State Stabilization in Open Quantum Systems

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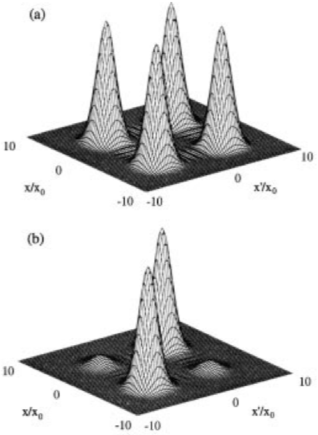
Quantum Information Science

- Computation power needs to keep up with the explosive increase in data/information.
- Discovery of quantum algorithms that can factor large integers (Shor's Algorithm).
- Quantum Simulation - molecules are computational complex.
- Quantum teleportation - we can transmit quantum information when sender and receiver share an entanglement.

Decoherence of a "Cat"

Real Part $\rho(x, x') = \langle x|\rho|x'\rangle$



"Cat" State

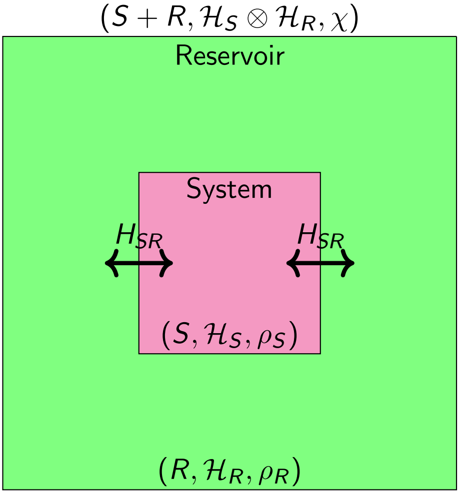
$$|\Psi\rangle \propto |\alpha\rangle + |-\alpha\rangle$$

After decoherence

$$\rho \propto |\alpha\rangle\langle\alpha| + |-\alpha\rangle\langle-\alpha|$$

Outline

Open Quantum System Framework



$$\dot{\chi} = -\frac{i}{\hbar}[H, \chi]$$

$$\chi = \sum_n p_n |\Psi_n\rangle\langle\Psi_n|$$

$$H = H_S + H_R + H_{SR}$$

Born Approximation

$$\tilde{\chi}(t) = \tilde{\rho}_{\mathcal{S}}(t) \otimes \rho_{\mathcal{R}}$$

Markov Approximation

$$\left\langle b_\alpha^\dagger(t) b_\alpha(s) \right\rangle \sim \delta(t-s)$$

Example: Driven-Qubit coupled to Thermal Reservoir

$$\frac{H}{\hbar} = \sum_n \omega_{cn} b_n^\dagger b_n + \frac{\epsilon}{2}\sigma_z + \Omega\cos(\omega_l t)\sigma_x + g\sigma_x \otimes \sum_n \left[b_n + b_n^\dagger\right]$$

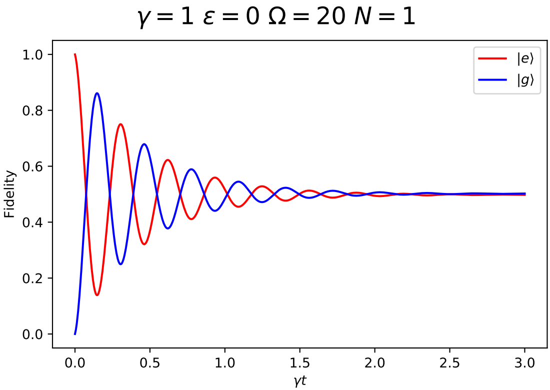
$$\Big\downarrow \quad \frac{H_{rot}}{\hbar} = \frac{\omega_l}{2}\sigma_z + \textstyle\sum_n \omega_{cn} b_n^\dagger b_n$$

$$\frac{H'}{\hbar} \approx \frac{\delta}{2}\sigma_z + \frac{\Omega}{2}\sigma_x + \boxed{\underbrace{g\sum_n \sigma b_n^\dagger + g^*\sum_n \sigma^\dagger b_n}_{=\frac{H_{SR}}{\hbar}}}$$

Post rotating-wave approximation, we get a Jaynes-Cummings interaction.

Example: Driven-Qubit coupled to Thermal Reservoir

$$\dot{\rho}_{\mathcal{S}} = -i\frac{\delta}{2}[\sigma_z, \rho] - i\frac{\Omega}{2}[\sigma_x, \rho] + \gamma(1 + N_{\omega_b})\left(\sigma\rho_{\mathcal{S}}\sigma^\dagger - \frac{1}{2}\left\{\sigma^\dagger\sigma, \rho_{\mathcal{S}}\right\}\right)$$
$$+ \gamma N_{\omega_b}\left(\sigma^\dagger\rho_{\mathcal{S}}\sigma - \frac{1}{2}\left\{\sigma\sigma^\dagger, \rho_{\mathcal{S}}\right\}\right)$$



Liouvillian

$$\dot{\rho} = \mathcal{L}\rho$$

$$\mathcal{L}\bullet = -\frac{i}{\hbar}[H, \bullet] + \sum_n \gamma_l \left(c_l \bullet c_l^\dagger - \frac{1}{2}\left\{ c_l^\dagger c_l, \bullet \right\} \right)$$

$$\mathcal{D}[c_l]\rho = \left(c_l \rho c_l^\dagger - \frac{1}{2}\left\{ c_l^\dagger c_l, \rho \right\} \right)$$

$$\rho(t) = e^{\mathcal{L}t}\rho = \rho_{ss} + \sum_n e^{\lambda_n}\rho_n$$

We use fidelity to measure “closeness” of the steady state, ρ_{ss}, to another, $|\Phi\rangle$, which is frequently entangled.

$$F_{|\Phi\rangle} = tr\{|\Phi\rangle\langle\Phi|\rho_{ss}\}$$

Rate of stabilization is called the “gap”

$$\Delta_{\mathcal{L}} = \min\{Re[\lambda_n]\} \tag{1}$$

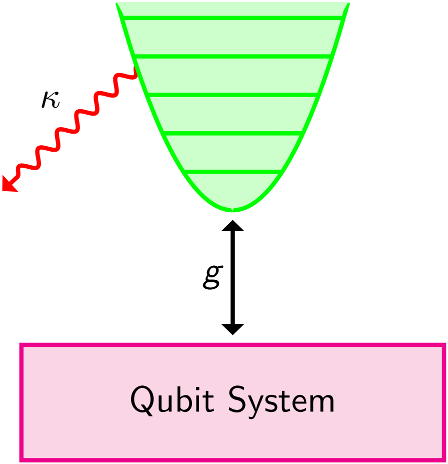
Dark States

Theorem

A pure state is stabilized, $\mathcal{L}\left(|\Phi\rangle\langle\Phi|\right) = 0$*, if and only if the following two conditions are satisfied:*
(1) $H'|\Phi\rangle = \Im(\lambda)|\Phi\rangle$ *for some* $\lambda \in \mathbb{C}$
(2) $c_l'|\Phi\rangle = 0$ *for some* $\lambda_l \in \mathbb{C}$ *with* $\sum_l g_l|\lambda_l|^2 = Re(\lambda)$*.*
When pure steady steady state meets these conditions it is called a **dark state***.*

$H' = H - i\sum_l g_l\lambda_l(c_l')^\dagger + i\sum g_l\lambda_l^* c_l'$
$c_l' = c_l - \lambda_l$

Adiabatic Elimination Problem



Quasi-static bath (low Q-value)

$$\chi \approx \rho_S(t) \otimes |0\rangle\langle 0|$$

"Engineered" a coupling between a qubit system and a harmonic oscillator to facilitate entanglement stabilization.

$$\begin{aligned} \frac{H_R}{\hbar} &= \Delta b^\dagger b \\ \frac{H_{SR}}{\hbar} &= g\left(b^\dagger \hat{S} e^{i\phi} + b\hat{S}^\dagger e^{-i\phi}\right) \end{aligned}$$

Adiabatic Elimination Problem

Quasi-static bath (low Q-value)

$$\chi \approx \rho_S(t) \otimes |0\rangle\langle 0|$$

Coupled weekly, $\kappa \gg g$, to a **single** harmonic oscillator

$$\begin{aligned} \frac{H_R}{\hbar} &= \Delta b^\dagger b \\ \frac{H_{SR}}{\hbar} &= g\left(b^\dagger \hat{S} e^{i\phi} + b\hat{S}^\dagger e^{-i\phi}\right). \end{aligned} \tag{2}$$

