with IDs 4612426329-4616681990 are not available and are immediate descendants of 2187#914

Groups          4717133005-4717651022          have rank 5 and pclass 4
Groups          4717651023-4717651101          have rank 5 and pclass 5
Groups          4717651102-5937761741          have rank 6 and pclass 2
Groups with IDs 4717651102-5937761741 are not available and are immediate descendants of 729#504

Groups          5937761742-5937874762          have rank 6 and pclass 3
Groups with IDs 5937761742-5937811556 are not available and are immediate descendants of 2187#930

Groups          5937874763-5937874872          have rank 6 and pclass 4
Groups          5937874873-5937876585          have rank 7 and pclass 2
Groups with IDs 5937874873-5937876585 are not available and are immediate descendants of 2187#931

Groups          5937876586-5937876632          have rank 7 and pclass 3
Groups with IDs 5937876593-5937876632 are not available and are immediate descendants of 6561#139

Groups          5937876633-5937876644          have rank 8 and pclass 2
Groups with IDs 5937876633-5937876644 are not available and are immediate descendants of 6561#139

Group           5937876645 has rank 9 and pclass 1
This library was created by David Burrell (2022).

```

## 2.1.2 GRPS39\_CheckoutDescendants

▷ GRPS39\_CheckoutDescendants(*arg*) (function)

## 2.1.3 GRPS39\_Heritage (for IsGroup)

▷ GRPS39\_Heritage(*G*) (attribute)

**Returns:** list

Returns as a list the following information for a group of order 19683 loaded from the library [ParentGroupID, ParentGroupOrder, Age]. The Age of a group is the position of the group among its siblings in the ordered list of their standard PC codes (the order is determined by the output of the p-group generation algorithm).

# References

- [Bur21] D. Burrell. On The Number of Groups of Order 1024. *Communications in Algebra*, 0(0):1–3, 2021. [4](#)
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- [EO99] B. Eick and E. A. O’Brien. Enumerating  $p$ -Groups. *Journal of the Australian Mathematical Society. Series A. Pure Mathematics and Statistics*, 67(2):191–205, dec 1999. [4](#)
- [GNOH19] G. Gamble, W. Nickel, E. O’Brien, and M. Horn. ANU  $p$ -Quotient, 2019. [4](#)
- [O’B90] E. A. O’Brien. The  $p$ -group generation algorithm. *Journal of Symbolic Computation*, 9(5):677–698, oct 1990. [4](#)



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