

# Causal Inference with Linear Regression: A Modern Approach

By CausAI



## Instructor



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## Linear Regression

Linear Regression is very well-understood, right...?

**Can you answer:**

- *If our Linear Regression coefficients are unbiased, then what exactly are they unbiased for?*
- *To obtain unbiased coefficient estimates in our model, is it necessary to include all variables that are part of the DGP for the outcome variable?*
- *Under what exact conditions can Linear Regression coefficients be interpreted causally, and under what conditions can they not?*



## Linear Regression

Linear Regression is very well-understood, right...?

**Can you answer:**

- *If regression coefficients do represent causal effects, how should we interpret them when our model includes quadratic, cubic, or interaction terms?*
- *What if the "No Omitted Variable Bias" assumption is violated?*



## Course Goals

- Understand when and how Linear Regression can be used for Causal Inference
- A modern approach using Judea Pearl's Causal Inference framework
- First part of a 2-part course series on Causal Inference with Linear Regression



## Course Content

- **Module 1:** Basics of Causal Inference
- **Module 2:** Mechanics of Linear Regression with OLS
- **Module 3:** Structural Causal Models, its parameters, and when Linear Regression can uncover Structural Causal Parameters
- **Module 4:** Robustness Tests and Sensitivity Analysis
- Clarify common misconceptions, discuss high-quality Causal Inference sources, coding examples & exercises



## Who is this course for?

- Everyone with basic Probability, Linear Algebra and Statistics knowledge
- It's beneficial if you know Python
- No prior knowledge on Causal Inference is required



## Why Causal Inference?

- Mainstream AI Methods don't help us understand the impact of different decisions on future outcomes
- They rely on patterns/associations in data to make predictions
- E.g. An airline company might predict higher demand during holidays because demand and holidays are positively associated
- E.g. A grocery store might predict higher sales of a tropical drink on a sunny day



## Why Causal Inference?

- But we don't just want to predict outcomes, we want to **change** them
- Airline company: we don't just want to predict demand, we want to **increase** demand
- Grocery store: we don't just want to predict sales, we want to **increase** sales
- Can association-focused models help here?



## Why Causal Inference?

- Airline company: higher prices are associated with higher demand, does this mean increasing the prices will increase demand?
- No... positive association exists because holiday increases both
- *Association/Correlation is not Causation*
- We can't tweak variable values in association-focused models and see how the outcome changes as a result to measure causal effects



## Why Causal Inference?

- Most business questions are about the effect of decisions on outcomes
- *What's the effect of marketing strategy X on sales?*
- *How will launching a loyalty program affect customer retention?*
- *What impact will introducing a new product line have on revenue?*
- *How will offering free delivery influence customer satisfaction?*

We need Causal Inference!



## Individual Treatment Effect

- Suppose you own a company that has multiple retail stores, and one of the stores struggles with high stockout rates
- You hire a consultant to optimize inventory management, and the stockout rate decreases from 20% to 8% over the next quarter
- You want to know: if you hadn't hired the consultant, would the stockout rate of the store still have improved?

**SOLD OUT**



## Individual Treatment Effect

- Let  $T$  denote the treatment variable, where  $T = 1$  means you hire the consultant, and  $T = 0$  means you don't
- Let  $Y$  be the outcome variable, representing the stockout rate (e.g.  $Y = 0.12$  means the stockout rate is 12% after the quarter,  $Y = 0.15$  means the stockout rate is 15% after the quarter etc.)
- Denote by  $Y(1)$  the stockout rate  $Y$  of the store if you hire the consultant, and  $Y(0)$  the stockout rate if you don't. Here,  $Y(t)$ , where  $t = 0$  or  $1$  are called potential outcomes

The quantity of interest is (ITE)  $Y(1) - Y(0)$



## Individual Treatment Effect

- Let  $T$  denote the variable representing different treatment options
- A *unit* is the basic entity being studied at a specific point in time (stores, countries, households,...)
- For each unit and treatment pair, the outcome that results from applying that treatment is the potential outcome. We define this as  $Y_i(t)$
- Each  $Y_i(t)$  is a realization of random variable  $Y(t)$ , where  $Y(t)$  can differ for different units



## Individual Treatment Effect

The *Individual Treatment Effect* for unit  $i$  is defined as (binary treatment)

$$Y_i(1) - Y_i(0)$$

- All potential outcomes have the potential to be observed, but only one is actually observed for a unit
- The observed outcome is called the *factual outcome*, unobserved potential outcomes are called *counterfactual outcomes*
- Since we can never observe counterfactuals, we can never compute ITEs exactly
- It's hard to obtain reliable estimates for ITE's...



