OFDM

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Orthogonal Frequency Division Multiplexing

While phase modulation outperforms frequency modulation, frequency can be used in a different way to increase data rates. Namely, by exploiting the orthogonality of sinusoidal functions with different frequencies.

Sinusoidal functions with frequencies that are a multiple of the data rate R=1/T are orthogonal; the L_2 inner product

$$\langle \sin(2\pi\, mR\cdot),\ \sin(2\pi\, nR\cdot) \rangle := \int_{[0,T]} \sin(2\pi\, nR\, t) \sin(2\pi\, nR\, t)\, dt = 0 \text{ if } m\neq n.$$

This means that signals sent over different frequencies f_1=mR and f_2=nR can be separated at the receiver (using Fourier decomposition, in particular the fast Fourier transform.)

Definition: Orthogonal frequency division multiplexing is the use of several different frequencies, with each frequency having a modulation scheme, for a data stream of a single data source.

Definition: The carrier frequency of a OFDM scheme is the center frequency, f_0.

Definition: The subcarriers of an OFDM scheme are the several different frequencies used.

Definition: The difference of frequencies between consecutive subcarriers in a OFDM scheme is the subcarrier spacing of the scheme.

SCS is short for subcarrier spacing.

Definition: Twelve consecutive frequencies spaced according to a subcarrier spacing constitute a physical resource block.

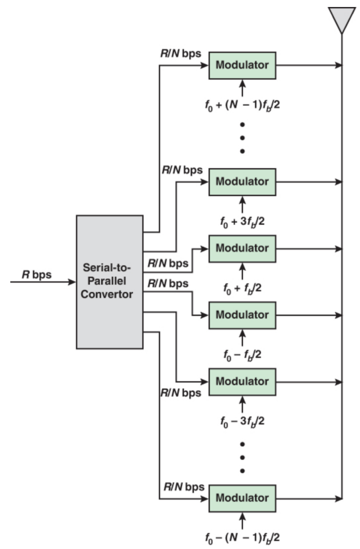
PRB is short for physical resource block.

Note that one, two, or more physical resource blocks (PRBs) can be used for one data stream.

For notation, say there are N sub-streams with frequencies spaced by the sub-carrier spacing f_b. Note that to maintain orthogonality of subcarriers the minimum subcarrier spacing is R and that f_b=kR for some positive integer k. The N subcarrier frequencies are

$$f_0 + R/2\ ,\ f_0 + 3R/2\ ,\ \ldots\ ,\text{ and } f_0 + (N-1)R/2$$

and
f_0 - R/2 , f_0 -3R/2 , … , and f_0 - (N-1)R/2.



In 4G the transmission rate required to be R = 1/1ms=1kHz and the subcarrier spacing was required to be 15kHz. This lack of options puts considerable limits on 4G OFDM.

5G Numerology

In 5G the there are 7 subcarrier spacing options indexed below by μ. Each subcarrier spacing option is accompanied by a slot duration; that is why the name is not just "subcarrier spacing type μ" .

Definition: A numerology is an indexed subcarrier spacing paired with a slot duration.

The term 3GPP applies only to OFDM schemes (despite what you will find if you google "numerology"). There are (as of release 17) just 7 numerologies: 0, 1, 2, 3, 4, 5, and 6.

μ	Subcarrier Spacing f_b	Slot Duration T	Symbols Per Slot S
0	15kHz	1 ms	7 or 14
1	30kHz	0.5 ms	7 or 14
2	60kHz	0.25 ms	7 or 14
3	120kHz	0.125 ms	14
4	240	62.5 µs	14
…	…	…	…

In general, numerology μ has (f_b,T) = (15 kHz 2^{μ}, 1 ms $2^{-\mu}$).

Let's unfold what is meant by "symbols per slot". An example of a symbol is the symbol `01` in the modulation scheme QAM. The statement that there are S symbols in a slot of duration T means that one symbol is transmitted in each time interval of length T/S. This happens for each subcarrier frequency.

