All in the family: Anna and Lisa Hahner's finishing times in the 2016 women's Olympic marathon

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"I invested all I had and 300 meters before the finish line, I was next to Lisa. It was a magical moment that we could finish this marathon together. We did not think about what we were doing." – Anna Hahner

Introduction

On August 14, 2016, at 9:30 in the morning, the Women's Olympic marathon kicked off in Rio de Janeiro, Brazil, when 156 runners from 80 countries across the world took off from the starting line en route to their destination 42.195 kilometers away. Two hours, twenty-four minutes, and four seconds later, Jemima Sumgong of Kenya would be the first to cross the finishline and take home gold; Sumgong was just 3.5 minutes behind her personal best time in the marathon. Approximately 21 minutes later, twin marathoners from Germany, Anna and Lisa Hahner, would cross the finish-line together, holding hands and celebrating a personal victory. Although the Hahners would finish 81st and 82nd, respectively, with times slightly more than 18 minutes slower than corresponding personal bests, Anna Hahner would describe their joint finish as a "magical moment." The media quickly picked up on the Hanher story as an image of the beaming twins finishing hand-in-hand captured a public audience. Many believed the moment was a reflection of the Olympic spirit.

Not everyone agreed with this rosy interpretation. The twins' happy facial expressions at the finish were portrayed as a bit contrived (smiling like "Honigkuchenpferde"—a Honigkuchenpferd is a German cookie in the shape of a horse), and the sports director of the German Athletics Federation, Thomas Kurschilgen, stirred up controversy when he suggested that the Hahners' photo-finish was no coincidence. Kurschilgen averred that the twins slowed down so as to finish simultaneously and sought to create a spectacle and "generate media attention." Kurschilgen justified his charge with the fact that the twins were at least 18 minutes behind their personal best times.[1] Not surpris-

[1] `https://www.nytimes.com/2016/08/17/sports/olympics/twins-finish-marathon-hand-in-hand-but-their-country-says-they-`

ingly, these accusations were denied by the Hahners, who claimed that their simultaneous finish was simply an unintended coincidence.

What happened in the women's Olympic marathon, and how might we assess whether the Hahner finish was coincidental or intentional? These two interpretations are clearly at odds. If the former, then the Hahner twins are to be celebrated and their finish treated as an expression of the spirit behind the Olympic games. If the latter, though, then the Hahner twins may have violated this spirt by not trying hard enough. It is perhaps too easy for us to write such a glib sentence—neither of us cannot fathom being able to complete a marathon anywhere in the vicinity of two and a half hours—but we nonetheless want to know what the data from the Olympic marathon data tell us. Was the women's Olympic marathon a lovely coincidence or something else?

Among female Olympic marathoners, the Hahner twins were not alone in their familial ties, and we draw on this in the analysis that follows. The marathon also featured twins from North Korea, Kim Hye-song and Kim Hye-gyong, who had identical times and finished 10th and 11th, respectively. The Kim finish, unlike the Hahner finish, appears devoid of post-race controversy. Moreover, three triplets from Estonia competed in the Rio marathon, although only two, Lily Luik and Leila Luik, finished it, in 97th and 114th place, respectively. The third Estonia triplet, Liina Luik, recorded what is known as a DNF—an abbreviation that means did not finish, a term that we will use throughout this article.

Marathon data

For each participant who started the women's Olympic marathon, we know several things: personal best marathon time prior to the 2016 Olympic games; split times from the Rio marathon course at 5 kilometers, 10 kilometers, and so forth; and, finishing time. We cannot directly observe the effort that an individual put into the race, and we do not know why some runners have DNF results;

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some runners may have injured themselves on the course and accordingly dropped out, and others may have dropped out, uninjured, in anticipation of an unsatisfactory result. Of the 156 marathon starters, 133 completed the race and 23 DNFed at various locations throughout the course. The overall DNF rate was thus $\frac{23}{156} \approx 0.15$, and the relatively small sample size at our disposal—156 runners—means that a 95% confidence for this rate is fairly wide, namely, $(0.098, 0.22)$.

Key in the analysis that follows is the deviation between a runner's Rio time and her prior personal best time in the marathon. Of course there is variance across runners in these deviations, and we want to know if patterns in the deviations support an honest effort by the Hanher twins or perhaps something else.

