CS 355 Homework #4:
Transformations and Matrices

Questions 1–5 are basic ones that you can answer in 10–20 minutes using transformations we have discussed in class. (You might need additional time to look up and review the necessary material if necessary.) Question 6 uses matrix composition for a particular application. Questions 7–8 have you construct matrices and matrix products to do the transformations you you will use in Lab #4.

General notes:

- You may leave your answers in terms of trigonometric functions where applicable.
- You do not need to multiply out matrices by other matrices but may instead write your answer as a product of multiple matrices where applicable.
- When asked to actually calculate coordinates, you do have to multiply vectors by matrices, calculate trigonometric functions, etc.

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1. Write the transformation matrix that rotates around the origin by a counterclockwise angle of $\theta = \pi/6$ radians. If I applied this to the point $(10, 20)$, what are the (x, y) coordinates of the resulting point?
2. Write the transformation matrix that translates by an offset of $(30, -50)$. If I applied this to the point $(10, 20)$, what are the (x, y) coordinates of the resulting point?
3. Write the transformation matrix that scales uniformly by a factor of 3. If I applied this to the point $(10, 20)$, what are the (x, y) coordinates of the resulting point?
4. Write the transformation matrix that scales nonuniformly by a factor of 2 horizontally and 5 vertically. If I applied this to the point $(10, 20)$, what are the (x, y) coordinates of the resulting point?
5. Write the transformation matrix that will apply a shearing transform where $x' = x$ and $y' = 3x + y$. If I applied this to the point $(10, 20)$, what are the (x, y) coordinates of the resulting point?

6. Rotation around an arbitrary center of rotation $\mathbf{c}$ can be done by applying a translation by an offset of $-\mathbf{c}$, rotating by the desired angle, and then translating by an offset of $+\mathbf{c}$. Write an equation that would give you a *single* linear transformation rotates counterclockwise by an angle of $\theta = \pi/4$ radians around the point $40, 50$. *You do not have to actually calculate this matrix—write it as the product of multiple other matrices.* If I applied this sequence to the point $(45, 50)$, what are the (x, y) coordinates of the resulting point?

7. For Lab 4, you will need to place images at various positions with different rotations and scaling. This can be accomplished by first scaling the image while still in its original coordinate system, then rotating the image around the origin in that system (i.e., the upper left corner), then translating that to place the image.

   Suppose that you have a 1024×1024 image and that you want to place a 256×256 copy of it so that the upper left corner is located at $(300, 400)$ and rotated clockwise by angle $\pi/6$. Write out the transformation as the composition of three separate matrices. *Make sure to keep the correct order of operations!*

   Where would the pixel at $(280, 312)$ map to using this composite transformation?

8. If you look closely at the code provided for the `compose` function in Lab 4, you'll see that it inverts the matrix you pass to it and uses that for the transformation. (We'll talk more about why this is when we get to Lab 9.) But it's not necessary to invert the matrix directly. You can compose the inverses of each of the component matrices in reverse order to achieve the same result. (Remember: $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$.)

   Write out as a sequence of composited matrices the inverse of the sequence you wrote for Question 7. Test it by applying it to your answer for the last part of that question to see if it maps back to the point $(280, 312)$. What point would $(200, 400)$ map *back* to using this sequence?

Note: Some of these rotations are subject to the ambiguity of which direction of rotation is which and which way you point the y axis. Latitude will be given for reversed minus signs on the $\sin$ functions.