Answers Coding Exercise Module 2

This notebook contains the answers for the coding exercise in Module 2 of the 2025 course "Causal Inference with Linear Regression: A Modern Approach" by CausAI.

Imports

```
import pandas as pd
import numpy as np

import matplotlib.pyplot as plt
import seaborn as sns

import itertools
import random

import statsmodels.api as sm
from scipy import stats

from sklearn.preprocessing import StandardScaler
from sklearn.model_selection import KFold
from sklearn.linear_model import LinearRegression
from sklearn.preprocessing import StandardScaler
from sklearn.metrics import mean_squared_error

import warnings

warnings.filterwarnings("ignore", category=UserWarning)
```

Question 1

```
data = pd.read_csv("EV_Miles.csv")
data
```

	age	gender	urban_rural	last_year_income
0	62	0	0	68469.13
1	43	1	0	108212.95
2	63	1	0	70245.48
3	48	0	0	72165.09
4	47	1	0	49918.35
...	...	...	...	...
89858	55	1	0	110483.31

	age	gender	urban_rural	last_year_income
89859	68	1	0	88936.62
89860	43	1	0	98950.97
89861	42	0	0	102328.84
89862	38	0	1	72786.16

	gas_price	owns_ev	miles_driven	fixed_expenses
0	2.97	0	11114	24229.50
1	2.88	1	14432	23737.42
2	3.08	0	14152	10602.99
3	2.98	0	12461	14083.11
4	3.19	0	11246	22264.88
...	...	...	...	...
89858	3.09	1	14239	22057.39
89859	2.96	0	12715	16675.73
89860	2.81	0	14419	26251.98
89861	3.06	1	11868	27382.49
89862	2.92	0	12471	18386.61

Question 2

```
print("\n--- Shape of Dataset ---")
print(data.shape)

print("\n--- Columns & Data Types ---")
print(data.dtypes)

print("\n--- Unique Values Per Column ---")
for col in data.columns:
    print(f"{col}: {data[col].nunique()} unique values")

data.describe(include="all")
```

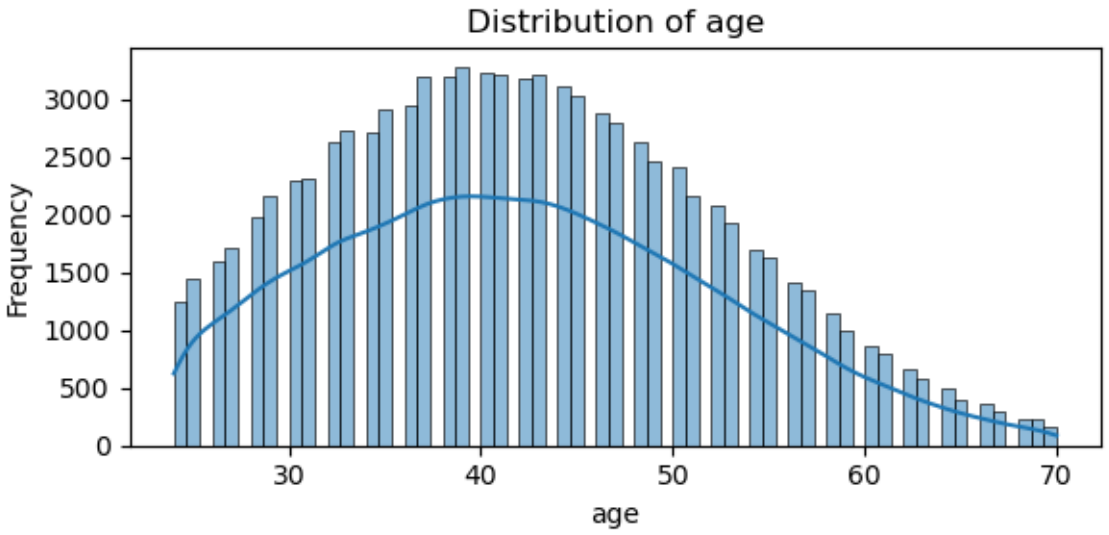
```
--- Shape of Dataset ---
(89863, 8)

--- Columns & Data Types ---
age                   int64
```

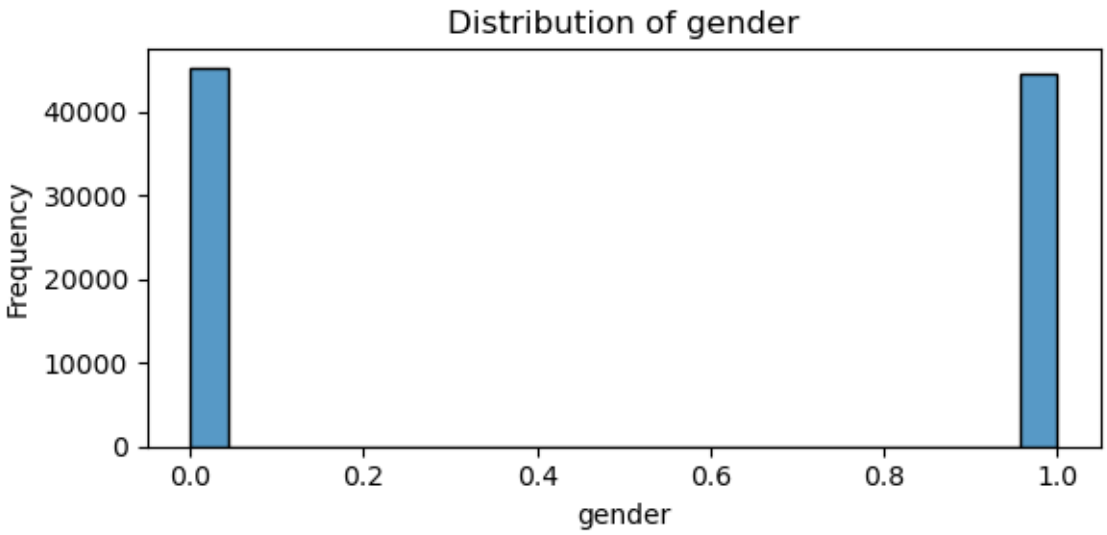
```
gender                int64
urban_rural           int64
last_year_income    float64
gas_price           float64
owns_ev               int64
miles_driven          int64
fixed_expenses      float64
dtype: object

--- Unique Values Per Column ---
age: 47 unique values
gender: 2 unique values
urban_rural: 2 unique values
last_year_income: 89282 unique values
gas_price: 95 unique values
owns_ev: 2 unique values
miles_driven: 7045 unique values
fixed_expenses: 87985 unique values
```

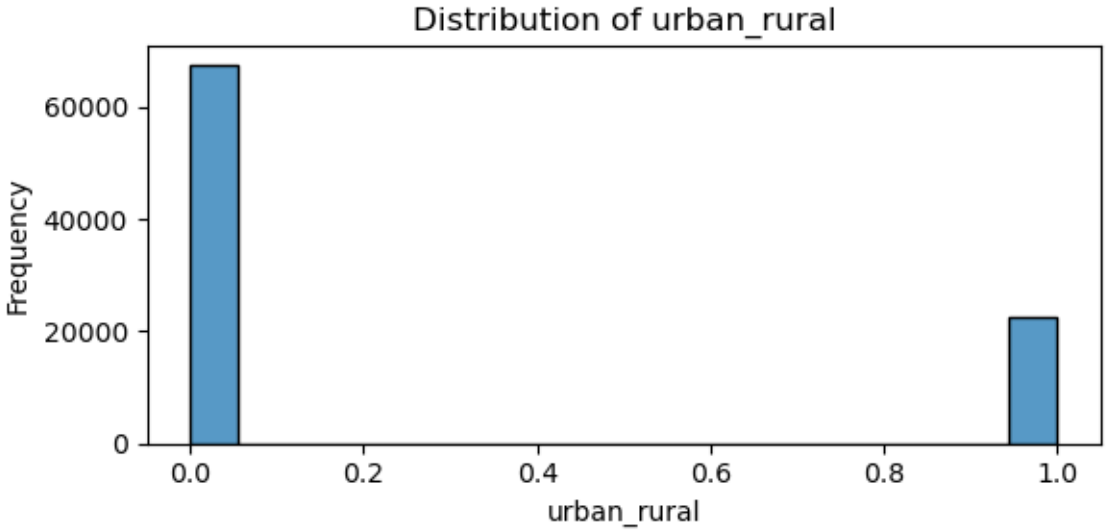
	age	gender	urban_rural	last_year_income
count	89863.000000	89863.000000	89863.00000	89863.000000
mean	42.196432	0.496879	0.24899	81093.812888
std	10.143660	0.499993	0.43243	19698.653565
min	24.000000	0.000000	0.00000	31114.510000
25%	34.000000	0.000000	0.00000	66862.575000
50%	42.000000	0.000000	0.00000	78659.890000
75%	49.000000	1.000000	0.00000	92770.785000
max	70.000000	1.000000	1.00000	198228.430000

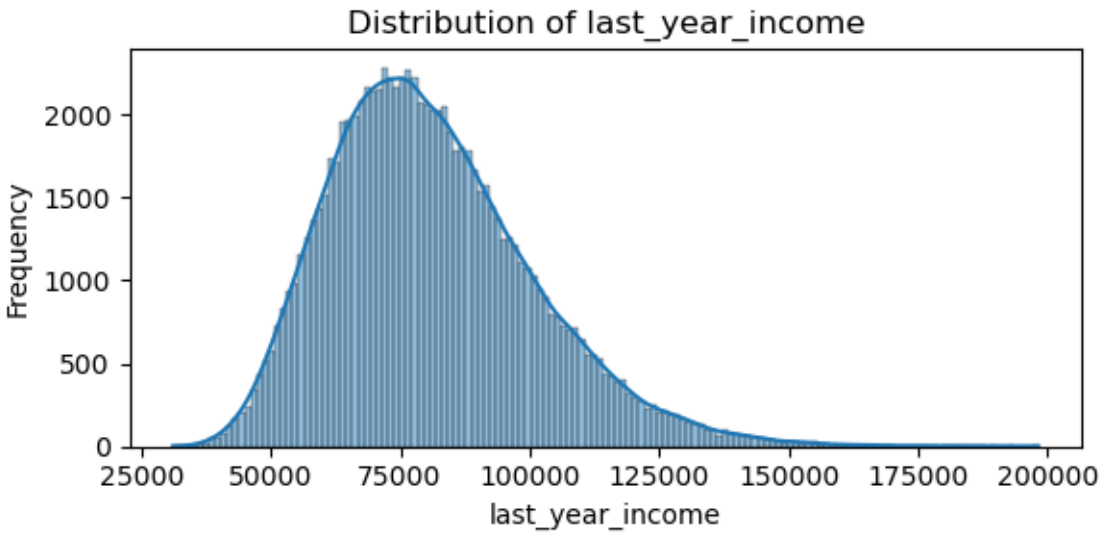
	gas_price	owns_ev	miles_driven	fixed_expenses
count	89863.000000	89863.000000	89863.000000	89863.000000
mean	3.000364	0.418526	13179.883322	20333.929052
std	0.114016	0.493320	1352.073811	6279.246484
min	2.500000	0.000000	9449.000000	5402.930000
25%	2.920000	0.000000	12238.000000	15778.330000
50%	3.000000	0.000000	13095.000000	19464.570000
75%	3.080000	1.000000	14053.000000	23956.975000
max	3.480000	1.000000	16926.000000	63361.360000

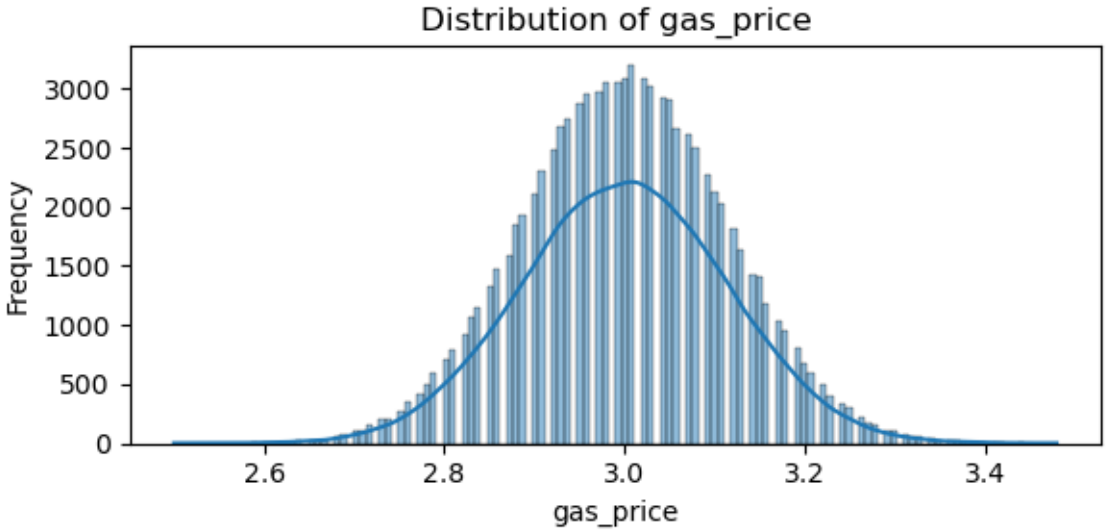
```
for col in data.columns:
    unique_vals = data[col].nunique(dropna=True)
    plt.figure(figsize=(6, 3))

    # Only show KDE if not binary
    sns.histplot(data[col], kde=(unique_vals > 2))

    plt.title(f"Distribution of {col}")
    plt.xlabel(col)
    plt.ylabel("Frequency")
    plt.tight_layout()
    plt.show()

# Box plots to identify outliers (only for non-binary numeric columns)
non_binary_num_cols = [
    col for col in data.columns if data[col].nunique(dropna=True) > 2
]

for col in non_binary_num_cols:
    plt.figure(figsize=(6, 3))
    sns.boxplot(x=data[col])
    plt.title(f"Boxplot of {col}")
    plt.xlabel(col)
    plt.tight_layout()
    plt.show()
```

