SEEM 5870: Computational Finance **2020-2021 Term2**

Homework 4 Solution

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In all the problems below, the asset price is assumed to follow a geometric Brownian motion $dS_t = rS_t dt + \sigma S_t dW_t$.

Problem 1[40'] Use the central difference method to estimate the following Greeks. The sample size is $n = 100000$, and the model parameters are $S_0 = 50, r = 0.03, \sigma = 0.2, T = 1$.

(a) [20'] Delta of a put option with maturity T and strike price K. Report your results for $K = 50$ and $h = 1, 0.1, 0.01$, respectively.

(b) [20'] Vega of a call option with maturity T and strike price K. Report your results for $K = 50$ and $h = 0.1, 0.01, 0.001$, respectively.

Solution:

(a) The payoff of put option is

$$e^{-rT}\left[K - S_0 \exp\left((r-\frac{1}{2}\sigma^2)T + \sigma\sqrt{T}Z\right)\right]^+, \quad Z \sim N(0,1),$$

therefore, the algorithm is

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for $i = 1, \ldots, n$ **do**
 Generate $Z_i \sim N(0,1)$
 Set $X_i = (S_0 + h) * \exp\left((r - \frac{1}{2}\sigma^2)T + \sigma\sqrt{T}Z_i\right)$
 Set $Y_i = (S_0 - h) * \exp\left((r - \frac{1}{2}\sigma^2)T + \sigma\sqrt{T}Z_i\right)$
 Set $H_i = \frac{1}{2h}\left[e^{-rT}(K - X_i)^+ - e^{-rT}(K - Y_i)^+\right]$
 Compute the estimator of Delta $\hat{\Delta} = \frac{1}{n}(H_1 + \ldots + H_n)$
 Compute the starndard error $\sqrt{\frac{1}{n(n-1)}\left(\sum_{i=1}^n H_i^2 - n\hat{\Delta}^2\right)}$
end for

Then show the results below

	h= 1	h=0.1	h=0.01
$\hat{\Delta}$	-0.40287199	-0.40008937	-0.40228522
S.E.	0.0012888	0.00133275	0.00133607

(b) The payoff of call option is

$$e^{-rT}\left[S_0 \exp\left((r-\frac{1}{2}\sigma^2)T + \sigma\sqrt{T}Z\right) - K\right]^+, \quad Z \sim N(0,1),$$

for $i = 1, \dots, n$ **do**
  Generate $Z_i \sim N(0,1)$
  Set $X_i = S_0 * \exp\left((r - \frac{1}{2}(\sigma + h)^2)T + (\sigma + h)\sqrt{T} Z_i\right)$
  Set $Y_i = S_0 * \exp\left((r - \frac{1}{2}(\sigma - h)^2)T + (\sigma - h)\sqrt{T} Z_i\right)$
  Set $H_i = \frac{1}{2h}\left[e^{-rT}(X_i - K)^+ - e^{-rT}(Y_i - K)^+\right]$
  Compute the estimator of Vega $\hat{V} = \frac{1}{n}(H_1 + \ldots + H_n)$
  Compute the starndard error $\sqrt{\frac{1}{n(n-1)}\left(\sum_{i=1}^{n} H_i^2 - n\hat{V}^2\right)}$
end for

Then show the results below

	h= 0.1	h=0.01	h=0.001
$\hat{V}$	19.23936779	19.35309767	19.2835303
S.E.	0.11828844	0.11787515	0.11799315

Problem 2[30'] Consider an average price call option with maturity T and strike price K, whose payoff is $(\bar{S} - K)^+$ with $\bar{S} = \frac{1}{m}\sum_{i=1}^{m} S_{t_i}$ and $t_1 < \ldots < t_m$. Derive the pathwise derivative estimator for the Delta. You do not need to write codes for this problem.

Solution: (Refer to Example 7.2.2 of Glasserman's book) Consider the payoff of the Asian option $Y = e^{-rT}(\bar{S} - K)^+$ with $\bar{S} = \frac{1}{m}\sum_{i=1}^{m} S_{t_i}$, we have

$$\frac{dY}{dS_0} = \frac{dY}{d\bar{S}}\frac{d\bar{S}}{dS_0} = e^{-rT} 1_{\{\bar{S} > K\}} \frac{d\bar{S}}{dS_0},$$

where

$$\frac{d\bar{S}}{dS_0} = \frac{1}{m}\sum_{i=1}^{m} \frac{dS_{t_i}}{dS_0} = \frac{1}{m}\sum_{i=1}^{m} \frac{S_{t_i}}{S_0} = \frac{\bar{S}}{S_0}.$$

Therefore, the pathwise estimator of the Delta of Asian option is $\frac{dY}{dS_0} = e^{-rT} 1_{\{\bar{S}>K\}} \frac{\bar{S}}{S_0}$.

Problem 3[30'] Estimate Delta of an European digital call option with payoff $1_{S(T) \geq K}$ by the likelihood ratio method. The sample size is $n = 100000$, and the model parameters are $S_0 = 50, K = 50, r = 0.03, T = 1$. Report your estimate and the standard error for $\sigma = 0.1, \sigma = 0.3, \sigma = 0.5$ respectively.

Solution: (Refer to Example 7.3.1 of Glasserman's book) The unbiased estimator of Delta of the option is

$$e^{-rT} 1_{S(T) \geq K} \left(\frac{\log(S_T/S_0) - (r - \frac{1}{2}\sigma^2)T}{S_0 \sigma^2 T}\right),$$

where S_T is obtained from geometric Brownian motion with $S_T = S_0 \exp\left((r - \frac{1}{2}\sigma^2)T + \sigma\sqrt{T} Z\right)$, then the estimator is simplied to

$$e^{-rT} 1_{S(T) \geq K} \frac{Z}{S_0 \sigma \sqrt{T}}.$$

	$\sigma = 0.1$	$\sigma = 0.3$	$\sigma = 0.5$
$\hat{\Delta}$	0.07504227	0.0258175	0.0151688
S.E.	3.6385e-04	1.1963e-04	7.2257e-05