MuMaX3 Tutorial Presentation

Nov 2021

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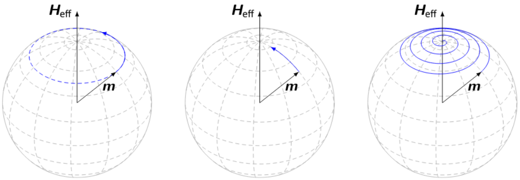
Continuum magnetization field: $\mathbf{M}(\mathbf{r},t) = M_{\mathrm{s}}\mathbf{m}(\mathbf{r},t)$

Energy

$$
\begin{aligned}
E = \int_V \mathrm{d}V \Big\{ & A \sum_{\alpha,\beta\in\{x,y,z\}} \left(\partial_\alpha m_\beta\right)^2 && \text{Exchange} \quad (1) \\
& + D\left(\mathcal{L}_{yz}^{(x)} + \ldots\right) && \text{DMI} \quad (2) \\
& - K\left(\mathbf{m}\cdot\hat{u}\right)^2 + K_{\mathrm{c}}\left(m_x^2 m_y^2 + \ldots\right) && \text{Anisotropies} \quad (3) \\
& - \mu_0 \mathbf{M}\cdot\mathbf{H} && \text{Zeeman} \quad (4) \\
& - \frac{\mu_0}{2}\mathbf{M}\cdot\mathbf{H}_{\mathrm{d}} \Big\} && \text{Demag} \quad (5)
\end{aligned}
$$

Effective field

$$\mathbf{H}_{\mathrm{eff}}(\mathbf{r}) = -\frac{1}{\mu_0 M_{\mathrm{s}}} \frac{\delta E(\mathbf{r})}{\delta \mathbf{m}} \tag{6}$$



LLG Precession, Damping and Full form[1].

- ☐ Dynamics (spin-wave, vortex dynamics, domain-wall motion, FMR, etc.):

Landau-Lifshitz-Gilbert equation

$$\frac{\partial \mathbf{M}}{\partial t} = -\frac{\gamma}{1+\alpha^2}\left(\mathbf{M} \times \mathbf{H}_{\text{eff}} + \frac{\alpha}{M_{\text{s}}}\mathbf{M} \times \mathbf{M} \times \mathbf{H}_{\text{eff}}\right) \tag{7}$$

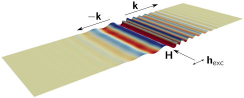
[1] Image taken from http://micromagnetics.org/micromagnetism/. Based on: C. Abert, Discrete Mathematical Concepts in Micromagnetic Computations. 2013.

- Static (hysteresis, local energy minima, phase diagrams, etc.):
- Energy minimization via gradient algorithms: steepest descent

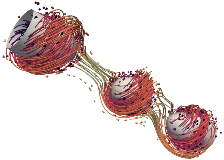
Exl et al. JAP 115, 17D118 (2014)

Minimizer stopping criteria

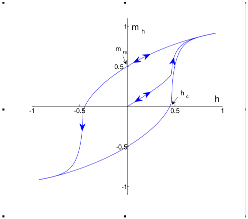
```
The minimization procedure stops when max(dM) < "MinimizerStop" for the last "MinimizerSamples" steps.
The default values are 1e-6 for MinimizerStop and 10 for minimizerSamples.
```



Spin waves

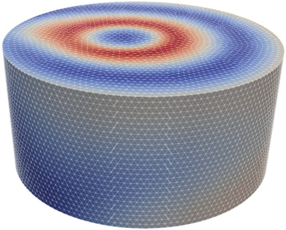


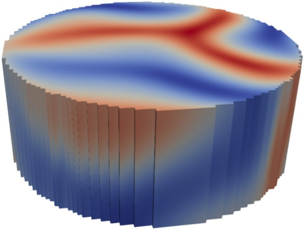
Local energy minimum states



Hysteresis

Numerical discretization





Characteristic lengths

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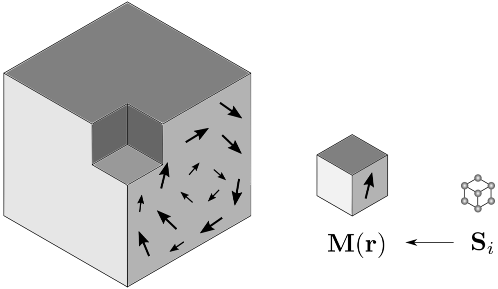


