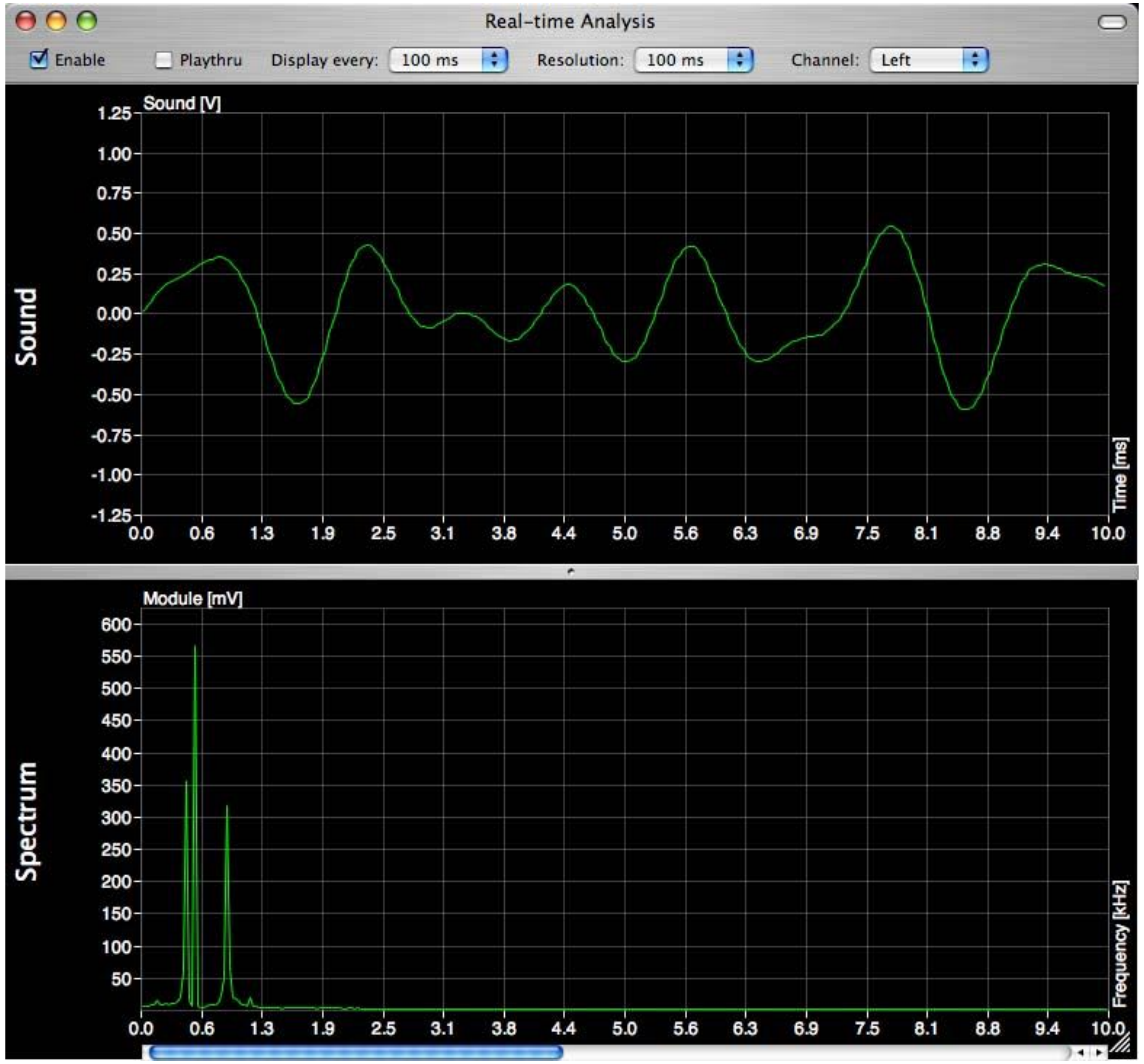
Frequency-Domain Processing

CS 355: Introduction to Graphics and Image Processing

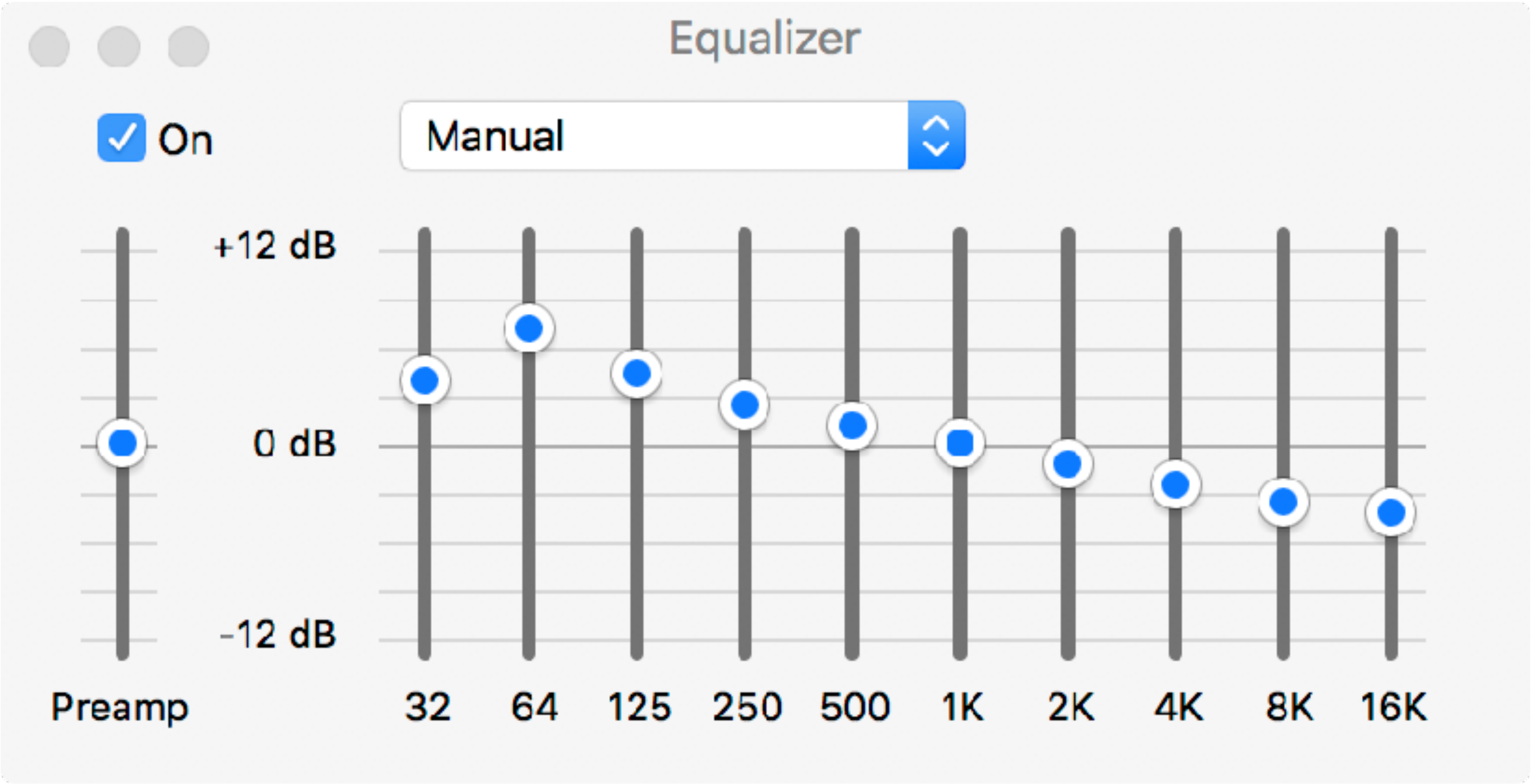
Time Domain vs. Frequency Domain

- For sampled functions of time, we talk about manipulating these samples in the **time domain**
- Earlier in the semester, we learned ways to manipulate images in the **spatial domain**
- Let's talk about manipulating them in the **frequency domain**

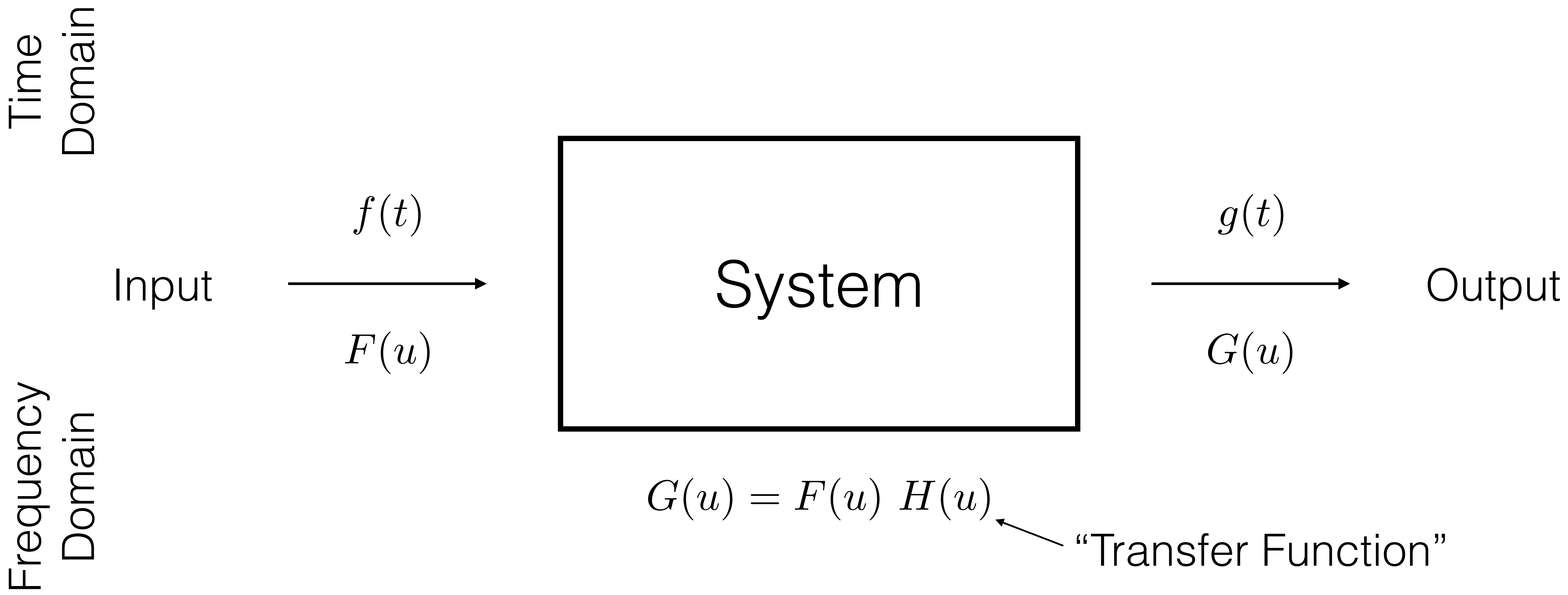


Frequency-Domain Processing

- The simplest way to process signals in the frequency domain is to
  - Selectively amplify (boost) certain frequencies
  - Selectively attenuate (weaken) other frequencies



Common Notation



The Transfer Function

- Intuition: how different frequencies “transfer” from input to output
- Remember:
  - Input F(u) is complex
  - Output G(u) is also complex
- If transfer function H(u) is **real-valued**, it multiplies both the real (cosine) part and the imaginary (sine) part equally
  - Magnitude/amplitude is scaled (amplification/attenuation)
  - Phase is left unchanged

$$G(u) = F(u)\ H(u)$$

The Transfer Function

- If H(u) is **complex-valued**, it multiplies the real (cosine) part and the imaginary (sine) part differently
  - The magnitudes multiply (amplification/attenuation)
  - The phases add (shifts different frequencies potentially differently)

$$G(u) = F(u)\ H(u)$$

The Transfer Function

$$G(u) = F(u)\ H(u)$$

Modulation Transfer Function

$$|H(u)|$$

Selectively amplifies/attenuates

Phase Transfer Function

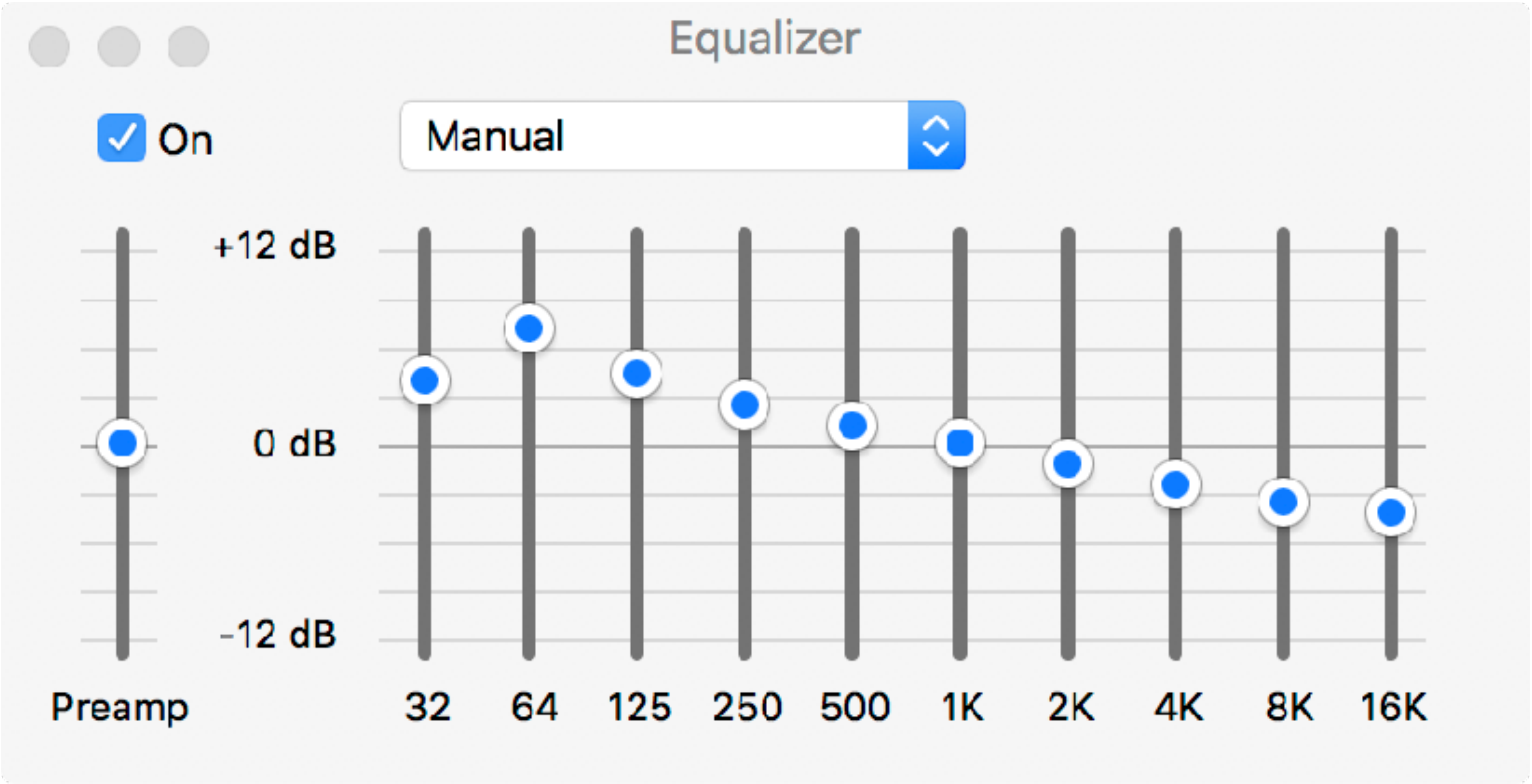
$$\phi(H(u))$$

Selectively shifts different frequencies

Filtering

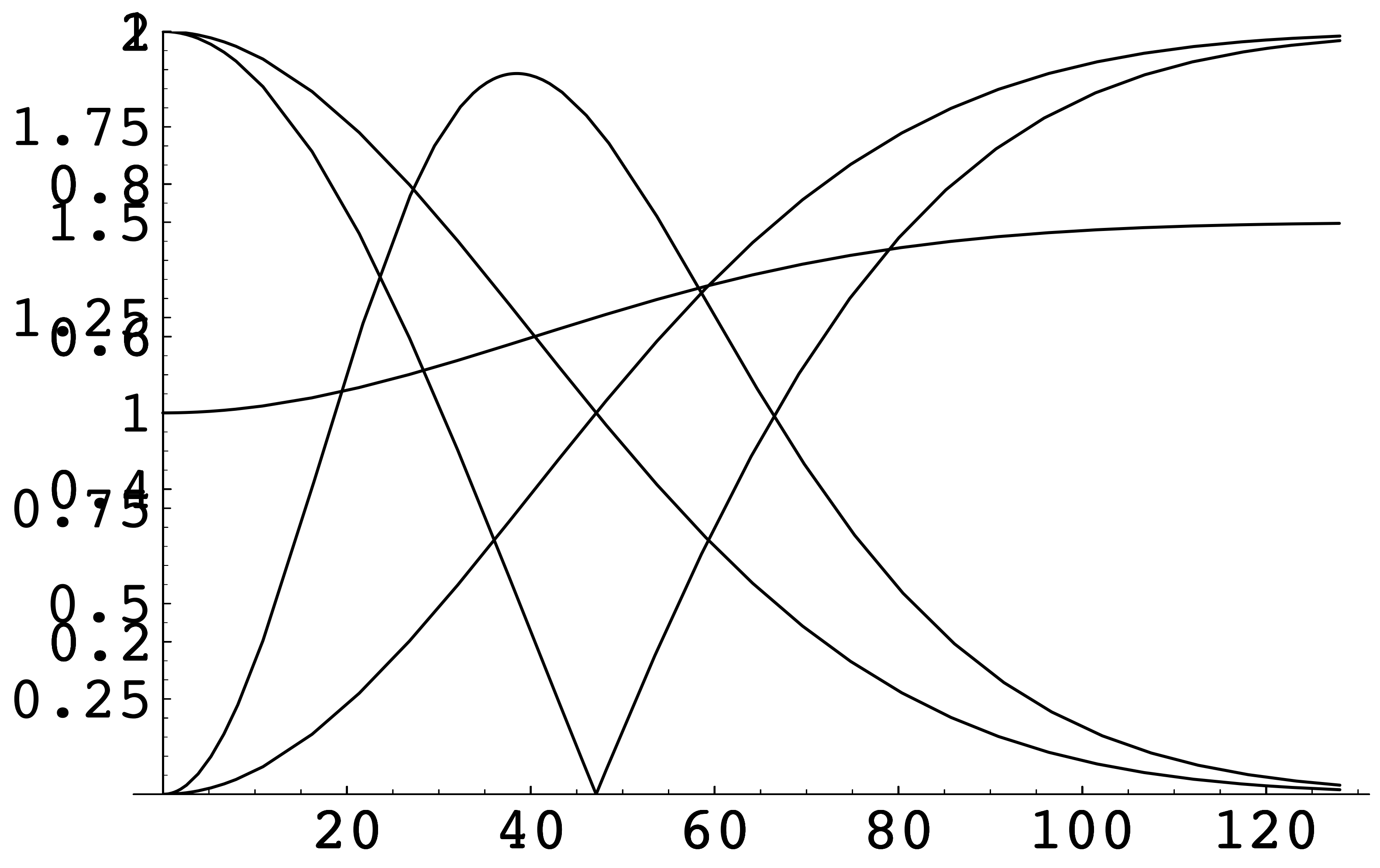
- Designing and applying a transfer function to a signal in the frequency domain it is called **filtering**

- You can design H(u) to be whatever you want it to be!



