SEEM 5870: Computational Finance	**2020-2021 Term2**

Homework 6 Solution

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Problem 1 [60', 15' each] In this exercise, you will compare the three finite difference methods (explicit scheme, implicit scheme, and Crank-Nicolson scheme) for pricing the European put option under the Black-Scholes model. The model parameters are as follows: the spot asset price is denoted by S_0, the strike $K = 50$, risk-free rate $r = 0.05$, Maturity $T = 0.5$ year, and the asset volatility is denoted by $\sigma = 0.2$. For all the finite difference methods, choose $S_{\min} = 0$ and $S_{\max} = 100$, and define $dt = T/N, dS = S_{\max}/M$ to form the grid. Define the pricing error of each finite difference scheme to be the difference between the price calculated from the Black-Scholes formula and the price computed from each finite difference scheme.

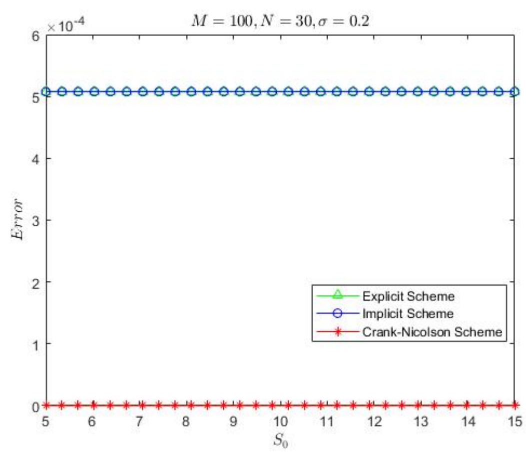
(a) Choose $M = 100, N = 30$. Plot (in one picture) the pricing error of each finite difference scheme as a function of S_0 from 5 to 15. You may use 30 equally distanced points for S_0 (e.g. linspace(5,15,30) in matlab)

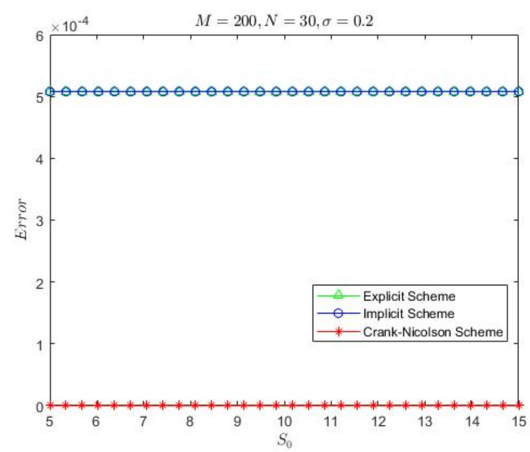
(b) Choose $M = 200, N = 30$. Plot the pricing error of each finite difference scheme as a function of S_0 from 5 to 15.

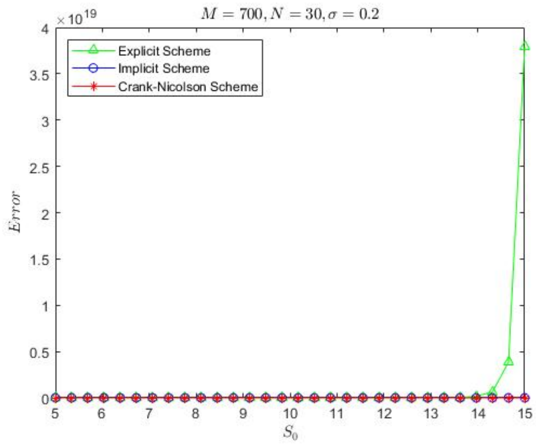
(c) Choose $M = 700, N = 30$. Plot the pricing error of each finite difference scheme as a function of S_0 from 5 to 15.

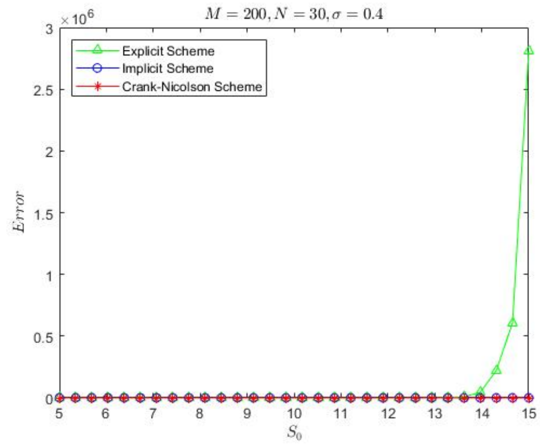
(d) Redo question (b) for $\sigma = 0.4$.

Solution: Implement the Matlab code given in Lecture 7,8,9. To better show the performance, we plot the abstract value of the difference between the price from BS formula and the simulated price. We can see that Crank-Nicolson scheme performs best.









Problem 2 [40'] Consider pricing an American put under the Black-Scholes model. Suppose $S_0 = 50, K = 50, r = 0.1, T = 5/12, \sigma = 0.4, S_{\max} = 100, N = 100, M = 100$. We have discussed the Crank-Nicolson scheme to price the American put. Using this method to plot the early exercise curve $S_f(t)$ for $t \in [0, T]$.

Solution: