Module 2

This notebook contains the code used for the coding examples in Module 2 of the 2025 course "Causal Inference with Linear Regression: A Modern Approach" by CausAI.

Imports

```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import statsmodels.api as sm
from scipy.stats import truncnorm
from statsmodels.iolib.summary2 import summary_col
```

Weight on Height Regression (Video 2.6)

Import Data

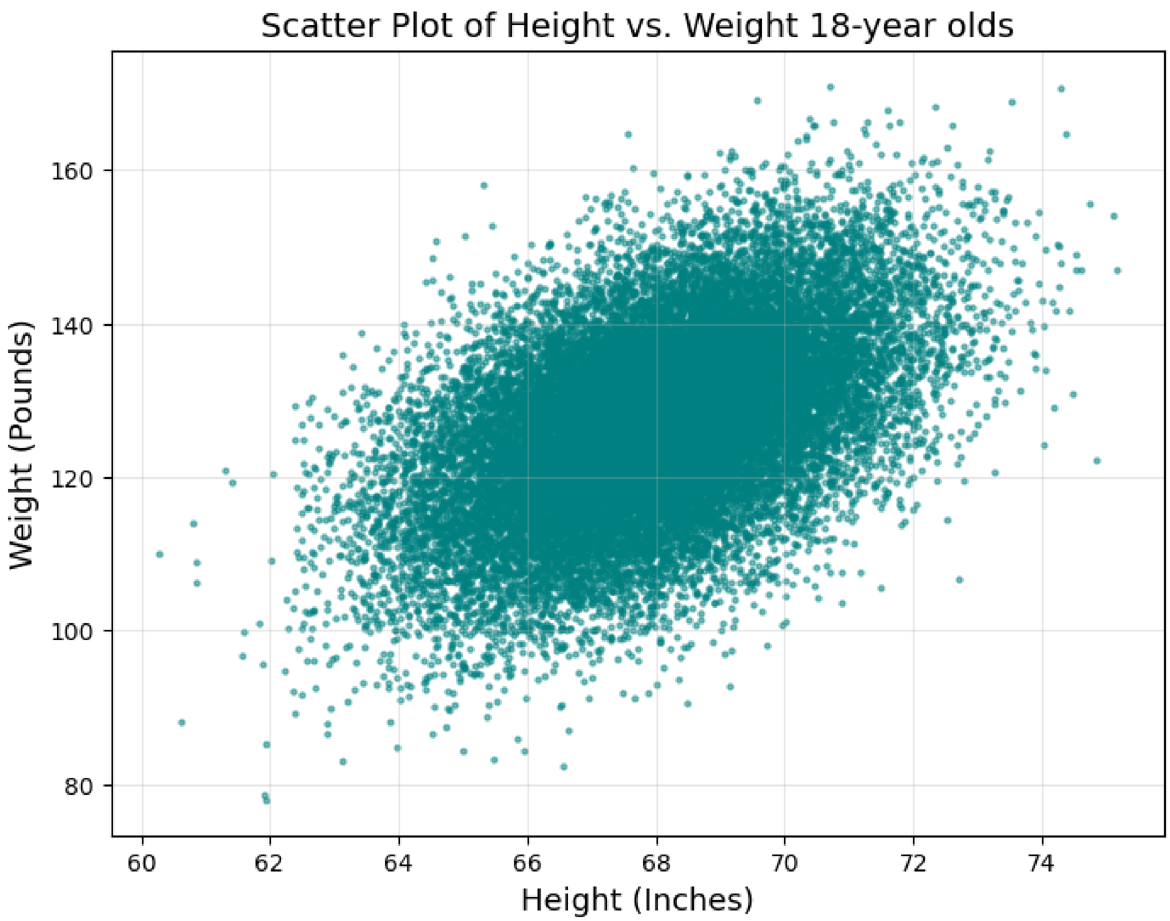
```
data_weight_height = pd.read_csv("SOCR-HeightWeight.csv")
data_weight_height[["Height(Inches)", "Weight(Pounds)"]]
```

	Height(Inches)	Weight(Pounds)
0	65.78331	112.9925
1	71.51521	136.4873
2	69.39874	153.0269
3	68.21660	142.3354
4	67.78781	144.2971
...	...	...
24995	69.50215	118.0312
24996	64.54826	120.1932
24997	64.69855	118.2655
24998	67.52918	132.2682
24999	68.87761	124.8742

Scatter Data

```
# Scatter plot for Height vs. Weight
plt.figure(figsize=(8, 6))
plt.scatter(
    data_weight_height["Height(Inches)"],
    data_weight_height["Weight(Pounds)"],
```

```
    alpha=0.5,
    s=5,
    color="teal",
)
plt.title("Scatter Plot of Height vs. Weight 18-year olds", fontsize=14)
plt.xlabel("Height (Inches)", fontsize=13)
plt.ylabel("Weight (Pounds)", fontsize=13)
plt.grid(alpha=0.3)
plt.show()
```



Regress Weight on Height using closed-form formula expressions

```
heights = data_weight_height["Height(Inches)"]
weights = data_weight_height["Weight(Pounds)"]
```

```python
# Step 1: Calculate means of X (height) and Y (weight)
mean_height = np.mean(heights)
mean_weight = np.mean(weights)

# Step 2: Calculate the slope (b1)

# Numerator
num = np.sum((heights - mean_height) * (weights - mean_weight))

# Denominator
denom = np.sum((heights - mean_height) ** 2)

b1 = num / denom  # Slope

# Step 3: Calculate the intercept (b0)
b0 = mean_weight - b1 * mean_height  # Intercept

# Output the results
print(f"Slope (b1): {b1}")
print(f"Intercept (b0): {b0}")

# Predicted regression equation:
print(f"Regression equation: w_i = {b0:.2f} + {b1:.2f} * h_i + eps_i")
```

```
Slope (b1): 3.0834764454029657
Intercept (b0): -82.57574306454087
Regression equation: w_i = -82.58 + 3.08 * h_i + eps_i
```

Regress Weight on Height using statsmodels

```python
# Step 1: Add a constant term for the intercept
X = sm.add_constant(heights)

# Step 2: Fit the OLS model
model = sm.OLS(weights, X).fit()

# Step 3: Print the summary of the model
summary_col(model, regressor_order=["Height(Inches)"], drop_omitted=False)
```

Standard errors in parentheses.