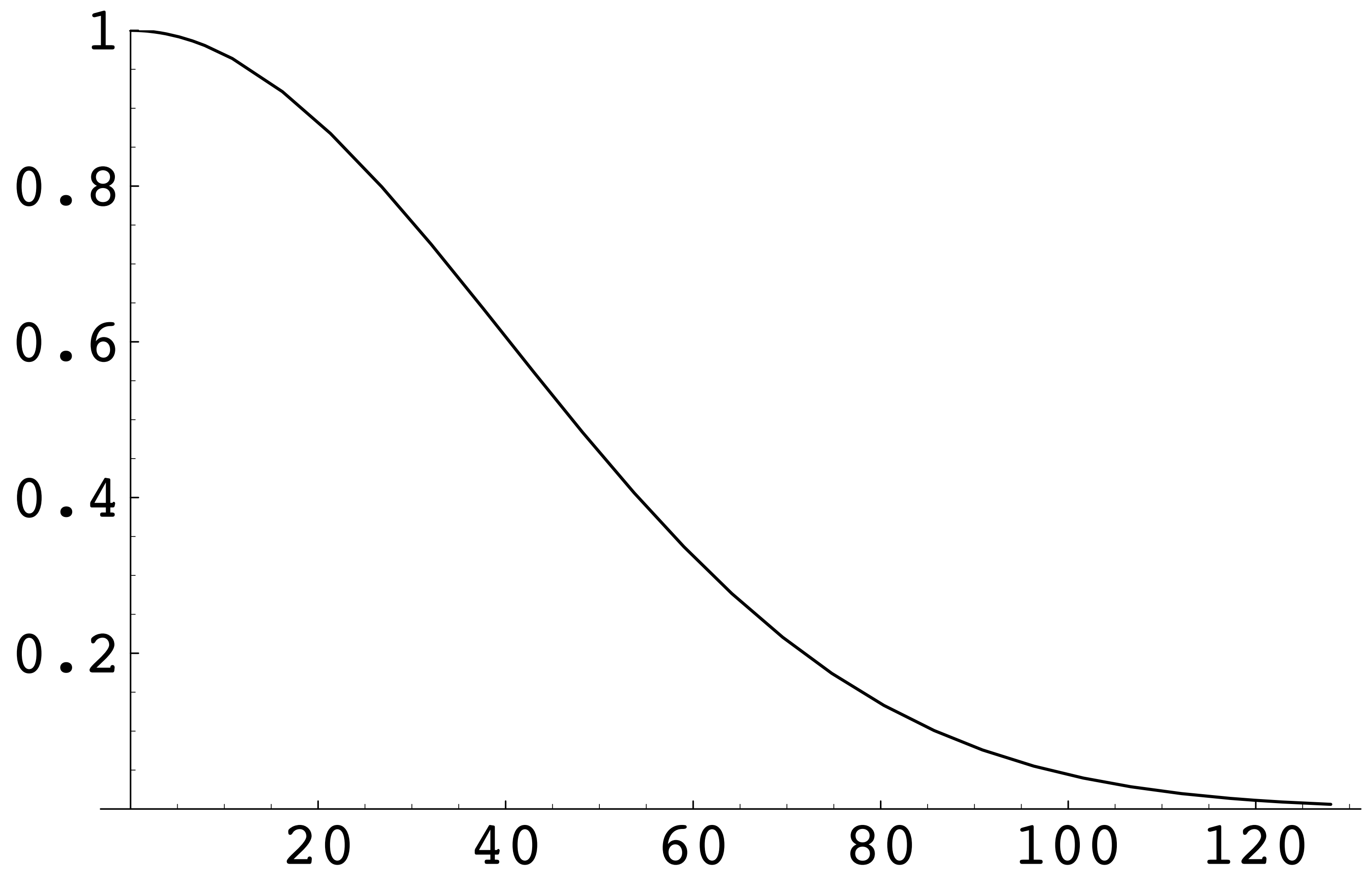
Types of Filters

Low-Pass	Keeps low frequencies, attenuates higher ones
High-Pass	Keeps high frequencies attenuates lower ones
High-Boost	Keeps lower frequencies, boosts higher frequencies
Band-Pass	Attenuates frequencies above or below a target band
Band-Suppress	Attenuates frequencies in a target band



Low-Pass Filters

- Lets **low** frequencies **pass** through
- Attenuates higher frequencies
- Examples:
  - Sound through air
  - Sound through other materials
  - RF signals through air
  - Electronic signals through wire

