



# Image Warping

CS 355: Introduction to Graphics and Image Processing

# Geometric Operations

- Transformations (Shift, Rotation, etc.)
- Resizing
- Adding/Correcting a Warp
- Texture Mapping
- Morphing

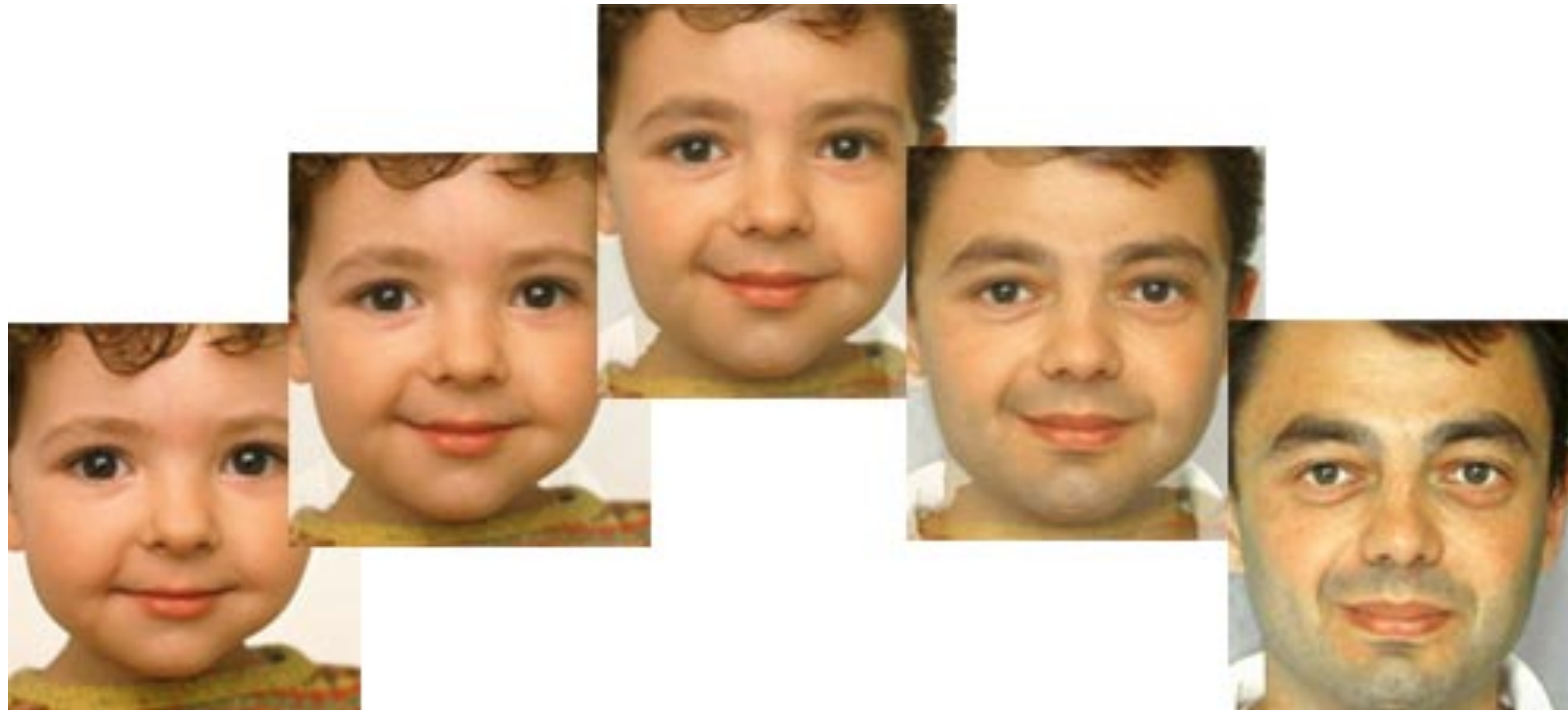
# Texture Mapping

- Mapping an image onto the surface of a geometric model
- Increased realism
- Requires warping image onto the object surface



# Image Morphing

- Warp a pair of images based on corresponding points
- Combination of warp/cross-dissolve





# Transformations

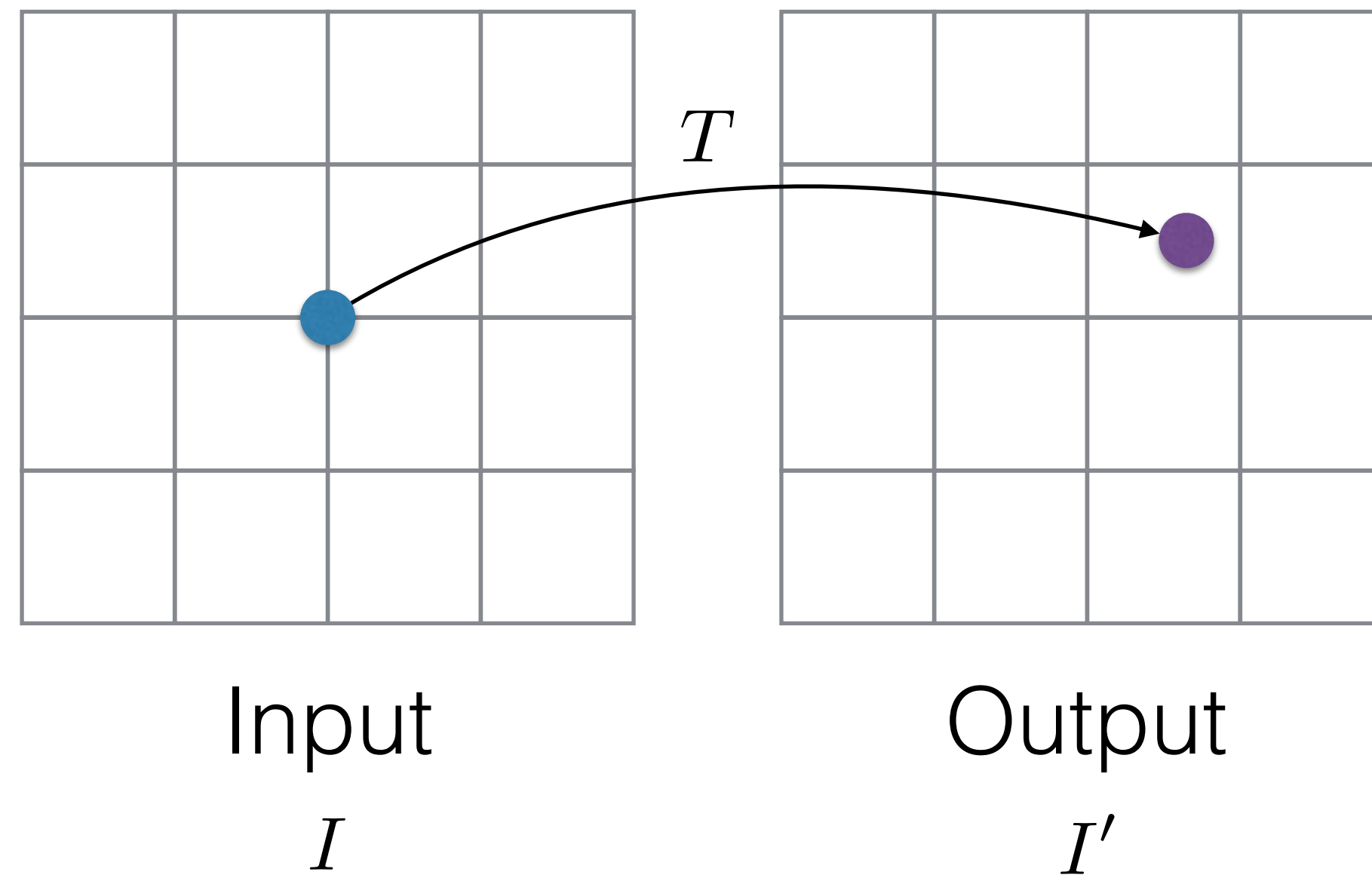
- Define what pixels go where
  - Translate (shift image)
  - Rotate around point
  - Scale (resize)
  - Affine
  - Perspective
  - Or anything else...



