Lines, Curves, and Surfaces

CS 355: Introduction to Graphics and Image Processing

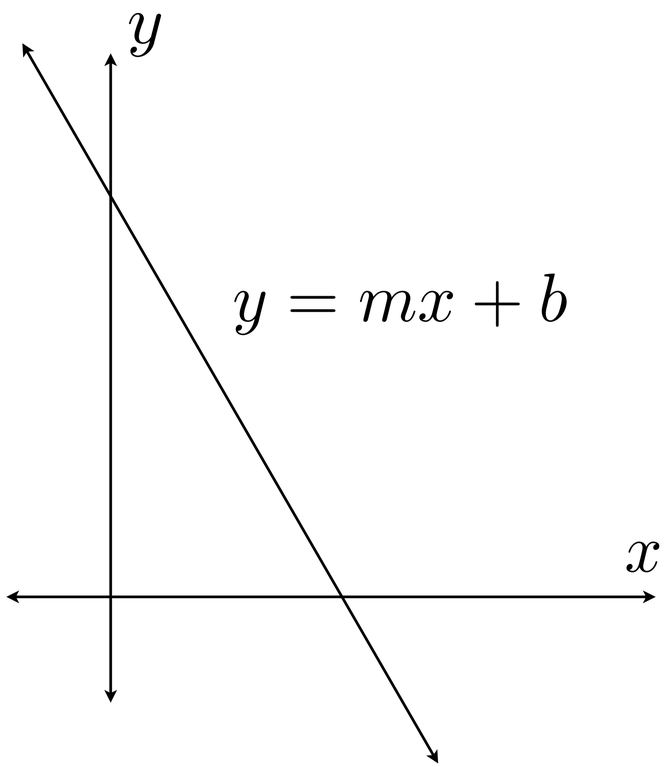
Lines

Lines extend infinitely in both directions

Line segments are finite

Rays extend infinitely in one direction

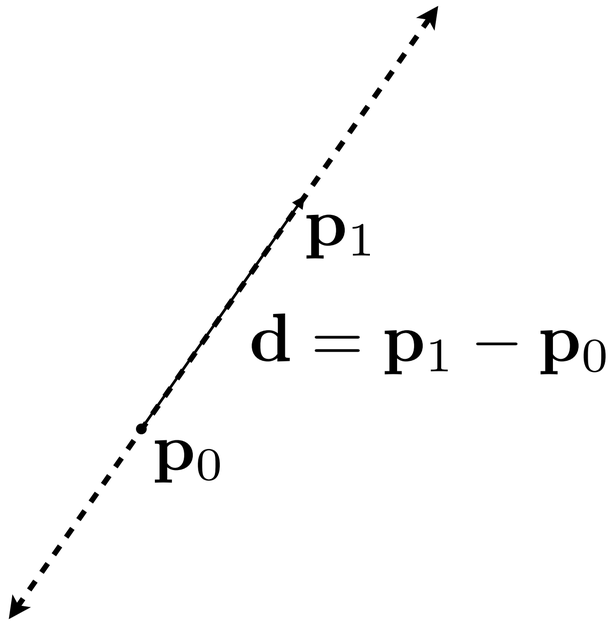
Slope / Intercept



Doesn’t work for vertical lines

Hard to extend to 3D

Parametric Representation



