Dipartimento di Informatica e
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Tools for Modeling and Analysis of Non-manifold Shapes

by

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Abstract

In this thesis, we address the effective representation of arbitrary shapes, called *non-manifold* shapes, discretized through simplicial complexes, and we introduce a set of tools for their modeling and analysis.

Specifically, we propose two dimension-independent data structures for simplicial complexes in arbitrary dimensions. The first contribution is the *Incidence Simplicial (IS)* data structure, based on the incidence relations for simplices of consecutive dimensions. The second contribution is the *Generalized Indexed Data Structure with Adjacencies (IA*)*, based on the adjacency relations for top simplices. The IS and IA* data structures are compact, support efficient navigation, and exhibit a small overhead, if restricted to manifolds. In the literature, there are several topological data structures for cell and simplicial complexes, thus a framework targeted to their fast prototyping is a valuable tool. Here, we introduce the dimension-independent and extensible *Mangrove Topological Data Structure (Mangrove TDS)* framework. This framework describes any data structure through a graph-based representation, which we call a *mangrove*. In this thesis, we provide extensive experimental comparisons for several data structures implemented in the Mangrove TDS framework, including the IS and IA* data structures. At the same time, we complete the definition of several data structures, previously proposed in the literature.

In the second part of the thesis, we decompose any non-manifold shape into almost manifold parts in order to deal with its intrinsic complexity. We consider a dimension-independent decomposition of a non-manifold shape, called *Manifold-Connected Decomposition (MC-Decomposition)*, previously investigated only for two- and three-dimensional complexes. Here, we propose several graph-based representations of such a decomposition, which can be combined with any topological data structure. We provide experimental comparisons about building times and storage costs of these data structures.

Recently, the computation of topological invariants, like the simplicial homology, has drawn much attention in several applications. Here, we design and implement the dimension-independent and modular *Mayer-Vietoris (MV)* algorithm, which exploits the MC-Decomposition for computing the simplicial homology of a non-manifold simplicial shape in arbitrary dimensions. The MV algorithm offers an elegant way for computing the homology of any simplicial complex from the homology of its MC-components and of their intersections.

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