Operational Research case study: Sweetland – What Merchant Navy?”

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1 Introduction

This case study was developed with the goal of calculating and solving some questions regarding the problem ”Sweetland - What Merchant Navy?”. In order to arrive to the conclusions presented in this report, the group applied some operational research knowledge and techniques, solving the main problem with linear programming. Some other concepts were applied and explained, like the sensitivity analysis of some results. The linear programming model used was built and solved using the Excel Solver.

This report thoroughly describes the problem and all the questions that needed to be answered, and all the assumptions the group made before solving them. Afterwards, each question is explained and answered in its own chapter, showcasing all formulas and calculations, and the overall thought process that originated the solution.

2 Problem

”Sweetland - What Merchant Navy?” describes a group of 3 countries, Sweetland, Fatland and Sealand, and their desire of exchanging products through sea, each having a set of products that needed to be exported or imported. The main problem to be solved is to **find out, for Sweetland, the number and type of vessels that should be purchased**, having into account various factors like the distances to other countries, the different types of vessels, the capacity of each vessel, the quantity of exports and imports for the country, among other things. The full list of restrictions and conditions of the problem are the following:

- Sweetland should **export wheat and corn** to Fatland and Sealand, and **import iron and copper** from these countries.
- The vessels leaving Sweetland should depart from one of its ports loaded with wheat or corn, discharge at the destination port in either Fatland or Sealand, load back with copper or iron in the harbor where they were unloaded or in any other port within Fatland and Sealand, and return loaded to the port of origin in Sweetland.
- There are, in total, 5 ports: Doce and Bom in Sweetland, SKY and Moon in Fatland, and MARS in Sealand.
- Each port has an allowed draft. The following image showcases all considered ports in the 3 countries, and their respective draft. Note that large vessels cannot go to MOON, as they do not satisfy the draft allowed.

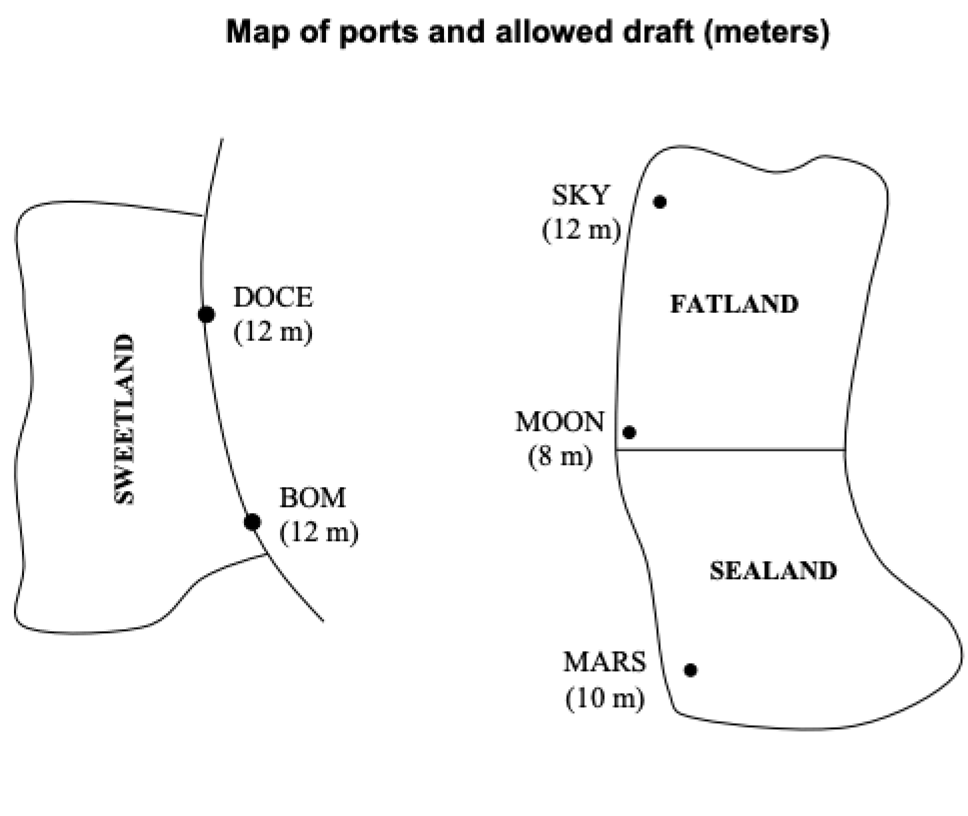

Figure 1: Information about each port, and their respective draft.

- We also know the distance between every pair of ports. Those distances are specified in the following distance matrix.

Distance matrix between ports (in km)

	DOCE	BOM	SKY	MOON	MARS
DOCE	-	na	6000	5000	5500
BOM		-	6000	5800	4800
SKY			-	500	2000
MOON				-	1000
MARS					-

Figure 2: Distance matrix showcasing the distance between every pair of considered ports.