The equation of motion of the full system+reservior is

$$\dot{\chi} = -\frac{i}{\hbar}[H, \chi] + \kappa\mathcal{D}[a]\chi. \tag{3}$$

Adiabatic Elimination Problem

Construct a perturbative solution and trace of the reservoir

$$\dot{\tilde{\rho}}_S = \int_0^t d\tau \; tr_R \left\{ \left[\tilde{H}_{SR}(t), \left[\tilde{H}_{SR}(\tau), \tilde{\rho}_S(t) \otimes |0\rangle_R \langle 0| \right] \right] \right\}.$$

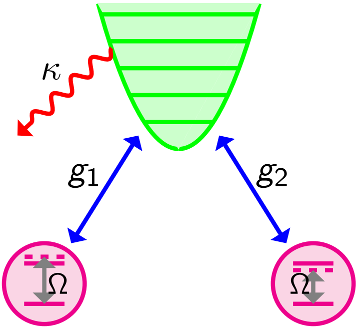
Evaluating the partial trace

$$tr_R\{\tilde{\mathcal{L}}_{SR}(t)\tilde{\mathcal{L}}_{SR}(\tau)\tilde{\chi}\} = g^2 e^{-\kappa(t-\tau)/2}\{\mathcal{S}_1'^{\dagger}(t)\mathcal{S}_1'(\tau) + \mathcal{S}_1'(t)\mathcal{S}_1'^{\dagger}(\tau) - \mathcal{S}_2'^{\dagger}(t)\mathcal{S}_1'^{\dagger}(\tau) - \mathcal{S}_2'(t)\mathcal{S}_1'(t)\}\tilde{\rho}_S(t).$$

Make the Markov approximation $e^{-\kappa(t-\tau)/2} \rightarrow 2\delta(t-\tau)/\kappa$ and $t \rightarrow \tau$. The punchline is,

$$\dot{\rho}_S = -\frac{i}{\hbar}[H_S, \rho_S] + \frac{4g^2}{\kappa}\mathcal{D}[\hat{S}]\rho_S.$$

Symmetric Scheme



$$\frac{H'_R}{\hbar} = 0$$

$$\frac{H'_S}{\hbar} \approx \sum_{i=1}^{2}\left(\frac{\delta_i}{2}\sigma_{zi} + \frac{\Omega_i}{2}\sigma_{x1}\right)$$

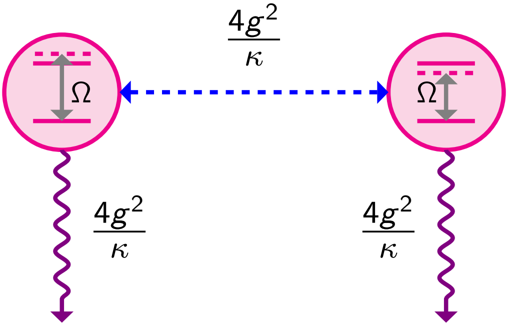
$$\frac{H'_{SR}}{\hbar} \approx b^\dagger \underbrace{(g_1\sigma_1 + g_2\sigma_2)}_{=g\hat{S}} + h.c.$$

Choose homogeneous couplings $g_1 = g_2 = g$.

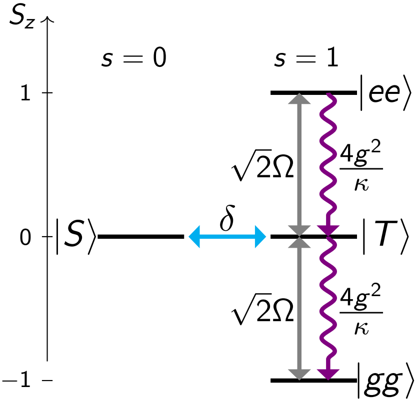
$$\boxed{\dot{\rho}_S = -\frac{i}{\hbar}[H'_S, \rho_S] + \frac{4g^2}{\kappa}\mathcal{D}[\sigma_1 + \sigma_2]\rho_S}$$