Example

As an example, say the center frequency is 700MHz and the numerology is 3. Then the subcarrier spacing is R=120kHz. If one PRB is used then there are N=12 subcarrier frequencies. They are

(700MHz - (12-1)/2 120kHz, 700MHz - (10-1)/2 120kHz , … , 700MHz - (12+1)/2 120kHz).

If QAM is the modulation coding scheme, each one of these subcarrier frequencies transmits one of the four symbols in (00,01,10,11) using one of four amplitude modulations in a time interval of width 0.125ms/14=8.9µs. In case you are thinking that this is a short time interval for transmitting a sinusoidal wave I'll point out that this is enough time for 6,244 periods of the subcarrier with the lowest frequency. The bit transmission rate for all 12 subcarriers is then 12(2b)/8.9µs = 2.7Mb/s=336kB/s.

Theory

Claude Shannon, the mathematician who invented information theory, proved that using a bandwidth B one can not transmit information oat a bit rate greater than 2B. For example, if a bandwidth of 15kHz is used then the bit rate can not exceed 30 bits per second. In numerology µ the bandwidth used by one PRB is $12(15\text{kHz})2^{\mu}$ and so the maximum bit rate is $2^{\mu}(180\text{kb/s})$. If one increased µ to get a enable a larger bandwidth, but the same slot duration was used, then the bit rate would not increase, thereby wasting bandwidth. 3GPP knew there was a way to get doubled bit rate out of doubled bandwidth thanks to Shannon's theorem. The way to get that additional bit rate from the additional bandwidth is to proportionally reduce slot duration.

On the other hand, slot duration can not be reduced indefinitely to squeeze more out of a fixed bandwidth; eventually reduction of slot duration would try to squeeze bitrate greater than 2B out of the bandwidth B, in violation of Shannon's theorem.

(By the way, Shannon invented the entire idea of reducing information to strings of 1s and 0s, and transmitting such strings; he is the official inventor of information theory and thereby the foundation for all computer technology.)

Claude Shannon

Practice

Pros

Besides providing a way to transmit data at close to the Shannon channel capacity (relative to other methods) OFDM is resistant to selective fading (frequency dependent signal intensity loss) and robust against temporal dispersion. It has proven so useful that it is used not only in wireless transmission, but also in wireline transmission of data (e.g. discrete multi-tone.)

