MAT1856/APM466 Assignment #1: The Value of Time

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Fundamental Questions (25 pts)

1. **Q1 (5 pts)**
   (a) Governments issue bonds to finance spending without causing inflation, whereas printing money increases the money supply and can devalue the currency.
   (b) If investors expect short-term rates to fall or anticipate slower growth, demand for long-term bonds rises and their yields fall, flattening the curve.
   (c) Quantitative easing is when a central bank buys government or mortgage-backed securities to inject liquidity; since COVID-19 the US Fed used QE by purchasing Treasury and mortgage-backed bonds to stabilize markets and support lending.
2. **Q2 (10 pts)** Ten bonds (0.5–5 y to maturity, 5 Jan 2026): Canadian government fixed-coupon, semiannual, from Markets Insider (Frankfurt listing), evenly spaced maturities, no near-duplicates.

CAN 4 Aug 26	CAN 1 Sep 26	CAN 3 Feb 27	CAN 2.75 May 27
CAN 2.5 Aug 27	CAN 2.5 Nov 27	CAN 2 Jun 28	CAN 4 Mar 29
CAN 2.75 Mar 30	CAN 0.5 Dec 30		

3. **Q3 (10 pts)** Eigenvalues give each mode's contribution to total variability; eigenvectors give the shape (e.g. parallel shift, tilt, curvature). The largest eigenvalue and its eigenvector identify the dominant movement of the curve over time.

Empirical Questions (75 pts)

4. **Q4**
   (a) **4(a) (10 pts)** YTM computed for each bond; 5-year yield curve per day superimposed. Interpolation: linear over 0.6–5 y (from 0.545 y); yields from dirty prices.
   (b) **4(b) (15 pts) Spot curve.** (1) Per date: sort bonds by maturity T. (2) First bond: $r = (CF/P)^{1/T} - 1$. (3) For each next bond: discount prior cash flows with known spots; residual $= P - \mathrm{PV(prev)}$; $r = (CF_{\text{last}}/\text{residual})^{1/t_{\text{last}}} - 1$. (4) Interpolate to 1–5 y; repeat for all dates.
   (c) **4(c) (15 pts) Forward curve.** (1) Per date: get spot curve; interpolate to 1–5 y. (2) $S_{\text{cont}} = 2\ln(1 + S_{\text{semi}}/2)$. (3) $F_{1,n} = (S_{1+n}(1+n) - S_1)/n$, $n = 1, \ldots, 4$. (4) Store and plot.
5. **Q5 (20 pts)** Covariance matrices for spot and forward rates: see Fig. 1(d)–(e).
6. **Q6 (15 pts)** PCA on spot and forward rates. All components (eigenvalues and eigenvectors):

	Spot rates		Forward rates	
	Eigenval.	Expl. (%)	Eigenval.	Expl. (%)
PC1	8.67×10^{-5}	59.24	1.17×10^{-4}	68.75
PC2	4.89×10^{-5}	33.42	4.16×10^{-5}	24.37
PC3	4.83×10^{-6}	3.30	7.28×10^{-6}	4.27
PC4	3.17×10^{-6}	2.17	4.46×10^{-6}	2.61
PC5	2.74×10^{-6}	1.87	–	–

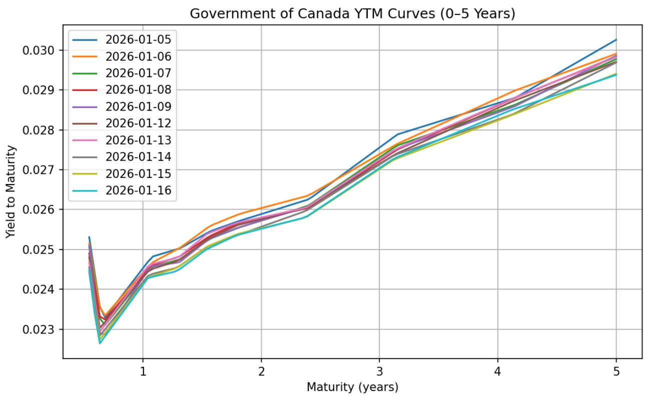
Table 1: PCA eigenvalues (all components).

Interpretation: PC1 captures ∼59% (spot) and ∼69% (forward) of variability; its eigenvector is a near-parallel shift (spot) and short/mid forwards moving together (forward).