Dissipative Interaction

$$\dot{\rho}_S = -\frac{i}{\hbar}[H'_S, \rho_S] + \frac{4g^2}{\kappa}\mathcal{D}[\sigma_1]\rho_S + \frac{4g^2}{\kappa}\mathcal{D}[\sigma_2]\rho_S$$
$$+ \frac{4g^2}{\kappa}\left[\left(\sigma_1\rho_S\sigma_2^\dagger - \frac{1}{2}\left\{\sigma_2^\dagger\sigma_1, \rho_S\right\}\right) + \left(\sigma_2\rho_S\sigma_1^\dagger - \frac{1}{2}\left\{\sigma_1^\dagger\sigma_2, \rho_S\right\}\right)\right]$$



Dark States (Symmetric Case)



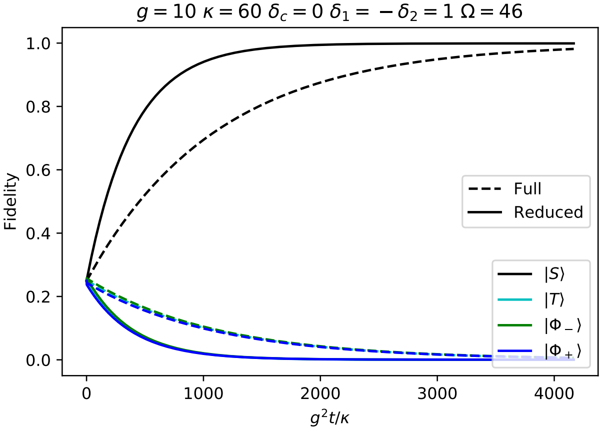
We consider the eigenvalue problem form **Theorem 1** for our engineered jump operator, $\hat{S} = \sigma_1 + \sigma_2$

$$|\Phi\rangle = \frac{1}{\sqrt{1+|\alpha|^2}}(|gg\rangle + \alpha|S\rangle) \tag{4}$$

Bell State Stabilization (Symmetric Case)

$$|\Phi\rangle \propto |gg\rangle + \alpha|S\rangle \quad H'_S|\Phi\rangle = 0 \Longrightarrow \alpha = \frac{\Omega}{\sqrt{2}\delta}$$

$$\delta_1 = -\delta_2 = \delta$$

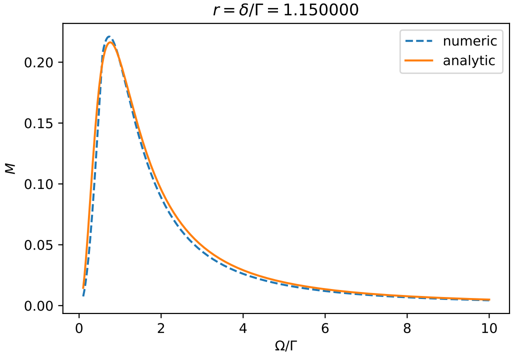


Performance Metric

We define a new performance metric: $M = F_{|S\rangle}\Delta_{\mathcal{L}}/\Gamma$ with $\Gamma = \frac{4g^2}{\kappa}$.

$$\dot{P}_{|S\rangle} = \Gamma_{|ee\rangle}P_{|ee\rangle} + \Gamma_{|S\rangle}P_{|S\rangle} + \Gamma_{|T\rangle}P_{|T\rangle} + \Gamma_{|ee\rangle}P_{|ee\rangle}$$

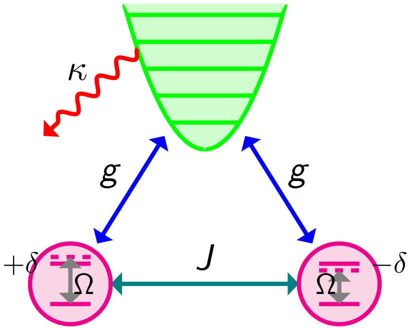
$$\Delta_{\mathcal{L}} \approx \Gamma_{|gg\rangle} \quad F_{|S\rangle} = \frac{|\alpha|^2|}{1+|\alpha|^2}$$

