Name ____________________ Date __________ Partners____________________

TA ________________ Section __________ ______________________

Lab 2: Measurement, Uncertainty, and Deviation

"The first principle is that you must not fool yourself – and you are the easiest person to fool."

--Richard Feynman

Objective: To understand how to report both a measurement and its uncertainty.

Learn how to propagate uncertainties through calculations

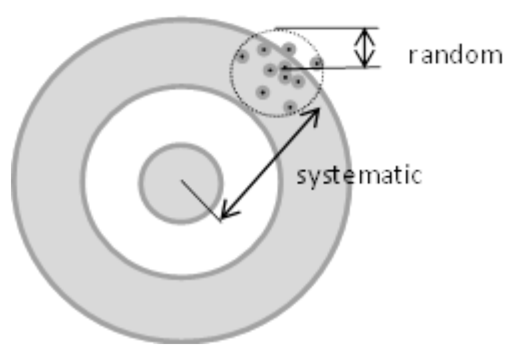
Define *mean*, *standard deviation*, and *standard deviation of the mean*.

Equipment: meter stick, 1 kg mass, ruler, caliper, short wooden plank, graduated cylinder

Types of uncertainties

Random uncertainties occur when the results of repeated measurements vary due to truly random processes. For example, random uncertainties may arise from small fluctuations in experimental conditions or due to variations in the stability of measurement equipment. These uncertainties can be estimated by repeating the measurement many times.

A *systematic uncertainty* occurs when all of the individual measurements of a quantity are biased by the same amount. These uncertainties can arise from the calibration of instruments or by experimental conditions. For example, slow reflexes while operating a stopwatch would systematically yield longer measurements than the true time duration.

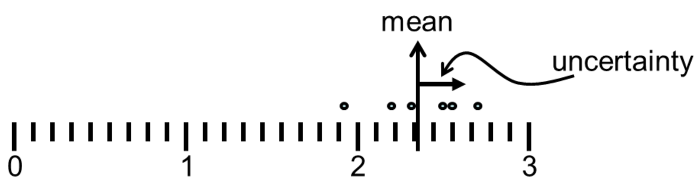


Note that **we are not talking about "*human error*"! We are not talking about *mistakes*!** Rather, uncertainty is inherent in the instruments and methods that we use **even when perfectly applied.** The goddess Athena cannot not read a digital scale any better than you.

Mistakes can be made in any experiment, either in making the measurements or in calculating the results. However, by definition, mistakes can also be avoided. Such blunders and major systematic errors can only be avoided by a thoughtful and careful approach to the experiment.

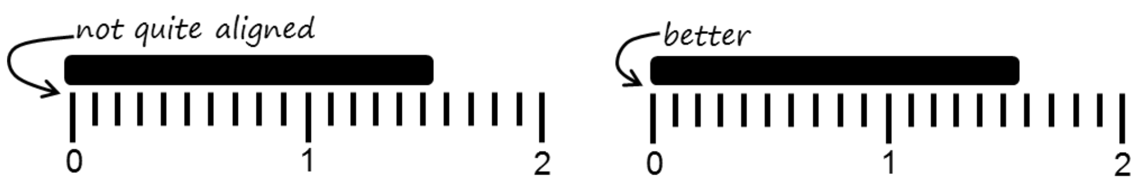
Determining the Uncertainty

Repeated observation: Suppose you make repeated measurements of something: say with a stopwatch you time the fall of a ball. Due to random variations, each measurement will be a little different. From the spread of the measurements, you can calculate the uncertainty of your results.



Shortly, we will describe the formal procedure to do this calculation. (Oddly enough, truly random uncertainties are the easiest to deal with.)

By eye or reason: Sometimes, repeated measurements are not relevant to the problem. Suppose you measure the length of something with a meter stick. Meter sticks are typically ruled to the mm; however, we can often read them more precisely that that.



Consider the figure above. Measuring from the left side of each mark and considering the position uncertainties of <u>both ends</u> of the bar, I can confidently say that the bar is (1.56 ± 0.03) cm. Perhaps your younger eyes could read it with more confidence, but when in doubt it is better to overestimate uncertainty.

Could I do a better job by measuring several times? Only if each measurement is independent of the others. In measuring the bar above, I can look at it at many time as I wish, but it is still comes down to "*Looks like (1.56 ± 0.03) cm to me.*" But that's ok. Your reasoned judgment is a valid measure of uncertainty. **Science is defined by rigorous honesty, not rigorous precision!**

Reducing random uncertainty by repeated observation

However, often we can take a large number of independent measurements. We can then use statistics to improve the measurement. For instance, suppose we want to determine the mass of a standard U.S. penny. We measure the mass of N = 3 pennies with a balance.

Penny 1: 2.43 g

Penny 2: 2.54 g

Penny 3: 2.49 g

a. The best estimate of the mass of a penny is the average mass.

$$M = \frac{1}{N}\sum_{i=1}^{N} m_i$$

b. We can then calculate the deviation of each of these pennies from the average?

$$d_i = m_i - M$$

Standard Deviation of a set of measurements is defined as:*.

$$\Delta m = \sqrt{\sum_{i=1}^{N} \frac{\left(m_i - M\right)^2}{\left(N-1\right)}}$$

Discussion Question: Suppose that instead of three pennies, we measured 10 pennies. What will happen to the standard deviation? Explain your reasoning.

Increase Decrease Stay the same

* Specifically, this is the expression for the Standard Deviation of a Sample (in Excel, this is the STDEV or STDEV.S function).
There are other ways to define the uncertainty of a measurement, but this is the most commonly used. If you wish to start a fist fight at a mathematics convention, a discussion of the virtues of Standard Deviation could be an effective provocation.

c. The Standard Deviation of the Mean ΔM is defined by:

$$\Delta M = \frac{\Delta m}{\sqrt{N}}$$

Discussion Question 2: What happens to ΔM as the number of measurements increases? Can we reduce the uncertainty of the mean value without limit?

Excel Exercises

The provided Excel file includes 17 measurements. Using Excel, calculate . . .

1. The mean value of the mass.
2. The standard deviation (STDEV) of the measurements.
3. The standard deviation of the mean.

Compare your results with others to verify that you have done the math correctly.

An Actual Experiment

Obtain a straight walled vessel of unknown volume. (A coffee cup will work.) You will also need a stopwatch or similar device. (Your phone should work fine.)

1) This activity will be done at your sink. We want to measure the time it takes to fill a small glass or cup in order to determine a flow rate.
    a. Turn on the faucet to a low stream. Do not change the flow rate once you start making measurements.
    b. If you have one, use the stop watch in the clock app on your phone. If you don't, use whatever method you have to measure seconds. You may need to adjust the flow rate to accommodate your measurement system.
    c. Make 20 repeated measurements of the time from when the cup first starts filling up to when water first overflows the edge of the cup (tedious, but having 20 data points will be useful as we study this unit.)
2) With Excel, find the Standard Deviation and the Standard Deviation of the Mean for the measured fill times.
3) Determine the volume of the interior of your cup or glass and the associated uncertainty. You can use a measuring cup or ruler to do this.
4) Calculate the flow rate of the water and the associated uncertainty.
5) Write a very brief report describing your pedicure, and results. Careful with your units. Also, include your Excel file. Clearly label your columns and deviation results in the spreadsheet.