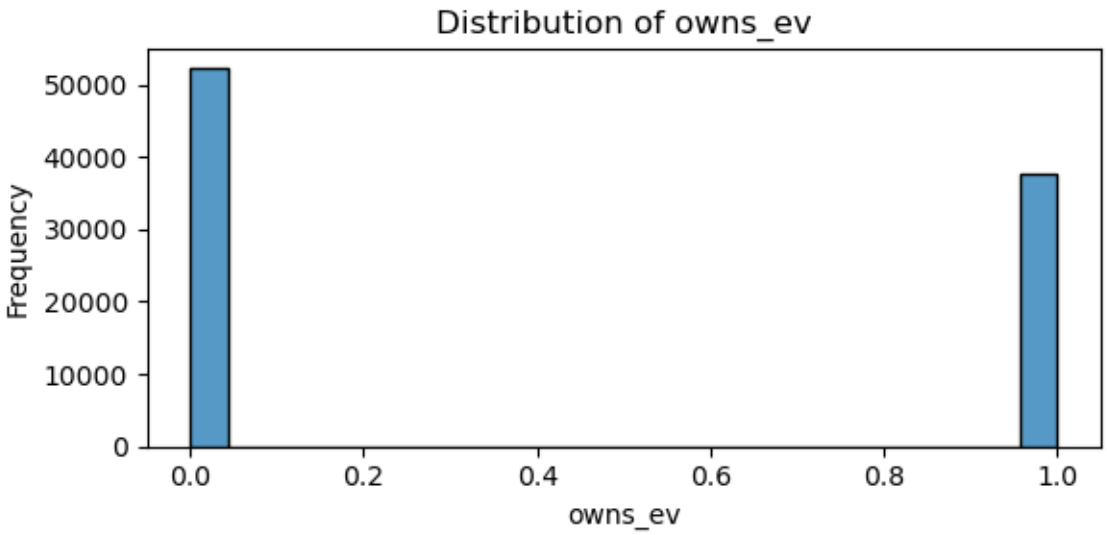


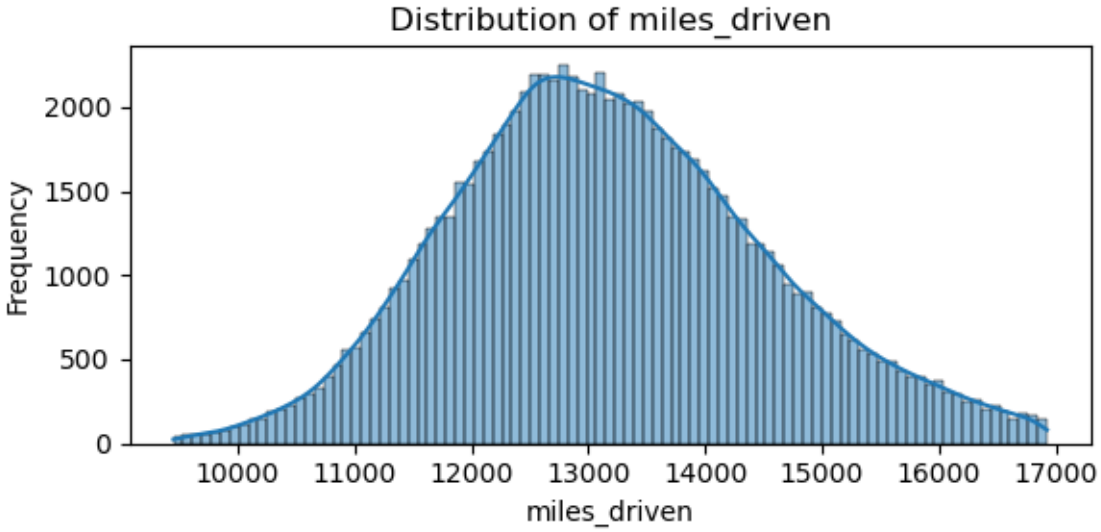

Distribution of gender
Frequency
40000
30000
20000
10000
0
0.0
0.2
0.4
0.6
0.8
1.0
gender

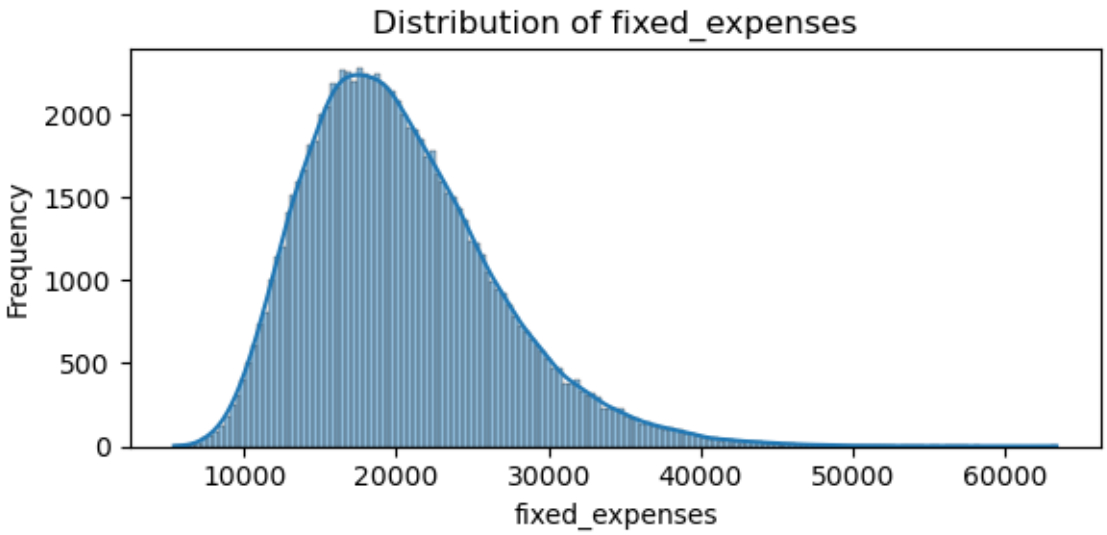

Distribution of urban_rural
Frequency
60000
40000
20000
0
0.0
0.2
0.4
0.6
0.8
1.0
urban_rural