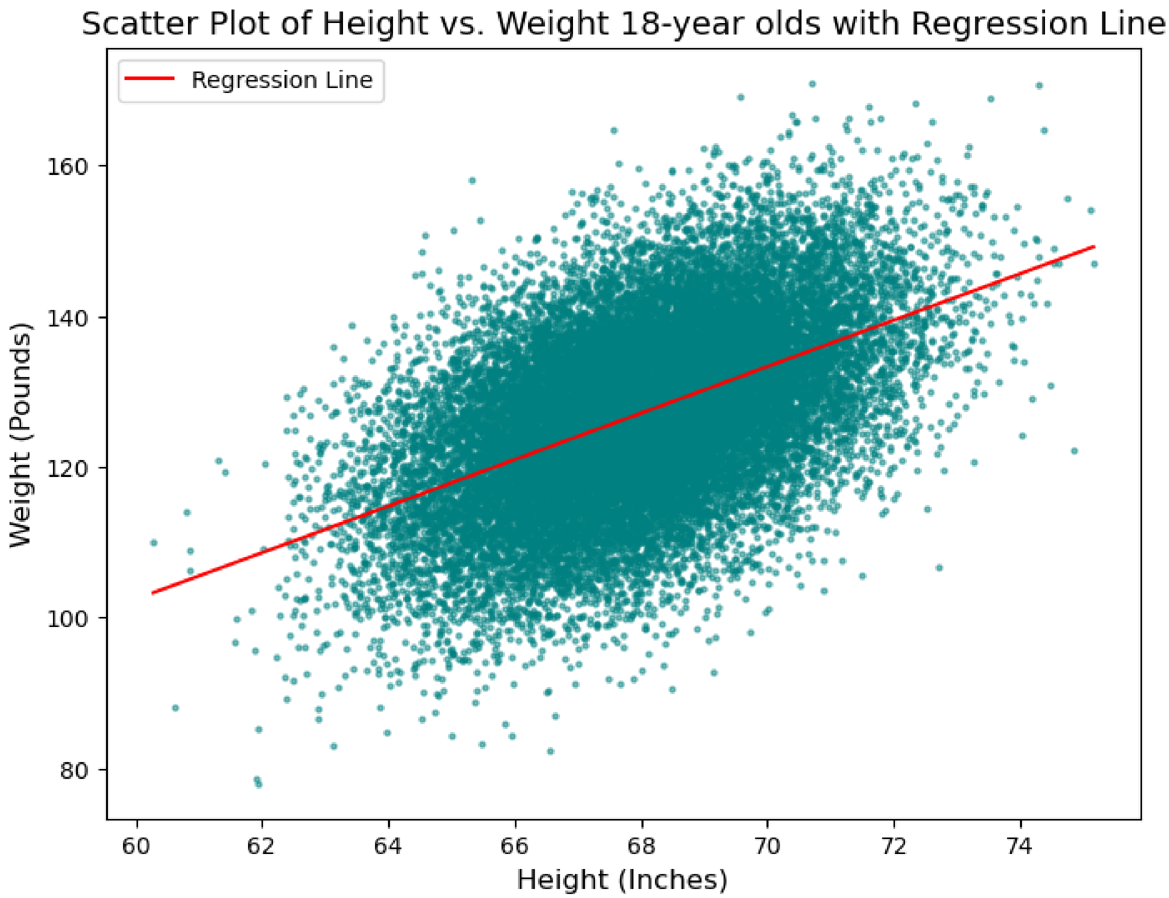
Table 2

	Weight(Pounds)
Height(Inches)	3.0835
	(0.0335)
const	-82.5757
	(2.2802)
R-squared	0.2529
R-squared Adj.	0.2528

Show regression line in scatter plot

```
# Extract regression coefficients
intercept = model.params[0]
slope = model.params[1]

# Scatter plot for Height vs. Weight
plt.figure(figsize=(8, 6))
plt.scatter(
    data_weight_height["Height(Inches)"],
    data_weight_height["Weight(Pounds)"],
    alpha=0.5,
    s=5,
    color="teal",
)
plt.title("Scatter Plot of Height vs. Weight 18-year olds", fontsize=14)
plt.xlabel("Height (Inches)", fontsize=12)
plt.ylabel("Weight (Pounds)", fontsize=12)

# Regression line
x_vals = np.linspace(heights.min(), heights.max(), 100)
y_vals = intercept + slope * x_vals
plt.plot(x_vals, y_vals, color="red", label="Regression Line")

# Plot settings

plt.title(
    "Scatter Plot of Height vs. Weight 18-year olds with Regression Line",
    fontsize=14,
)
plt.xlabel("Height (Inches)", fontsize=12)
```

```
plt.ylabel("Weight (Pounds)", fontsize=12)
plt.legend()
```



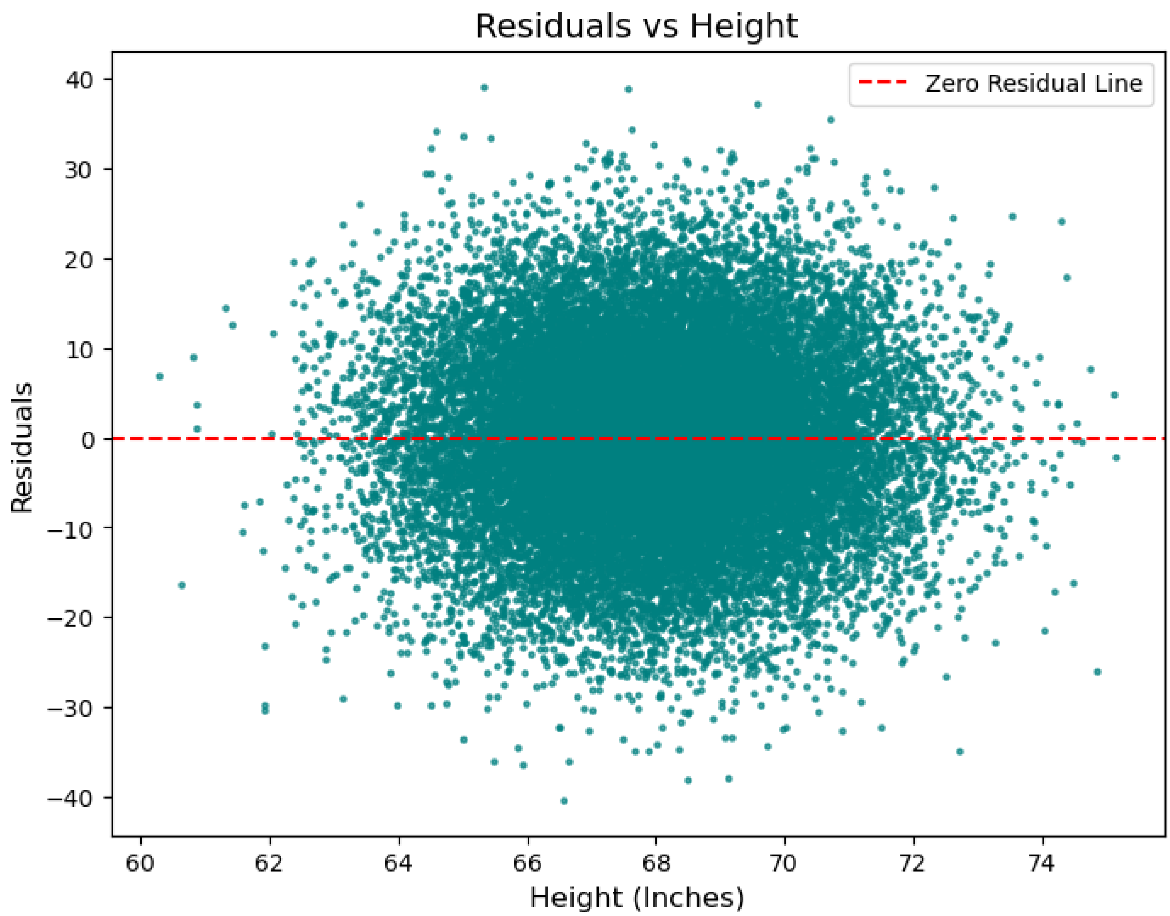
Show residuals are by construction uncorrelated with regressors

```
# get model residuals
residuals = weights - (b0 + b1 * heights)

# Check sample analog E(e) ~ 0
mean_residual = np.mean(residuals)
print(f"\nMean of residuals): {mean_residual:.5f}")

# Check sample analog E(X * e) ~ 0
mean_x_e = np.mean(heights * residuals)
print(f"\nMean of X*e: {mean_x_e:.5f}")
```

```python
print(
    f"\nCorrelation coefficient between X and e {np.corrcoef(residuals, heights)[0,1]:.5f}"
)
```

```
Mean of residuals): -0.00000

Mean of X*e: -0.00000

Correlation coefficient between X and e -0.00000
```

```python
# Plot residuals against height
plt.figure(figsize=(8, 6))
plt.scatter(heights, residuals, alpha=0.7, s=5, color="teal")
plt.axhline(0, color="red", linestyle="--", label="Zero Residual Line")
plt.xlabel("Height (Inches)", fontsize=12)
plt.ylabel("Residuals", fontsize=12)
plt.title("Residuals vs Height", fontsize=14)
plt.legend()
```



Compute correlation coefficient between the regression residuals and the independent variable

```
np.corrcoef(residuals, heights)[0, 1]
```

```
-3.9621948871994\u200b26e-17
```

FWL Theorem (Video 2.12)

```
# Set the seed and generate arbitrary data
np.random.seed(3)

X_2 = np.random.normal(0, 2, 100)
X_1 = 2 * X_2 + np.random.normal(0, 1, 100)
Y = 8 * X_1 + 2 * X_2 + np.random.normal(0, 3, 100)
```

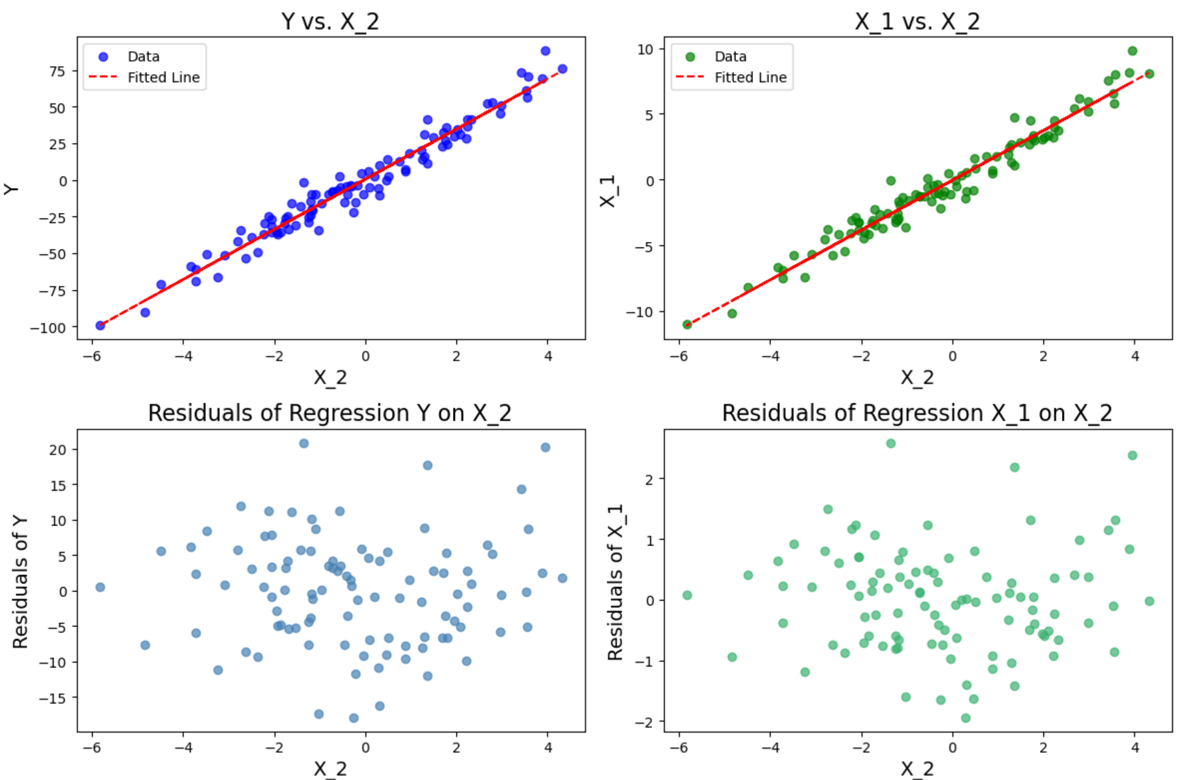
```python
df = pd.DataFrame({"X_1": X_1, "Y": Y, "X_2": X_2})

# Full Regression
independent_variables = sm.add_constant(df[["X_1", "X_2"]])
model = sm.OLS(Y, independent_variables).fit()

# FWL Step 1: Regress Y on X_2 and obtain residuals
X_2_const = sm.add_constant(df["X_2"])
model_y_on_x2 = sm.OLS(df["Y"], X_2_const).fit()
residuals_y = model_y_on_x2.resid

# FWL Step 2: Regress X_1 on X_2 and obtain residuals
model_x1_on_x2 = sm.OLS(df["X_1"], X_2_const).fit()
residuals_x1 = model_x1_on_x2.resid

# FWL Step 3: Regress residuals from step 1 on residuals from step 2
model_fwl = sm.OLS(residuals_y, residuals_x1).fit()

print(f"Coefficient for X_1 from full regression: {model.params['X_1']:.5f}")
print(f"Coefficient for X_1 from FWL theorem: {model_fwl.params[0]:.5f}")
```

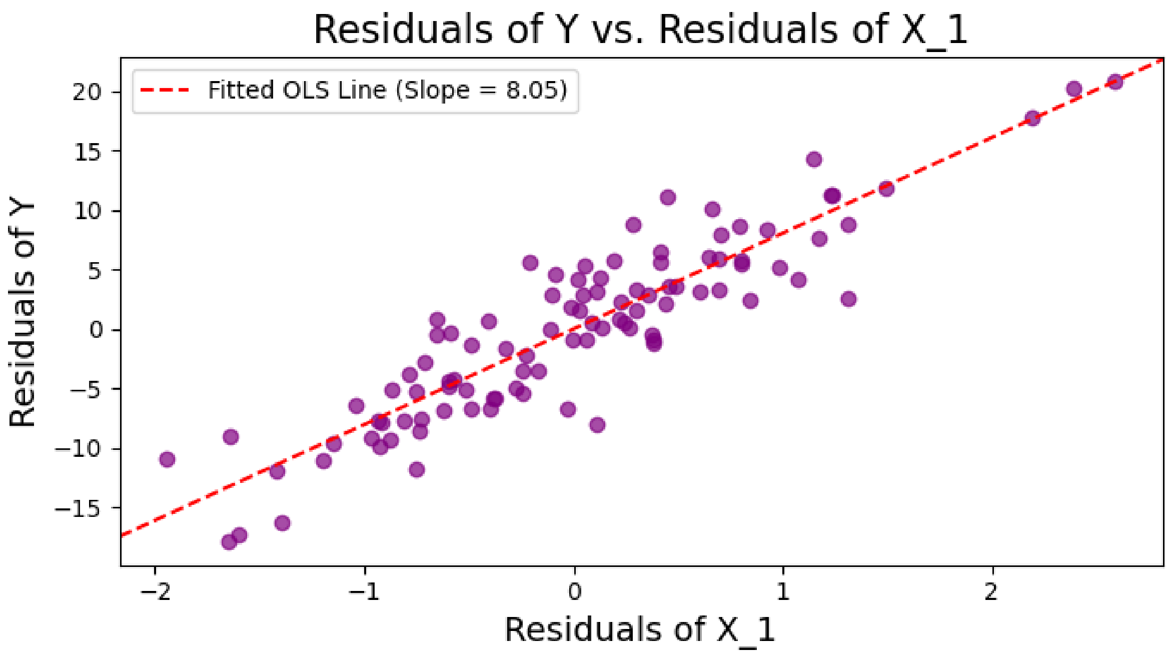
```
Coefficient for X_1 from full regression: 8.05158
Coefficient for X_1 from FWL theorem: 8.05158
```

```python
# Plot visualizations
plt.figure(figsize=(12, 8))

# Step 1: Scatter plot of Y vs. X_2 with regression line
plt.subplot(2, 2, 1)
plt.scatter(df["X_2"], df["Y"], alpha=0.7, color="blue", label="Data")
plt.plot(
    df["X_2"],
    model_y_on_x2.predict(X_2_const),
    color="red",
    linestyle="--",
    label="Fitted Line",
)
plt.title("Y vs. X_2", fontsize=16)
plt.xlabel("X_2", fontsize=14)
plt.ylabel("Y", fontsize=14)
plt.legend()
```

```python
# Step 2: Scatter plot of X_1 vs. X_2 with regression line
plt.subplot(2, 2, 2)
plt.scatter(df["X_2"], df["X_1"], alpha=0.7, color="green", label="Data")
plt.plot(
    df["X_2"],
    model_x1_on_x2.predict(X_2_const),
    color="red",
    linestyle="--",
    label="Fitted Line",
)
plt.title("X_1 vs. X_2", fontsize=16)
plt.xlabel("X_2", fontsize=14)
plt.ylabel("X_1", fontsize=14)
plt.legend()

# Step 3: Scatter plot of residuals of Y on X_2 (Partial Y)
plt.subplot(2, 2, 3)
plt.scatter(df["X_2"], residuals_y, alpha=0.7, color="steelblue")
plt.title("Residuals of Regression Y on X_2", fontsize=16)
plt.xlabel("X_2", fontsize=14)
plt.ylabel("Residuals of Y", fontsize=14)

# Step 4: Scatter plot of residuals of X_1 on X_2 (Partial X_1)
plt.subplot(2, 2, 4)
plt.scatter(df["X_2"], residuals_x1, alpha=0.7, color="mediumseagreen")
plt.title("Residuals of Regression X_1 on X_2", fontsize=16)
plt.xlabel("X_2", fontsize=14)
plt.ylabel("Residuals of X_1", fontsize=14)

plt.tight_layout()
plt.show()

# Step 5: Scatter plot of residuals of Y vs. residuals of X_1 (FWL regression)
plt.figure(figsize=(7, 4))
plt.scatter(residuals_x1, residuals_y, alpha=0.7, color="purple")
plt.title("Residuals of Y vs. Residuals of X_1", fontsize=16)
plt.xlabel("Residuals of X_1", fontsize=14)
plt.ylabel("Residuals of Y", fontsize=14)
plt.axline(
    (0, 0),
    slope=model_fwl.params[0],
    color="red",
```

```
    linestyle="--",
    label=f"Fitted OLS Line (Slope = {model_fwl.params[0]:.2f})",
)
plt.legend()
plt.tight_layout()
plt.show()
```





Exercise & Weight Loss (Videos 2.13 & 2.14)