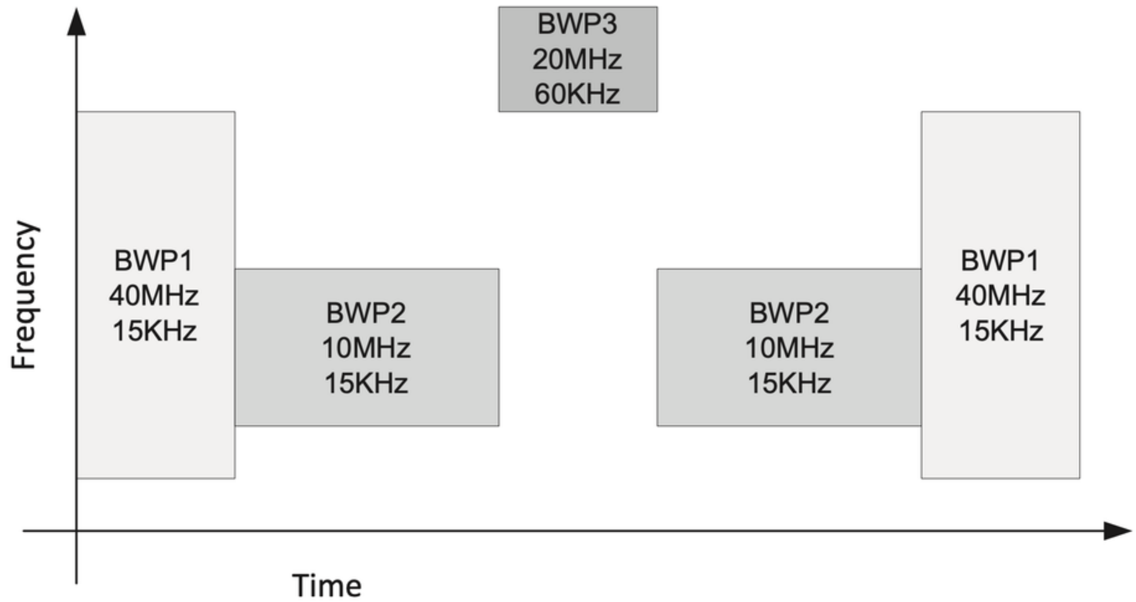
Cons

There is a problem with OFDM; since multiple frequencies of electromagnetic wave are sent on top of each other, there are times of constructive interference; the resulting high peak to average power ratio (PAPR) is high, which is a difficulty for transmitters and receivers.

Bandwidth Part

In 5G, the set of frequencies used in an OFDM scheme by a UE is called a bandwidth part (BWP). It represents the subset of the cell's radio resources allocated to the UE. The BWP can change in time to adapt to needs through coordination between the UE and gNB. This is called bandwidth adaption (BA). For example the diagram below shows BA for a UE through 5 time intervals of arbitrary width.

1. in the first time interval, a 40MHz bandwidth and 15kHz subcarrier spacing (SCS) is used,
2. the data needs of the UE drop, and in the second time interval, a 10MHz bandwidth and 15kHz SCS is used.
3. the data needs of the UE go up, and a high center frequency bandwidth of 20MHz and 60kHz spacing (μ=2) is used.
4. the data needs of the UE drop, and in the fourth time interval, a 10MHz bandwidth and 15kHz SCS is used.
5. the data needs of the UE increase, and in the fifth time interval, a 40MHz bandwidth and 15kHz SCS is used.



The gNB gives the UE 4 or less BWPs and updates about which to use in which intervals of time. There is a default BWP; when the physical downlink control channel is not used by the UE, a BWP inactivity timer runs out and the UE is told to switch to the default BWP.

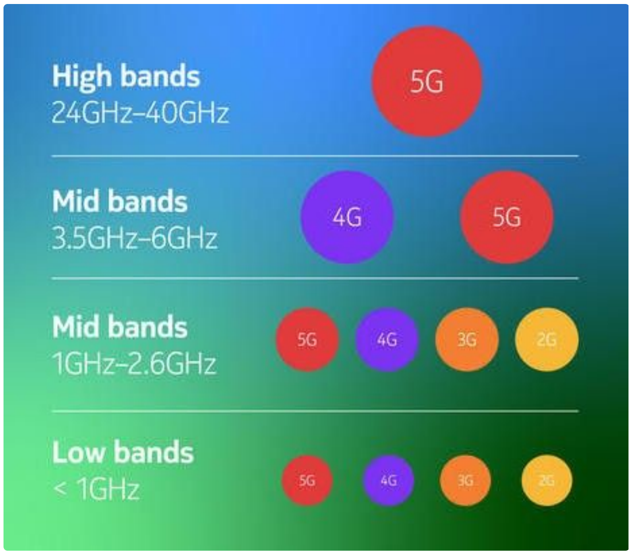
By the way, the number of subcarrier frequencies used for a particular combination of bandwidth and SCS is not determined by simple division. For example, you might expect that at a 30MHz bandwidth and 30kHz SCS you'd have 30M/30k = 1,000 subcarriers. The table below shows otherwise. I do not know why 3GPP chose this, or where the missing frequencies are in the interval of frequencies.

Number of 5G/NR occupied subcarriers for 30KHz subcarrier spacing for different RF bandwidths.

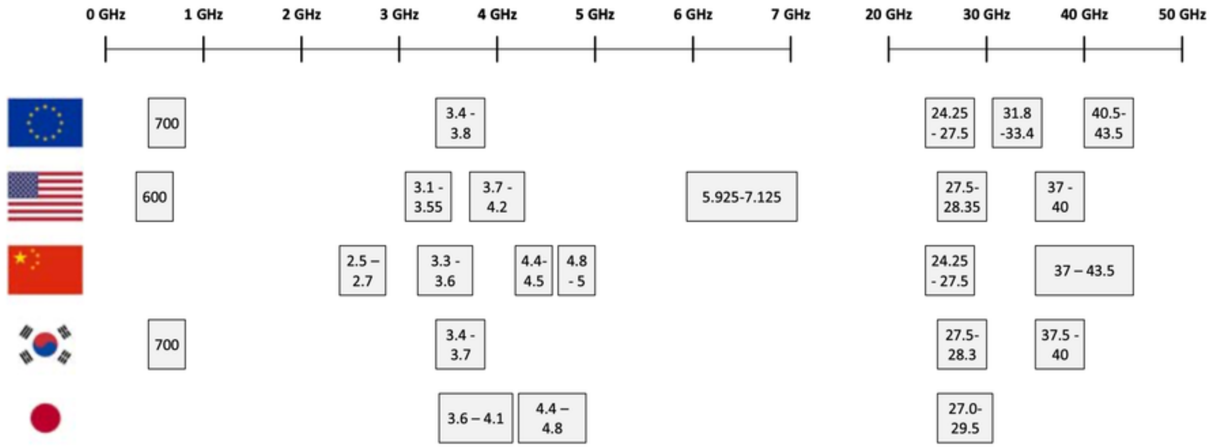
5G b/w	5	10	15	20	25	30	40	50	60	80	90	100
Carriers	132	288	456	612	780	936	1272	1596	1944	2604	2940	3276

Reason

5G introduces larger subcarrier spacings that 4G's only option of 15kHz. The reason is that much higher frequencies have been allocated for use in 5G than were in 4G, 3G, or 2G. Phase noise (and thus error rate) increases with carrier frequency, but decreases with carrier spacing. Thus, at higher frequencies one should use larger spacings. To take advantage of the higher possible data rates for these frequencies, shorter slot durations are used. To ensure that enough periods of a wave appear in a slot duration and to ensure sufficient carrier separation, shorter slot durations are coupled with proportionally larger subcarrier spacings.

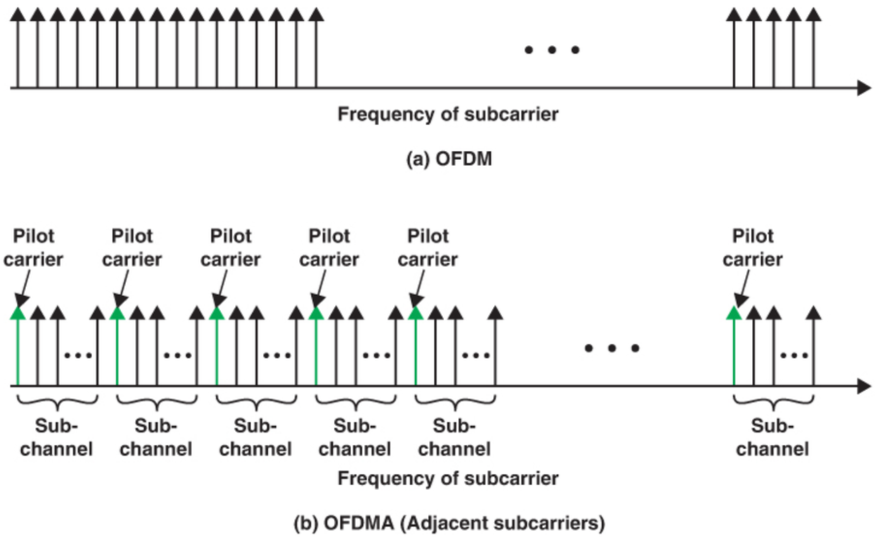


The specific frequencies used for 5G vary from one country to another. The diagram below lists 5G frequencies in a few countries and the European Union.