## Average Treatment Effect

Given binary treatment  $T$ , the *Average Treatment Effect* is defined as

$$ATE = \mathbb{E}[Y(1) - Y(0)] = \mathbb{E}[Y(1)] - \mathbb{E}[Y(0)]$$

- We no longer look at the treatment effect for one specific unit, but rather at the average effect for an entire population
- In the store example: we look at the effect hiring a consultant on average has on the stockout rates of all stores in the chain
- The *ATE* is the "best guess" for the treatment effect of a randomly selected unit from the population
- It hides individual differences...



## Average Treatment Effect

- When the treatment effect differs among units, we say that the treatment effects are *heterogeneous*
- The *ATE* can't capture heterogeneity which results in information loss, but the *ITE* can't be estimated reliably...
- A midway approach: Conditional Average Treatment Effects!



## Conditional Average Treatment Effect

Given binary treatment  $T$ , the *Conditional ATE* is defined as

$$CATE(x) = \mathbb{E}[Y(1) - Y(0)|X = x] = \mathbb{E}[Y(1)|X = x] - \mathbb{E}[Y(0)|X = x]$$

where  $X$  represents a set of features describing the population units

- The  $CATE(x)$  is just an *ATE* in a subgroup of the population, defined by characteristics  $X = x$
- Since  $CATE(x)$  can differ for different values of  $X$ , it can capture heterogeneity



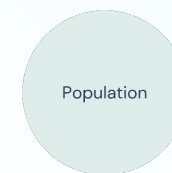
## Treatment Effects

In our store example:

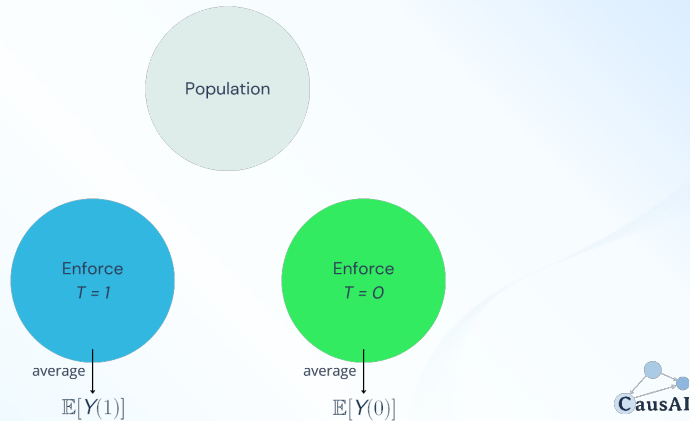
- The *ATE* focuses on populations, so it calculates the average effect of hiring a consultant on all of our stores
- The  $CATE(x)$  focuses on subpopulations, so it calculates the average effect of hiring a consultant on stores with similar characteristics  $X = x$
- The *ITE* focuses on individual units, so it calculates the effect of hiring a consultant for one specific store



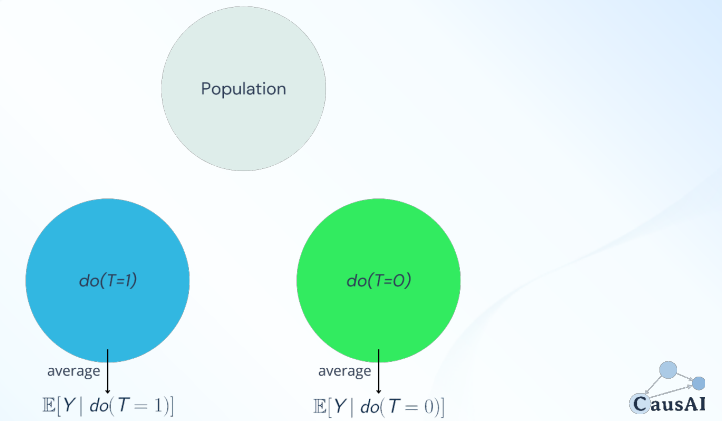
$$ATE = \mathbb{E}[Y(1)] - \mathbb{E}[Y(0)]$$



$$ATE = \mathbb{E}[Y(1)] - \mathbb{E}[Y(0)]$$



$$ATE = \mathbb{E}[Y(1)] - \mathbb{E}[Y(0)]$$



## Average Treatment Effect

The *ATE* can be defined in do-operator notation as (binary treatment)