- The quantity of each product that Sweetland needs to import and export is also specified. The following table shows the necessary quantities and the various products available in the various ports.

Import / Export Map (tons)

SWEETLAND

	Export		Import	
	Wheat	Corn	Copper	Iron
BOM	50000	-	-	50000
DOCE	-	40000	20000	20000

SEALAND

	Export		Import	
	Copper	Iron	Wheat	Corn
MARS	-	30000	20000	10000

FATLAND

	Export		Import	
	Copper	Iron	Wheat	Corn
SKY	10000	40000	30000	-
MOON	10000	-	-	30000

Figure 3: Information about the import and export quantities to be satisfied, for each port.

- The following table presents some characteristics of each type of vessel available. It is worth noting that neither of these vessels can make the transport of two different products at the same time.

Characteristics of the vessels

TYPE OF VESSEL	TONNAGE (tons)	DRAFT (meters)		SPEED (km/h)		FUEL CONSUMPTION (liters / 1000km)		COST OF VESSEL PURCHASE	COST OF CREW
		Loaded	Empty	Loaded	Empty	Loaded	Empty	($x10^3$ $)	($x10^3$ $ / year)
Vessel 1	35	8	6	25	30	50	42	1000	70
Vessel 2	70	10	6	20	24	40	30	1500	75

Figure 4: Information about each vessel type.

- Each ship can operate continuously, 24 hours a day for 345 days a year. The remaining 20 days are used for the annual maintenance of the units.
- Finally, we need to consider other expenses related to each ship. The annual maintenance cost of each vessel is 10% of its initial cost. The fuel needed for each one obviously depends on the distance traveled and the type of unit, being that the cost of fuel is $0.8/liter. Finally, the labor needed (crew members) only depends on the type of unit. The vessel and crew member prices are present in Figure 4.
- It is also worth noting that the purchase of the individual vessel units is done in fixed installments, without interest, over 25 years, which means that the yearly purchase cost for each vessel is $\frac{1}{25}$ of its total cost.

3 Assumptions

What follows is a list of assumptions made during this case study:

1. All vessels always carry as much as possible;
2. The amount of import/exports is all that is available during the year and must be transported during the year;
3. There is no downside of exceeding the amount exported/imported (related with 1, the indirect downside is that more trips require more costs);
4. The distance matrix presented in Figure 2 specification represents the shortest distance between two cities (no need to account for composite paths);
5. The departure port of a vessel from Sweetland is the same as the return port.

4 Question 1: Itineraries

Given the assumptions we found a total of nineteen itineraries, seen in Table 1. Columns *Origin 1* and *Dest 1* represent the original departure port from Sweetland and the arrival port at Fatland or Sealand. *Origin 2* represents the departure port at Fatland or Sealand whereas *Dest 2* represents the arrival port at Sweetland. The types of cargo transported on the outgoing and return journeys are represented by the *Export* and *Import* columns respectively. To improve readability we refer to type 1 and 2 vessels as S and L, respectively.

We chose to label the itineraries as idK where K is either S or L depending on the type of vessel, e.g. itineraries $5S$ and $5L$ represent the same itinerary with small and large vessels, respectively.

In the supplemental excel (*Itineraries* page) you can find a more complete description of the itineraries such as fuel costs, distances, etc..

ID	Origin 1	Dest 1	Origin 2	Dest 2	Export	Import	Type
$1S$	Doce	Moon	Sky	Doce	corn	copper	S
$2S$	Doce	Moon	Sky	Doce	corn	iron	S
$3S$	Doce	Moon	Moon	Doce	corn	copper	S
$4S$	Doce	Moon	Mars	Doce	corn	iron	S
$5S$	Doce	Mars	Sky	Doce	corn	copper	S
$6S$	Doce	Mars	Sky	Doce	corn	iron	S
$7S$	Doce	Mars	Moon	Doce	corn	copper	S
$8S$	Doce	Mars	Mars	Doce	corn	iron	S
$9S$	Bom	Sky	Sky	Bom	wheat	iron	S
$10S$	Bom	Sky	Mars	Bom	wheat	iron	S
$11S$	Bom	Mars	Sky	Bom	wheat	iron	S
$12S$	Bom	Mars	Mars	Bom	wheat	iron	S
$5L$	Doce	Mars	Sky	Doce	corn	copper	L
$6L$	Doce	Mars	Sky	Doce	corn	iron	L
$8L$	Doce	Mars	Mars	Doce	corn	iron	L
$9L$	Bom	Sky	Sky	Bom	wheat	iron	L
$10L$	Bom	Sky	Mars	Bom	wheat	iron	L
$11L$	Bom	Mars	Sky	Bom	wheat	iron	L
$12L$	Bom	Mars	Mars	Bom	wheat	iron	L

Table 1: Itineraries between the Sweetland, Fatland and Sealand.