$$\ell_{\mathrm{ex}} = \sqrt{\frac{2A}{\mu_0 M_{\mathrm{s}}^2}} \qquad \text{Exchange length} \tag{8}$$

$$\delta_{\mathrm{DW}} = \sqrt{\frac{A}{K}} \qquad \text{Domain Wall width} \tag{9}$$

(10)

Simulation codes

Ubermag

- Micromagnetic

Nmag

OOMMF

mumax^3
GPU-accelerated micromagnetism
Home Download Examples API

finmag

FIDIMAG

FIDIMAG

- Atomistic

VAMPIRE

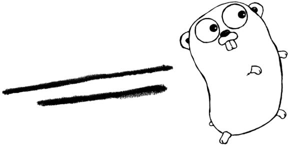
spirit

Boris Computational Spintronics

mumax^3

GPU-accelerated micromagnetism

NVIDIA



Golang

https://mumax.github.io/

https://mumax.github.io/examples.html

Script language based on **Go**: https://mumax.github.io/api.html https://mumax.github.io/api.html

Aex

Exchange stiffness (J/m)

```
methods: Average( )   EvalTo( Slice )   GetRegion( int )   IsUniform( )
MSlice( )   Region( int )   Set( float64 )   SetRegion( int ScalarFunction )
SetRegionFuncGo( int func() float64 )   SetRegionValueGo( int float64 )

examples: [1] [2] [3] [6] [7] [8] [9] [10] [11] [12] [13] [14] [15]
```

Mumax geometries

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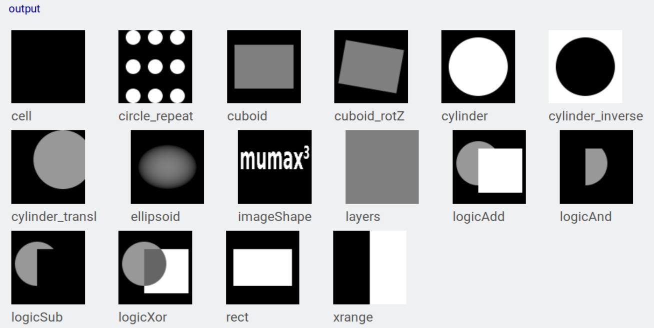
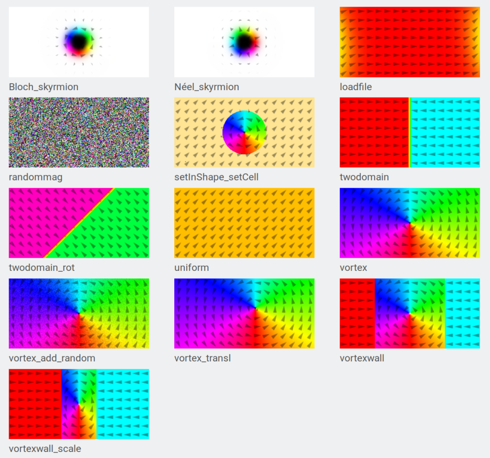
https://mumax.github.io/examples.html



Mumax initial states



Example: Standard problem 2

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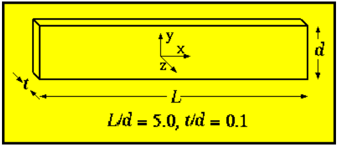
```
// Standard problem 2

lex := 5e-9
d := 30 * lex

lx := 5 * d
ly := 1 * d
lz := 0.1 * d

Ms := 1.0 / mu0  // 1 Tesla
Msat = Ms  // Or: Msat.Set(Ms)
Aex = 0.5 * mu0 * lex * lex * pow(Ms, 2)

nx := pow(2, ilogb(lx / (0.5 * lex)) + 1)
ny := pow(2, ilogb(ly / (0.5 * lex)) + 1)
Print(nx, ny)
SetGridSize(nx, ny, 1)
SetCellSize(lx / nx, ly / ny, lz / 1.)

// alpha = 0.02
m = uniform(1., 1., 0.9)
SaveAs(m, "m_initial")

// ...
```



