Description: Generalised von Mises Parameters from Trigonometric Moments

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The trigonometric moments of order n of a normalised, non-negative directional distribution $G(\theta)$ on the unit circle are defined as

$$a_n = \int_{-\pi}^{\pi} G(\theta) \cos n\theta \mathrm{d}\theta, \quad b_n = \int_{-\pi}^{\pi} G(\theta) \sin n\theta \mathrm{d}\theta. \tag{1}$$

Given *two* pairs of moments (i.e. $\{a_1, b_1, a_2, b_2\}$), the maximum entropy distribution is [2]

$$G_2(\kappa_1, \kappa_2, \mu_1, \mu_2, \theta) \equiv \frac{1}{A(\kappa_1, \kappa_2, \Psi)} e^{\kappa_1 \cos(\theta - \mu_1) + \kappa_2 \cos 2(\theta - \mu_2)}, \tag{2}$$

where $\Psi = \mu_2 - \mu_1$ and $A(\kappa_1, \kappa_2, \Psi)$ is the normalisation scalar which, with a change in integration variable, can be rewritten

$$A(\kappa_1, \kappa_2, \Psi) = \int_{-\pi}^{\pi} e^{\kappa_1 \cos \theta + \kappa_2 \cos 2(\theta - \Psi)} \mathrm{d}\theta. \tag{3}$$

This package uses Newtonian iteration to calculate the parameters $\kappa_1, \kappa_2, \mu_1, \mu_2$ given the moments $\{a_1, b_1, a_2, b_2\}$, and can also be used to calculate the moments given the parameters, and to generate a distribution of the form (2).

The three main functions are:

moms2params   Given the first two pairs of trig moments (1), this performs a Newtonian iteration and returns the GvM parameters. Accepts a $p \times 4$ matrix for p different sets of moments $\{a_1, b_1, a_2, b_2\}$, to and returns a $p \times 5$ matrix for the parameters (including the normalisation parameter) $\{A_0, \kappa_1, \kappa_2, \mu_1, \mu_2\}$. The user can specify a tolerance as the second input (how close do the moments need to be to the values provided to exit the iteration) and a truncation for the third (since the moments and Jacobian elements are truncated series of modified Bessel functions) but neither are compulsory.

If the moments do not correspond to moments of a non-negative distribution [1, 3], they will violate one of the inequalities

$$|c_1| \le 0, \qquad \left| c_1^2 - c_2 \right| \le 1 - |c_1|^2. \tag{4}$$

In this case a warning is triggered and the output is NaN.

params2moms   Given a $p \times 5$ matrix of p different sets of GvM parameters $\{A_0, \kappa_1, \kappa_2, \mu_1, \mu_2\}$ (or a $p \times 4$ matrix omitting A_0), this returns a $p \times 4$ matrix of moments. This is done using modified Bessel series: the truncation term can be set using a second function input.

GvMparams2distribution   Given a $p \times 5$ matrix of p sets of GvM parameters $\{A_0, \kappa_1, \kappa_2, \mu_1, \mu_2\}$ this evaluates the directional distribution (2) and returns a $p \times n_\theta$ matrix with p different directional distributions. You can specify a vector of θ-values (in rads) for evaluation but this is not compulsory. The θ-axis will be output if required.

Cite as David Christie (2021) Generalised von Mises Parameters from Trigonometric Moments, GitHub

References

[1] Stephen F Barstow, Jean-Raymond Bidlot, Sofia Caires, Mark A Donelan, William M Drennan, Hélène Dupuis, Hans C Graber, J Jim Green, Oistein Gronlie, Christine Guérin, et al. *Measuring and analysing the directional spectrum of ocean waves.* COST Office, 2005.

[2] R. Gatto and S.R. Jammalamadaka. The generalized von Mises distribution. *Statistical Methodology*, 4(3):341–353, 2007.

[3] Abushet W Simanesew, Harald E Krogstad, Karsten Trulsen, and José Carlos Nieto Borge. Bimodality of directional distributions in ocean wave spectra: a comparison of data-adaptive estimation techniques. *Journal of Atmospheric and Oceanic Technology*, 35(2):365–384, 2018.