Orthogonal Frequency Division Multiple Access

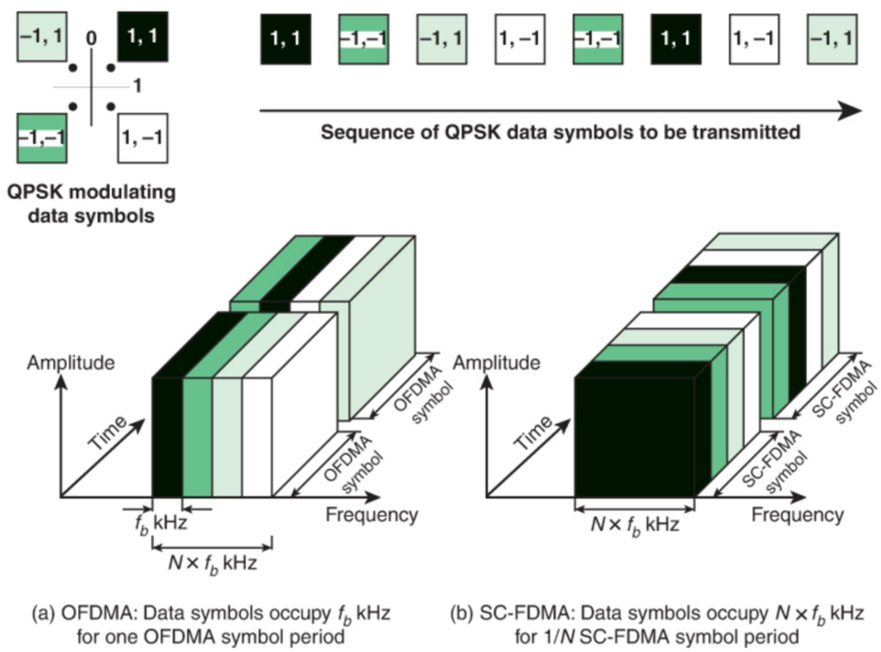
Unlike OFDM, OFDMA serves multiple data streams. To do this, the subcarriers are grouped into sets of (not necessarily adjacent) subcarriers called sub-channels. This allows multiple applications, multiple UE, etc to use the same bandwidth (as in range of frequencies).



Single Carrier FDMA

There is a problem with OFDM; the subcarriers are waves that combine to sometimes form large peaks. This creates a large peak to average power ratio (PAPR). Large PAPR is energy consuming for amplifiers.

A simple way to address this problem has been invented; the discrete fast Fourier transform is applied to the digital signal before it is sent to the subcarrier mapping. Since the Fourier transform maps the time domain to the frequency domain, this has the effect of turning the modulation symbols sideways in the diagram below.



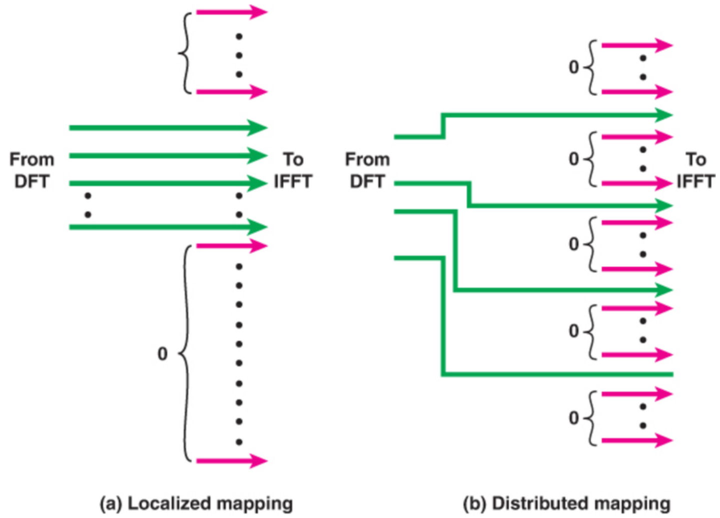
At any given moment in time, the data being sent is a modulation symbol for a single UE or app. This motivates the name “single carrier”. However, over time data for all subchannels are sent, so the multiple access nature of the signal is maintained.