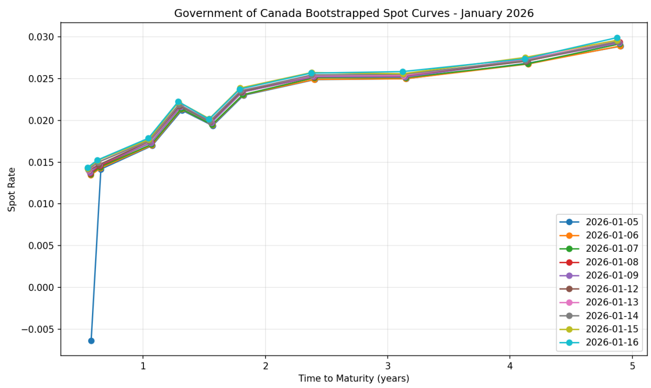
References and GitHub

[1] Bond data scraper (Jaspreet Khela):
`https://colab.research.google.com/drive/1kCYtYmExgO7-iXjSc_2Pj87BsRBJGZnp?usp=sharing`

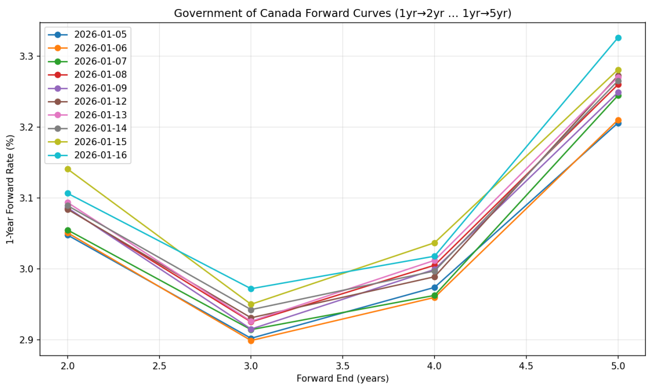
GitHub: `https://github.com/davidcagoh/bond-yield-calculator`



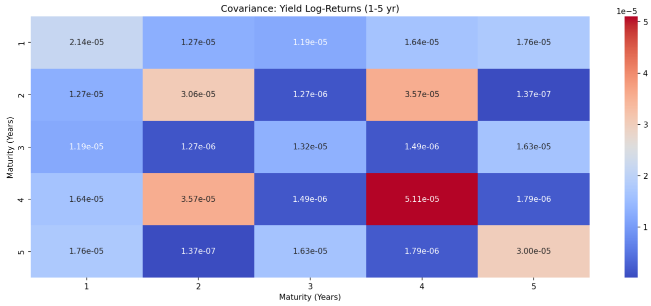
(a) Yield curves (YTM) per date.



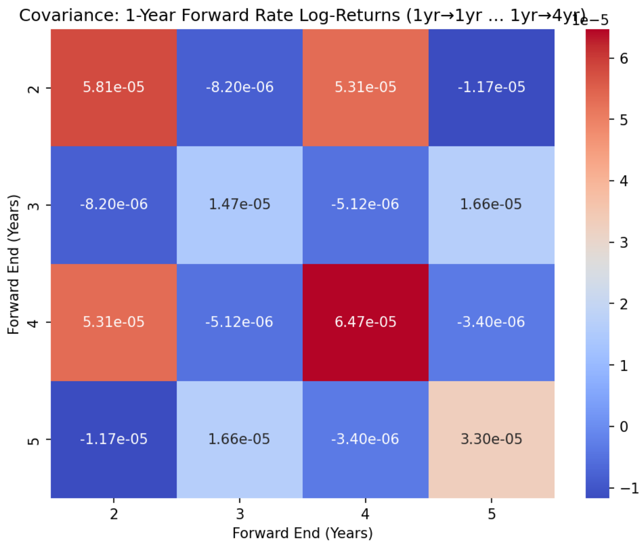
(b) Spot curves per date.



(c) 1-year forward curves per date.



(d) Covariance: spot rates.



(e) Covariance: forward rates.

Figure 1: Q4–Q5: Yield, spot, forward curves and covariance matrices.

	M1	M2	M3	M4	M5
PC1	−0.358	−0.548	−0.120	−0.727	−0.170
PC2	−0.394	0.220	−0.445	0.271	−0.725
PC3	0.554	0.369	0.253	−0.470	−0.521
PC4	0.573	−0.672	−0.216	0.321	−0.263
PC5	0.285	0.251	−0.823	−0.269	0.325

	F2	F3	F4	F5
PC1	0.681	−0.113	0.708	−0.146
PC2	−0.015	0.474	0.263	0.840
PC3	−0.689	−0.401	0.603	0.024
PC4	0.247	−0.776	−0.254	0.522

Table 2: PCA eigenvectors (all PCs): spot (left), forward (right).