Generate Data

```
n = 18000
np.random.seed(3)

# Define the parameters for weekly weight loss goal.
mean = 0.3
std_dev = 0.25
lower, upper = 0.1, 2

# Calculate the normalized truncation limits
a, b = (lower - mean) / std_dev, (upper - mean) / std_dev

weekly_goal = truncnorm.rvs(
    a, b, loc=mean, scale=std_dev, size=n
)  # Weekly weight loss goal in kg
exercise_hours = (
    4 + 7 * weekly_goal + np.random.normal(0, 1.2, n)
)  # Number of hours exercised in a week
calories = (
    22000
    - 8000 * weekly_goal
```

```
    + 200 * exercise_hours
    + np.random.normal(0, 1600, n)
)  # Number of calories consumed in a week
realized_loss = (
    1.6
    - 0.00007 * calories
    + 0.05 * exercise_hours
    + np.random.normal(0, 0.03, n)
)  # Realized weight loss in kg
```

```
data = pd.DataFrame(
    {
        "Weekly Goal": weekly_goal,
        "Hours Exercised": exercise_hours,
        "Calorie Intake": calories,
        "Realized Weightloss": realized_loss,
    }
)
```

```
data
```

	Weekly Goal	Hours Exercised	Calorie Intake	Realized Weightloss
0	0.393612	6.382600	21025.145689	0.422768
1	0.484694	8.049785	20006.825929	0.604361
2	0.262974	7.518145	24634.020099	0.279398
3	0.372742	6.235215	22480.177068	0.349943
4	0.644061	7.376843	17679.198245	0.705492
...	...	...	...	...
17995	0.200150	6.253155	23075.349318	0.258785
17996	0.133683	4.374678	22186.336640	0.244023
17997	0.826869	12.942140	19400.541905	0.917214
17998	0.112123	4.079003	21120.831374	0.344001
17999	0.472024	6.825330	20855.407851	0.488243

Create method to create various scatters of the data

```
def scatter_plot(
    data,
    Y_col,
    X_col,
```

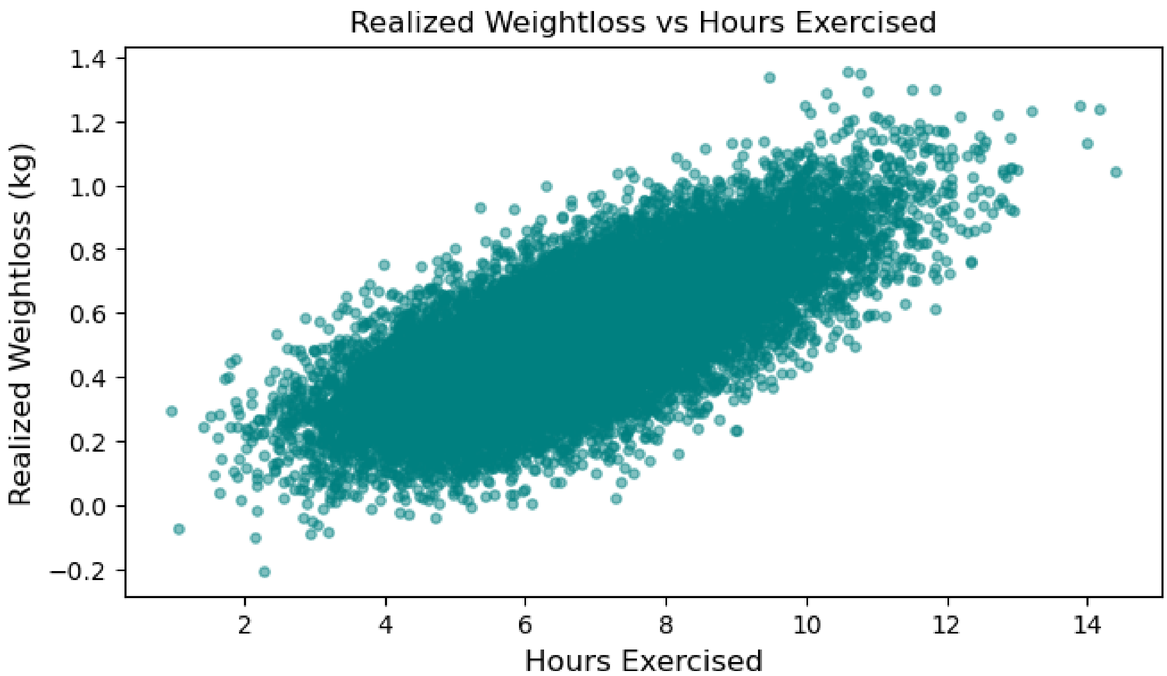
```
    colored_variable_col="",
    bucket_col="",
    regression=False,
    bucket_amount=0,
):

    plt.figure(figsize=(7.5, 4))

    if bucket_col == "":  # Data isn't bucketed

        if (
            regression
        ):  # If regression is true, then also plot the regression line

            X = data[[X_col]]
            y = data[Y_col]
            X = sm.add_constant(X)

            model = sm.OLS(y, X).fit()
            coef = model.params[X_col]

            # Regression line
            x_vals = np.linspace(data[X_col].min(), data[X_col].max(), 100)
            x_vals_with_const = sm.add_constant(x_vals)
            y_vals = model.predict(x_vals_with_const)

            plt.plot(
                x_vals,
                y_vals,
                linestyle="--",
                linewidth=2,
                color="red",
                label="Fitted OLS Line",
            )

            plt.legend()

        if (
            colored_variable_col != ""
        ):  # Data should be colored based on a variable value

            scatter = plt.scatter(
```

```
            data[X_col],
            data[Y_col],
            c=data[colored_variable_col],
            s=12,
            cmap="terrain",
        )
        plt.colorbar(scatter, label=colored_variable_col)

    else:  # No coloring based on a 3th variable
        plt.scatter(data[X_col], data[Y_col], s=15, color="teal", alpha=0.5)

    plt.title(f"{Y_col} vs {X_col}")
    plt.xlabel(X_col, fontsize=12)
    plt.ylabel(Y_col + " (kg)", fontsize=12)

else:  # If the plot should be color based on different buckets

    i = 0

    colors = ["dodgerblue", "lightgreen", "khaki"]

    for bucket in range(bucket_amount):

        if bucket in [
            6,
            8,
            10,
        ]:  # only plot for a few buckets to avoid cluttered plot

            subset = data[data[bucket_col] == bucket]

            if (
                regression
            ):  # If regression is true, then also plot regression line

                X = subset[[X_col]]
                y = subset[Y_col]
                X = sm.add_constant(X)

                model = sm.OLS(y, X).fit()
                coef = model.params[X_col]
```

```
                # Regression line
                x_vals = np.linspace(
                    subset[X_col].min(), subset[X_col].max(), 100
                )
                x_vals_with_const = sm.add_constant(x_vals)
                y_vals = model.predict(x_vals_with_const)

                plt.scatter(
                    subset[X_col],
                    subset[Y_col],
                    label=f"Bucket {bucket+1} (Coef: {coef:.3f})",
                    color=colors[i],
                    s=12,
                )
                plt.plot(
                    x_vals, y_vals, color="red", linestyle="--", linewidth=1
                )
            else:
                plt.scatter(
                    subset[X_col],
                    subset[Y_col],
                    alpha=0.8,
                    label=f"Bucket {bucket+1}",
                    color=colors[i],
                    s=12,
                )

            i += 1

        plt.title(f"{Y_col} vs {X_col} (colored by {bucket_col})")
        plt.xlabel(X_col, fontsize=12)
        plt.ylabel(Y_col, fontsize=12)
        plt.legend(
            title=f"{bucket_col} Buckets",
            loc="upper left",
            bbox_to_anchor=(1, 1),
        )

    plt.show()
```