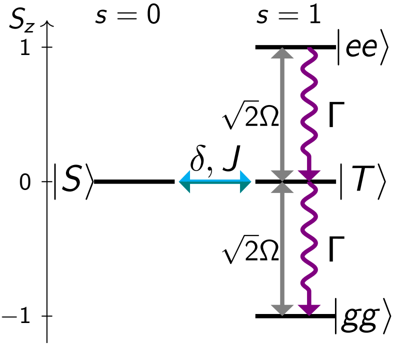


Chiral Case

Addition of qubit-qubit couplings.

$$\frac{H_S'}{\hbar} = \sum_{i=1}^{2}\left(\frac{\delta_i}{2}\sigma_{zi} + \frac{\Omega_i}{2}\sigma_{x1}\right) + J\sigma_1\sigma_2^\dagger + J^*\sigma_1^\dagger\sigma_2$$





Chiral Case

$$\dot{\rho}_S = -\tfrac{i}{\hbar}[H'_S, \rho_S] + \Gamma\mathcal{D}[\sigma_1 + \sigma_2]\rho_S \qquad \Gamma = \tfrac{4g^2}{\kappa}$$

$$\begin{aligned}
\frac{d}{dt}\langle\sigma_1\rangle &= -i\Delta_1\langle\sigma_1\rangle + i\Omega_1\langle\sigma_{z1}\rangle + \left(iJ^* + \frac{\Gamma}{2}\right)\langle\sigma_{z1}\sigma_2\rangle - \frac{\Gamma}{2}\langle\sigma_1\rangle \\
\frac{d}{dt}\langle\sigma_2\rangle &= -i\Delta_2\langle\sigma_2\rangle + i\Omega_2\langle\sigma_{z2}\rangle + \left(iJ + \frac{\Gamma}{2}\right)\langle\sigma_1\sigma_{z2}\rangle - \frac{\Gamma}{2}\langle\sigma_2\rangle
\end{aligned}$$

$$\boxed{J = \pm i\frac{\Gamma}{2}}$$

Chirality Condition: One qubit can see the other but not vise-versa.

Symmetric vs. Chiral

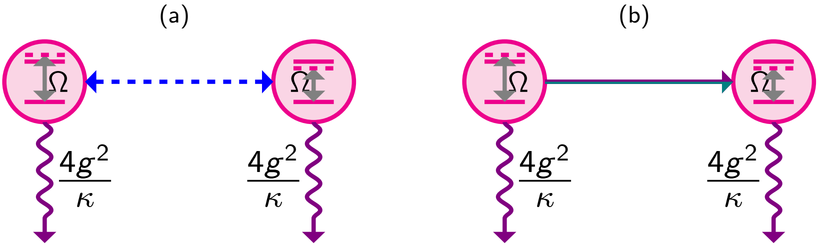
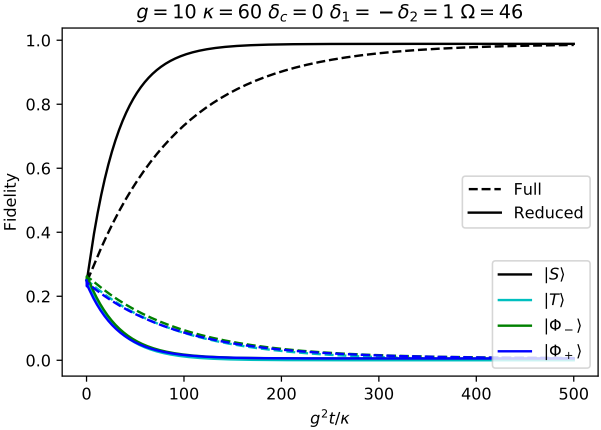


Figure: (a) Symmetric dissipative interaction.
(b) Uni-direction interaction do to inference of dissipative interaction and qubit-qubit couplings.