$$ATE = \mathbb{E}[Y | do(T=1)] - \mathbb{E}[Y | do(T=0)]$$

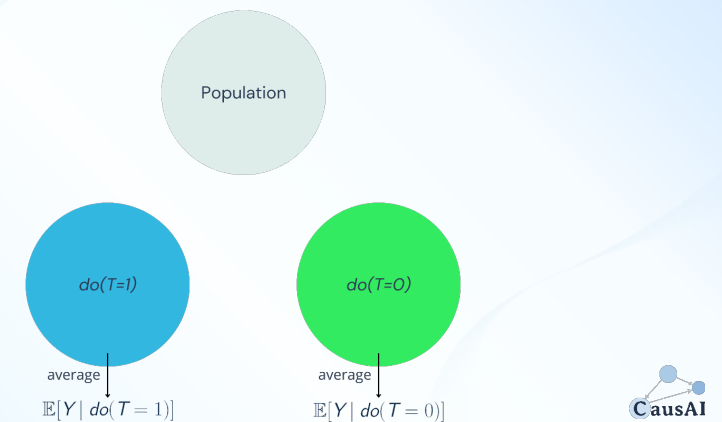
This looks similar to familiar statistical expression

$$\mathbb{E}[Y | T=1] - \mathbb{E}[Y | T=0]$$

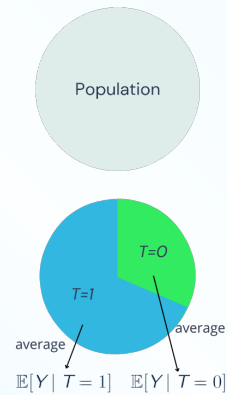
But they are not the same...



## Average Treatment Effect



## Average Treatment Effect



## Example

- Imagine a company that offers a paid subscription service providing additional benefits to its customers
- The company wants to reduce churn (the rate at which customers cancel their subscriptions) by sending out discounts
- But before they do, they want to understand the effect they can expect of sending a discount on the churn rate



## Example

	CustomerID	ShoppingFrequency	DiscountSent	Churn	
	0	1	medium	0	1
	1	2	medium	1	0
	2	3	low	1	1
	3	4	medium	1	0
	4	5	high	0	0
	...	...	...	...	...
	49995	49996	medium	0	1
	49996	49997	medium	0	1
	49997	49998	low	1	1
	49998	49999	low	0	0
	49999	50000	medium	1	1

50000 rows × 4 columns



## Example

	CustomerID	ShoppingFrequency	DiscountSent	Churn
	1	2	medium	1
	2	3	low	1
	3	4	medium	1
	7	8	low	1
	8	9	low	1
	...	...	...	...
	49990	49991	medium	1
	49991	49992	high	1
	49993	49994	medium	1
	49997	49998	low	1
	49999	50000	medium	1
26476 rows × 4 columns				

26476 rows × 4 columns

$$\mathbb{E}[Churn | DiscountSent = 1]$$

	CustomerID	ShoppingFrequency	DiscountSent	Churn	
	0	1	medium	0	1
	4	5	high	0	0
	5	6	high	0	0
	6	7	low	0	0
	11	12	medium	0	0
	...	...	...	...	...
	49992	49993	high	0	1
	49994	49995	medium	0	0
	49995	49996	medium	0	1
	49996	49997	medium	0	1
	49998	49999	low	0	0

23524 rows x 4 columns

23524 rows × 4 columns

$$\mathbb{E}[Churn | DiscountSent = 0]$$





## Example

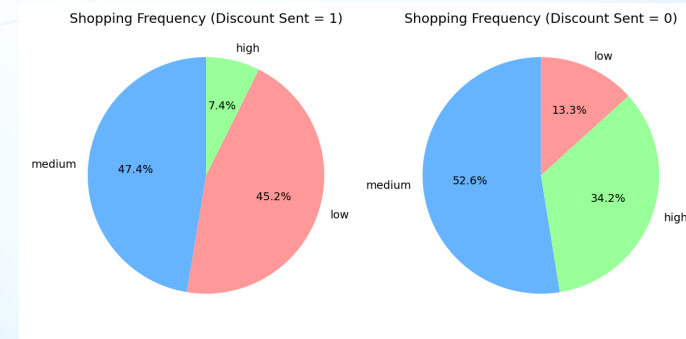
Surprisingly

$$\mathbb{E}[\text{Churn} \mid \text{DiscountSent} = 1] - \mathbb{E}[\text{Churn} \mid \text{DiscountSent} = 0]$$

is positive



## Example

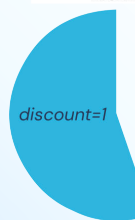


## Example

- The marketing team tended to send discounts sooner to customers with lower historical shopping frequency
- But historical shopping frequency also directly affects churn probability
- For customers who got discounts we are more likely to see higher churn rates compared to those who didn't get discounts, not necessarily because of whether they got a discount or not, but rather because of differences in their historical shopping frequency (bias!)



