Introduction to the Frequency Domain

CS 355: Introduction to Graphics and Image Processing

Let’s revisit the idea of transformations…

Basis Sets

- Remember that a basis set is a minimal set of vectors that span a space of vectors
- That means any vector in the space can be represented by a unique sum of the basis vectors
- **All vectors are represented with respect to some basis set**

$$\{\mathbf{e}_i\}$$

$$\mathbf{v} = \sum_i a_i \ \mathbf{e}_i$$

Change of Basis

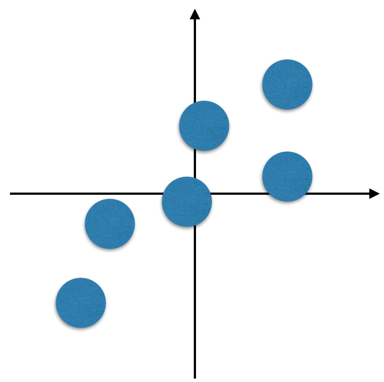
- Can change from
  - representation in terms of one basis set to
  - representation in terms of another basis set
- If basis vectors are orthonormal, this is just simple dot products (you've seen this already)

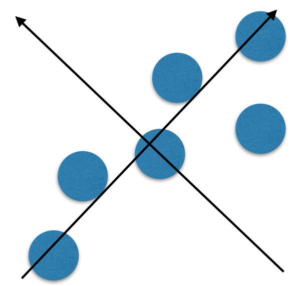
$$\mathbf{v} = \sum_i a_i \; \mathbf{e}_i$$

$$a_i = \mathbf{v} \cdot \mathbf{e}_i$$

Change of Basis

- For many problems, *analyzing points and vectors may be easier in a different coordinate system*
- Key is often to find the right coordinate system
    - Can be based on the problem
    - Can adapt to the specific data



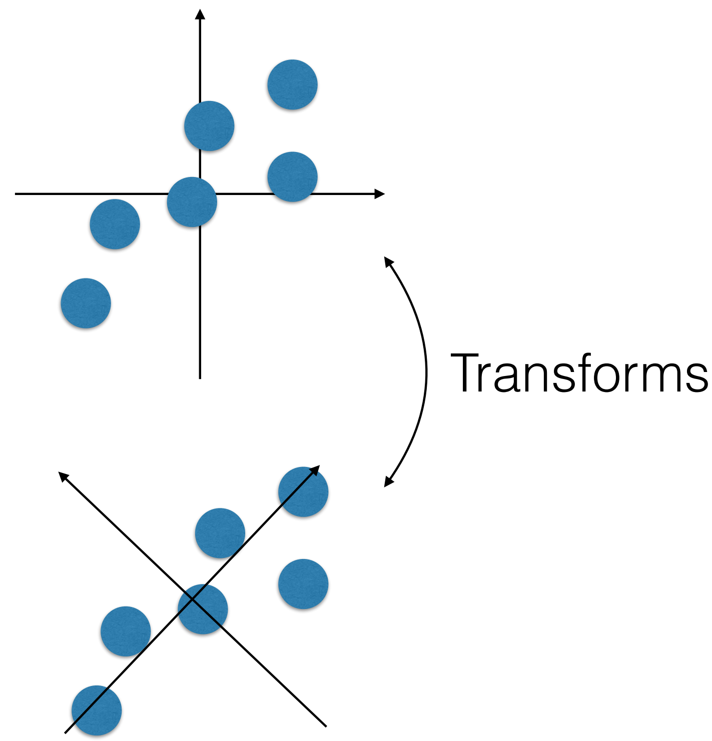


Data as Vectors

Lots of things can be thought of as points/vectors in some space

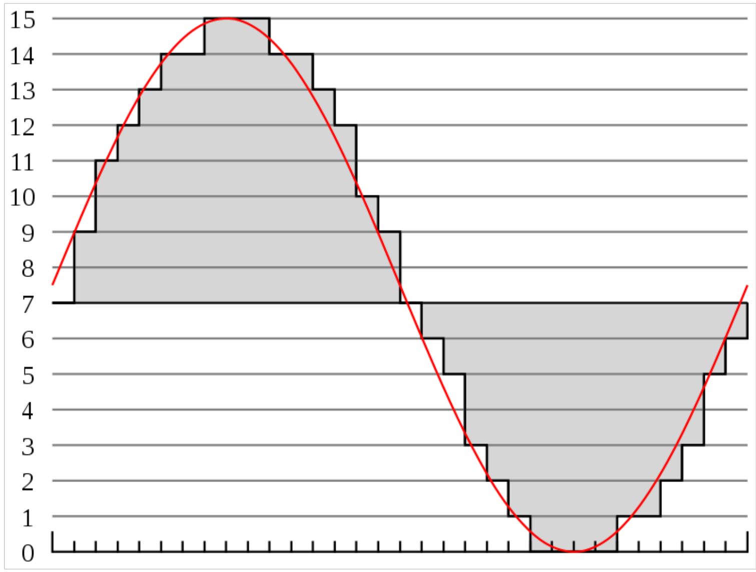
Data Transforms

- Common pattern in data analysis:
  - Represent data as vectors
  - Convert to a different coordinate system
  - Analyze (or change!) while in that coordinate system
  - Convert back (if needed)



Digital Audio

- Raw digital audio is stored as a series of sampled values (Pulse Code Modulation)
- One second of (one channel of) music on a CD is 44,100 samples
- Is this any different from a vector in a 44,100 dimensional space?
- Can we represent it differently?



Fourier Analysis

- Sampled sines and cosines of different frequencies form an orthonormal basis set
- Can decompose any waveform into a weighted sum of sines and cosines of different frequencies
- Great for analysis, manipulation, etc.

$$c[u] = \sum_k f[k] \; \cos(2\pi u t)$$

$$s[u] = \sum_k f[k] \; \sin(2\pi u t)$$

(OK, there's a bit more to it, but this is the basic idea.)

Let’s hear it…

Key Ideas from Today

- Transformations used for geometry extend to higher-dimensional data
- Can represent any point/vector as a weighted sum of basis vectors
- Choose the right set of basis vectors for your problem:
  - If the basis is orthonormal, get the weights by projecting (dot product)
  - Get back the original using a weighted sum of the basis vectors
- Sines and cosines are orthonormal basis set for 1-D functions like sound
- Can represent sound in the **time domain** or the **frequency domain**
- Can be used for analysis, but can also use it to manipulate the sound!

Coming up...

- A bit of review on complex numbers and sinusoids
- The Fourier Transform
- The Fast Fourier Transform
- How to feed data into FFT code and how to interpret the results
- Filtering in the frequency domain
- Sampling revisited
- Image filtering