Scatter total dataset with and without fitted OLS line

```
scatter_plot(data, "Realized Weightloss", "Hours Exercised")
scatter_plot(data, "Realized Weightloss", "Hours Exercised", regression=True)
```



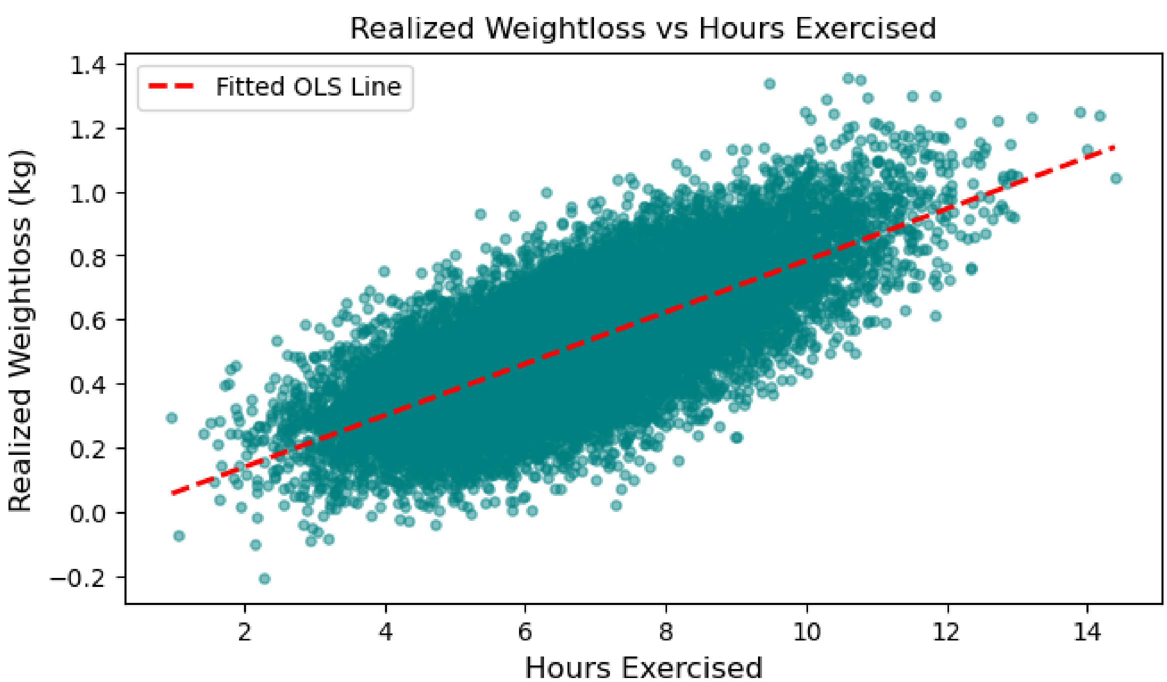


Table 4

	Realized Weightloss
Hours Exercised	0.0806
	(0.0006)
const	-0.0215
	(0.0040)
R-squared	0.5246
R-squared Adj.	0.5246

Perform regressions of Realized Weight Loss on Exercise and various other variables

```
# Model 1: Realized Weightloss on Exercise Hours

X1 = sm.add_constant(data["Hours Exercised"])
model1 = sm.OLS(data["Realized Weightloss"], X1).fit()

print("\nModel 1: Realized Weightloss on Exercise Hours \n\n")
summary_col(model1, regressor_order=["Hours Exercised"], drop_omitted=False)
```

```
Model 1: Realized Weightloss on Exercise Hours
```

Standard errors in parentheses.

```
# Model 2: Realized Weightloss on Exercise Hours and Weekly Goal

X2 = sm.add_constant(data[["Hours Exercised", "Weekly Goal"]])
model2 = sm.OLS(data["Realized Weightloss"], X2).fit()

print("\nModel 2: Realized Weightloss on Exercise Hours and Weekly Goal\n\n")
summary_col(model2, regressor_order=["Hours Exercised"], drop_omitted=False)
```

```
Model 2: Realized Weightloss on Exercise Hours and Weekly Goal
```

Table 5

	Realized Weightloss
Hours Exercised	0.0370
	(0.0007)
const	0.0551
	(0.0035)
Weekly Goal	0.5552
	(0.0067)
R-squared	0.6551
R-squared Adj.	0.6550

Standard errors in parentheses.

```
# Model 3: Realized Weightloss on Exercise Hours and Calorie Intake

X3 = sm.add_constant(data[["Hours Exercised", "Calorie Intake"]])
model3 = sm.OLS(data["Realized Weightloss"], X3).fit()


print("\nModel 3: Realized Weightloss on Exercise Hours and Calorie Intake\n\n")
summary_col(model3, regressor_order=["Hours Exercised"], drop_omitted=False)
```

```
Model 3: Realized Weightloss on Exercise Hours and Calorie Intake
```