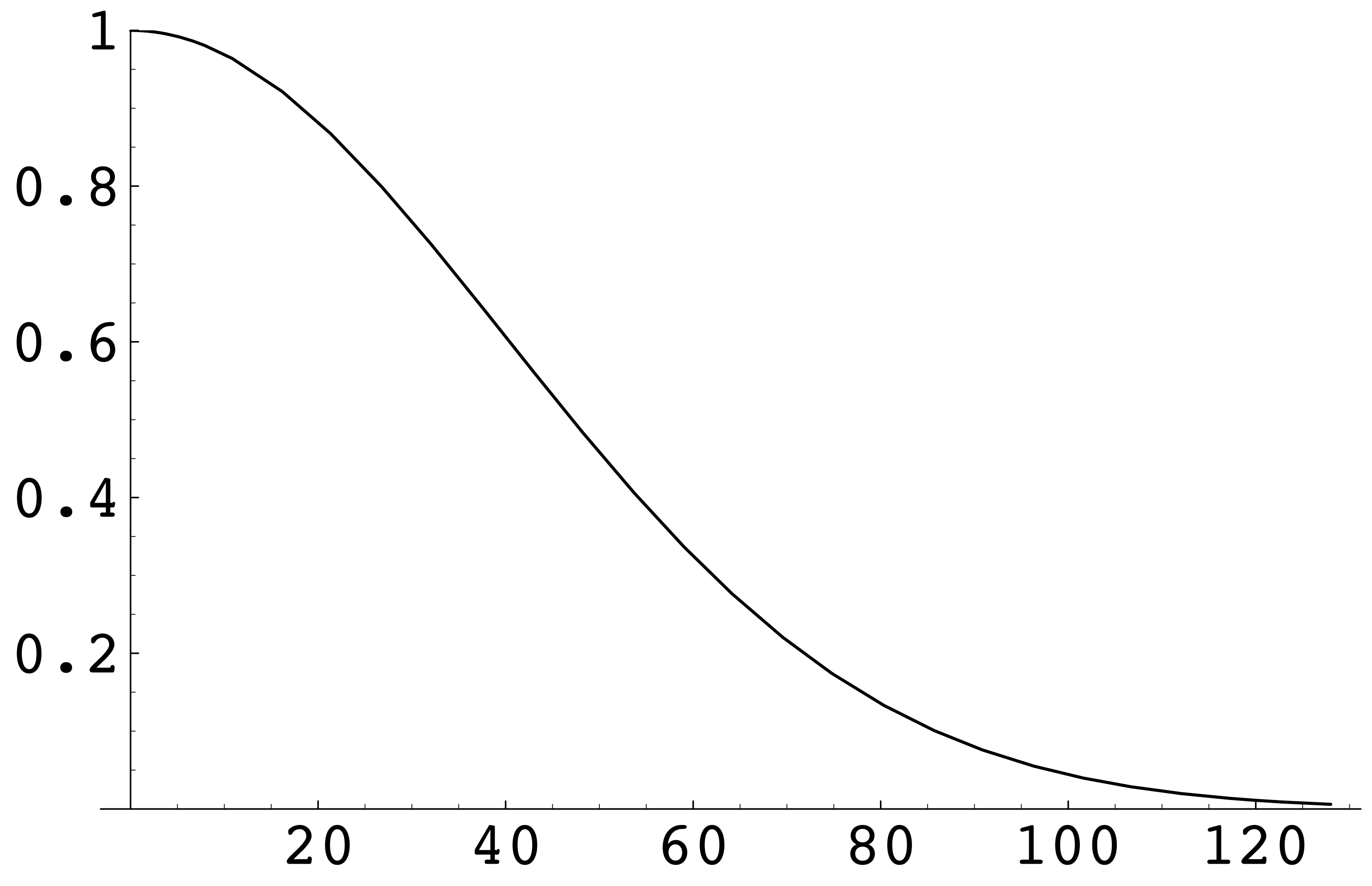


Example: Gaussian Low-Pass Filter

- One common form of a low-pass filter is the Gaussian

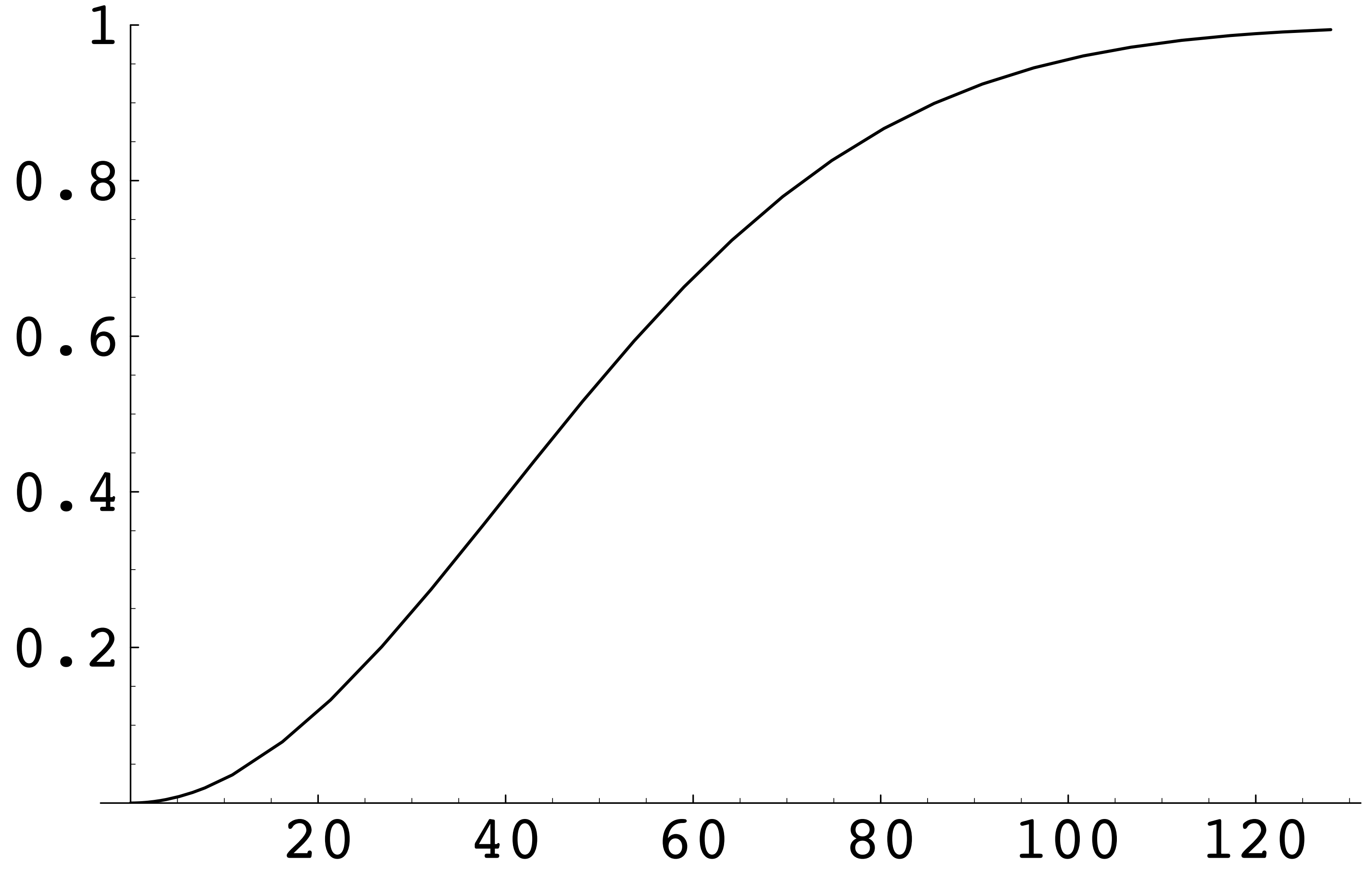
$$H(u) = e^{-\frac{1}{2}u^2/{u_c}^2}$$

Cutoff frequency



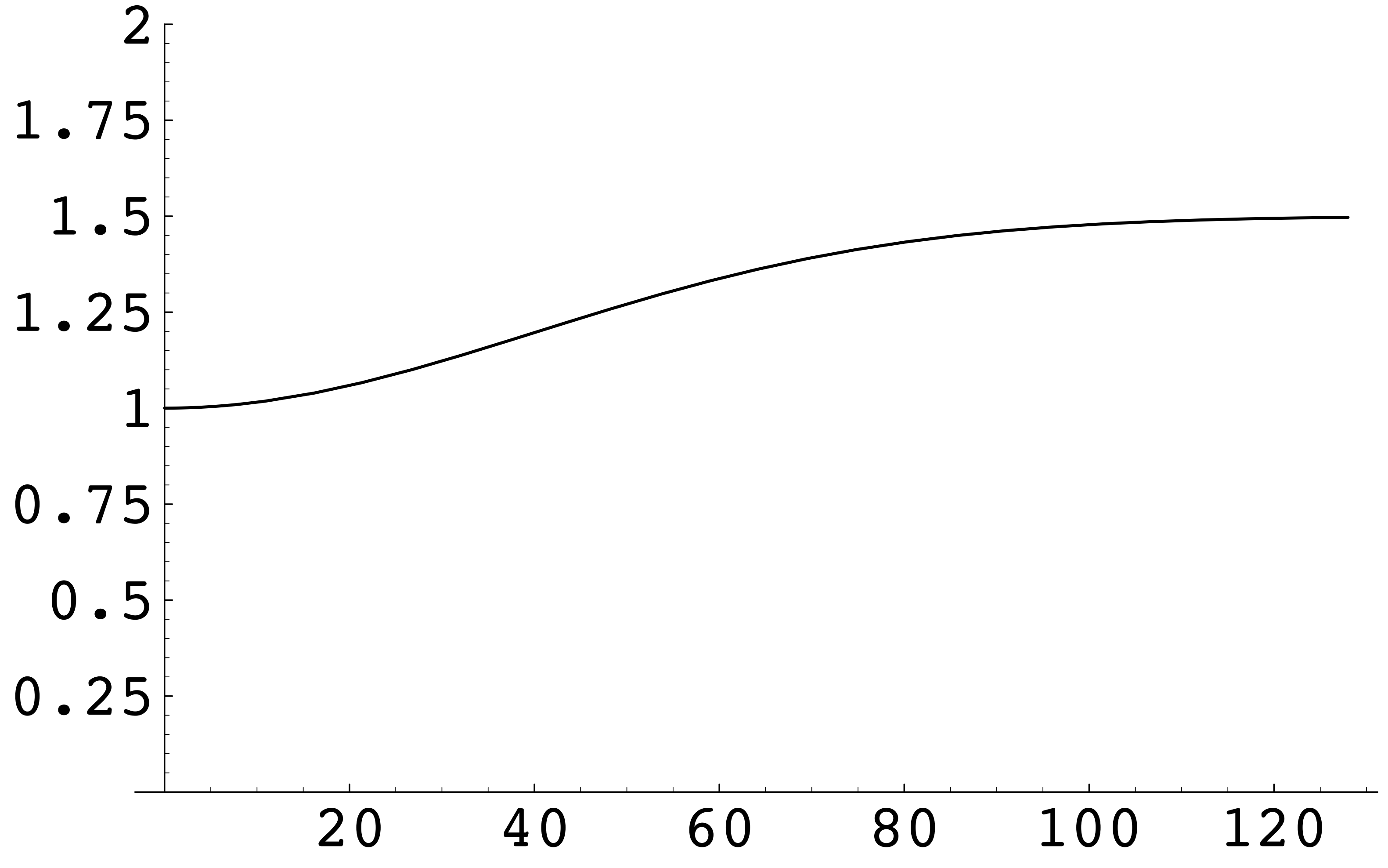
High-Pass Filters

- Lets **high** frequencies **pass** through
- Attenuates lower frequencies
- Not very common in nature but useful in electronics or signal processing
- Often 1.0 minus a low-pass filter



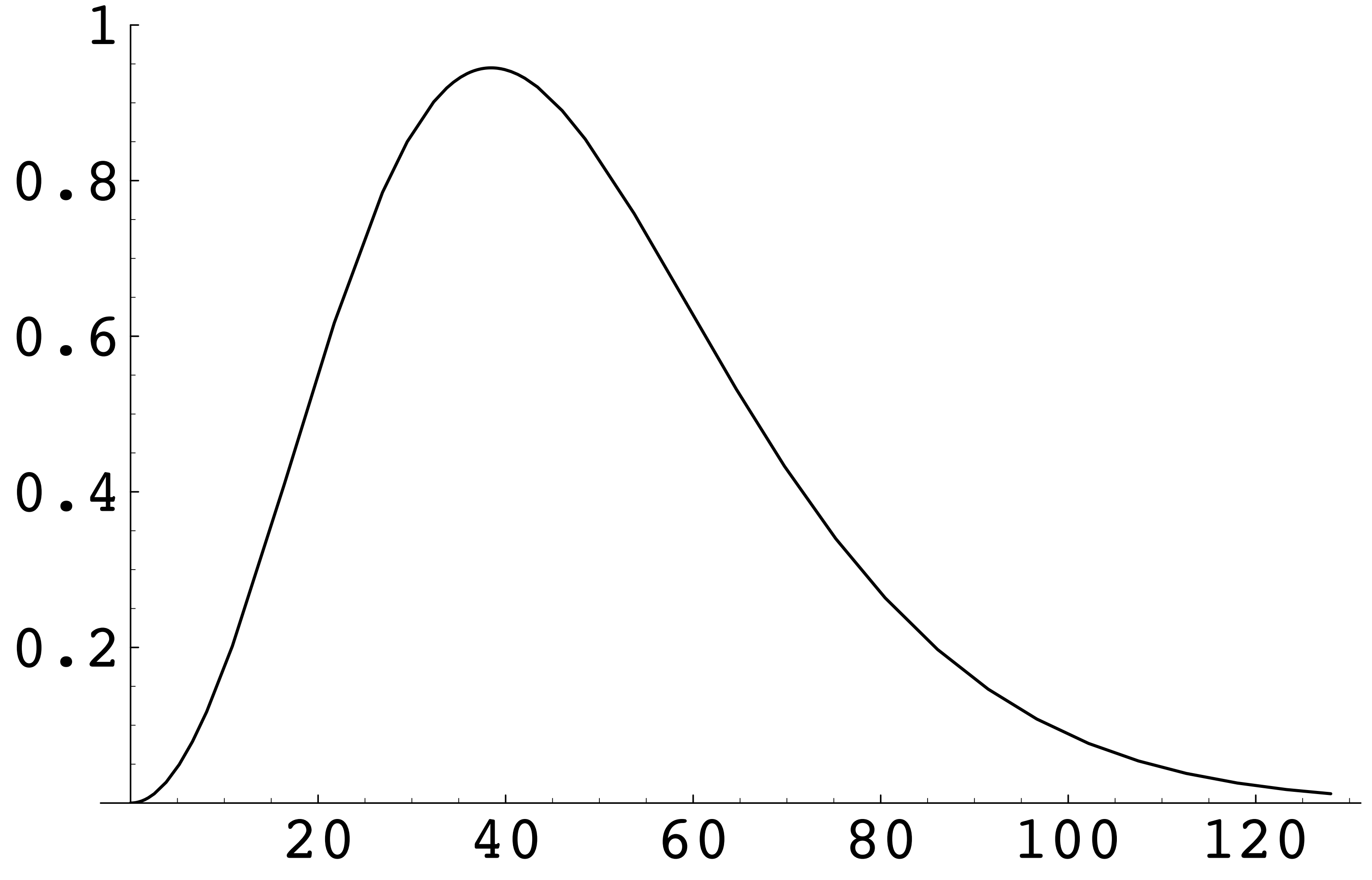
High-Boost Filters

- Boosts high frequencies
- Useful for “undo-ing” low-pass filters
- Can boost any other part of the frequency domain as well
- Often 1.0 plus a high-pass filter



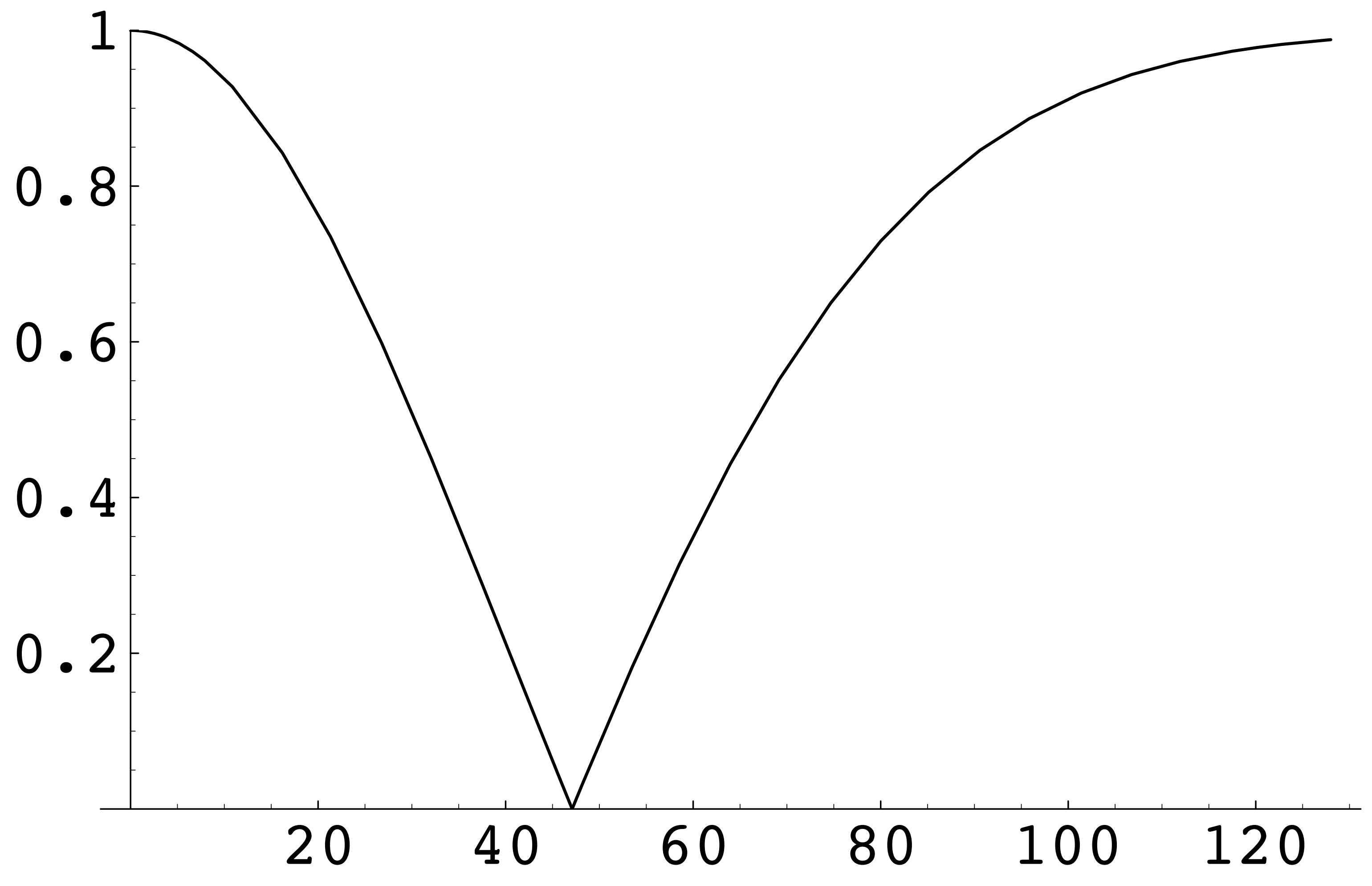
Band-Pass Filters

- Lets frequencies in some target **band pass** through
- Attenuates higher/lower frequencies
- Eg., difference of two low-pass filters



Band-Suppresses Filters

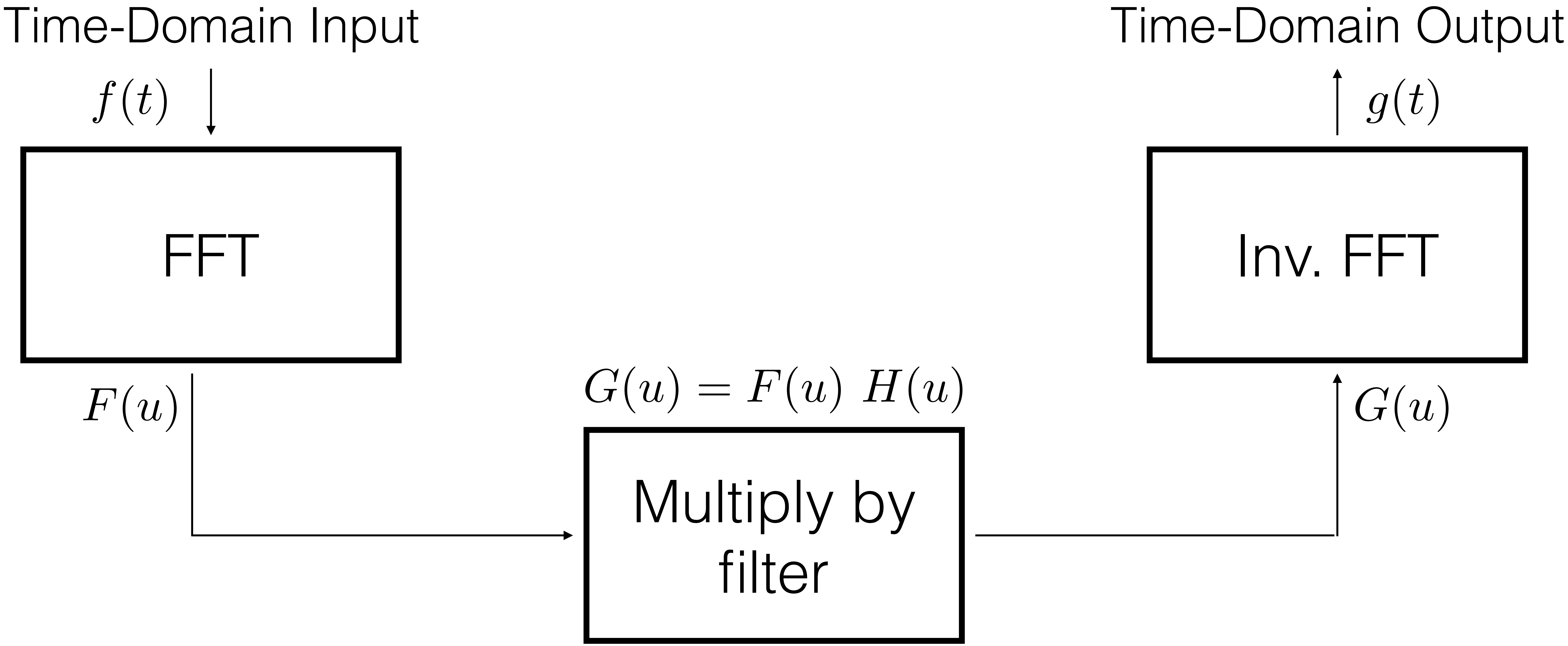
- **Suppresses** frequencies in a target **band**
- Preserves higher/lower frequencies