Discrete Fourier Transform Single OFDM

A slight variation of SC-FDMA is one of the two 3GPP specified OFDM mechanisms for 5G. The discrete fourier transform across the time direction is applied to the N digital substreams first to obtain the frequency domain representation of the signal, just as in SC-FDMA. This can still be visualized as the rotation of a slot, as in the diagram above. However, the next step is different than SC-FDMA; the number of subcarrier frequencies K is then chosen, and K > N.

The advantages to DFT-S-OFDM are

- reduced PAPR (peak to average power ratio) for reduced power consumption
- the option to distribute incoming signals into non-contiguous subcarrier frequencies with zero amplitude for the remaining subcarrier frequencies.



The latter option allows for great flexibility in the 5G system.

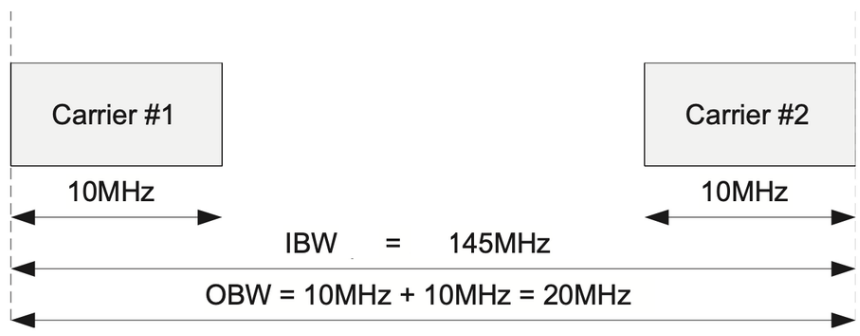
Instantaneous vs Occupied Bandwidth

Definition: The instantaneous bandwidth of a bandwidth part is the difference between the highest and lowest frequency used.

IBW is short for instantaneous bandwidth.

Definition: The occupied bandwidth of a bandwidth part is the sum of the products of subcarrier spacing and number of carrier frequencies used.

OBW is short for occupied bandwidth.

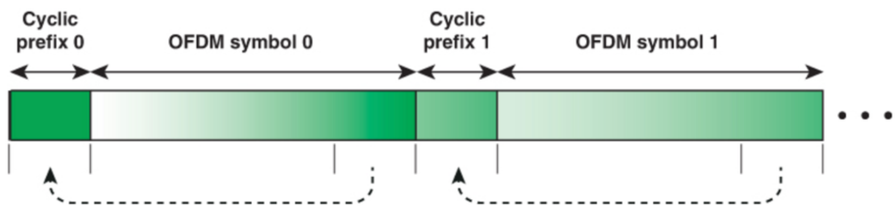


Cyclic Prefixing

Slot durations for a subcarrier are not entirely occupied with the combined sinusoidal waves for a symbol. For example in numerology $\mu = 0$ the slot duration of 1 ms is occupied with

- 66.67 µs of OFDM symbol transmission, but also
- a prefix of 4.69 µs that is identical to the last

These combine to form 71.35 µs signal for one QAM symbol, and 14 symbols are sent at once; 14(71.35µs) = 1 ms.



The reason to use cyclic prefixing in OFDM is that it reduces inter-symbol interference.

4G also used cyclic prefixes in its 15 kHz spaced slot duration 1ms system

5G MCS by OFDM

This is the list of modulation coding schemes (MCS) supported in 5G, broken down by the kind of OFDM used:

MCS's supported for Downlink:

- QPSK
- 16, 64, and 256-QAM

MCS's supported for Uplink with CP-OFDM :

- QPSK
- 16, 64, and 256-QAM

MCS's supported for Uplink with CP-DFT-s-OFDM:

- π/2-BPSK,
- QPSK,
- 16, 64, and 256-QAM