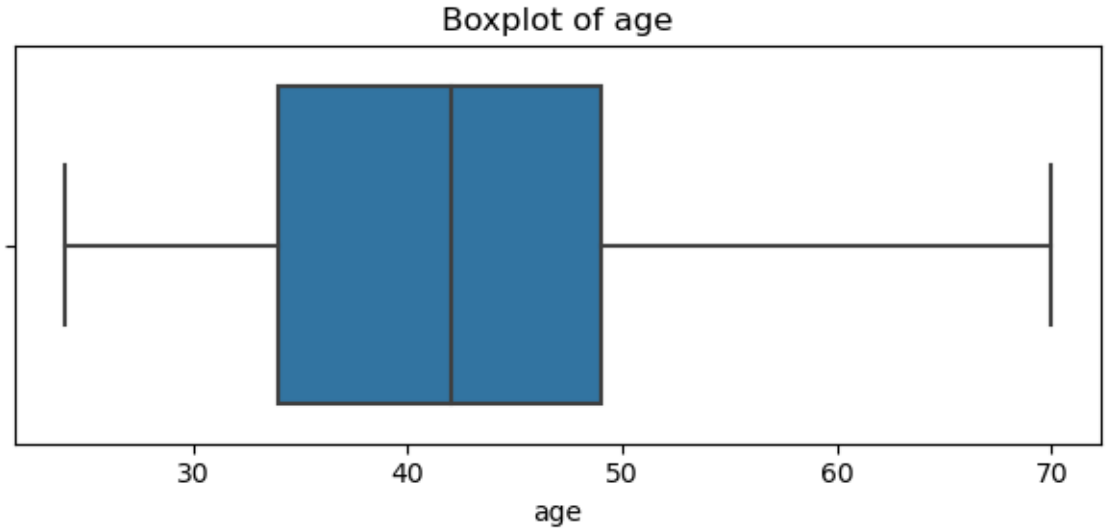

Distribution of last_year_income
Frequency
2000
1500
1000
500
0
25000
50000
75000
100000
125000
150000
175000
200000
last_year_income

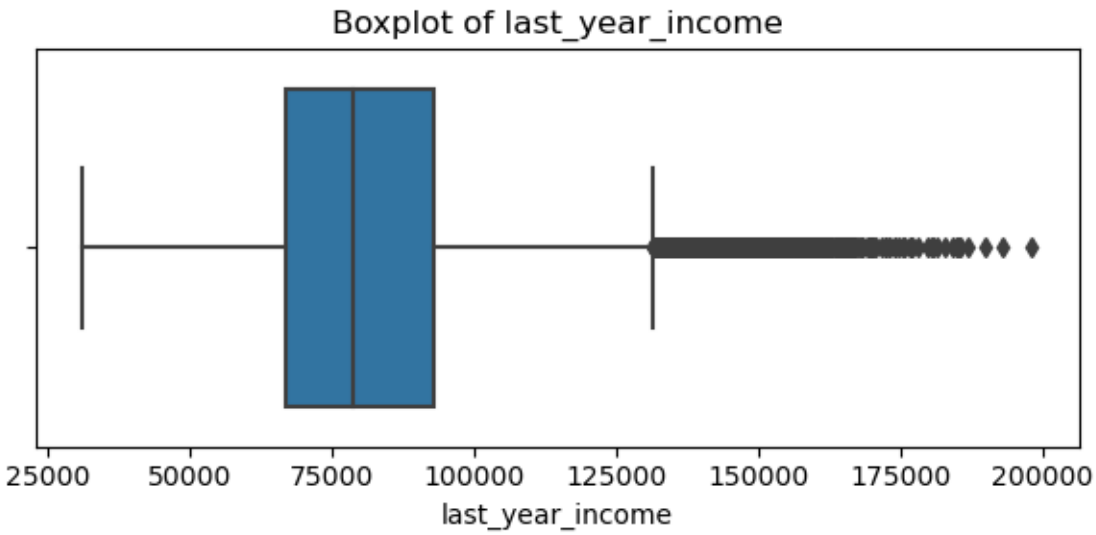

Distribution of gas_price
Frequency
3000
2500
2000
1500
1000
500
0
2.6
2.8
3.0
3.2
3.4
gas_price

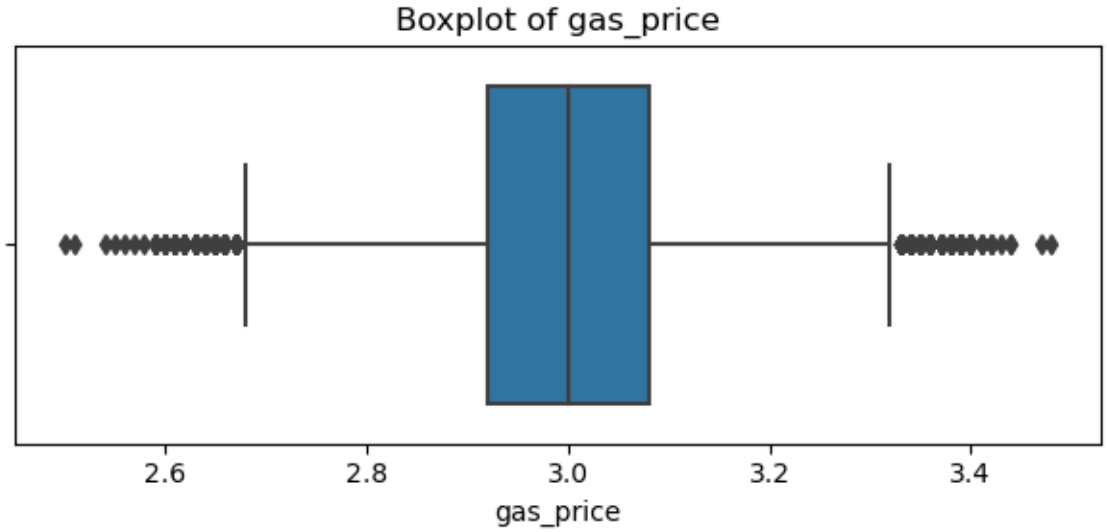

Distribution of owns_ev
Frequency
50000
40000
30000
20000
10000
0
0.0
0.2
0.4
0.6
0.8
1.0
owns_ev