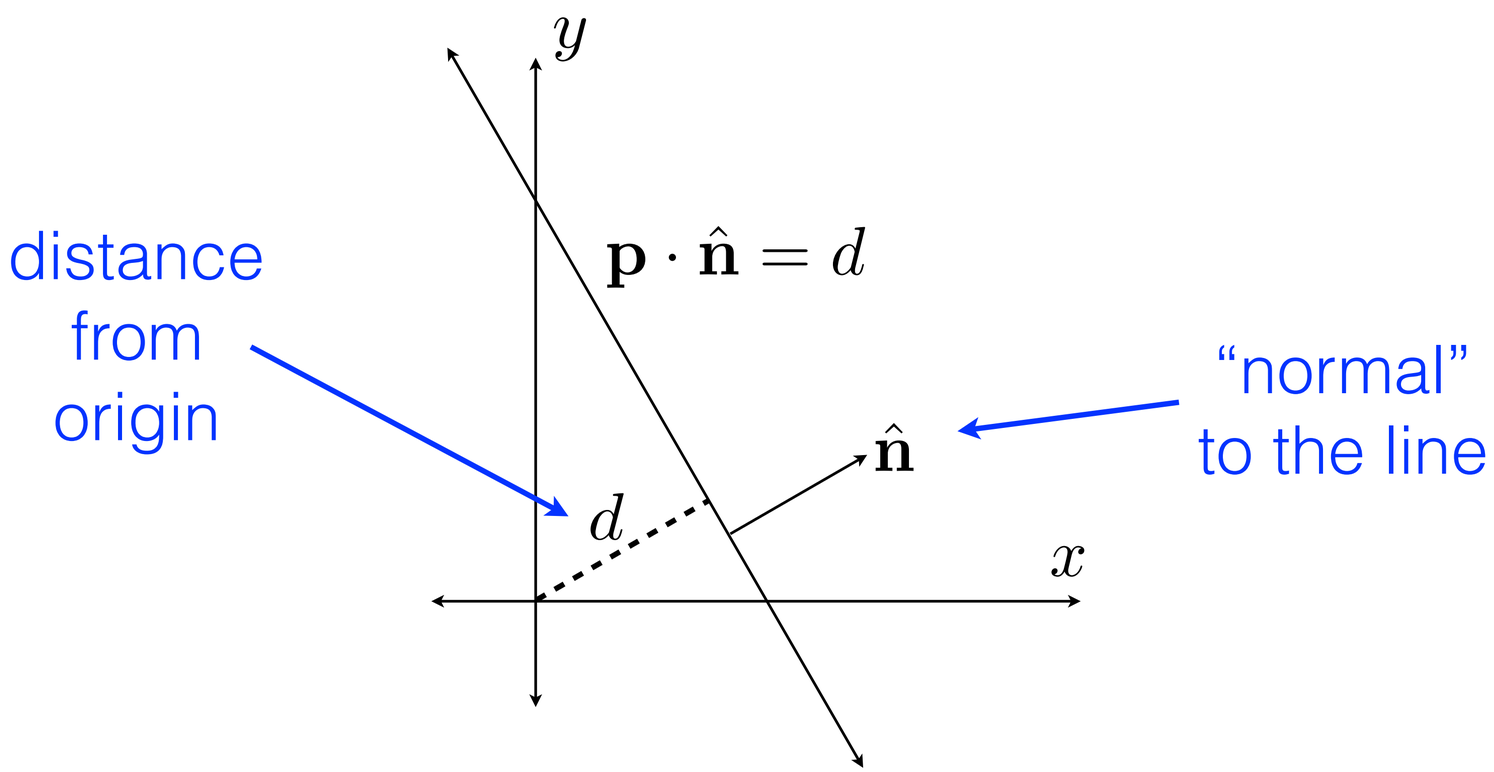
$$\mathbf{p}_0 + t\ \mathbf{d}$$

Line: $-\infty < t < \infty$

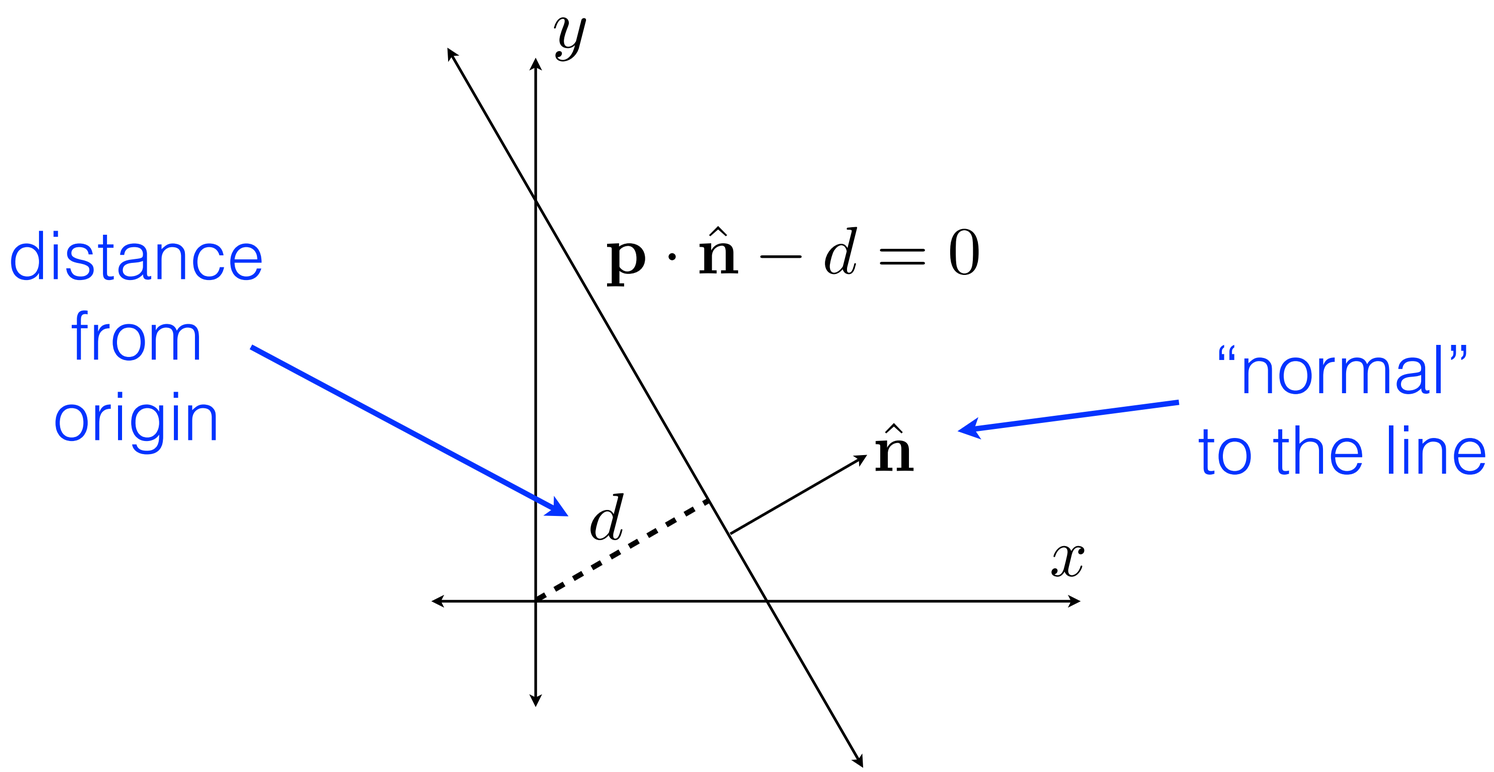
Line segment: $0 \leq t \leq 1$

Ray: $0 \leq t < \infty$

Normal + Distance



Implicit Representation



Aside: Lines and Distance

You've seen this before:

$$ax + by + c = 0$$

normal

negative distance

Normal and distance:

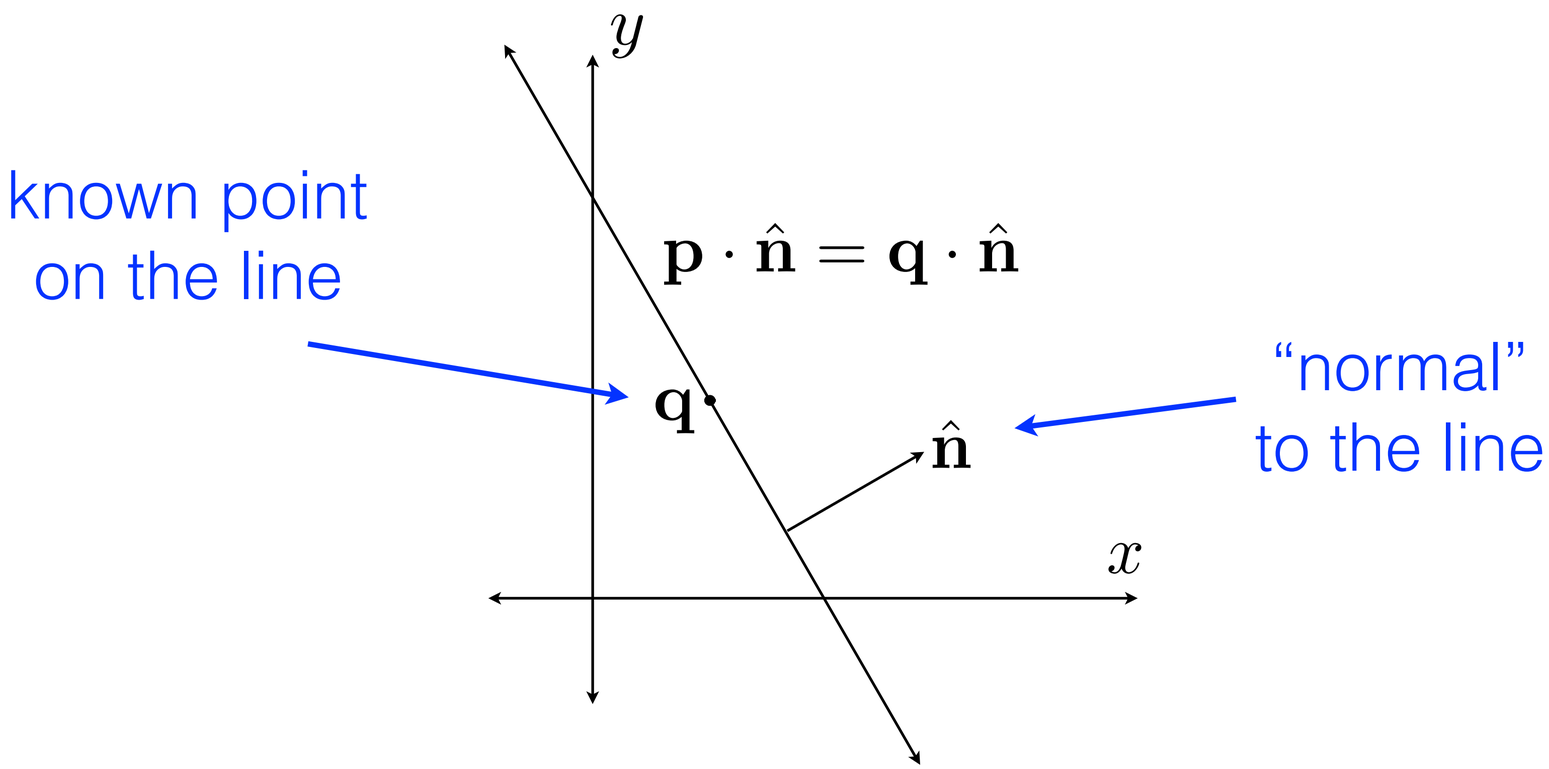
$$\mathbf{p} \cdot \hat{\mathbf{n}} = d$$

$$\mathbf{p} = \begin{bmatrix} x \\ y \end{bmatrix} \quad \hat{\mathbf{n}} = \begin{bmatrix} a \\ b \end{bmatrix}$$