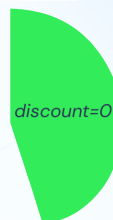
## Example



$$\mathbb{E}[\text{Churn} \mid \text{DiscountSent} = 1]$$

$$- \mathbb{E}[\text{Churn} \mid \text{DiscountSent} = 0]$$

is positive





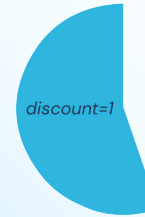
## Example



Even if the customers with discount would have actually had discount = 0, we would still observe a nonzero difference in average outcomes



## Example



Even if the customers without discount would have actually had discount = 1, we would still observe a nonzero difference in average outcomes



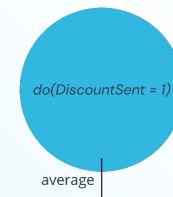
## Example

$$ATE = \mathbb{E}[Churn \mid do(DiscountSent = 1)] - \mathbb{E}[Churn \mid do(DiscountSent = 0)]$$

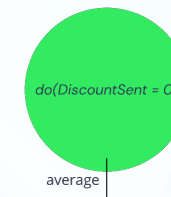
$$\neq$$

$$\mathbb{E}[Churn \mid DiscountSent = 1] - \mathbb{E}[Churn \mid DiscountSent = 0]$$

## Example



average  
 $\mathbb{E}[Churn \mid do(DiscountSent = 1)]$



average  
 $\mathbb{E}[Churn \mid do(DiscountSent = 0)]$

## Example

- To make the associational difference equal to the *ATE*, we need that any characteristics that affect the outcome are equally distributed among the groups with different treatments
- Most intuitive way to ensure this: randomize the treatment



## Example



The only difference between the groups on average is the difference in treatment (discount sent yes/no)



## Example

If discounts are randomized, then

$$\begin{aligned}
 ATE &= \mathbb{E}[Churn \mid do(DiscountSent = 1)] - \mathbb{E}[Churn \mid do(DiscountSent = 0)] \\
 &= \\
 &\mathbb{E}[Churn \mid DiscountSent = 1] - \mathbb{E}[Churn \mid DiscountSent = 0]
 \end{aligned}$$



## Ignorability

The *Ignorability* condition states that  
 $Y(1), Y(0) \perp\!\!\!\perp T$

$$\begin{aligned}
 \mathbb{E}[Y(0) \mid T = 0] &= \mathbb{E}[Y(0) \mid T = 1] = \mathbb{E}[Y(0)] \\
 \mathbb{E}[Y(1) \mid T = 0] &= \mathbb{E}[Y(1) \mid T = 1] = \mathbb{E}[Y(1)]
 \end{aligned}$$

It doesn't matter which group received which treatment, if the groups were switched we would get the exact same result

The groups are *exchangeable*



## Ignorability

Under Ignorability (and SUTVA) we can rewrite

$$\begin{aligned} ATE &= \mathbb{E}[Y \mid do(T = 1)] - \mathbb{E}[Y \mid do(T = 0)] \\ &= \mathbb{E}[Y(1)] - \mathbb{E}[Y(0)] \\ &= \mathbb{E}[Y(1) \mid T = 1] - \mathbb{E}[Y(0) \mid T = 0] \\ &= \mathbb{E}[Y \mid T = 1] - \mathbb{E}[Y \mid T = 0] \end{aligned}$$

We can use observational data to estimate/measure causal effects!

Unfortunately, ignorability is unlikely to hold in observational studies...



## Conditional Ignorability

The *Conditional Ignorability* condition states that  
 $Y(1), Y(0) \perp\!\!\!\perp T \mid Z$

The potential outcomes are independent of the treatment assignment, conditional on a set of variables  $Z$



## Example (revisited)

- We couldn't directly compare churn rates between customers who received a discount and those who didn't because they also differed in their historical shopping frequency (which also affects churn probabilities)
- But what if we would compare churn rates between customers who received vs didn't receive a discount **only among those with the same shopping frequency**?
- That's what *conditional ignorability* is about: If we look at **subpopulations** where  $Z$  attains a fixed value, then the treatment groups are comparable or exchangeable



## Example



discount=1



discount=0

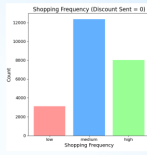
$$\begin{aligned} &\mathbb{E}[\text{Churn} \mid \text{DiscountSent} = 1] \\ &\quad - \\ &\mathbb{E}[\text{Churn} \mid \text{DiscountSent} = 0] \\ &\quad \neq ATE \end{aligned}$$



## Example



Conditional ignorability given *ShoppingFrequency* means it's valid to compute subgroup ATEs by comparing averages among customers with the same shopping frequency



## The Adjustment Formula

The *Adjustment Formula* states that under Conditional Ignorability, we can rewrite

$$ATE = \mathbb{E}[Y \mid do(T = 1)] - \mathbb{E}[Y \mid do(T = 0)] \\ = \mathbb{E}_Z[\mathbb{E}[Y \mid T = 1, Z] - \mathbb{E}[Y \mid T = 0, Z]]$$

$$CATE(x) = \mathbb{E}[Y \mid do(T = 1), X = x] - \mathbb{E}[Y \mid do(T = 0), X = x] \\ = \mathbb{E}_Z[\mathbb{E}[Y \mid T = 1, X = x, Z] - \mathbb{E}[Y \mid T = 0, X = x, Z]]$$