$$\mathbf{p}' = T(\mathbf{p})$$

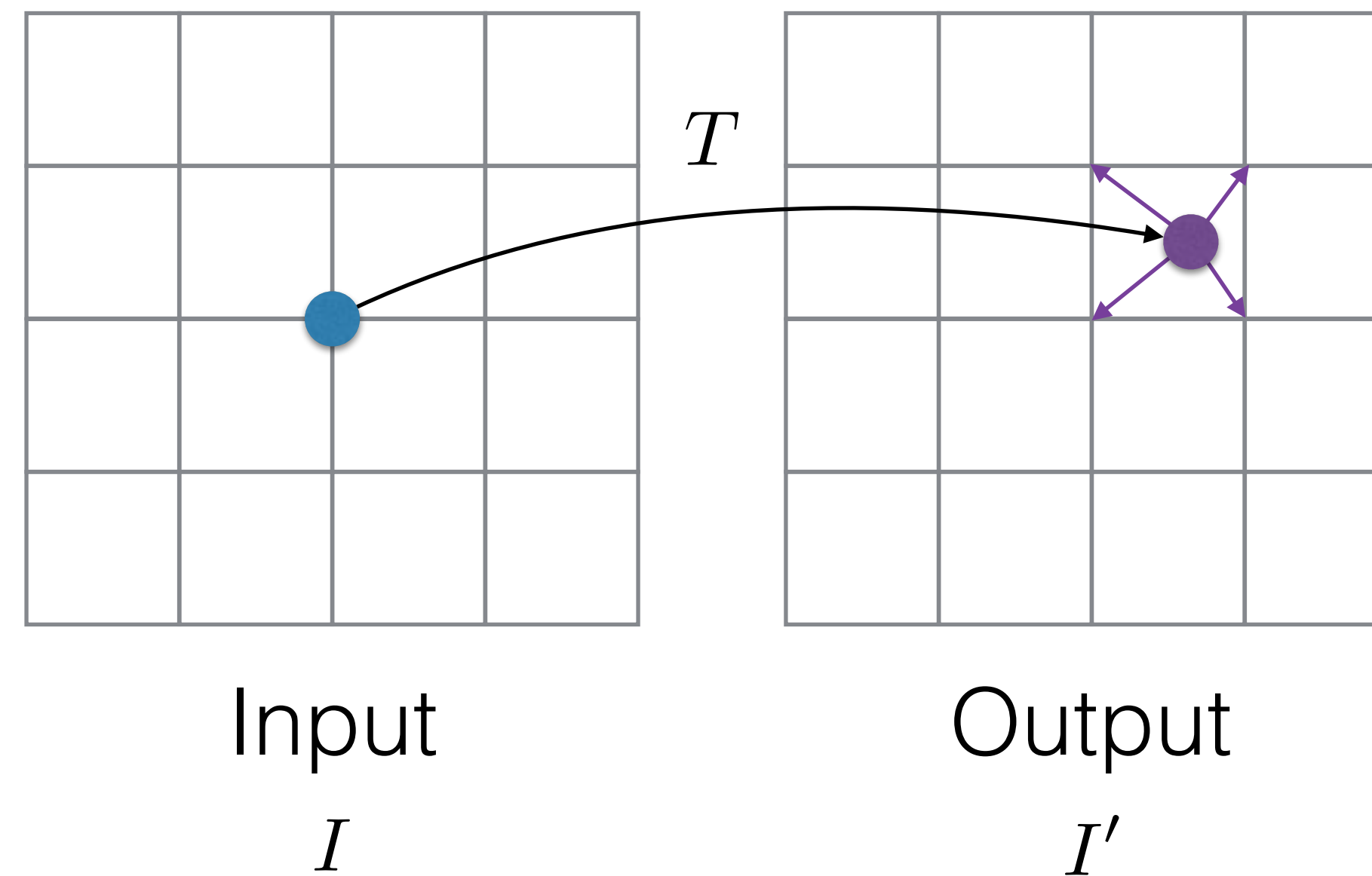
# Forward Mapping

$$I'(T(\mathbf{p})) = I(\mathbf{p})$$



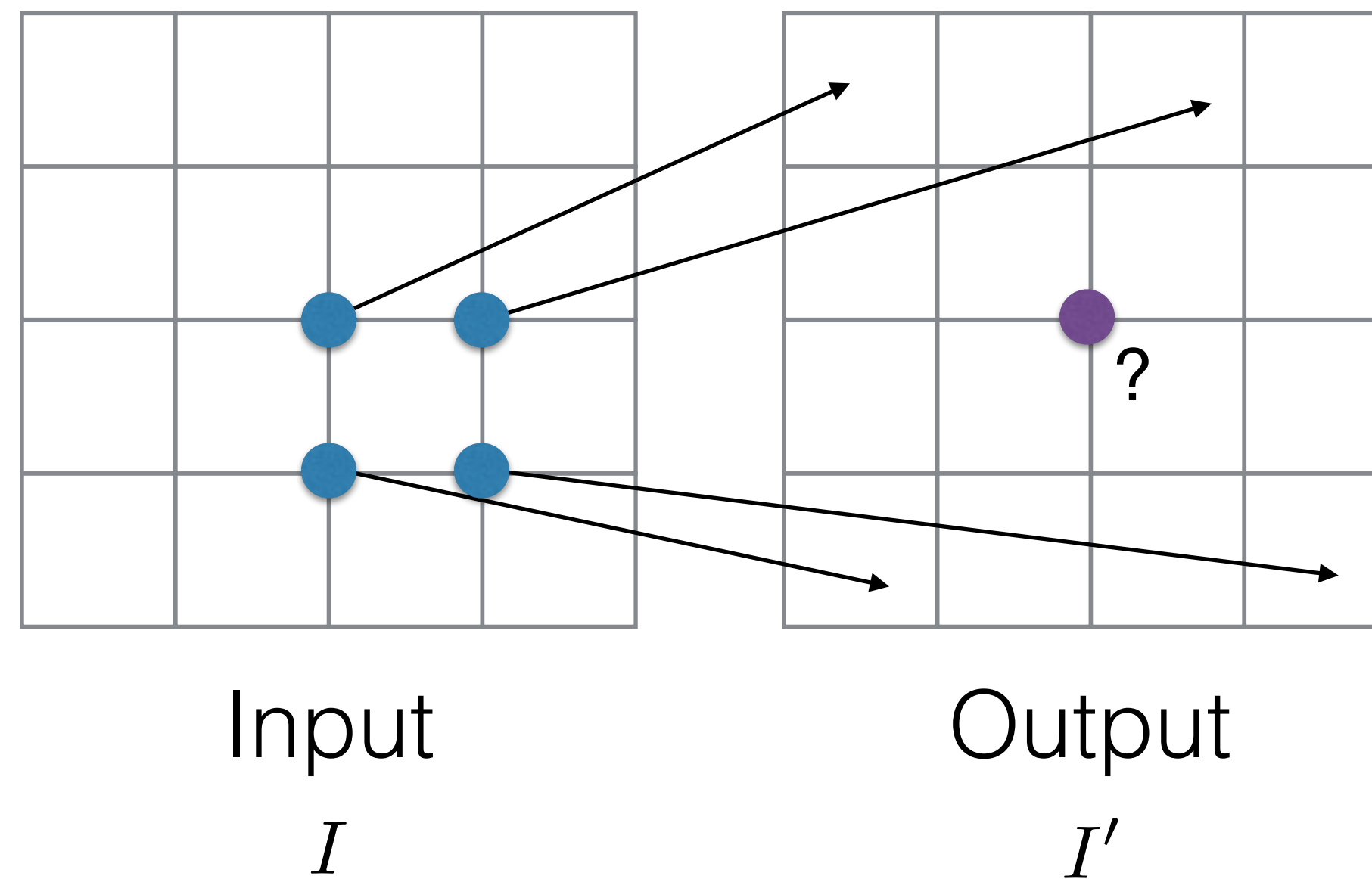