5 Question 2: Linear program model formulation

5.1 Decision variables

We created one variable (x_{idK}) for each itinerary described in section 4. The value of these variables represents how many trips of the respective itinerary need to be done during the year.

$$\overline{x} = [x_{1S}, x_{2S}, ..., x_{5L}, ..., x_{nK}] \in (\mathbb{R}_0^+)^n \quad \text{number of trips of each itinerary} \tag{1}$$

We also created two variables, n_S and n_L, which represent the number of small and large vessels that need to be bought by Sweetland's government.

$$n_S, n_L \in \mathbb{R}_0^+ \quad \text{number of small and large vessels} \tag{2}$$

5.2 Objective function

The objective is to **minimize** the yearly cost of running the fleet while also ensuring that time and import/export constraints are met (see subsection 5.3). For clarity, we divided the cost function into fixed costs and variable costs. Fixed costs are related to the amount of vessels in the fleet and cover the yearly instalments for the ship, the maintenance and the crew costs.

$$fixedCostsS(n_S) = n_S \times (\frac{S_{shipCost}}{25} + S_{shipCost} \times 0.1 + S_{crewCost]}) \tag{3}$$

$$fixedCostsL(n_L) = n_L \times (\frac{L_{shipCost}}{25} + L_{shipCost} \times 0.1 + L_{crewCost]}) \tag{4}$$

$$fixedCosts = fixedCostsS + fixedCostsL \tag{5}$$

where $K_{shipCost}$ represents the total cost of the ships, and $K_{crewCost}$ is the crew cost of the vessels. The term $K_{shipCost} \times 0.1$ represents the maintenance costs.

The variable costs are the costs associated with the chosen itineraries (x_{idK}), in particular, they are related to the fuel costs. As is expressed in the original problem, the fuel consumption of the vessels depends on whether they are full or not. For this, we calculated the distances that each itinerary goes through without ($distEmpty$) and with cargo ($distFull$):

$$emptyFuelCost(\overline{x}) = \sum_{id}^{itineraries} x_{id} \times distEmpty_{id} \times emptyFuelCons \times fuelCost \quad (6)$$

$$fullFuelCost(\overline{x}) = \sum_{id}^{itineraries} x_{id} \times distFull_{id} \times fullFuelCons \times fuelCost \quad (7)$$

$$variableCosts(\overline{x}) = emptyFuelCost(\overline{x}) + fullFuelCost(\overline{x}) \quad (8)$$

where $emptyFuelCons$ and $fullFuelCons$ represent the fuel consumption of the type of vessel associated with the itinerary. The values of $distEmpty_{id}$ and $distFull_{id}$ are calculated from the distance matrix of annex 3 of the specification and can be consulted in the *Itineraries* page of the supplemental spreadsheet.

As such we can define the cost function as:

$$cost(n_S, n_L, \overline{x}) = fixedCosts(n_S, n_L) + variableCosts(\overline{x}) \quad (9)$$

And the problem's goal as:

$$\min cost(n_S, n_L, \overline{x}) \qquad , n_S, n_L \in \mathbb{R}_0^+, \overline{x} \in (\mathbb{R}_0^+)^n \quad (10)$$

5.3 Constraints

We identified two types of constraints for the problem: time constraints and import/export constraints. The **time** constraints relate the chosen itineraries with the number of boats, ensuring that time time needed to complete the itineraries is within the time available for the chosen number of boats:

$$\begin{aligned}
&\sum_{id}^{itinerariesS} totalTime_{idS} \times x_{idS} \leq n_S \times (365 - maintenanceDays) \times 24 \leftrightarrow \\
&\sum_{id}^{itinerariesS} totalTime_{idS} \times x_{idS} - n_S \times (365 - maintenanceDays) \times 24 \leq 0
\end{aligned} \tag{11}$$

$$\begin{aligned}
&\sum_{id}^{itinerariesL} totalTime_{idL} \times x_{idL} \leq n_L \times (365 - maintenanceDays) \times 24 \leftrightarrow \\
&\sum_{id}^{itinerariesL} totalTime_{idL} \times x_{idL} - n_L \times (365 - maintenanceDays) \times 24 \leq 0
\end{aligned} \tag{12}$$

Where $maintenanceDays$ is equal to 20 in the current problem, and $totalTime_{idK}$ represents the total time needed to complete itinerary idK. We multiply the right side by 24 in order to convert it to hours. The mathematical formulation of these constraints would be too extensive, so we'll resort to simplified versions.