This standard micromagnetic problem includes both magnetostatic and exchange energies, but has the advantage of only one scaled parameter. If crystalline anisotropy is neglected and the geometry is fixed, scaling of the static micromagnetic equations (Brown's equations) yield a hysteresis loop which depends only on the scaled geometry to the exchange length when expressed as M/Ms versus H/Hm, where Hm = Ms (SI) or 4piMs (cgs emu). The exchange length is lex = (A/Km)1/2, where A is the exchange stiffness constant and Km is a magnetostatic energy density, Km = 1/2μ0Ms2 (SI) or 2piMs2 (cgs emu).

The field should be applied in the [1,1,1] direction, approximately 54.74 degrees (arccos[1/root(3)]) from each of the coordinate axes. This field orientation is chosen to avoid potential symmetry breaking problems.

Geometry:

Let us take a thin film of thickness t, width d, and length L. We suggest to make the problem virtually 2D by choosing t/d = 0.1, and to obtain interesting non-uniform reversal modes, L/d = 5.

Material parameters:

```
The magnetostatic exchange length, lex
Zero magnetocrystalline anisotropy
```

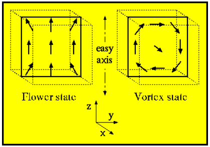
Desired output for comparison:

Calculated as a function of d/lex, with aspect ratios held constant at t/d = 0.1 and L/d = 5.0,

- Coercivity (Hc/Hm, the magnitude of the field at which the projection of the magnetization along the field, Mx+ My+ Mz, is zero.)
- Remanence (Mx/Ms, My/Ms, Mz/Ms, at H = 0)

Example: Standard problem 3


```
// Standard problem 3

N := 64

SetGridSize(N, N, N)
// Setting the initial state

Lmin := 8.0
Lmax := 9.0
Lstep := 0.05
L := Lmin
setcellsize(L/N, L/N, L/N)

// Initial magnetization (xy-plane)
m = vortex(1, -1)

// Parameters in reduced units -> lex = 1.0
Msat = sqrt(2 / mu0)
Aex = 1.0
Ku1 = 0.1
anisU = vector(1, 0, 0)

tableadd(E_total)
tableaddvar(L, "L", "")

for L = Lmin ; L <= Lmax; L += Lstep {
    ...
}
```



This problem is to calculate the single domain limit of a cubic magnetic particle. This is the size L of equal energy for the so-called flower state (which one may also call a splayed state or a modified single-domain state) on the one hand, and the vortex or curling state on the other hand. Material parameters:

Geometry:

A cube with edge length, L, expressed in units of the intrinsic length scale, lex = (A/Km)1/2, where Km is a magnetostatic energy density, Km = 1/2μ0Ms2 (SI).

Material parameters:

Uniaxial anisotropy Ku with Ku = 0.1 Km, and with the easy axis directed parallel to a principal axis of the cube.

Desired output for comparison:

- ☐ The indicated “single domain limit,” L **where the energy of the flower state is equal to the energy of the vortex state.**
- ☐ The partial energies (exchange, stray field, anisotropy) all in units of Km.
- ☐ The average magnetization along the three axes.