# Forward Mapping

Problem: doesn't map to discrete pixel locations



# Forward Mapping

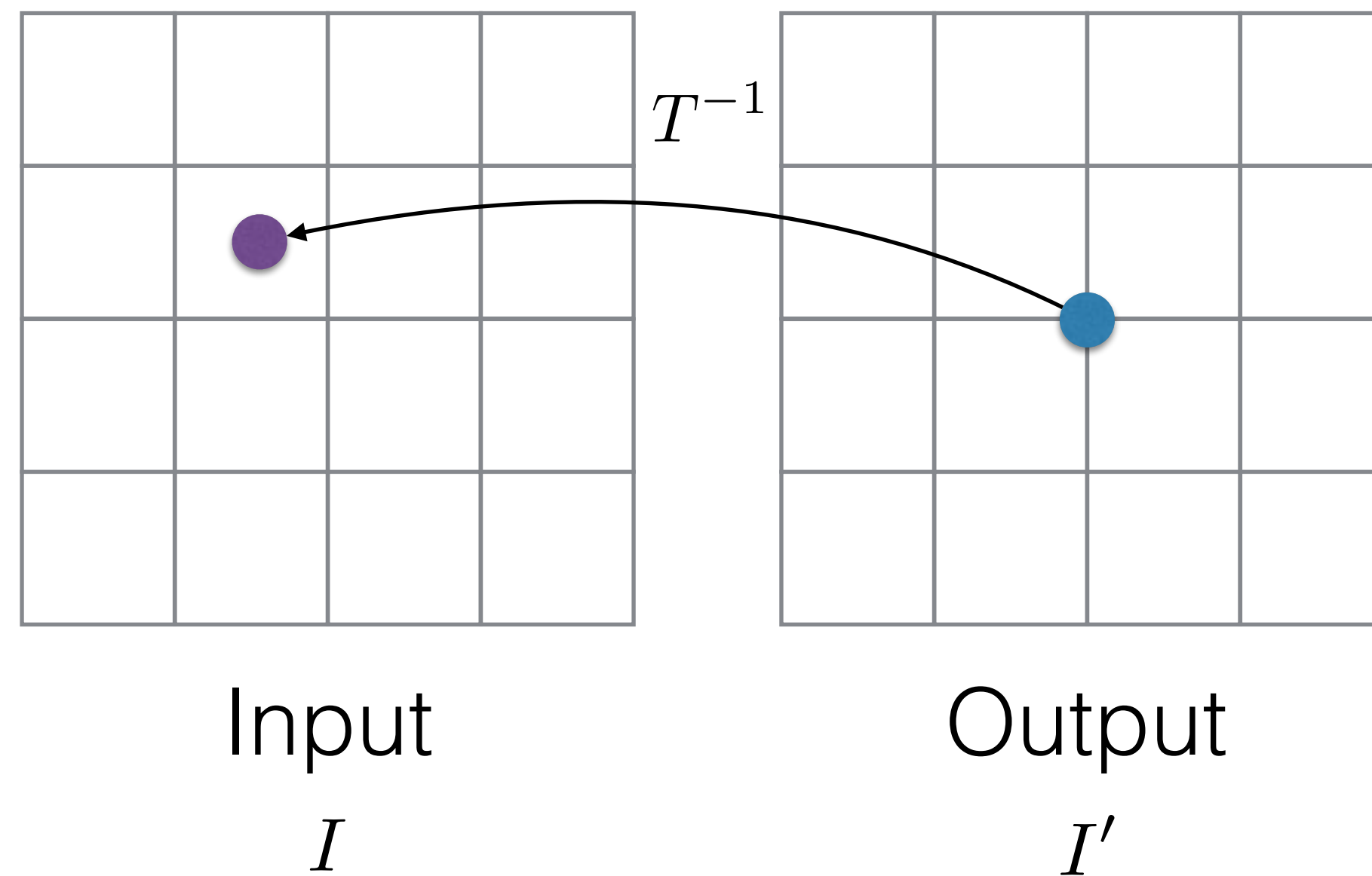
Problem: may produce holes in the output