Bell State Stabilization (Chiral Case)

$$|\Phi\rangle \propto |gg\rangle + \alpha|S\rangle \quad H'_S|\Phi\rangle = 0 \Longrightarrow \alpha = \frac{\sqrt{2}\Omega}{2\delta + i\Gamma}$$

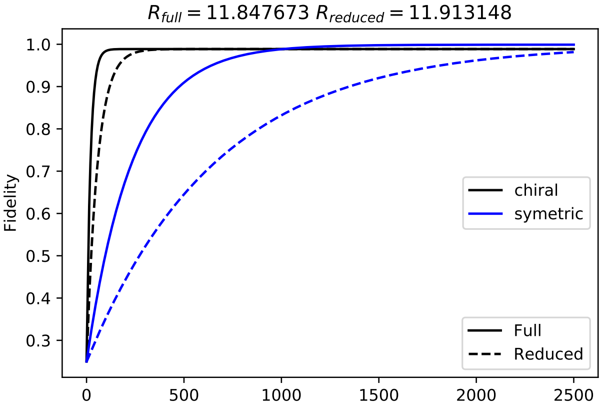
$$\delta_1 = -\delta_2 = \delta$$



Symmetric vs. Chiral

Ratio of respective performance metrics $R = \frac{M_{chiral}}{M_{sym}}$

$$g = 10 \ \kappa = 60 \ \delta_1 = -\delta_2 = 1 \ \Omega = 46$$



Four-Qubit Entanglement

$$H_{SR} = gb^{\dagger}(\sigma_1 + \sigma_2 + \sigma_3 + \sigma_4) + h.c.$$

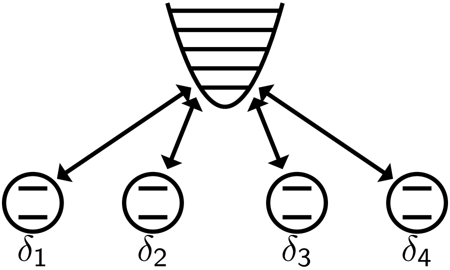


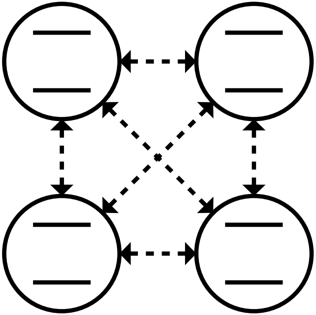
Figure: Four qubits coupled to a mutual resonator mode with an associated detuning pattern $\Delta = (\delta_1, \delta_2, \delta_3, \delta_4)$.

"Engineered" jump operator is $\hat{S} = \sum_{i=1}^{4} \sigma_i$.

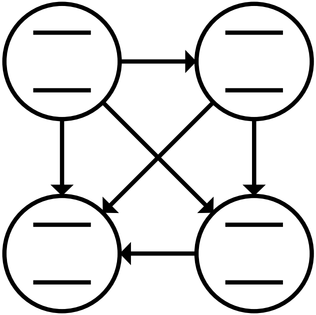
$$\dot{\rho}_S = -\frac{i}{\hbar}[H_S, \rho] + \Gamma \mathcal{D}[\hat{S}]; \qquad \Gamma = \frac{4g^2}{\kappa}$$

Four-Qubit Entanglement: Symmetric vs. Chiral

$$H_S = \underbrace{\sum_n \left(\frac{\delta_i}{2}\sigma_{zi} + \frac{\Omega}{2}\sigma_{xi} \right)}_{=H_{sym}}$$



$$H_S = H_{sym} + \frac{i\Gamma}{2}\sum_{l>j}\left(\sigma_l^\dagger \sigma_j - \sigma_l \sigma_j^\dagger\right)$$



Entanglement Identification

We define the purity of the density matrix : $tr(\rho^2)$.

$$
\begin{aligned}
tr(\rho^2) &= 1 \quad \text{for a pure state} \\
tr(\rho^2) &< 1 \quad \text{for a mixed state}
\end{aligned}
$$