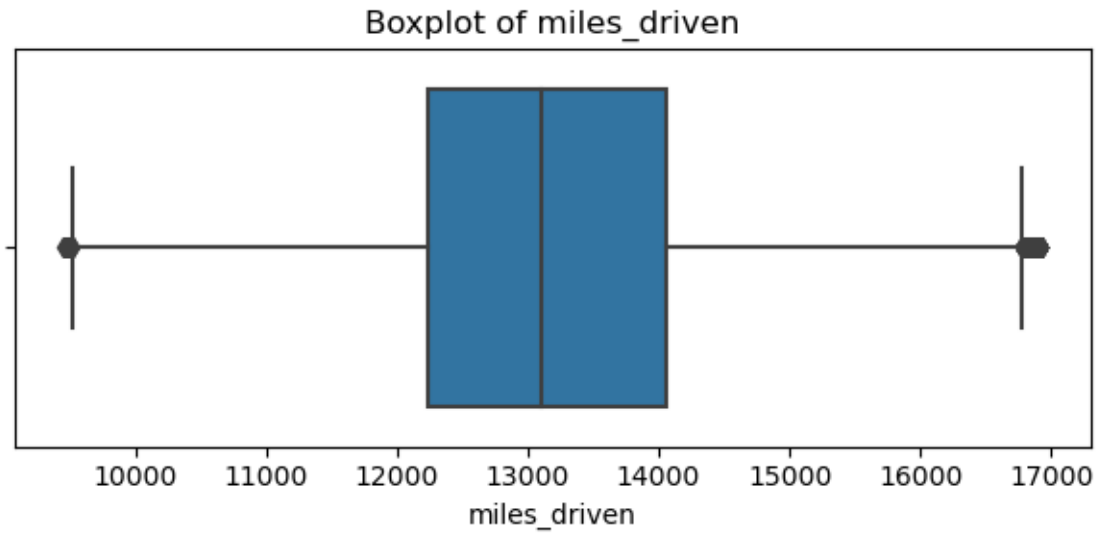

Distribution of miles_driven
Frequency
2000
1500
1000
500
0
10000
11000
12000
13000
14000
15000
16000
17000
miles_driven

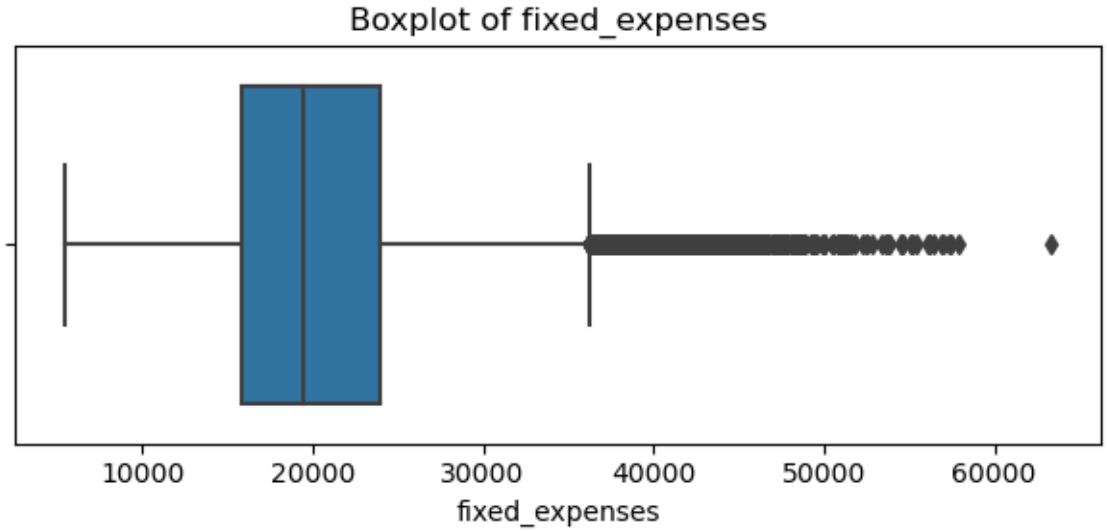

Distribution of fixed_expenses
Frequency
2000
1500
1000
500
0
10000
20000
30000
40000
50000
60000
fixed_expenses


Boxplot of age
30
40
50
60
70
age


Boxplot of last_year_income
25000
50000
75000
100000
125000
150000
175000
200000
last_year_income