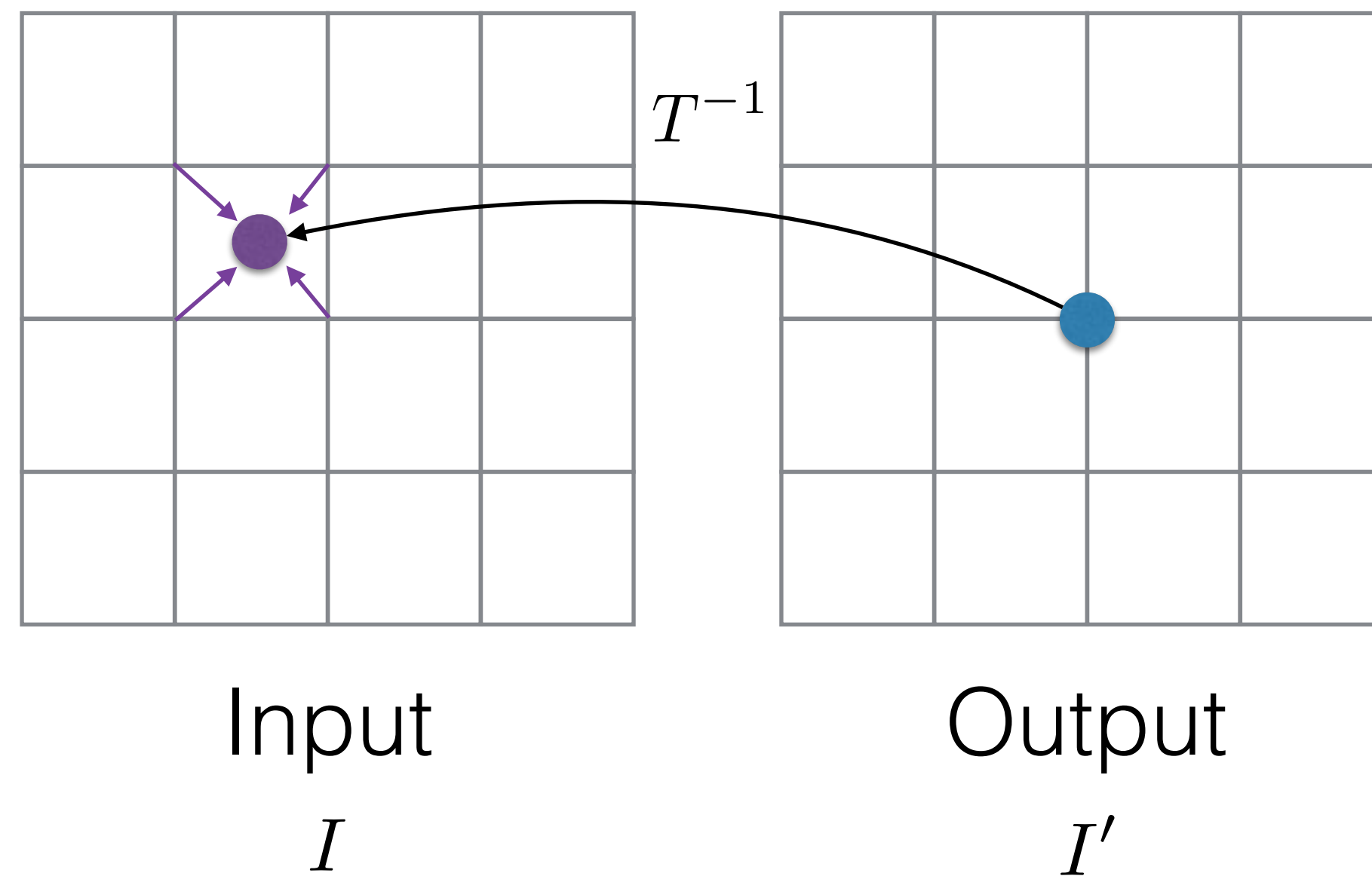
# Backward Mapping

$$\mathbf{p} = T^{-1}(\mathbf{p}')$$



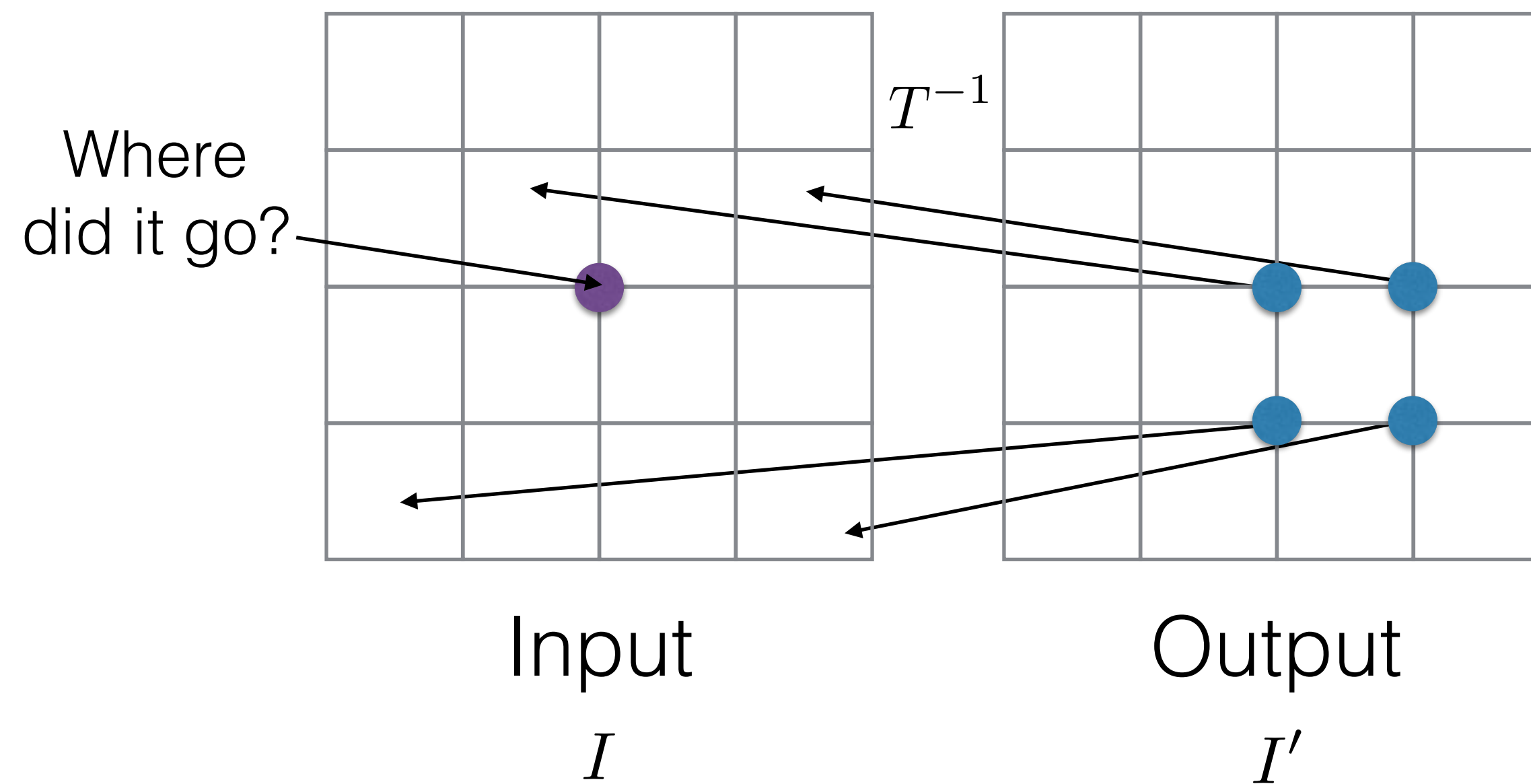
# Backward Mapping

Problem: doesn't map *from* discrete pixel locations



# Backward Mapping

Problem: may miss things in the input



# Backward Mapping

- Reducing an image (even locally) can cause aliasing and Moiré patterns just like insufficient sampling
- May have to blur first



# Forward vs. Backward

Forward mapping:

- Uses forward transformation
- Have to spread effect (“splatting”)
- Parallel writes (slow)
- May cause holes in output

Backward mapping:

if you can!

- Requires inverse transformation
- Standard interpolation (easy)
- Parallel reads (fast)
- May miss fine detail and/or cause aliasing

# Repeated Warping

- Each spatial transformation requires interpolation, which means some error (blurring, jaggies, etc.)
- Repeated transformations accumulate this error
- Better idea:
  - keep track of transformations
  - composite transformations
  - warp image once