## Example (revisited)

- *ShoppingFrequency* produced a positive bias in the association measured between *DiscountSent* and *Churn*
- At the same time, it could be that *DiscountSent* has a causal effect on *Churn*, which produces a (negative) association between them

*Total Association = Causal Association + Non-causal Association (Bias)*

- By conditioning on *ShoppingFrequency*, we 'removed' the bias part:

*Total Association = Causal Association*

## Example (revisited)

$$\mathbb{E}[\text{Churn} \mid \text{DiscountSent} = 1, \text{ShoppingFrequency} = z]$$

—

$$\mathbb{E}[\text{Churn} \mid \text{DiscountSent} = 0, \text{ShoppingFrequency} = z]$$

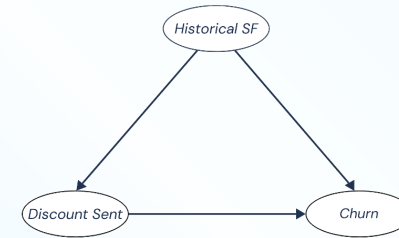
gives us the ATE of a subgroup with historical shopping frequency  $z, z \in \{\text{low}, \text{medium}, \text{high}\}$ , because conditional on *ShoppingFrequency*, any association between *DiscountSent* and *Churn* is of causal nature

## Causal Graphs

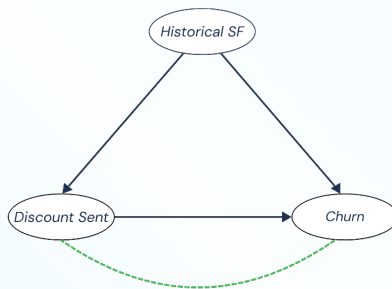
- *Causal Directed Acyclic Graphs* (Causal Graphs) are visual representations of the existence of cause-and-effect relationships between variables
- *Nodes* represent variables
- *Edges* representing causal relationships between variables
- An edge from node *X* to *Y* means changing *X* could potentially directly effect *Y*
- The absence of an edge from node *X* to *Y* means changing *X* does not directly affect *Y*
- Causal DAGs are *acyclic*: there's no sequence of edges that forms a loop



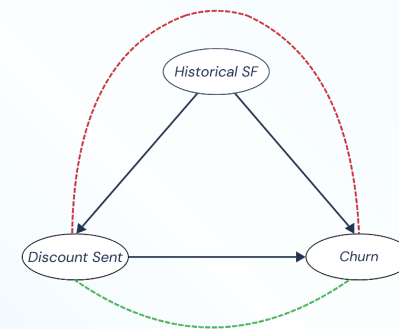
## Causal Graphs



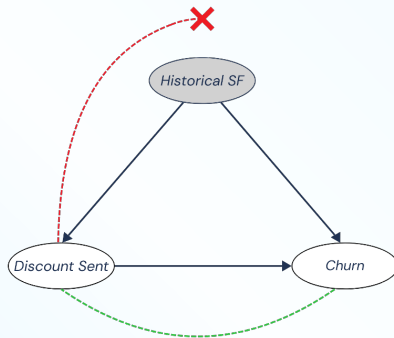
## Causal Graphs



## Causal Graphs

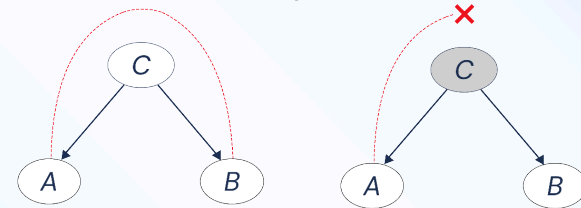


## Causal Graphs



## Graph Patterns

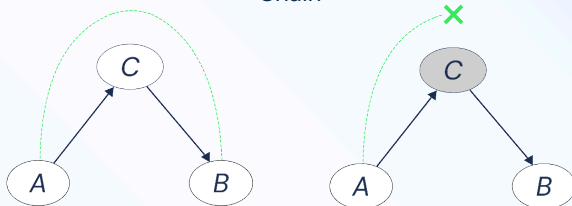
*Fork*



If we don't condition on  $C$ ,  $A$  and  $B$  are dependent (bias),  
if we do,  $A$  and  $B$  are independent