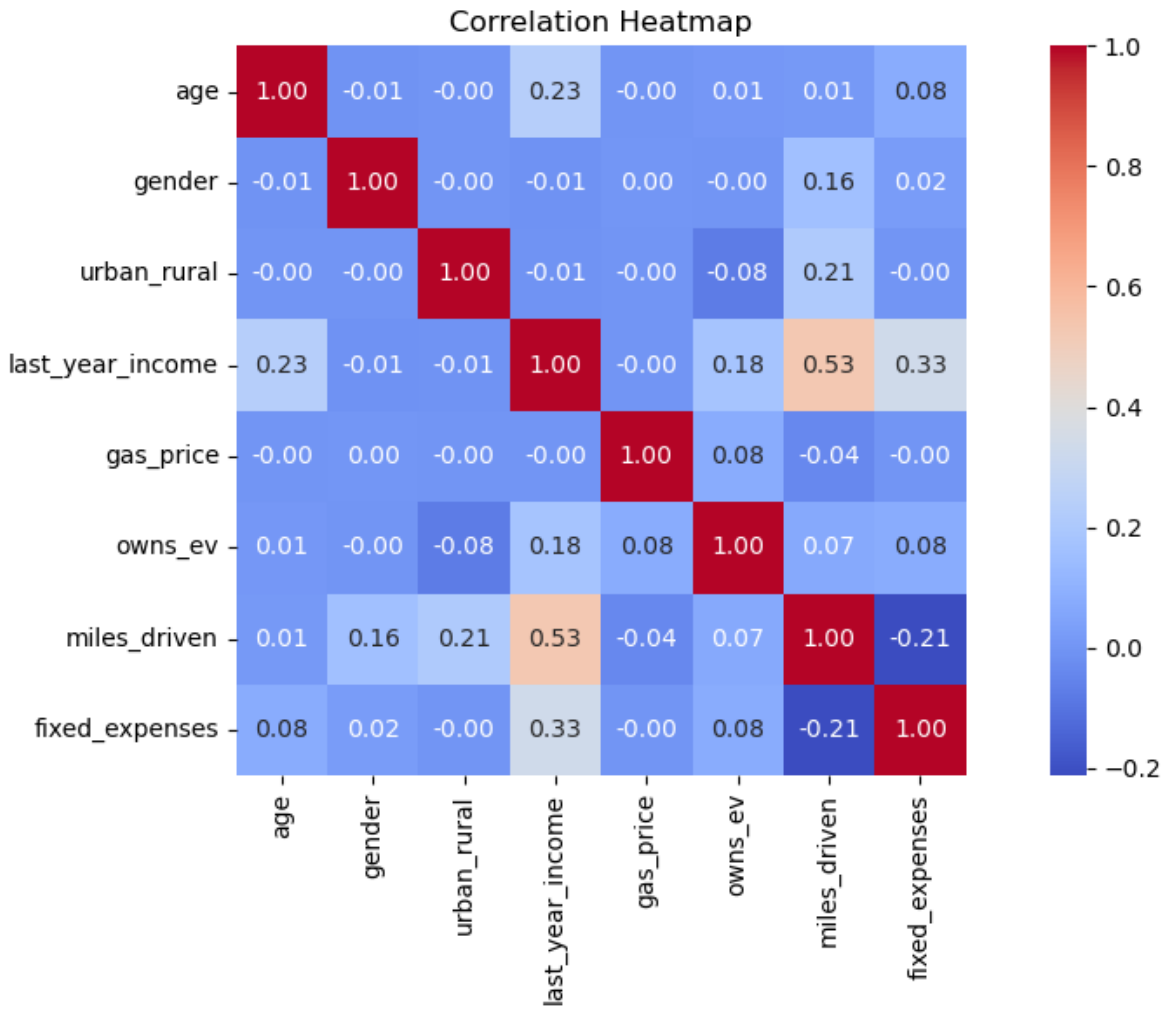

Boxplot of gas_price
2.6
2.8
3.0
3.2
3.4
gas_price

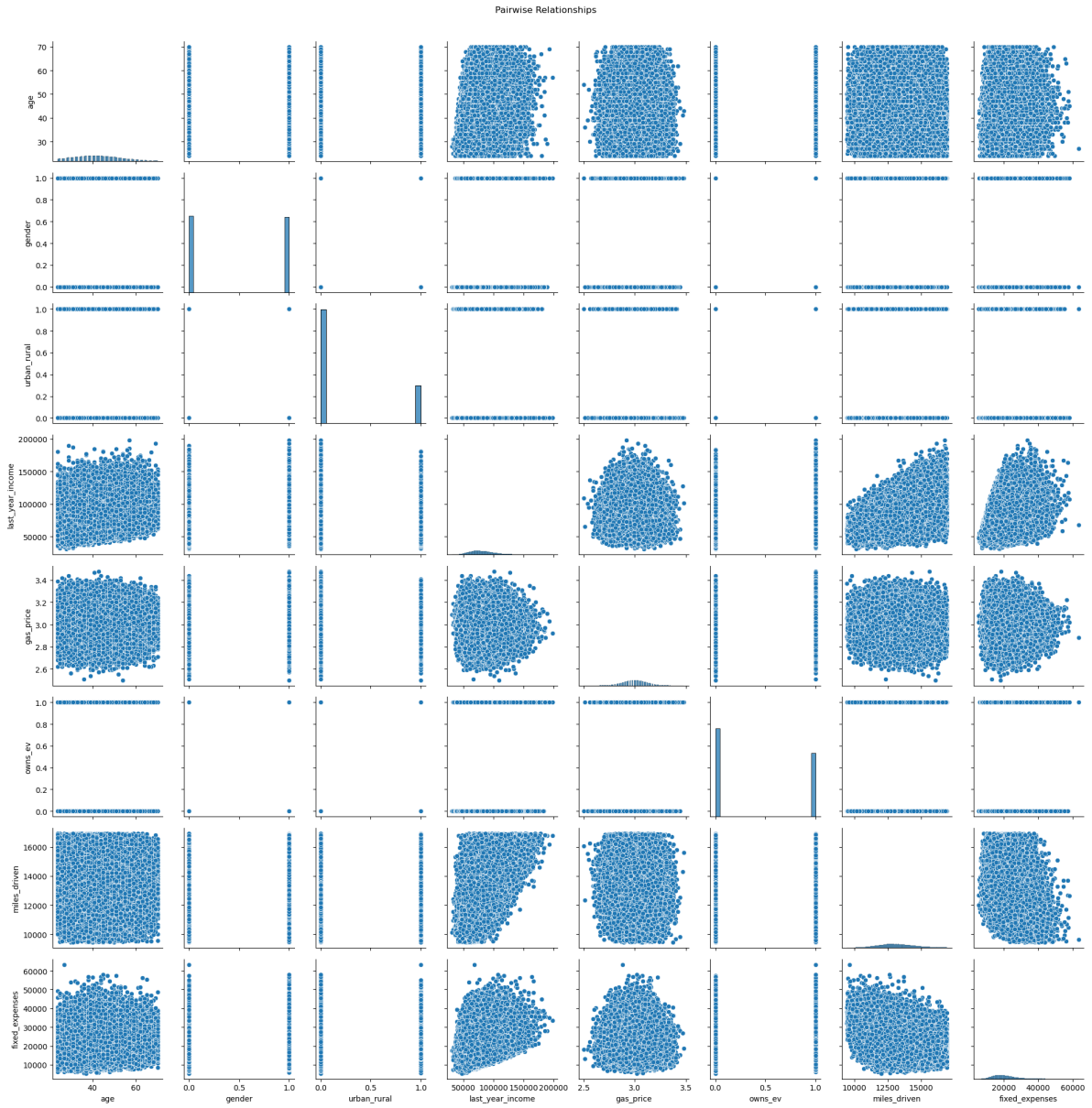





```
corr_matrix = data.corr()

# Heatmap of correlations
plt.figure(figsize=(10, 6))
sns.heatmap(corr_matrix, annot=True, cmap="coolwarm", fmt=".2f", square=True)
plt.title("Correlation Heatmap")
plt.tight_layout()
plt.show()

sns.pairplot(data)
```

```
plt.suptitle("Pairwise Relationships", y=1.01)
plt.tight_layout()
plt.show()
```





Question 3

```
mean_difference = np.mean(data["miles_driven"][data["owns_ev"] == 1]) - np.mean(
    data["miles_driven"][data["owns_ev"] == 0]
)
mean_difference
```

```
190.71781349717457
```

The mean difference is 190.72. This means that on average, owning an EV is associated with 190.72 more annual miles compared to not owning an ev. This difference probably does not reflect the average treatment effect of owning an EV on annual miles driven. There are likely several common causes of owning an EV and miles driven, for example gas prices, income, or age.