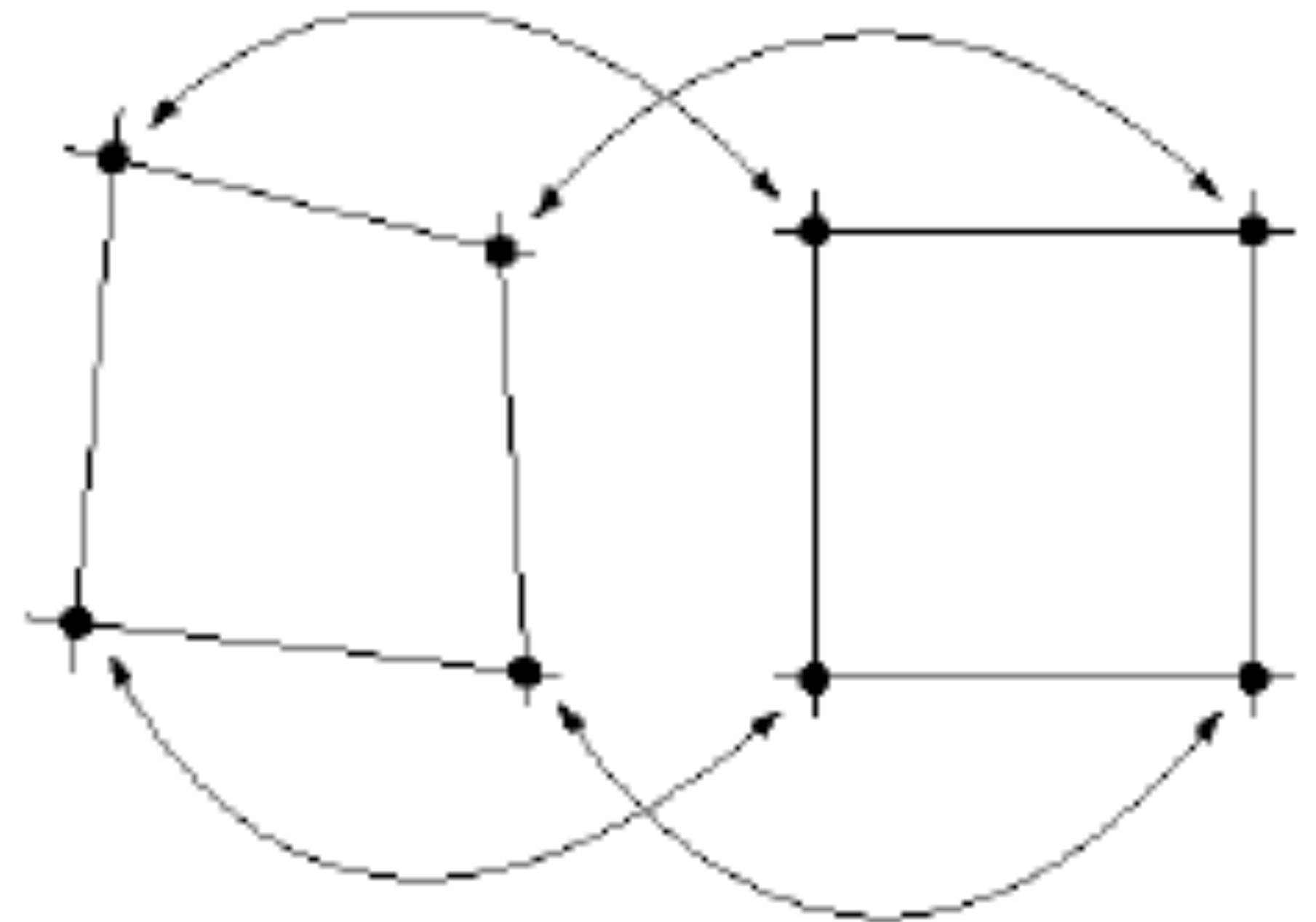


# Transformations Revisited

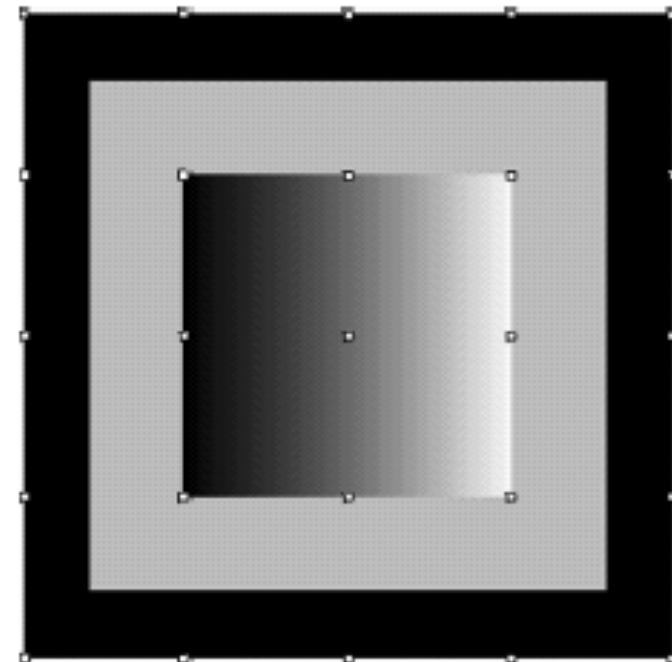
- Can be a single, closed-form transformation:
  - Scale, translation, rotation, skew, other affine, or even perspective
  - But all of the pixels undergo the same global transformation
- What about locally different transformations?

# Warp Meshes

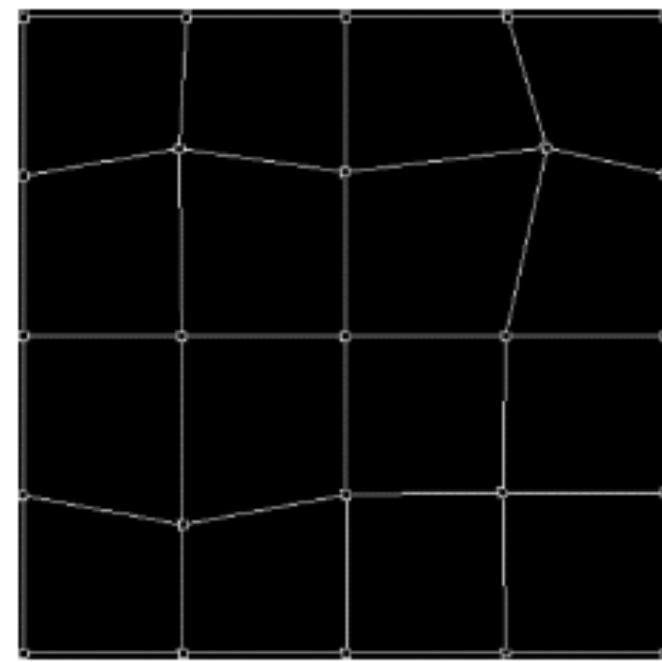
- Break source/destination images into meshes of quadrilaterals
- Map between corresponding “tiepoints”