## Graph Patterns

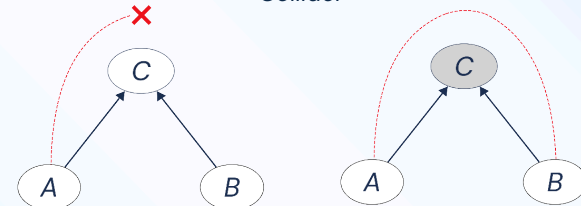
*Chain*



If we don't condition on  $C$ ,  $A$  and  $B$  are dependent (causal),  
if we do,  $A$  and  $B$  are independent

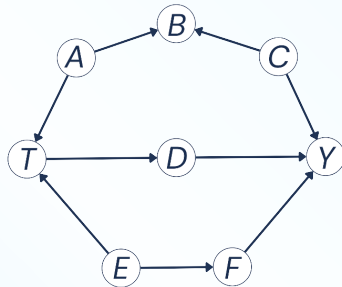
## Graph Patterns

*Collider*



If we don't condition on  $C$ ,  $A$  and  $B$  are independent,  
if we do,  $A$  and  $B$  become dependent (bias)

## Blocking Paths

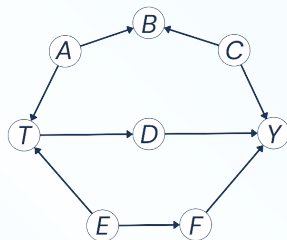


## Blocking Paths

- A *path* in a Causal Graph is a sequences of edges that connect two nodes, regardless of the direction of the edges
- A *causal path* between  $T$  and  $Y$  is a directed path where every edge points in the same direction, starting with an arrow out of  $T$  and ending with an arrow into  $Y$
- A *non-causal path* is any other path connecting  $T$  and  $Y$



## Blocking Paths



There are 3 paths connecting  $T$  and  $Y$

$T \leftarrow A \rightarrow B \leftarrow C \rightarrow Y$ ,  $T \rightarrow D \rightarrow Y$ ,  $T \leftarrow E \rightarrow F \rightarrow Y$



## Blocking Paths

- Each path in a Causal Graph can produce some kind of *association flow* between  $T$  and  $Y$
- Along causal paths, either a causal association or no association is produced
- Along a non-causal path, either a biased association or no association is produced
- When there is no flow going along a path, we say that a path is *blocked*
- We can use the properties of forks, chains and colliders to block/open paths





## Blocking Paths

A path between nodes  $T$  and  $Y$  is *blocked* by a (potentially empty) conditioning set  $Z$  if either of the following is true:

- Along the path, there is a chain  $\dots \rightarrow W \rightarrow \dots$  or a fork  $\dots \leftarrow W \rightarrow \dots$  where  $W \in Z$
- Along the path, there is a collider  $\dots \rightarrow W \leftarrow \dots$  that is not conditioned on ( $W \notin Z, de(W) \notin Z$ )

When all paths between  $T$  and  $Y$  are blocked by conditioning on a set  $Z$ , we say that  $Z$  *d-separates*  $T$  and  $Y$  in the graph



## Blocking Paths

**Path 1:**  $T \leftarrow A \rightarrow B \leftarrow C \rightarrow Y$

Contains patterns  $T \leftarrow A \rightarrow B$ ,  $A \rightarrow B \leftarrow C$ ,  $B \leftarrow C \rightarrow Y$

Condition 2 of the definition of blocking paths holds by not conditioning on anything, and so the path is blocked (due to collider  $B$ )



## Blocking Paths

**Path 2:**  $T \rightarrow D \rightarrow Y$

Contains simple chain pattern  $T \rightarrow D \rightarrow Y$

If we don't condition on anything, this path isn't blocked, since both condition 1 and 2 are not satisfied. The path produces a causal association



## Blocking Paths

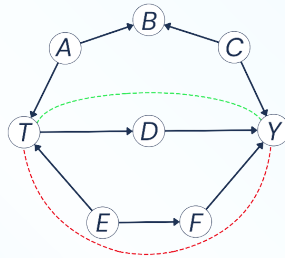
**Path 3:**  $T \leftarrow E \rightarrow F \rightarrow Y$

Contains patterns  $T \leftarrow E \rightarrow F$ ,  $E \rightarrow F \rightarrow Y$

If we don't condition on anything, this path isn't blocked, since both condition 1 and 2 are not satisfied. The path produces a non-causal association



## Blocking Paths



If we don't condition on anything, the total association between  $T$  and  $Y$  consists of a causal and non-causal part



## Backdoor Criterion

The *backdoor criterion* is a method that helps identify a set of variables  $Z$  that, if conditioned on, ensure conditional ignorability w.r.t.  $T$  and  $Y$

Formally:

Given a pair of variables  $(T, Y)$  in a Causal DAG, a set of variables  $Z$  satisfies the backdoor criterion relative to  $(T, Y)$  if  $Z$  blocks every path between  $T$  and  $Y$  that contains an arrow into  $T$  and no node in  $Z$  is a descendant of  $T$



## Backdoor Criterion

*"Z blocks every path between  $T$  and  $Y$  that contains an arrow into  $T$ "*

Causal paths must start with an outgoing arrow from  $T$ , and so any path with an ingoing arrow into  $T$  can only produce a biased association flow and must be blocked

*"No node in  $Z$  is a descendant of  $T$ "*

Any path between  $T$  and  $Y$  with an outgoing arrow of  $T$  will always only contain either causal flow or no flow if we don't condition on anything

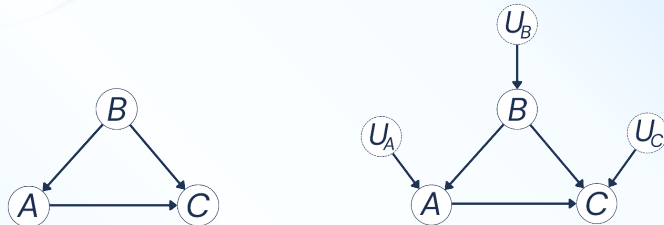


## Backdoor Criterion

- If a set of variables  $Z$  satisfies the backdoor criterion relative to  $(T, Y)$ , we have that conditional on  $Z$ , all non-causal paths between  $T$  and  $Y$  are blocked and all causal paths are kept open
- We then have conditional ignorability given  $Z$  and we can use the adjustment formula to rewrite the *ATE* or *CATE* into purely statistical quantities



## Double-Headed Arrows

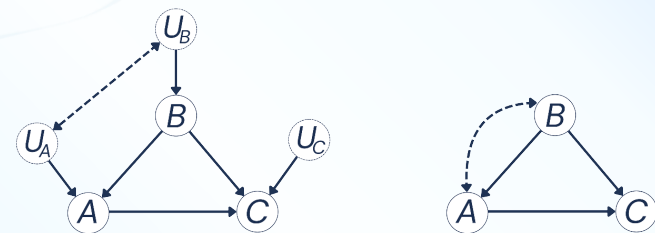


$U_A, U_B, U_C$  represent omitted causes of  $A, B$  and  $C$ , respectively

By drawing the graph this way, we implicitly assume that  $U_A, U_B, U_C$  are independent of each other



## Double-Headed Arrows

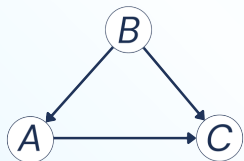


If we believe that two unobserved factors are dependent, we draw a double-headed arrow between them



## Double-Headed Arrows

How does the existence of double-headed arrows change the way in which we analyze paths in a graph?



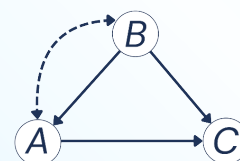
Paths between  $A$  and  $C$ :

$A \leftarrow B \rightarrow C$  (blocked by set  $\{B\}$ )  
 $A \rightarrow C$  (unblocked causal path)



## Double-Headed Arrows

How does the existence of double-headed arrows change the way in which we analyze paths in a graph?



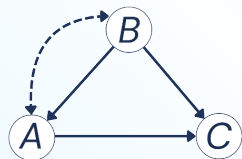
Paths between  $A$  and  $C$ :

$A \leftarrow B \rightarrow C$  (blocked by set  $\{B\}$ )  
 $A \rightarrow C$  (unblocked causal path)  
 $A \leftrightarrow B \rightarrow C$



## Double-Headed Arrows

How does the existence of double-headed arrows change the way in which we analyze paths in a graph?



Paths between A and C:

$A \leftarrow B \rightarrow C$  (blocked by set  $\{B\}$ )  
 $A \rightarrow C$  (unblocked causal path)  
 $A \leftarrow \dots \rightarrow B \rightarrow C$  (blocked by set  $\{B\}$ )



## Double-Headed Arrows

In general, if there is a path with  $\leftrightarrow$ , just replace each  $\leftrightarrow$  by  $\leftarrow \dots \rightarrow$  and apply blocking path techniques like usual

For example, rewrite a path

$A \rightarrow B \rightarrow C \leftrightarrow D \leftrightarrow E \rightarrow F$

as

$A \rightarrow B \rightarrow C \leftarrow \dots \rightarrow D \leftarrow \dots \rightarrow E \rightarrow F$