Question 4

I picked some arbitrary sets of variables

```
# All variables we can choose to add to the model
all_vars = [
    "age",
    "gender",
    "urban_rural",
    "last_year_income",
    "gas_price",
    "fixed_expenses",
]

# Standardize non-binary variables for improved numerical stability
to_standardize = ["age", "last_year_income", "gas_price", "fixed_expenses"]

data_scaled = data.copy()
scaler = StandardScaler()
data_scaled[to_standardize] = scaler.fit_transform(data[to_standardize])

# Generate random adjustment sets
random.seed(3)

all_combos = []

for r in range(2, len(all_vars) + 1):
    combos = list(itertools.combinations(all_vars, r))
    all_combos.extend(combos)

# Shuffle and take 20 unique random ones
random.shuffle(all_combos)
selected_combos = all_combos[:20]

# Run regressions and print owns_ev coefficient
for i, adjustment_set in enumerate(selected_combos, 1):
    predictors = ["owns_ev"] + list(adjustment_set)
    X = data_scaled[predictors]
```

```
    y = data_scaled["miles_driven"]

    X = sm.add_constant(X)
    model = sm.OLS(y, X).fit()

    coeff = model.params["owns_ev"]
    std_err = model.bse["owns_ev"]
    p_val = model.pvalues["owns_ev"]

    print(f"\nmiles_driven ~ owns_ev + {', '.join(adjustment_set)}")
    print(
        f"owns_ev coefficient: {coeff:.4f} | Std Err: {std_err:.4f} | p-value: {p_val:.4f}"
    )
```

```
miles_driven ~ owns_ev + urban_rural, gas_price, fixed_expenses
owns_ev coefficient: 296.0122 | Std Err: 8.7389 | p-value: 0.0000

miles_driven ~ owns_ev + gender, urban_rural, gas_price, fixed_expenses
owns_ev coefficient: 297.2670 | Std Err: 8.6078 | p-value: 0.0000

miles_driven ~ owns_ev + age, urban_rural, gas_price, fixed_expenses
owns_ev coefficient: 296.0858 | Std Err: 8.7351 | p-value: 0.0000

miles_driven ~ owns_ev + urban_rural, gas_price
owns_ev coefficient: 247.6860 | Std Err: 8.9443 | p-value: 0.0000

miles_driven ~ owns_ev + gender, last_year_income
owns_ev coefficient: -67.0465 | Std Err: 7.7449 | p-value: 0.0000

miles_driven ~ owns_ev + age, gas_price
owns_ev coefficient: 200.6857 | Std Err: 9.1371 | p-value: 0.0000

miles_driven ~ owns_ev + age, urban_rural, last_year_income
owns_ev coefficient: -32.1564 | Std Err: 7.5941 | p-value: 0.0000

miles_driven ~ owns_ev + age, urban_rural, gas_price
owns_ev coefficient: 247.5256 | Std Err: 8.9439 | p-value: 0.0000

miles_driven ~ owns_ev + age, last_year_income, gas_price
owns_ev coefficient: -70.3397 | Std Err: 7.8418 | p-value: 0.0000
```

```
miles_driven ~ owns_ev + gender, last_year_income, gas_price, fixed_expenses
owns_ev coefficient: -33.6449 | Std Err: 6.7170 | p-value: 0.0000

miles_driven ~ owns_ev + gender, urban_rural, gas_price
owns_ev coefficient: 248.1662 | Std Err: 8.8244 | p-value: 0.0000

miles_driven ~ owns_ev + gender, gas_price
owns_ev coefficient: 201.3084 | Std Err: 9.0208 | p-value: 0.0000

miles_driven ~ owns_ev + gender, last_year_income, gas_price
owns_ev coefficient: -58.4921 | Std Err: 7.7607 | p-value: 0.0000

miles_driven ~ owns_ev + age, gender, urban_rural, gas_price
owns_ev coefficient: 247.9900 | Std Err: 8.8238 | p-value: 0.0000

miles_driven ~ owns_ev + urban_rural, fixed_expenses
owns_ev coefficient: 285.2038 | Std Err: 8.7251 | p-value: 0.0000

miles_driven ~ owns_ev + age, gender, gas_price
owns_ev coefficient: 201.1345 | Std Err: 9.0204 | p-value: 0.0000

miles_driven ~ owns_ev + last_year_income, fixed_expenses
owns_ev coefficient: -42.3215 | Std Err: 6.9028 | p-value: 0.0000

miles_driven ~ owns_ev + urban_rural, last_year_income, gas_price, fixed_expenses
owns_ev coefficient: 13.5017 | Std Err: 6.6399 | p-value: 0.0420

miles_driven ~ owns_ev + age, gender, urban_rural, last_year_income, gas_price, fixed_expens
owns_ev coefficient: 0.9733 | Std Err: 6.3469 | p-value: 0.8781

miles_driven ~ owns_ev + age, gender, last_year_income, gas_price, fixed_expenses
owns_ev coefficient: -45.9296 | Std Err: 6.6338 | p-value: 0.0000
```

In each regression, you can interpret the coefficient for *owns_ev* as "Holding all other variables in the model constant, owning an EV instead of a gasoline car is linearly associated with (coefficient) more annual miles, on average". You can clearly see that this coefficient varies widely among the different regressions, from negative values like -58 to positive values like 297.