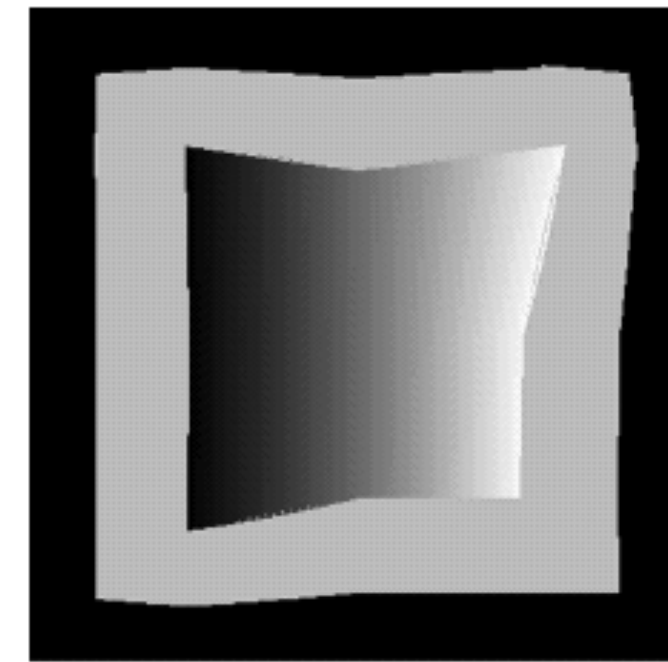
# Warp Meshes



Source with  
tie points

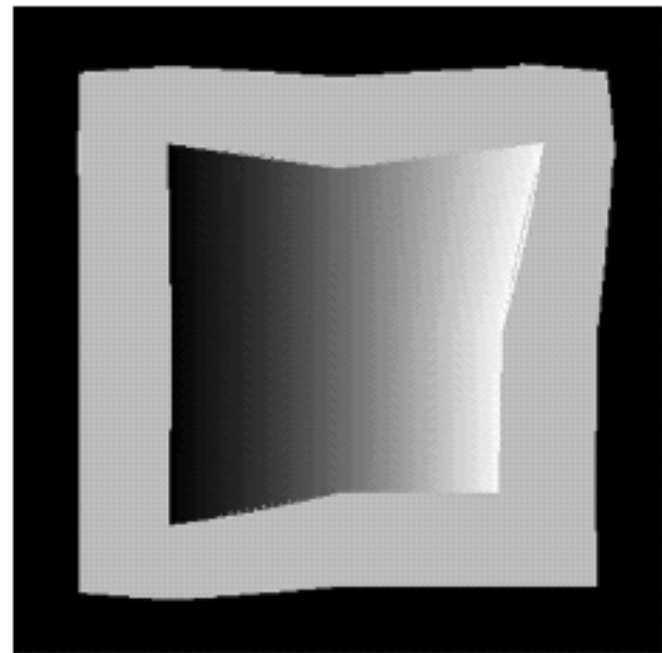


Destination  
mesh

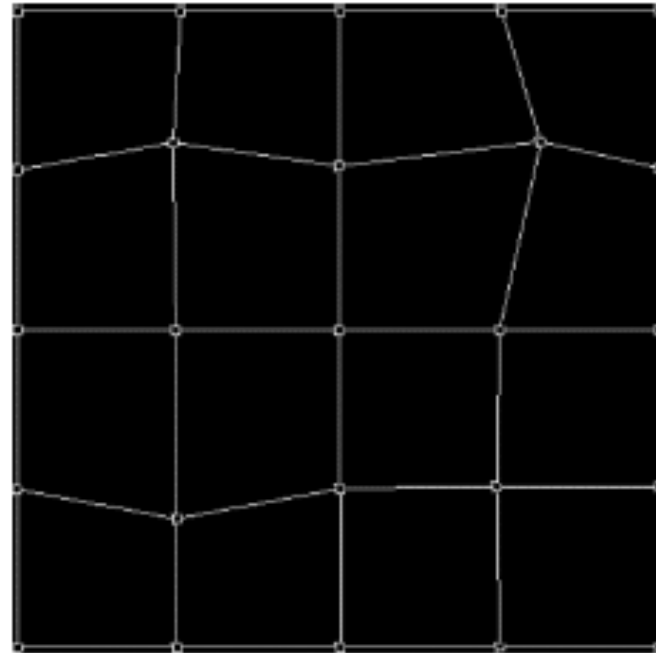


Result

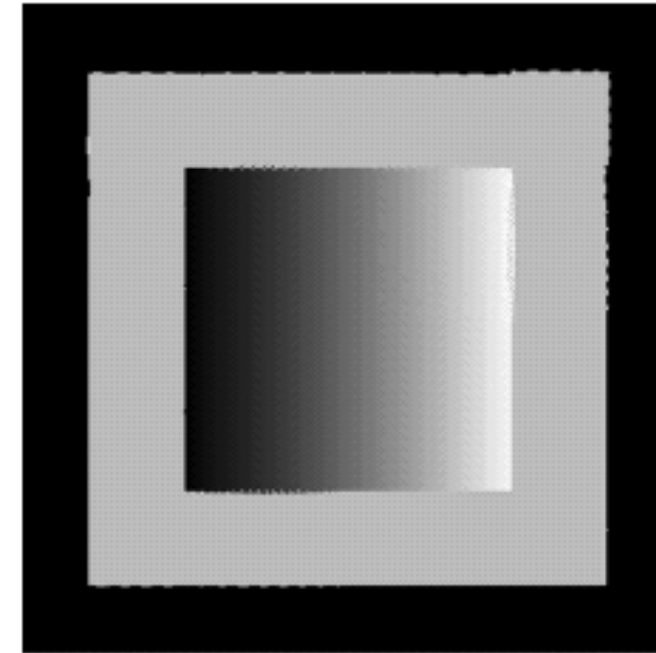
# Correcting Distortion



Distorted  
source



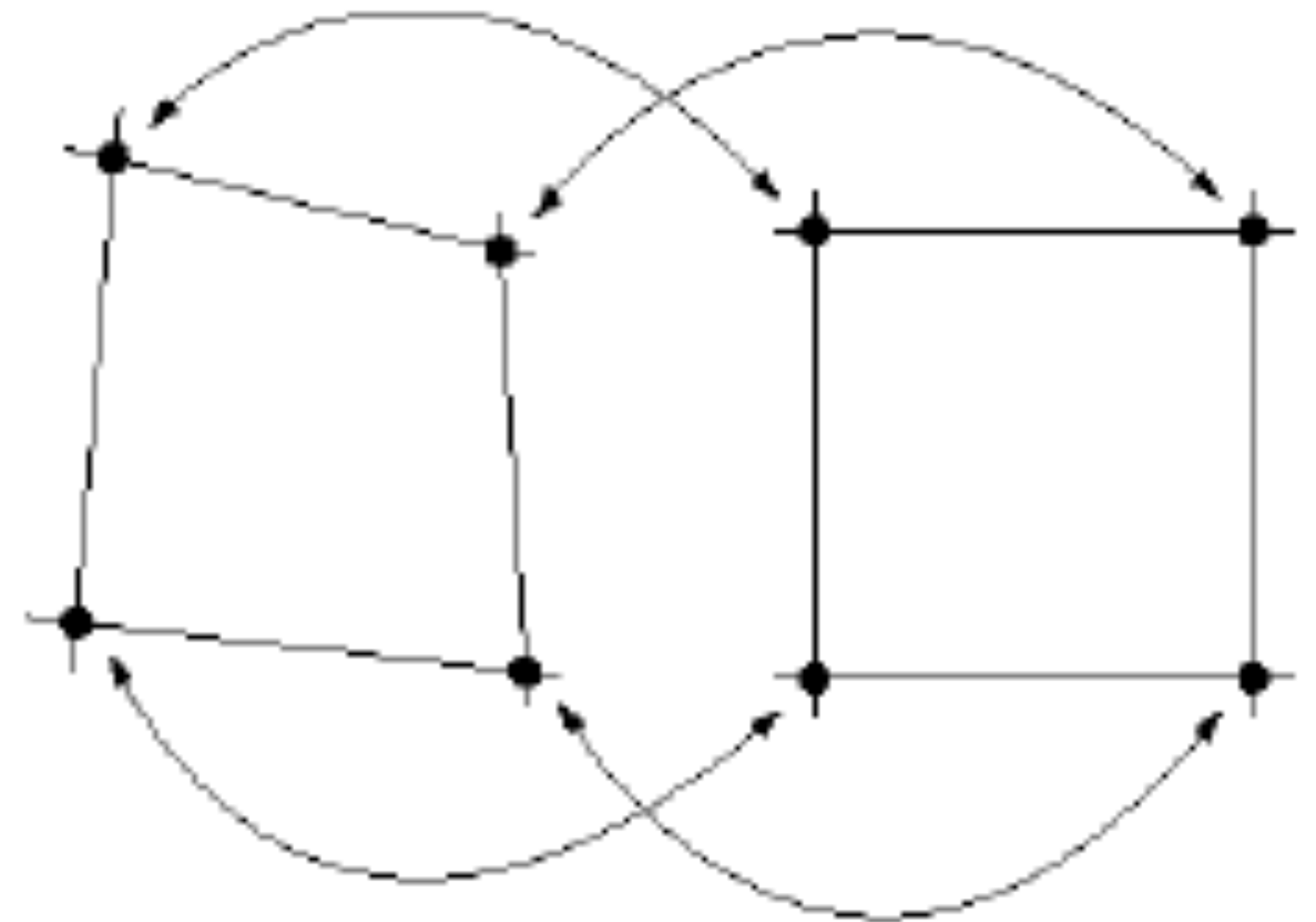
Overlaying  
source mesh



Restored  
result

# Interpolating Transformations

- To get the transformation from one quadrilateral to the corresponding one, interpolate the transformation
- Same idea as interpolating *values*, but for *positions*  $(x,y)$
- Already seen how to do bilinear interpolation on quadrilaterals

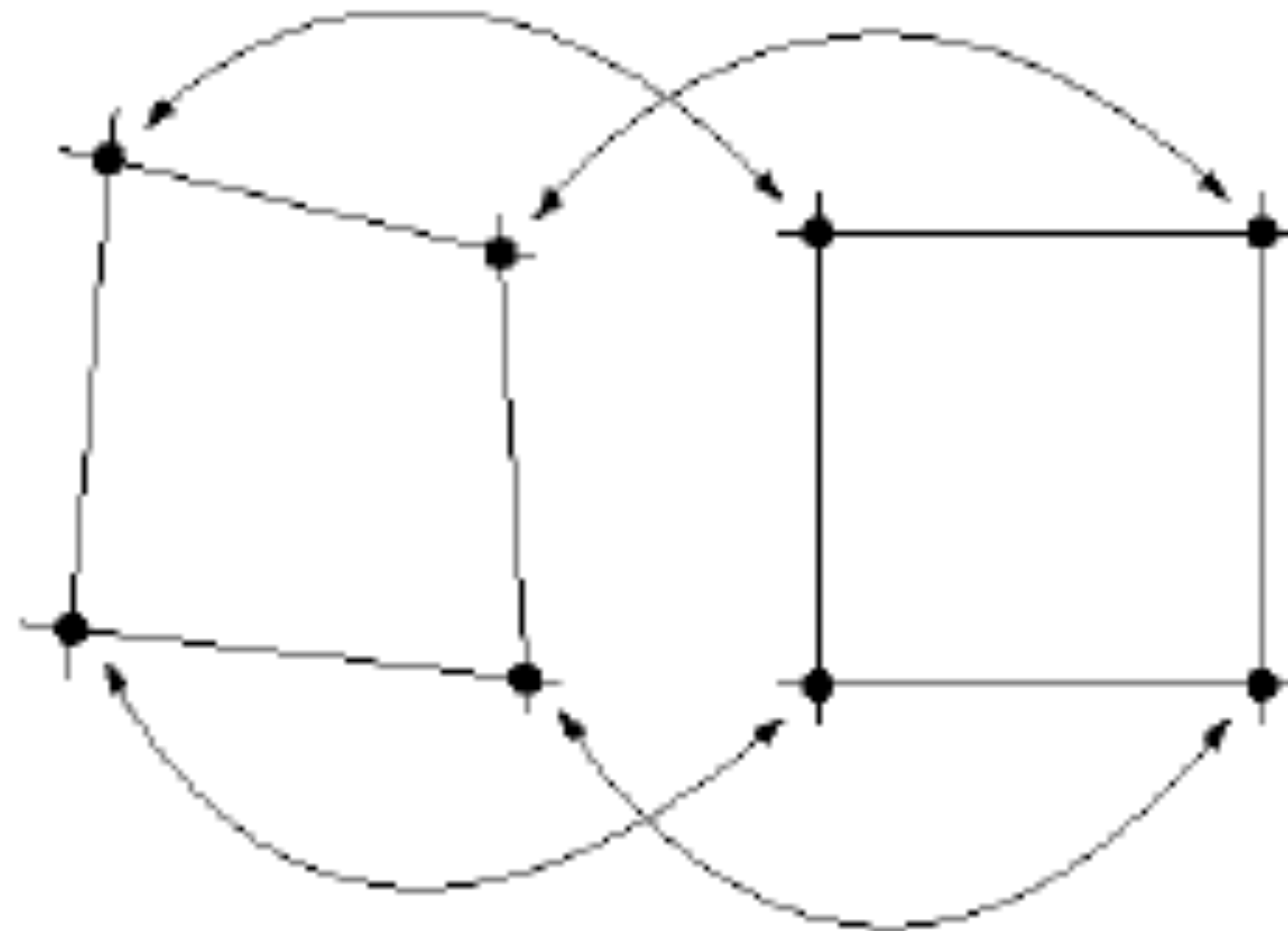


# Bilinear Transformations

Solve two systems of four equations each:

$$x' = a_1x + b_1y + c_1xy + d_1$$

$$y' = a_2x + b_2y + c_2xy + d_2$$



Key idea:  
If you can write the transformation in terms of some number of unknowns, you can solve for it with enough corresponding points



# Higher-Order Warps

- Can extend idea of bilinear interpolation of the transformation to higher-order
- More neighboring mesh points
- More computation
- Smoother results
- See Photoshop's "Liquify" tool

# Coming up...

- Perspective warping
- Back to more geometry:
  - Representing lines, curves, surfaces
  - Geometric tests