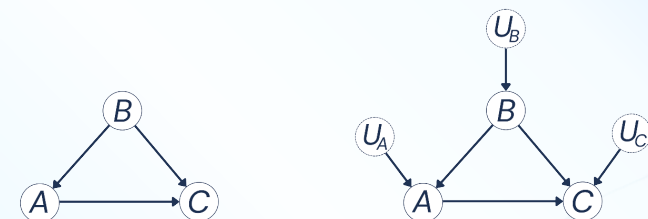
## Do-operator in Causal Graphs

- $do(X = x)$  means that we enforce the value of  $X = x$  over a population
- The natural mechanism that determines the value of  $X$  is no longer relevant. It is completely replaced by our intervention  $do(X = x)$
- We can show this in a Causal Graph by removing all incoming arrows into  $X$



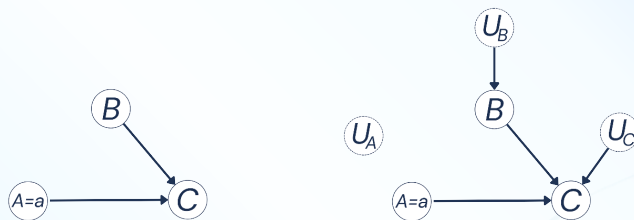
## Do-operator in Causal Graphs

Causal Graph corresponding to natural mechanisms



## Do-operator in Causal Graphs

Causal Graph corresponding to mechanisms after intervention  $do(A = a)$



## Continuous Treatments

The *Average Treatment Effect* of going from treatment level  $T = t_1$  to  $T = t_2$  is defined as

$$ATE(t_1, t_2) = \mathbb{E}[Y \mid do(T = t_2)] - \mathbb{E}[Y \mid do(T = t_1)]$$

For example, the average effect on sales of increasing the price of some product from 10\$ to 15\$ is defined as

$$ATE(10, 15) = \mathbb{E}[\text{sales} \mid do(\text{price} = 15)] - \mathbb{E}[\text{sales} \mid do(\text{price} = 10)]$$



## Continuous Treatments

The *Conditional Average Treatment Effect* of going from treatment level  $T = t_1$  to  $T = t_2$  is defined as

$$CATE(x, t_1, t_2) = \mathbb{E}[Y \mid do(T = t_2), X = x] - \mathbb{E}[Y \mid do(T = t_1), X = x]$$

where  $X$  represents a set of features describing the population units



## Continuous Treatments

- With continuous treatments, the treatment effect can be different for any different pair  $(t_1, t_2)$
- This is known as *non-linearity* in treatment effects
- For example, the effect of increasing a credit limit on people's spending behaviour (stronger effect at lower levels compared to at higher levels)



## Continuous Treatments

The *Conditional Ignorability assumption* under continuous treatment is no longer defined as  $Y(0), Y(1) \perp\!\!\!\perp T \mid Z$ , but instead as

$$Y(t) \perp\!\!\!\perp T \mid Z$$

where  $Y(t)$  is the potential outcome under treatment  $T = t$

The intuition remains the same: It still means that conditional on the variables  $Z$ , any group of units with different treatment values only differ on average in their treatment



## Continuous Treatments

The *Adjustment Formula* for *ATE* and *CATE* of going from treatment level  $T = t_1$  to  $T = t_2$  is defined as

$$\begin{aligned} ATE(t_1, t_2) &= \mathbb{E}[Y \mid do(T = t_2)] - \mathbb{E}[Y \mid do(T = t_1)] \\ &= \mathbb{E}_Z[\mathbb{E}[Y \mid T = t_2, Z] - \mathbb{E}[Y \mid T = t_1, Z]] \end{aligned}$$

$$\begin{aligned} CATE(x, t_1, t_2) &= \mathbb{E}[Y \mid do(T = t_2)] - \mathbb{E}[Y \mid do(T = t_1)] \\ &= \mathbb{E}_Z[\mathbb{E}[Y \mid T = t_2, X = x, Z] - \mathbb{E}[Y \mid T = t_1, X = x, Z]] \end{aligned}$$



## Why we need models

- In the coding example, we approximated  $\mathbb{E}[Y \mid T = t, Z]$  for different values of  $Z$  using subgroup averages
- This doesn't work for continuous treatments, or with many adjustment variables
- Imagine trying to find many other units with the exact same value  $T = 0.285$  and  $Z = 0.987$
- But even if we don't have exact matches, we might find other data points with *similar* values for the treatment and adjustment variables
- We rely on models to find such 'similar' data points, understand the relationships between the variables, and approximate  $\mathbb{E}[Y \mid T = t, Z]$



## Recap

- the *ATE* and *CATE* compare outcomes under different treatment conditions at a certain point in time of a specific (sub)population
- Under conditional ignorability, we can express causal quantities like the *ATE* and *CATE* in terms of purely statistical or observational quantities
- To find an adjustment set  $Z$  to achieve conditional ignorability, we can use Causal Graphs and the Backdoor Criterion
- Double-headed arrows in Causal Graphs imply dependence due to omitted common cause(s)