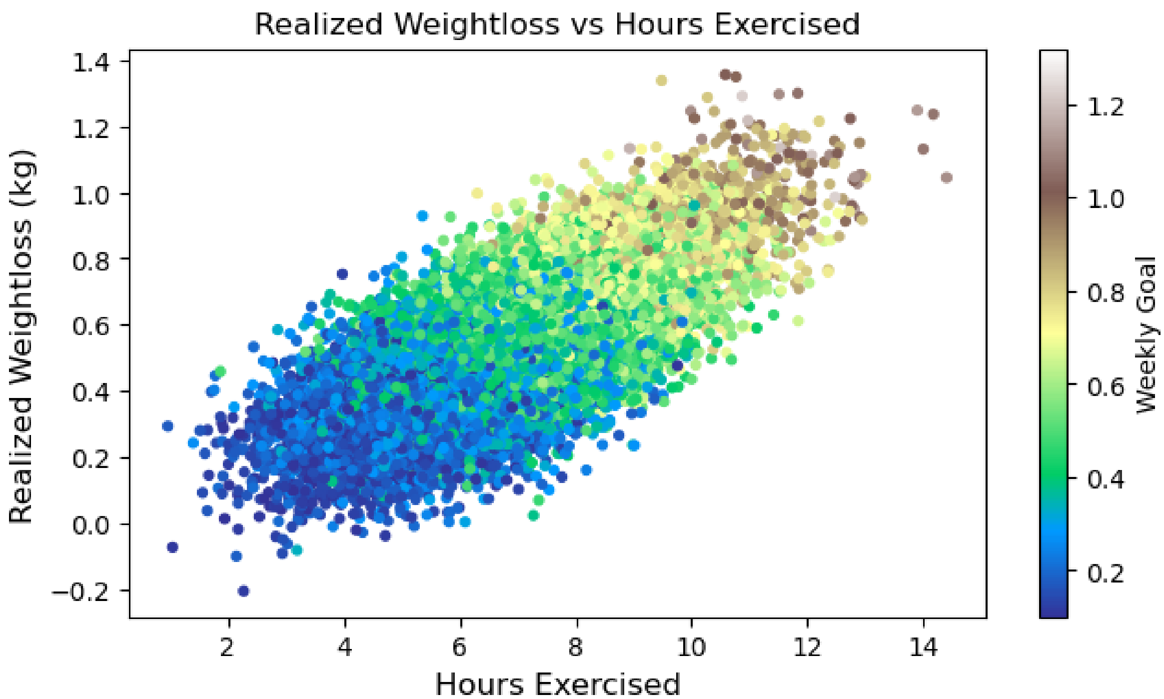
Standard errors in parentheses.

Scatter Exercise and Realized Weight loss again, but now colored based on the value of Weight Loss Goals

```
scatter_plot(
    data,
    "Realized Weightloss",
    "Hours Exercised",
    colored_variable_col="Weekly Goal",
)
```

Table 6

	Realized Weightloss
Hours Exercised	0.0502
	(0.0001)
const	1.5975
	(0.0029)
Calorie Intake	-0.0001
	(0.0000)
R-squared	0.9770
R-squared Adj.	0.9770

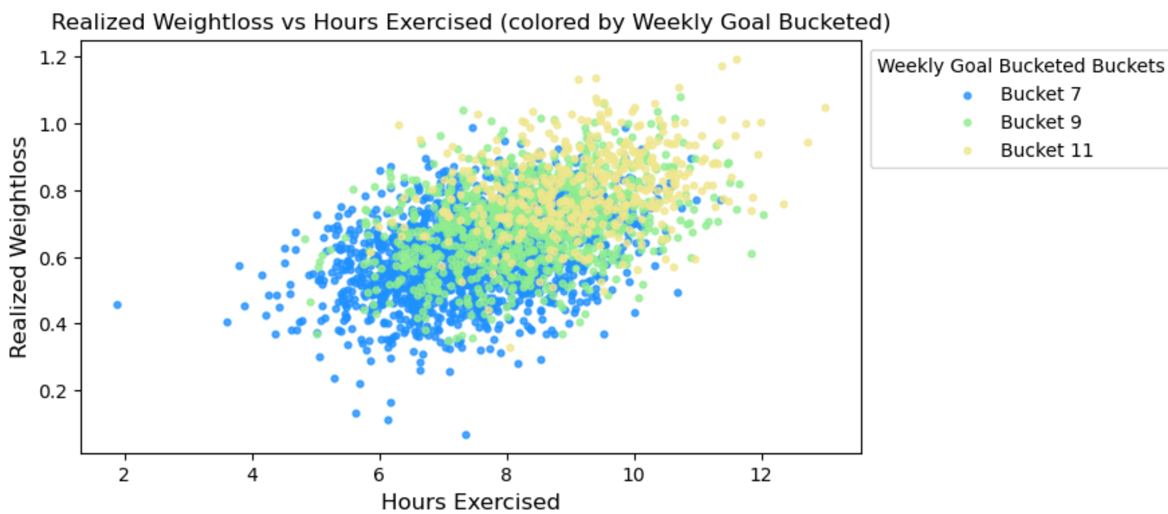


Bin the variable Weekly Weight Loss Goal into 20 equal sized buckets

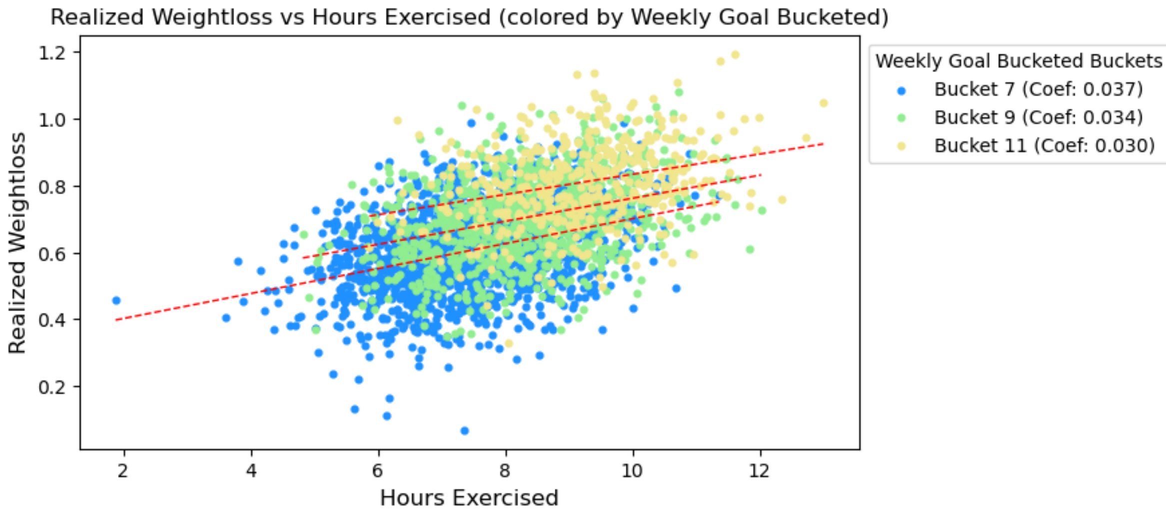
```
buckets = 20
data["Weekly Goal Bucketed"] = pd.cut(
    data["Weekly Goal"], bins=buckets, labels=False
)
```