Question 5

```python
# Cross-validation setup
kf = KFold(n_splits=5, shuffle=True, random_state=3)
results = []

# Evaluate each 4-variable combination
combos = list(itertools.combinations(all_vars, 4))

for combo in combos:
    predictors = ["owns_ev"] + list(combo)
    X = data_scaled[predictors].values
    y = data_scaled["miles_driven"].values

    mse_scores = []

    for train_idx, test_idx in kf.split(X):
        X_train, X_test = X[train_idx], X[test_idx]
        y_train, y_test = y[train_idx], y[test_idx]

        model = LinearRegression()
        model.fit(X_train, y_train)
        preds = model.predict(X_test)
        mse = mean_squared_error(y_test, preds)
        mse_scores.append(mse)

    avg_mse = np.mean(mse_scores)
    results.append({"combo": combo, "avg_mse": avg_mse})

# Find the best model
best_model = min(results, key=lambda x: x["avg_mse"])

# Output
print("\nBest 4-variable adjustment set (with owns_ev):\n")
print("Combo:", best_model["combo"])
print("Average CV MSE:", round(best_model["avg_mse"], 2))
```

```
Best 4-variable adjustment set (with owns_ev):

Combo: ('gender', 'urban_rural', 'last_year_income', 'fixed_expenses')
Average CV MSE: 868677.37
```

```python
# Use the best combo from previous step
optimal_combo = best_model["combo"]
predictors = ["owns_ev"] + list(optimal_combo)

# Prepare X and y
X = data_scaled[predictors]
y = data_scaled["miles_driven"]

# Add intercept
X = sm.add_constant(X)

# Fit the final model
final_model = sm.OLS(y, X).fit()

# Print summary
print("\nFinal Regression Model Summary (Best variable set in terms of MSE):\n")
print(f"Independent Variables: owns_ev, {', '.join(optimal_combo)}" + "\n")
print(final_model.summary())
```

```
Final Regression Model Summary (Best variable set in terms of MSE):

Independent Variables: owns_ev, gender, urban_rural, last_year_income, fixed_expenses

                            OLS Regression Results
==============================================================================
Dep. Variable:           miles_driven   R-squared:                       0.525
Model:                            OLS   Adj. R-squared:                  0.525
Method:                 Least Squares   F-statistic:                 1.985e+04
Date:                Thu, 27 Mar 2025   Prob (F-statistic):               0.00
Time:                        14:02:13   Log-Likelihood:            -7.4193e+05
No. Observations:               89863   AIC:                         1.484e+06
Df Residuals:                   89857   BIC:                         1.484e+06
Df Model:                           5
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          1.277e+04      5.516   2315.978      0.000    1.28e+04    1.28e+04
owns_ev          4.1864      6.424      0.652      0.515      -8.405      16.778
gender          475.9527      6.221     76.510      0.000     463.760     488.145
urban_rural     671.8651      7.212     93.161      0.000     657.730     686.000
```

```
last_year_income   916.8499      3.343    274.245      0.000     910.297     923.402
fixed_expenses    -599.3389      3.301   -181.553      0.000    -605.809    -592.869
==============================================================================
Omnibus:                    17197.669   Durbin-Watson:                   1.995
Prob(Omnibus):                  0.000   Jarque-Bera (JB):            32678.967
Skew:                           1.184   Prob(JB):                         0.00
Kurtosis:                       4.767   Cond. No.                         3.38
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

We can interpret the coefficient for *owns_ev* in this model as follows: "For individuals with the same gender, living location (urban vs rural), last year's income and fixed expenses, owning an EV is linearly associated with 4.19 more annual miles compared to not owning an EV, on average"

Question 6

```python
# Cross-validation setup
kf = KFold(n_splits=5, shuffle=True, random_state=3)
results = []

# Evaluate each 5-variable combination

combos = list(itertools.combinations(all_vars, 5))

for combo in combos:
    predictors = ["owns_ev"] + list(combo)
    X = data_scaled[predictors].values
    y = data_scaled["miles_driven"].values

    mse_scores = []

    for train_idx, test_idx in kf.split(X):
        X_train, X_test = X[train_idx], X[test_idx]
        y_train, y_test = y[train_idx], y[test_idx]

        model = LinearRegression()
        model.fit(X_train, y_train)
        preds = model.predict(X_test)
        mse = mean_squared_error(y_test, preds)
        mse_scores.append(mse)
```

```
    avg_mse = np.mean(mse_scores)
    results.append({"combo": combo, "avg_mse": avg_mse})

# Find the best model
best_model = min(results, key=lambda x: x["avg_mse"])

# Output
print("\nBest 5-variable adjustment set (with owns_ev):\n")
print("Combo:", best_model["combo"])
print("Average CV MSE:", round(best_model["avg_mse"], 2))
```

```
Best 5-variable adjustment set (with owns_ev):

Combo: ('age', 'gender', 'urban_rural', 'last_year_income', 'fixed_expenses')
Average CV MSE: 844479.02
```

```
# Use the best combo from previous step
optimal_combo = best_model["combo"]
predictors = ["owns_ev"] + list(optimal_combo)

# Prepare X and y
X = data_scaled[predictors]
y = data_scaled["miles_driven"]

# Add intercept
X = sm.add_constant(X)

# Fit the final model
final_model = sm.OLS(y, X).fit()

# Print summary
print("\nFinal Regression Model Summary (Best variable set in terms of MSE):\n")
print(f"Independent Variables: owns_ev, {', '.join(optimal_combo)}" + "\n")
print(final_model.summary())
```

```
Final Regression Model Summary (Best variable set in terms of MSE):

Independent Variables: owns_ev, age, gender, urban_rural, last_year_income, fixed_expenses
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:           miles_driven   R-squared:                       0.538
Model:                            OLS   Adj. R-squared:                  0.538
Method:                 Least Squares   F-statistic:                 1.745e+04
Date:                Thu, 27 Mar 2025   Prob (F-statistic):               0.00
Time:                        14:02:14   Log-Likelihood:            -7.4066e+05
No. Observations:               89863   AIC:                         1.481e+06
Df Residuals:                   89856   BIC:                         1.481e+06
Df Model:                           6
Covariance Type:            nonrobust
==============================================================================
                     coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          1.278e+04      5.440   2349.500      0.000    1.28e+04    1.28e+04
owns_ev          -8.0784      6.339     -1.274      0.203     -20.502       4.345
age            -160.2235      3.156    -50.774      0.000    -166.408    -154.038
gender          474.5957      6.134     77.378      0.000     462.574     486.617
urban_rural     669.8451      7.111     94.201      0.000     655.908     683.782
last_year_income  955.5196      3.383    282.440      0.000     948.889     962.150
fixed_expenses  -599.4554      3.255   -184.174      0.000    -605.835    -593.076
==============================================================================
Omnibus:                    18700.899   Durbin-Watson:                   1.994
Prob(Omnibus):                  0.000   Jarque-Bera (JB):            37398.372
Skew:                           1.254   Prob(JB):                         0.00
Kurtosis:                       4.924   Cond. No.                         3.38
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
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We can interpret the coefficient for *owns_ev* in this model as follows: "For individuals with the same gender, living location (urban vs rural), last year's income, fixed expenses and age, owning an EV is linearly associated with 8.08 less annual miles compared to not owning an EV, on average"

Question 7

No, we can't. Linear regression coefficients are purely measures of association, and we have no idea whether these measured associations are biased or not (they likely are) or to which extent. And although the final models are optimal for predictive accuracy, this doesn't necessarily mean that they are also optimal for causal inference.

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