Phase Filtering

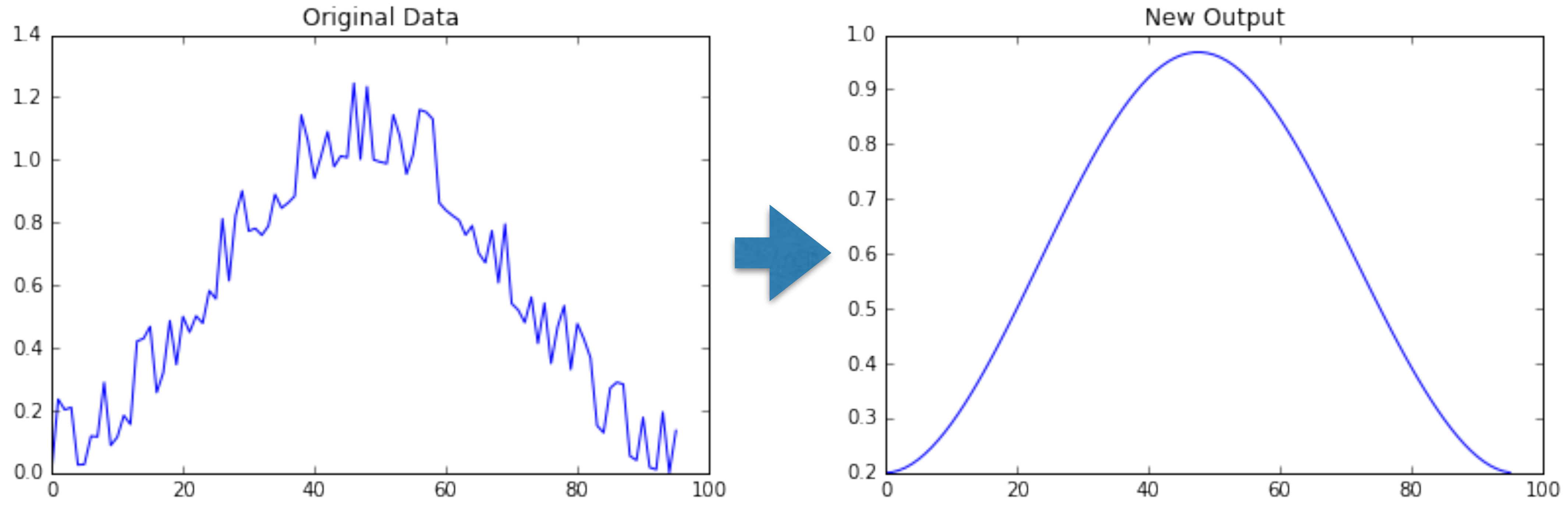
- Time delay
- Example: transmission of electronic signals through wires
    - Different frequencies pass at different speeds through the same media
    - Can become noticeable over long distances
    - Solution: periodically gather, re-sync the phases, retransmit

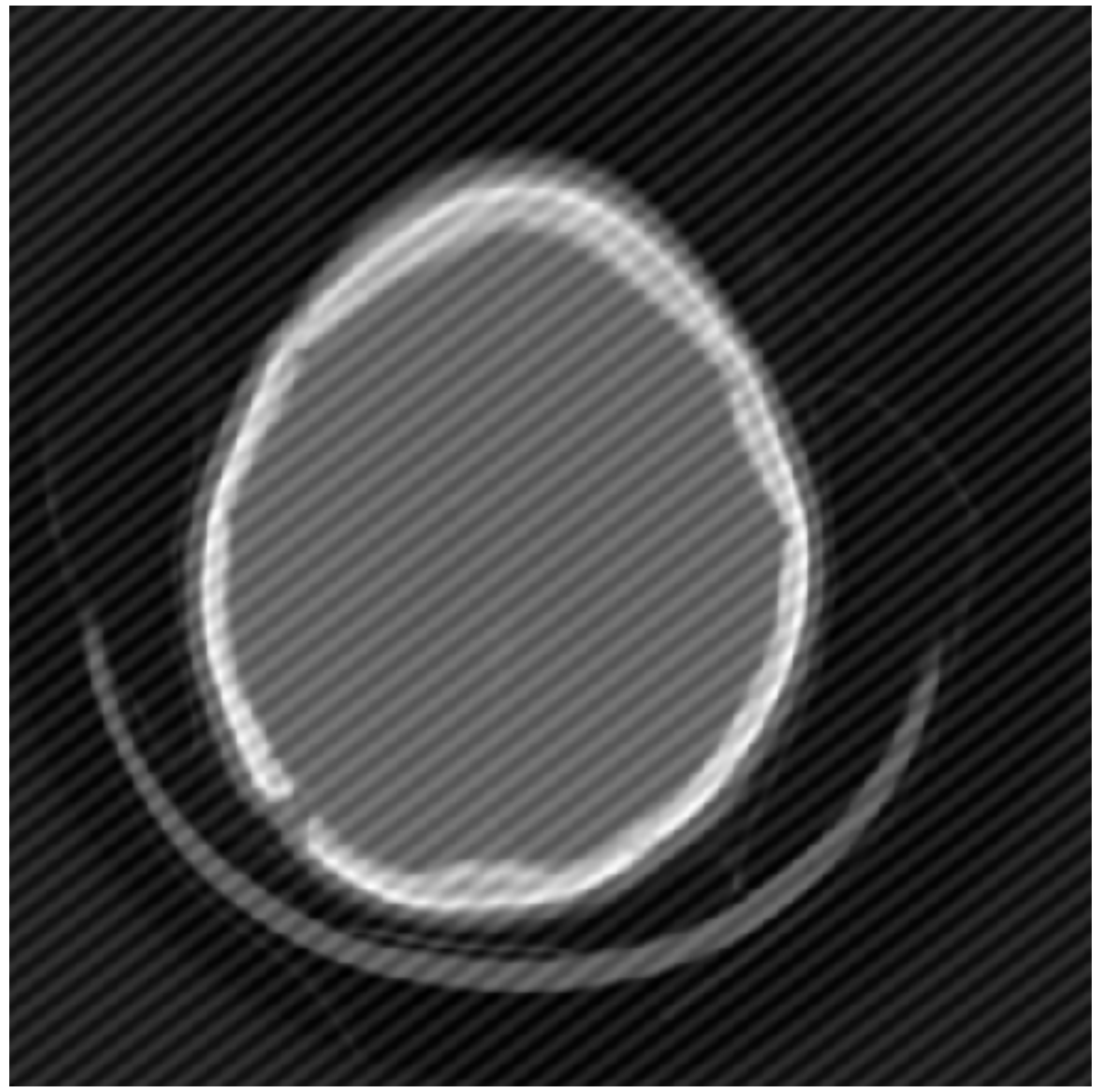
Implementing Filtering Digitally



Lab 10

- Filtering 1-D signals
  - Synthetic example (noisy function)
  - Audio example
- Filtering 2-D images
  - Smoothing/sharpening
  - Isolating and removing a single-frequency interference pattern





Coming up…

- 2-D FFT and image filtering in the frequency domain
- And then we're done!