Scatter Exercise against Realized Weight Loss again, now colored based on the bucketed values for Weight Loss Goal, both with and without sub-group regression lines

```
scatter_plot(
    data,
    "Realized Weightloss",
    "Hours Exercised",
    colored_variable_col="",
    bucket_col="Weekly Goal Bucketed",
    regression=False,
    bucket_amount=buckets,
)
```

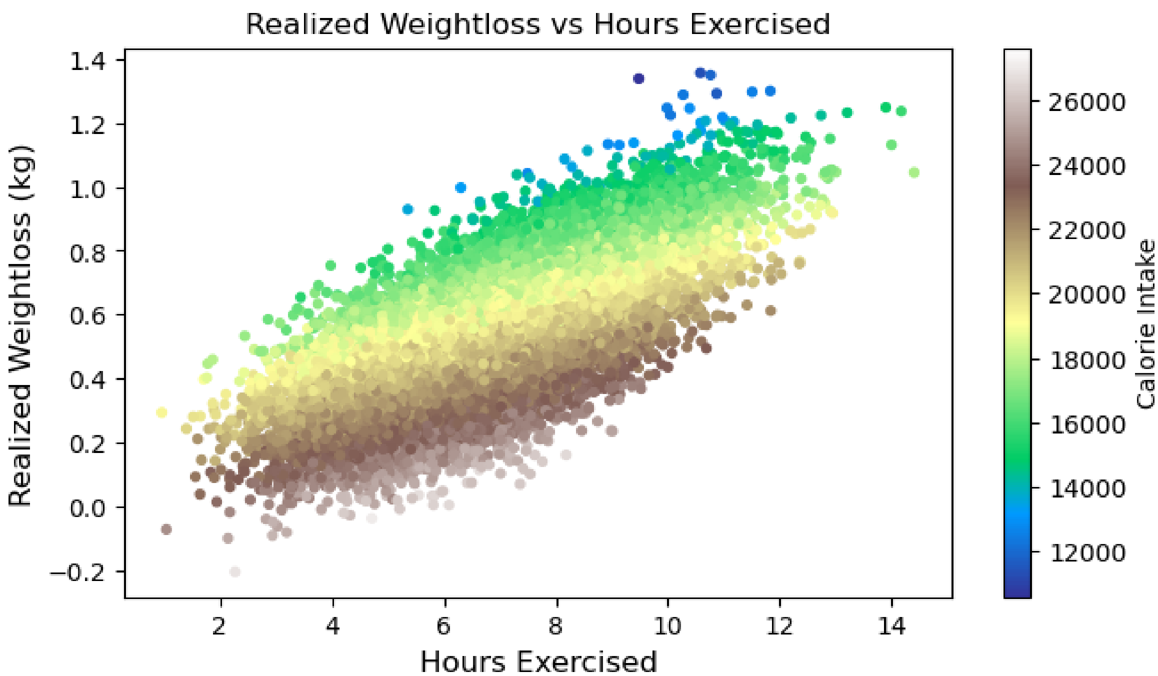



```
scatter_plot(
    data,
    "Realized Weightloss",
    "Hours Exercised",
    colored_variable_col="",
    bucket_col="Weekly Goal Bucketed",
    regression=True,
    bucket_amount=buckets,
)
```



Scatter Exercise and Realized Weight loss again, but now colored based on the value of Calorie Intake

```
scatter_plot(
    data,
    "Realized Weightloss",
    "Hours Exercised",
    colored_variable_col="Calorie Intake",
)
```

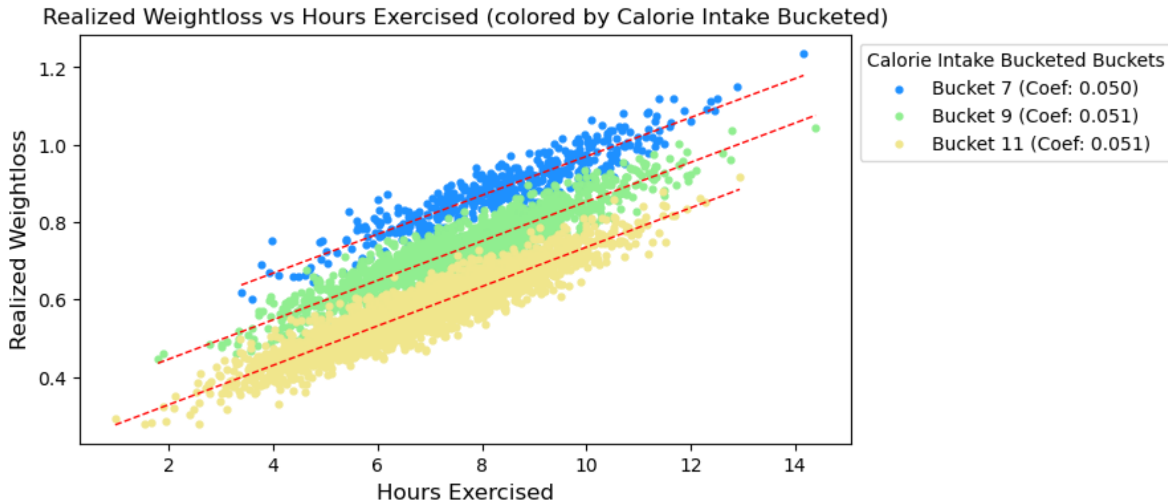


Bin the variable Calorie Intake into 15 equal sized buckets

```
data["Calorie Intake Bucketed"] = pd.cut(
    data["Calorie Intake"], bins=buckets, labels=False
)
```

Scatter Exercise against Realized Weight Loss again, now colored based on the bucketed values for Calorie Intake

```
scatter_plot(
    data,
    "Realized Weightloss",
    "Hours Exercised",
    colored_variable_col="",
    bucket_col="Calorie Intake Bucketed",
    regression=True,
    bucket_amount=buckets,
)
```



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