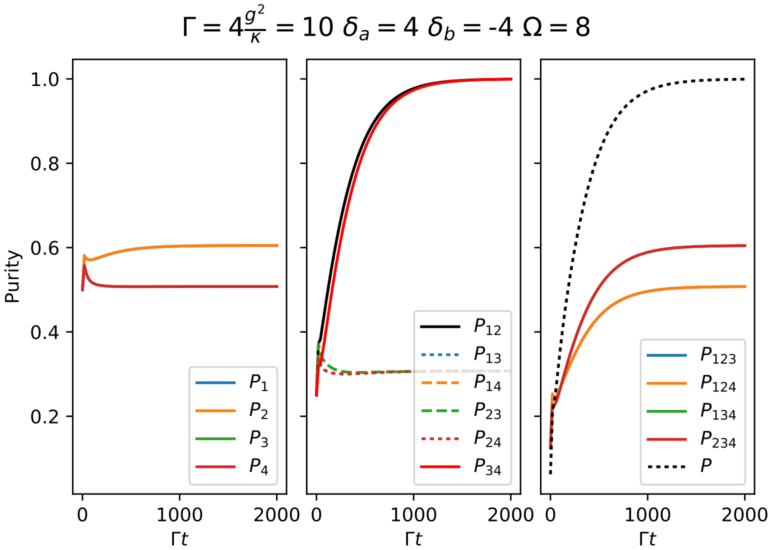
Suppose I have an entangled state $|\Phi\rangle$ between two subsystems. Then by definition

$$|\Phi\rangle \neq |\phi_1\rangle \otimes |\phi_2\rangle. \tag{5}$$

When I trace over one of the subsystem the result is not a pure state so the purity has to be less than one.

Alternating Detunings: Symmetric $\delta_a \; -\delta_a \; \delta_b \; -\delta_b$

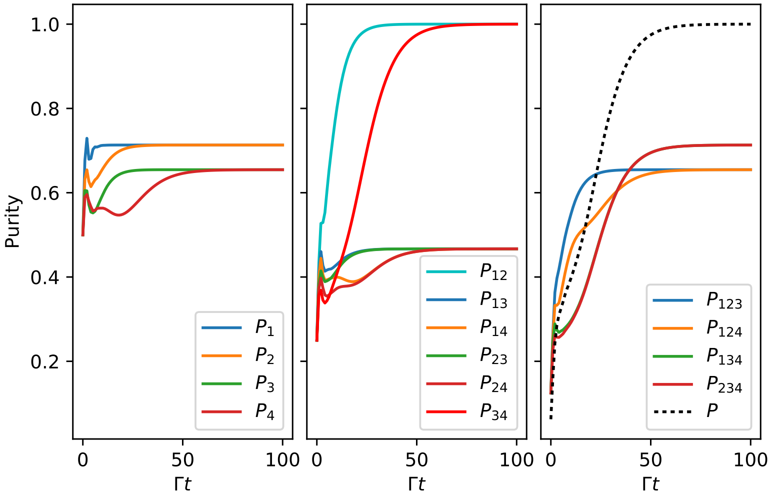
Two qubit entanglement



Alternating Detunings: Chiral $\delta_a \; -\delta_a \; \delta_b \; -\delta_b$

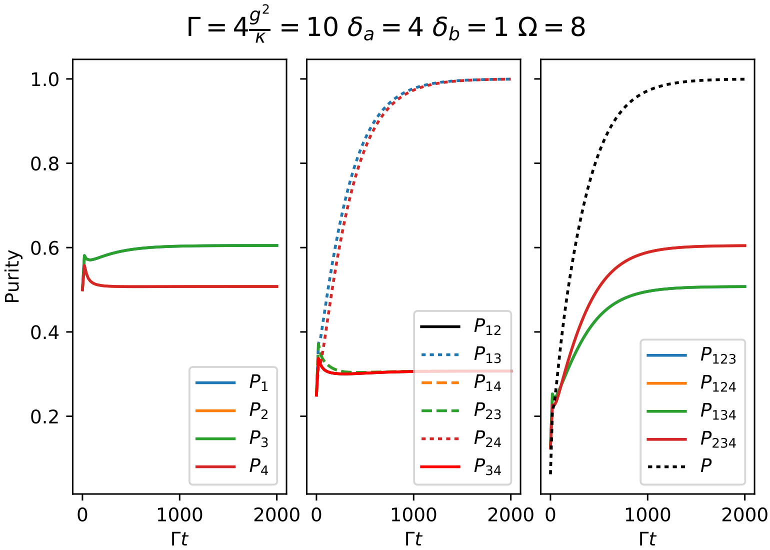
Two qubit entanglement

$$\Gamma = 4\frac{g^2}{\kappa} = 10 \; \delta_a = 4 \; \delta_b = 1 \; \Omega = 8$$



Staggered Detunings: Symmetric $\delta_a\ \delta_b\ -\delta_a\ -\delta_b$

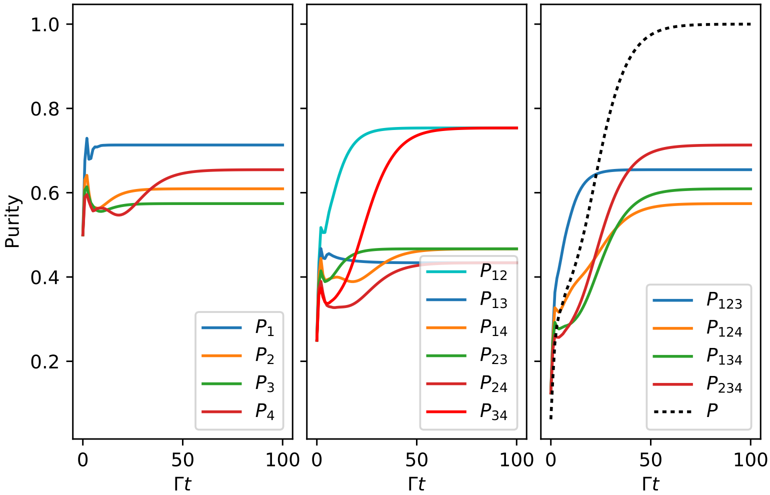
Non-local two qubit entanglement



Staggered Detunings: Chiral δ_a δ_b $-\delta_a$ $-\delta_b$

Four-way entanglement: **Tetramer**

$$\Gamma = 4\frac{g^2}{\kappa} = 10 \;\delta_a = 4 \;\delta_b = 1 \;\Omega = 8$$



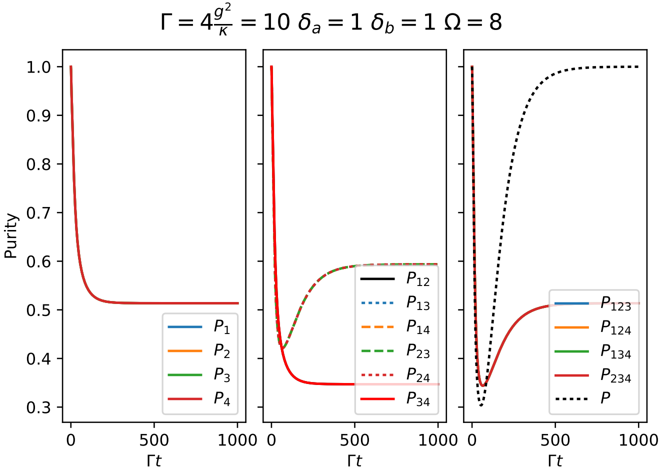
Caveat: four-qubit entanglement in symmetric case?

Homogeneous detunings ($\delta\ \delta\ -\delta\ -\delta$)
$+$ pure initial state preparation, $\rho_S(0) = |gggg\rangle\langle gggg|$

Detuning Pattern	Stabilized State
$(\delta, -\delta, \delta, -\delta)$	$\lvert S\rangle_{12}\lvert S\rangle_{34} - \lvert S\rangle_{14}\lvert S\rangle_{23}.$
$(\delta, -\delta, -\delta, \delta)$	$\lvert S\rangle_{12}\lvert S\rangle_{34} + \lvert S\rangle_{13}\lvert S\rangle_{24}$
$(\delta, \delta, -\delta, -\delta)$	$\lvert S\rangle_{13}\lvert S\rangle_{24} + \lvert S\rangle_{14}\lvert S\rangle_{23}$

Table: Table of detuning patterns and stabilized tetramer (four-way entangled state).

Example: Stabilization of $|S\rangle_{13}|S\rangle_{24} + |S\rangle_{14}|S\rangle_{23}$



Conclusions

- Don't hide from dissipation, use it!
- Adibatic elimination provides a way to identify engineered collapse operators
- Chirality improves performance metric (fidelity $\times$ rate of Bell state stabilization) by 10x
- "Engineered" dissipation can be used to stabilize four-qubit entanglement.
  - The time scale and dynamics is different with and without chirality
  - Chirality is essential to purify mixtures into genuine multipartite entangled states

Thank you!