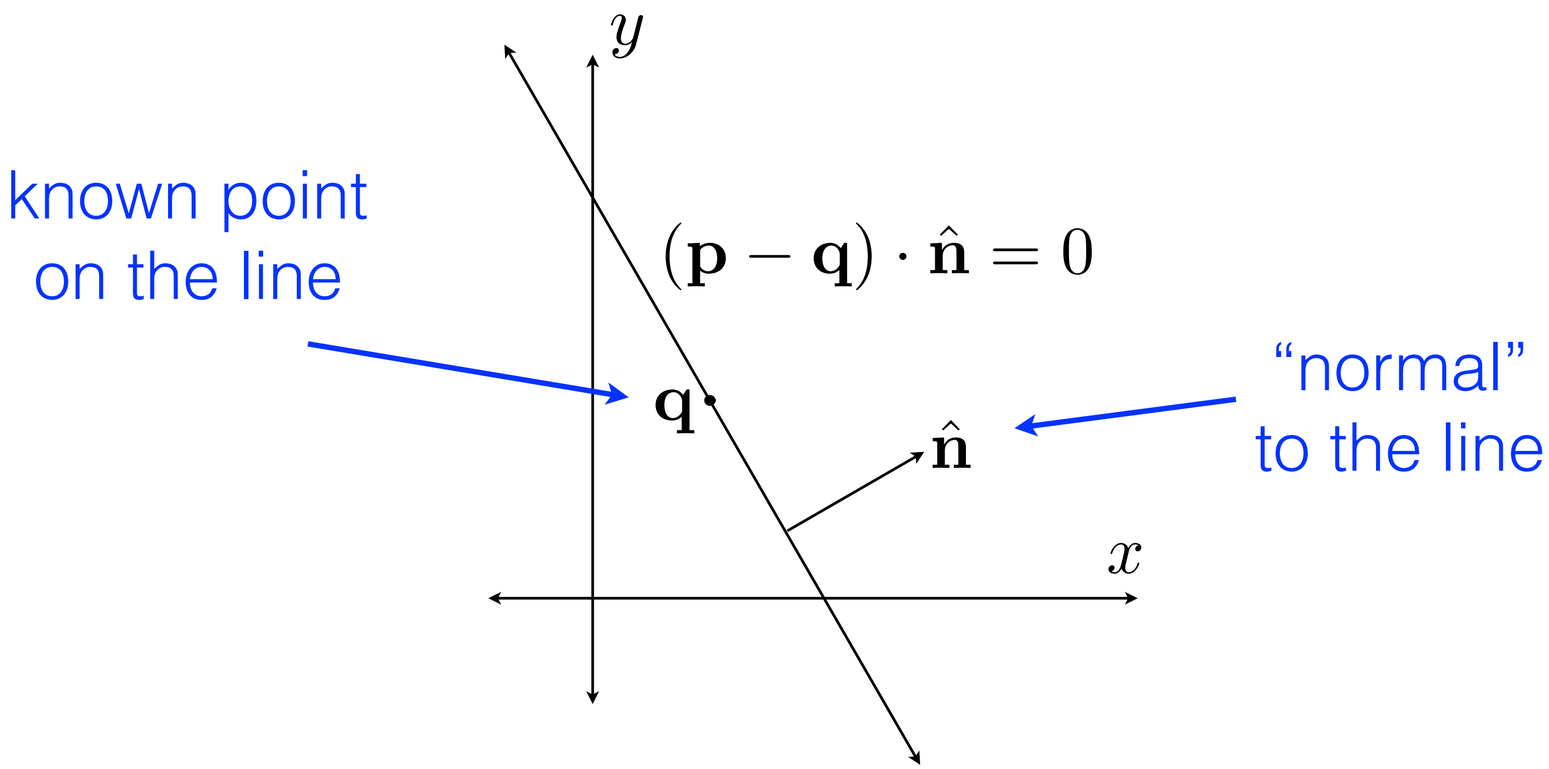
$$d = -c$$

This is really what you've seen years ago,
just in linear algebra form

Normal + Point



Implicit Representation



Representing Shapes

- Parametric
  (sweeps out the shape as a function of some parameters)

$$\mathbf{p}(t)$$

- Implicit
  (meets some test)

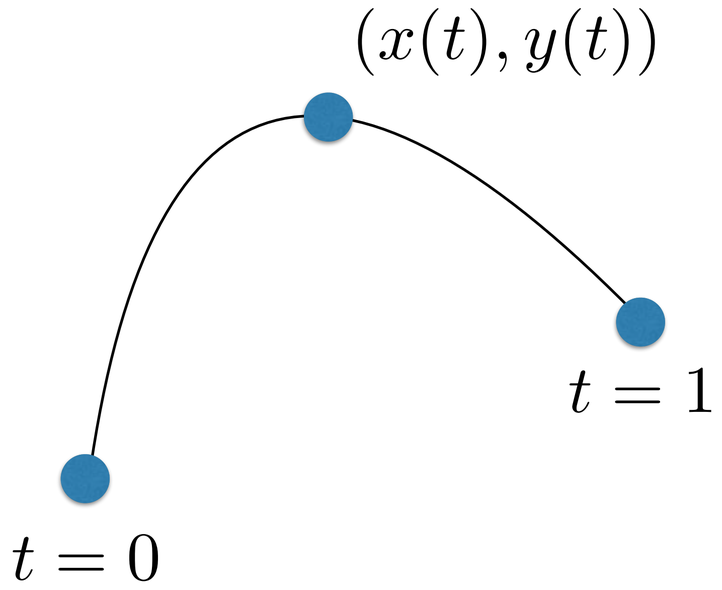
$$f(\mathbf{p}) = 0$$

- Others...

Can usually convert between representations

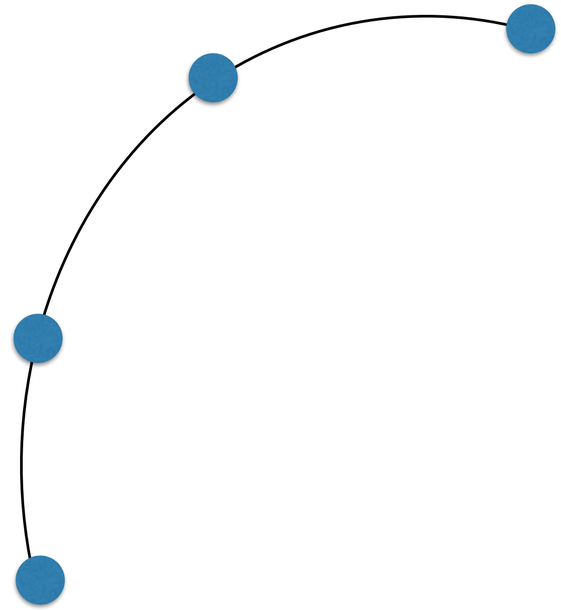
2D Curves

- Parametric curves
  - Parameter *t* traces the curve
  - One function for each dimension
  - Can extend to a space curve in any 3D or higher