Example: Standard problem 3

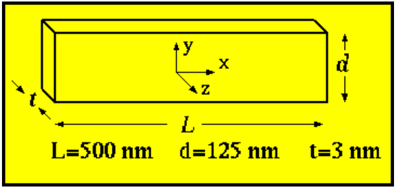
Normalize micromagnetic equations:

$$x \to \ell_{\mathrm{ex}} x \qquad A \to 1 \qquad K \to K/K_m \qquad M_{\mathrm{s}} \to \sqrt{2/\mu 0}$$

$$\begin{aligned} E = \int_V \mathrm{d}V \Bigg\{ & A \sum_{\alpha,\beta \in \{x,y,z\}} \left(\partial_\alpha m_\beta\right)^2 && \text{Exchange} \\ & - K \left(\mathbf{m}_z\right)^2 && \text{Anisotropies} \\ & - \mu_0 \mathbf{M} \cdot \mathbf{H} && \text{Zeeman} \\ & - \frac{\mu_0}{2} M_{\mathrm{s}}^2 \mathbf{m} \cdot \int \mathbf{N}(\mathbf{r} - \mathbf{r}') \cdot \mathbf{m}(\mathbf{r}') \mathrm{d}V' \Bigg\} && \text{Demag} \end{aligned}$$

Example: Standard problem 4 - mumax³


```
SetGridsize(128, 32, 1)
SetCellsize(500e-9 / 128,
            125e-9 / 32,
            3e-9 / 1)

Msat  = 800e3
Aex   = 13e-12
alpha = 0.02

m = uniform(1, .1, 0)
relax()
save(m)     // relaxed state

autosave(m, 200e-12)
tableautosave(10e-12)

B_ext = vector(-24.6E-3, 4.3E-3, 0)
run(1e-9)
```



Standard problem #4 is focused on dynamic aspects of micromagnetic computations. The initial state is an **equilibrium s-state** such as is obtained after applying and slowly reducing a **saturating field along the [1,1,1] direction to zero**.

Geometry: A film of thickness, t=3 nm, length, L=500 nm and width, d=125 nm will be used.

Material parameters similar to Permalloy: A = 1.3e-11 J/m, Ms = 8.0e5 A/m, K = 0.0, α = 0.02, γ_0 = 2.21e5 (m / A s)

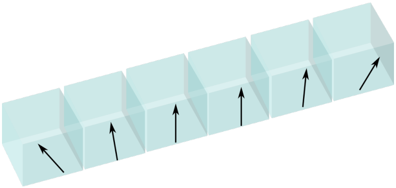
Applied Fields: Two switching events will be calculated using fields applied in the x-y plane of different magnitude and direction.

Field 1: μ0Hx=-24.6 mT, μ0Hy= 4.3 mT, μ0Hz= 0.0 mT which is a field approximately 25 mT, directed 170 degrees counterclockwise from the positive x axis

Field 2: μ0Hx=-35.5 mT, μ0Hy=-6.3 mT, μ0Hz= 0.0 mT which is a field approximately 36 mT, directed 190 degrees counterclockwise from the positive x axis

Example: DMI standard problem 1D

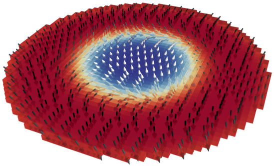

```
SetMesh(100, 1, 1, 1e-09, 1e-09, 1e-9,
        0, 0, 0)

Msat         = 0.86e6
Aex          = 13.0e-12
Ku1          = 0.4e6
anisU        = vector(0, 0, 1)
Dbulk        = 3.0e-3
NoDemagSpins = 1

// Initial state
m.setRegion(0, uniform(0, 0, 1))

OutputFormat = OVF2_TEXT

// Relax with specific torque:
// alpha        = 0.9
// RunWhile(MaxTorque > 1e-4)

// Relax with conjugate gradient:
minimize();
SaveAs(m, "one_dim_bulk")
```



Standard Problem DMI paper

Example 4: DMI standard problem 2D


```
// Disk radius
R  := 50e-9

// Number of cells
nx := 50
ny := 50
nz := 1
// Mesh discretisation
dx := 2e-9
dy := 2e-9
dz := 2e-9
// Set mesh and disk geometry
SetGridSize(nx, ny, nz)
SetCellSize(dx, dy, dz)
SetGeom(Circle(2 * R))
DefRegion(1, circle(R / 2))

Msat        = 860e3
Aex         = 13e-12
Ku1         = 0.4e6
anisU       = vector(0, 0, 1)
Dind        = -3e-3
// No Demag:
NoDemagSpins = 1

// Initial state
m.setRegion(0, uniform(0, 0, -1))
m.setRegion(1, uniform(0, 0, 1))

OutputFormat = OVF2_TEXT
// relax()
minimize()
SaveAs(m, "Skyrmion-Inter")
Snapshot(m)
```