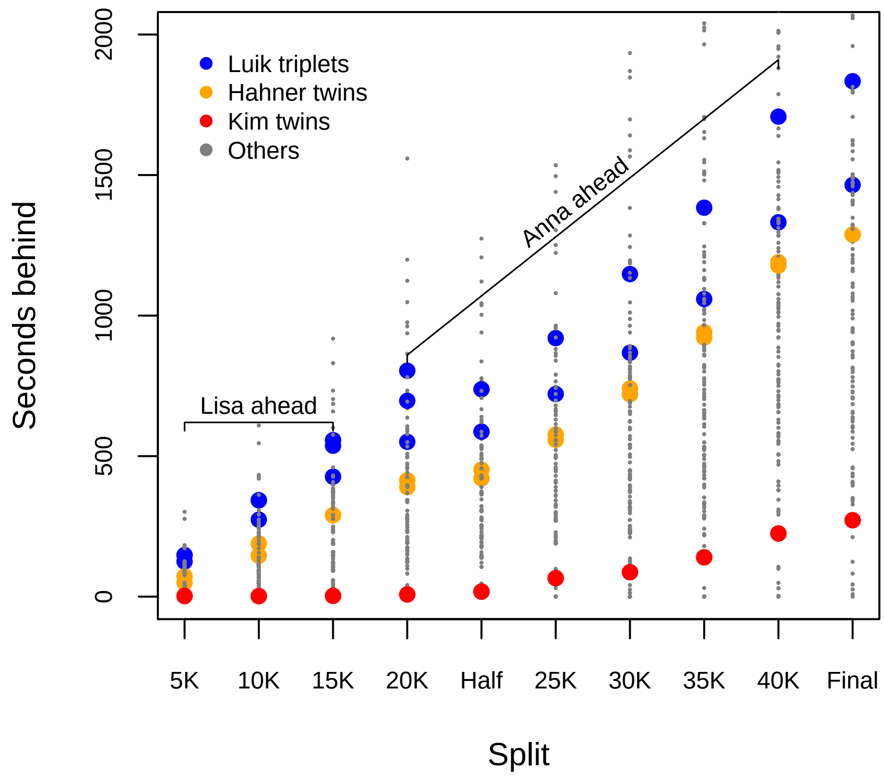
As a brief overview of the race, Figure 1 describes each runner's status at the various split times on the marathon course. Each dot in the figure depicts a recorded split and the number of seconds each runner was behind the race leader at the time. There are more dots at earlier splits due to the accumulation of DNFs. Regarding twins and triplets, the Estonian Luik triplets are denoted with blue dots; there are three blue dots through the 20 kilometer split, but the third triplet did not record a half-marathon split. The orange dots in the figure represent the Hahner twins; Lisa Hahner was ahead of her sister through 15 kilometers at which point Anna surged ahead. There are two red dots representing the North Korean Kim twins although this is not visually apparent; the Kim twins had identical split times during the marathon.

Figure 1 does not directly shed any light on our motivating research question, but it does show that the Hahner twins finished amidst many other runners. We will come back to this point later.

The probability of an unintentional simultaneous finish

Thomas Kurschilgen's accusations of an intentional finish stems from two observations. First, the Hahner twins finished slower than he expected given respective personal bests. Second, the twins finished together at the exactly same time. Kurschilgen clearly believed that neither of these events

Figure 1: Runner status by split



Note: each dot represents one runner at a split. DNFs are not pictured, and splits are not to scale.

would have occurred had both Hahner run the marathon independently, absent coordination. By his logic, the German twins should have run faster and not have finished simultaneously.

On the other hand, only a handful of runners completed the Rio marathon with times that were faster than their recorded best times. Thus, 18 minutes behind a personal record may not be the outlier that Kurschilgen claimed it to be. Moreover, there is good reason to suspect that an unintended simultaneous finish was more likely for the Hahner twin than it would be for any other set of runners. After all, these two women are twins with presumably similar abilities. Not only do they train together, but their pre-Rio best times are less than two minutes apart. While Anna may be slightly faster than Lisa measured by personal bests, we might expect the difference in their Rio result to be just as close as the differences in their recorded bests. And given random variation in finishing times, a simultaneous finish might not be out of the ordinary.

Even though they seem similar, finishing at very similar times and finishing together are different phenomena. If, for example, both Hahners were of similar ability to each other and also to many other runners, then we might expect similar finishing times yet not necessarily similar placements. The latter will be a function of the extent to which all runners on the Rio course have similar talent levels. This point is an important one and will be evident in the results that follow.

To test the assertion that the Hahner twins paced themselves to finish at the same time and with back-to-back placements, we need to compute the probability that such a result would have occurred unintentionally. Hence we need to know the probability distribution of Anna and Lisa's finishing times if their runs had been independent of each other. The challenge, of course, is that this distribution is unknown.

Modeling

The Hahner twins either acted independently I or coordinated their times C; these are the two possible states of the race $\psi = \{I, C\}$. Given that we observed Anna and Lisa's finishing times, $Y_A = y_A$ and $Y_L = y_L$, we want to know the probability that they ran independently, as they say they did. Hence, we are looking to determine,

$$P(\psi = I \mid y_A \cap y_L)$$

However, to determine this, we need to have some understanding of a likelihood function that specifies Rio finishing times. We want to know the likelihood of Anna's and Lisa's final times given independence $P(y_A \cap y_L \mid \psi = I)$. We can estimate this function with a few assumptions.

First, we assume that under independence, any given runner's final time Y_i is conditional on his/her running ability plus noise. Specifically, we assume that the Y_i is a linear function of the runner's ability X_i plus a normally distributed error term $e_i \sim N(0, \sigma)$. Second, we assume that every runner shares the same linear relationship - meaning the slope and intercept remain constant across runners. Third, we assume that the error term is drawn from a common distribution across runners. Hence, luck and misfortune are drawn from the same distribution. Therefore,

$$Y_i \sim N(\beta_0 + X_i\beta_1 + e_i, \sigma)$$

We also assume that a runner's ability X_i can be measured precisely by her best marathon performance leading up to the Olympics.

If Anna and Lisa intentionally slowed down as a result of coordination then we would likely observe $y_A > E(Y_A \mid \psi = I)$ and $y_L > E(Y_L \mid \psi = I)$. In other words, the final times that Anna and Lisa recorded in the race would be greater than we would expect if they had run independently.

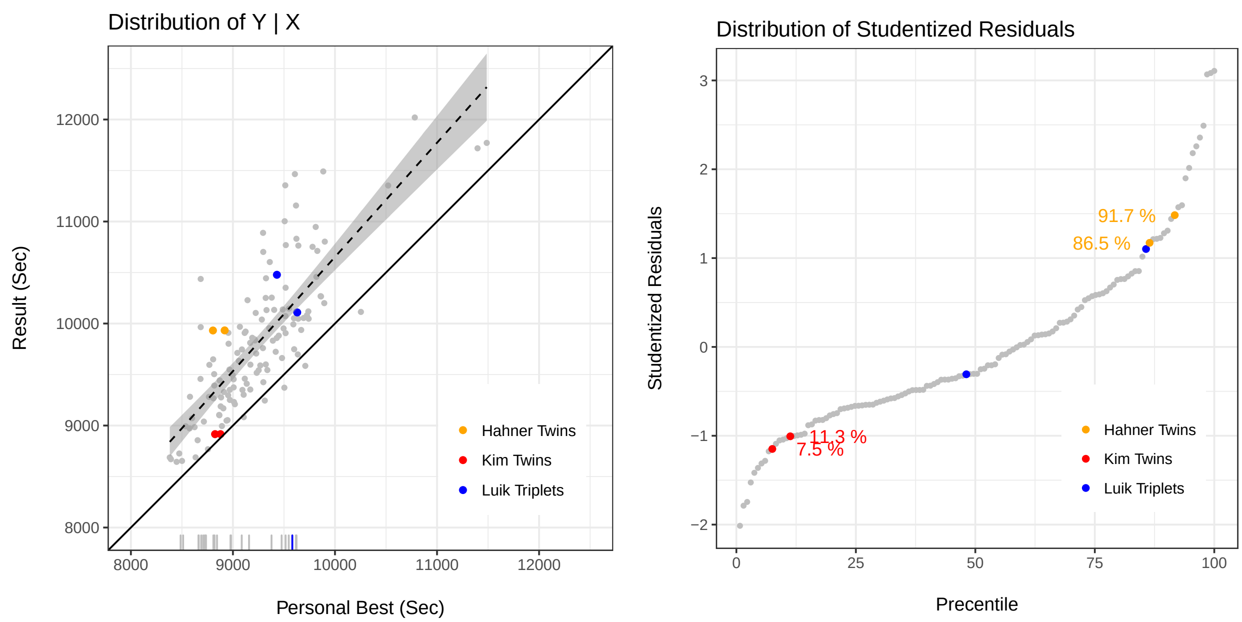
Moreover, if Anna and Lisa coordinated to finish simultaneously, then the difference between the two sisters' final times would be less that the expected difference had they run independently. Therefore, under a coordinated finish we would expect $|y_A - y_L| < E(|Y_A - Y_L| \mid \psi = I)$.

Did Anna and Lisa intentionally slow down?

According to Kurschilgen, Anna and Lisa underperformed in the Rio marathon. He claimed that because their goal was to finish simultaneously rather than finish at their fastest pace, their times were slower than they otherwise would have been. He claimed that the twins were simply trading speed for a photo-finish.

If this were the case, the function generating Anna and Lisa's final times would deviate from the function that generated everyone else's final times. Given everyone else would draw their times from the independent distribution $Y_i \sim N(\beta_0 + X_i\beta_1 + e, \sigma)$, Anna and Lisa would draw from a distribution of times that are slower in expectation. Hence, they would lie well-above the line that links a runner's performance in Rio to their previous best performance.

Figure 2: Relationship between Personal Best and Result – caption?



(a) Rio finishing times and personal best times

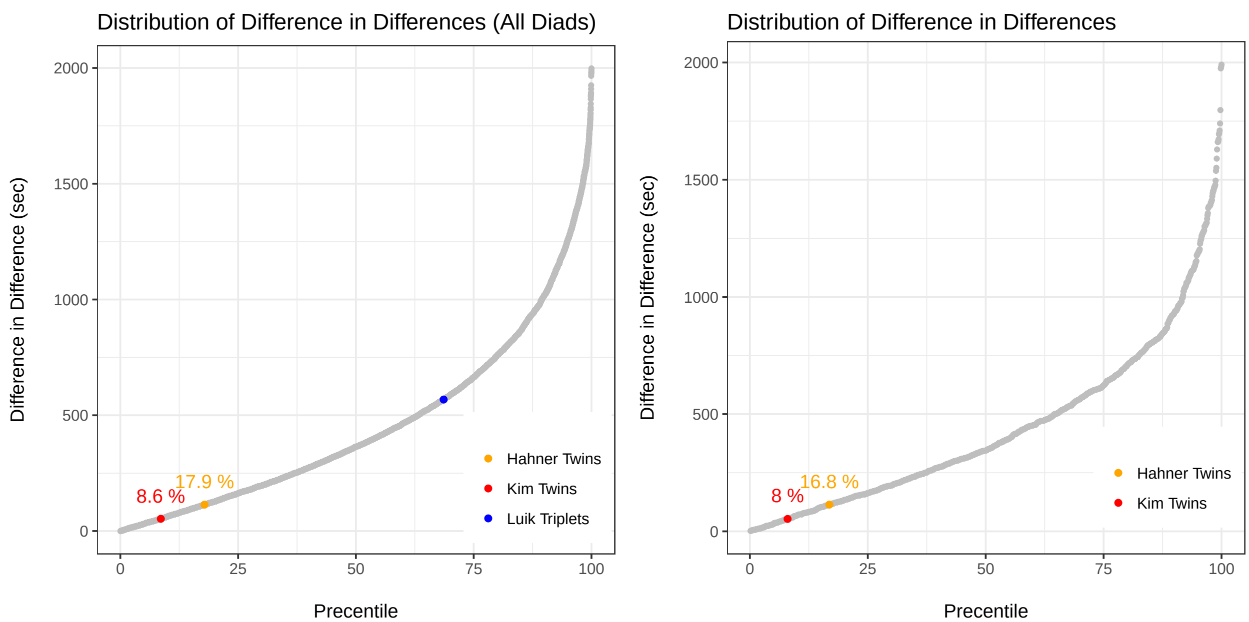
(b) Studentized residuals

Figure 2a displays the relationship between a runner's personal best (horizontal axis) and her final result (vertical axis). The 133 finishers are plotted as grey points, and the 23 DNFs are depicted below along the horizontal axis with tick-marks. The dashed black line with grey confidence intervals represent ordinarily least squares estimates of the linear relationship between racer's finishing and personal best times.

One can see in Figure 2 that there is a clear relationship between a runner's personal best time and her performance in the Olympics. While there is a significant amount of noise in the relationship, the Hahner twins' dots are well above the regression's fitted line notwithstanding this point. They finished the Rio marathon much slower than they should have, per their pre-Olympic personal best times. In fact, if we sort studentized residual, we can see in Figure 2b that the Hahner twins are in the tail-end of the residual distribution. Anna Hahner's residual deviation is greater than 91.7% of the runners who completed the marathon while Lisa Hahner's deviation is greater than 86.5%. Hence, the twins appear to have finished at a much slower pace than expected, which is what we might expect if they were coordinating their runs. The North Korean Kim twins are also in the tail of the residual distribution—but the left tail. These twins finished much faster than expected.

Did Anna and Lisa intentionally finish together?

Figure 3: Relationship between Difference in Personal Best and Difference in Result



Steps of the simulation

1. Removing the two sets of twins and triplets from the sample, estimate a linear model that predicts a runner's final time (Y_i) based on her pre-Olympic personal best time (X_i).

2. Extract the coefficients and the variance-covariance matrix from this model.

3. For each runner, draw β_0 and β_1 from a multivariate normal distribution with mean equal to the coefficients and variance equal to the variance-covariance matrix.

4. For each runner, draw an error term (e) from a normal distribution with mean equal to zero and a standard deviation equal to the standard deviation of the model's residuals (σ).

5. For each runner, predict the final result by combining the randomly generated beta coefficients and error terms, $\hat{y} = \beta_0 + \beta_1 + e$.

6. Eliminate each runner from the race with a probability equal to the percent of the total runners who did not finish in the actual race. This account for the likelihood of a DNF.

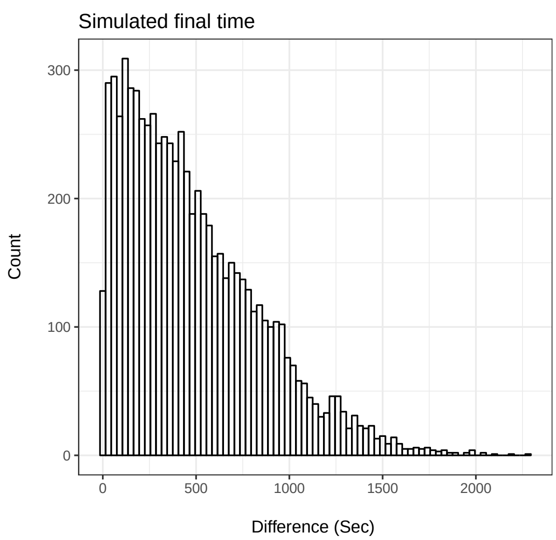
7. Calculate the difference in time between Anna Hahner and Lisa Hahner.

8. Calculate the difference in ranking between Anna Hahner and Lisa Hahner.

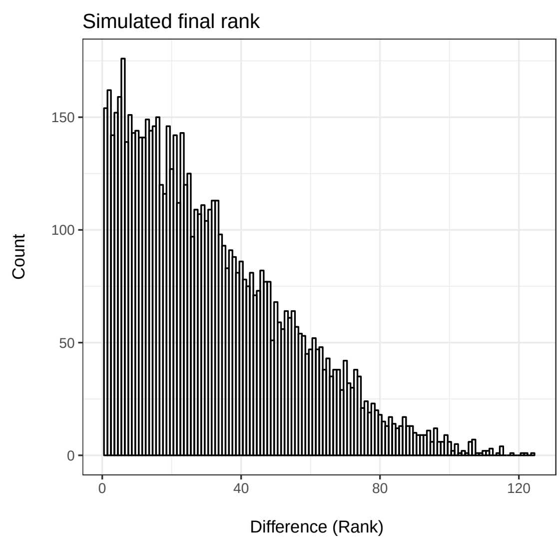
9. Repeat steps 3 through 8 ten thousand times.

10. Plot a histogram of the results in Figure 4

Figure 4: Distribution of simulated results for the Hahner twins



(a) Simulated differences in finishing times



(b) Simulated differences in places