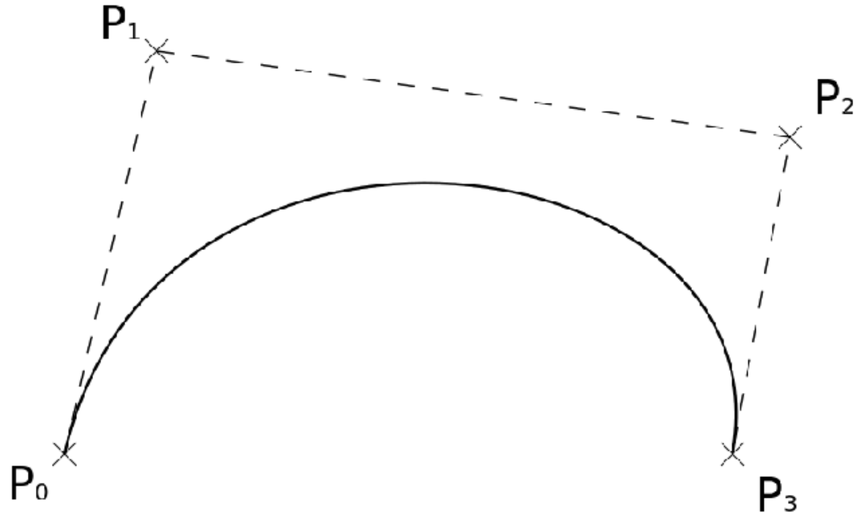
Cubic Curves

- Lots of types
  - Exact-fit curves (splines, etc.)
  - Bezier
  - B-splines
  - …



Bezier Curves

- Specify *end points* and a set of *control points* in between
- Curve doesn't (usually) pass through the control points
- Control points determine tangents at certain points on the curve
- Common in 2D drawing programs

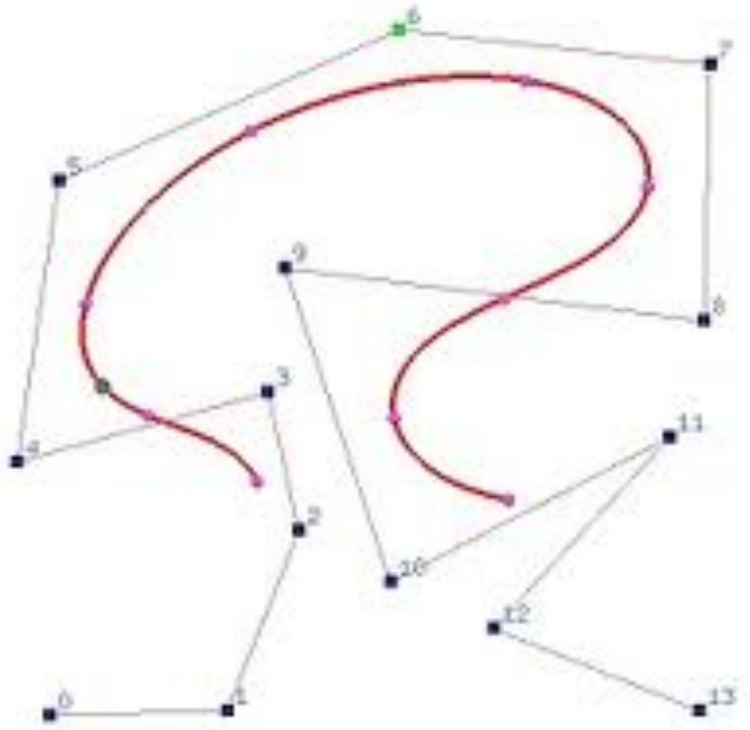


Complicated Curves

- Use higher-degree polynomials
  - Can make an nth-order Bezier curve out of two end points and *n-1* control points
  - Cumbersome and slow for long curves
  - Non-local control!
- Piecewise cubic segments
  - Can get 0th-order continuity by sharing endpoints
  - Can get 1st-order continuity by aligning control points

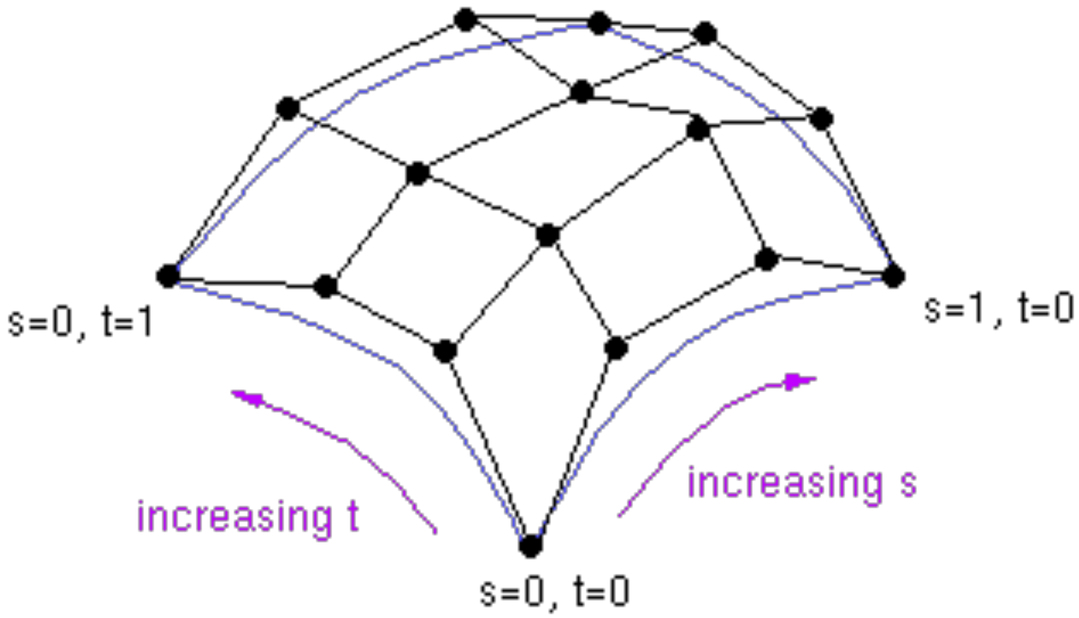
B-Splines

- More common to use *basis splines* (B-splines)
- Each local portion of the curve is a *weighted blend* of nth-order basis functions (usually cubic)



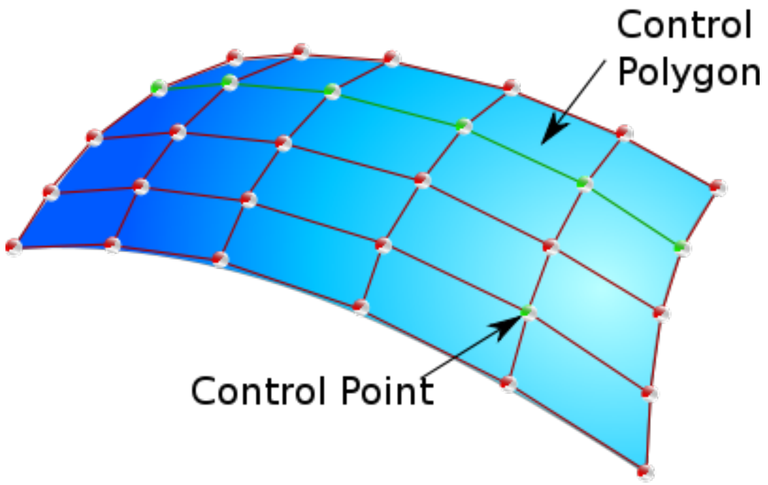
3D Surfaces

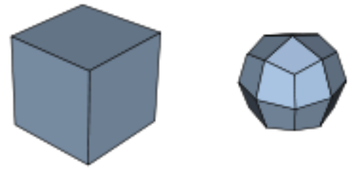
- Extend idea of Bezier or B-spline curves to a mesh or grid of control points
- Two parameters across surface: s, t
- Similar in principle to bicubic interpolation



3D Surfaces

- Bezier
- B-spline
- Non-Uniform Rational B-splines (NURBs)
- Catmull-Rom and other spline variations
- Subdivision surfaces





See CS 455 for more…

Coming up...

- Geometric tests