Example 5: Skyrmions on the track

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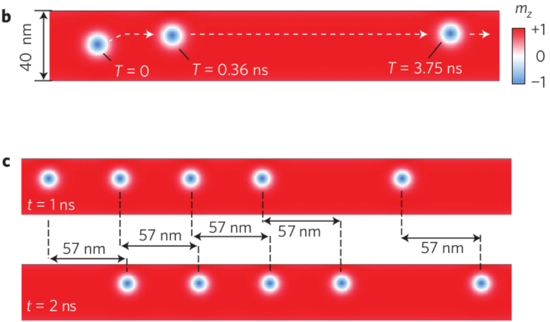

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$$\boldsymbol{\tau}_{\text{ZL}} = \frac{(1+\alpha\xi)}{1+\alpha^2}\mathbf{m}\times(\mathbf{m}\times(\mathbf{u}\cdot\nabla)\mathbf{m}) + \frac{(\xi-\alpha)}{1+\alpha^2}\left[\mathbf{m}\times(\mathbf{u}\cdot\nabla)\mathbf{m}\right] \quad (11)$$

$$\mathbf{u} = \frac{\mu_B P}{2e\gamma_0 M_{\text{s}}(1+\xi^2)}\mathbf{j} \quad (12)$$

Notice that for the current density $\mathbf{j}$ the electron velocity $\mathbf{u}$ is in the opposite direction.

The design and verification of MuMax3. AIP Advances 4, 107133 (2014).

Example 5: Skyrmions on the track


```
// Track length
Lx := 400e-9
Ly := 40e-9
Lz := 0.4e-9

// Number of cells (power of 2) -> discretization of 1.25
nx := pow(2, 6) * 5
ny := 32
nz := 1

// Set mesh and disk geometry
SetGridSize(nx, ny, nz)
SetCellSize(dx, dy, dz)

Msat        = 580e3
Aex         = 15e-12
Ku1         = 0.8e6
anisU       = vector(0, 0, 1)
Dind        = 3e-3
// No Demag:
// NoDemagSpins = 1
```

```
// Initial state
m = NeelSkyrmion(1, -1).scale(3, 3, 1).translate()

OutputFormat = OVF2_BINARY
minimize()
SaveAs(m, "sk_track_relax")
SnapshotAs(m, "sk_track_relax.png")

alpha = 0.3
// Now set the current motion: in-plane -> Zhang-Li model
// (see eq in Landau-Lifshitz form)
// non-adiabaticity:: (0.15, 0.30 and 0.60)
pol  = 0.4
j    = vector(-0.1e12, 0, 0)  // 10 MA cm^-2 in the x-direction
xi   = 0.15

autosave(m, 5e-10)
tableAutosave(1e-11)
tableAdd(ext_bubblepos)

run(10e-9)
```

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Non-adiabaticity parameter β (0.15, 0.30 and 0.60)

Current densities: 0.01 - 100 MA cm^-2

Polarization $P = 0.4$

Example 5: Skyrmions on the track

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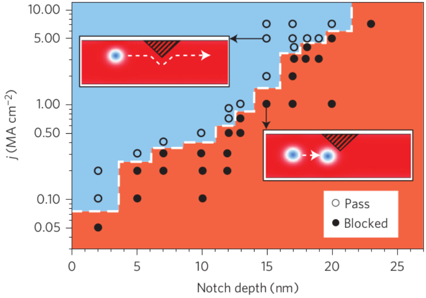

Figure 4 | Pinning of the current-induced motion of skyrmions by defects in a 40-nm-wide nanotrack. The defect (notch) is a triangular region of higher perpendicular anisotropy ($K = 1.2$ MJ m^{-3} instead of 0.8 MJ m^{-3} for the rest of the track), and the motion has been simulated for different notch

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