The **import** constraints need to guarantee that the total amount of a given type of cargo that enters each port is larger than the threshold defined in Annex 3 of the specification. We interpret these values as the minimum amount that a country must import, which means it can import more than the threshold (the downside is that it would have more costs by doing more trips). The amount imported is calculated using the number of trips of each itinerary that brings materials to the port and the tonnage of the type of vessel associated with the itinerary:

$$totalImported(\overline{x}) \geq importThreshold \qquad \text{For each import material of each port} \tag{13}$$

The **export** constraints need to guarantee that the total amount of a given type of cargo that leaves each port is not larger than threshold defined in Annex 3 of the specification. We interpreted the value as the maximum amount of that type of cargo that the port is able to export. The amount exported is calculated using the number of trips of each itinerary that take materials from the port and the tonnage of the type of vessel associated with the itinerary:

$$totalExported(\overline{x}) \leq exportThreshold \qquad \text{For each export material of each port} \tag{14}$$

One thing to note is that the import/export scheme is equivalent to using = for these constraints, however, using $\leq$ and $\geq$ is more meaningful given the assumptions.

5.4 A note about the formulation

According to the real world meaning of the variables, they should be whole numbers (positive integers), however, solving this as an integer problem (using branch-and-bound for example) proved intractable with the solver we used. We considered a meaning for the x_{idK} variables using float value where the fractional part represented a trip without the full cargo, e.g. if $x_{1S} = 2.1$ would mean that 3 ($ceiling(x_{1S})$) trips were made where one of those trips only used 10% of the cargo. However, this assumption would invalidate the time constraints of the obtained solution. Furthermore, simply rounding the values would not satisfy either the time or the import/export constraints.

Note that, this doesn't mean that the obtained solution isn't useful, because the obtained values can still serve Sweetland's government as a good estimate of the number of vessels that need to be bought, given the constraints and the best itineraries to follow. As we are dealing with large monetary values, small differences from the LP solution to the integer programming solutions aren't as relevant.

6 Question 3: Model solution 1

After formulating a linear programming model with the decision variables, constraints and objective function specified in the above section, the team solved the model using the Excel Solver, arriving at the following results:

Decision Variable	Value	Quantity
n_S	45.31861054	
n_L	57.10835059	
x_{1S}	285.7142857	10,000
x_{2S}	285.7142857	10,000
x_{3S}	285.7142857	10,000
x_{8L}	142.8571429	10,000
x_{9L}	428.5714286	30,000
x_{12L}	285.7142857	10,000
Objective function (Cost)	44,678,502.42	—

Table 2: Values of the decision variables obtained using the simplex method solver. For brevity, we omitted the zero valued ones. The right-most column contains the quantity (tons) imported/exported by all the trips of the itinerary.

According to our calculations, the optimal number of small vessels (n_S) is **45.31861**, whereas the number of large vessels (n_L) is **57.10835**.

Regarding the itineraries made by the small vessels, only the routes 1S, 2S and 3S are used, with the rest having the value 0. The number of times these 3 routes are taken is the same for each one, **285.7143**. This means that the usage of small vessels, when it comes to exporting, consists of transporting corn from Doce to Moon. Regarding imports, boats taking itineraries 1S and 3S import copper from Sky and Moon, respectively, and vessels taking path 2S will import iron from Sky. In the end, all boats return to the port of origin, Doce.

In order to calculate the total exported/imported amount of these products, we need to recall the vessel tonnage, that indicates how many tons of product can each boat carry at a time. For small vessels, that value is **35 tons**. Considering there is no shortage of corn to be exported, the amount of corn exported in small vessels, for each of the 3 taken itineraries, is 35 * 285.7143 ≈ 10000 tons, which gives a total of 30000 tons of corn exported in small vessels to Moon, thus fully satisfying their corn needs.

Concerning the imports from small vessels, and considering the calculations made for the exports, we can conclude that small vessels import 20000 tons of copper (from paths 1S and 3S) and 10000 tons of iron (from path 2S) back to Doce. Doce's copper necessities are fully satisfied with these trips.

Let's now consider the large vessels, that have a tonnage of **70 tons**. The itineraries made by large vessels are the paths 8L (**142.85714** times), 9L (**428.5714** times) and 12L (**285.714286** times). Path 8L delivers corn from Doce to Mars, and comes back from Mars to Doce with iron. That is a total of 70 * 142.85714 ≈ 10000 tons, which is the remaining quantity of corn from Doce that needed to be exported. 10000 tons of iron are imported, which, together with the amount imported by small boats taking path 2S, satisfies Doce's iron necessities. We can conclude that, with the itineraries explained thus far, all Doce's export and import necessities have been satisfied.

Paths 9L and 12L start from Bom, exporting wheat to Sky and Mars, respectively, and coming back with iron, also respectively from Sky and Mars, to Bom. Again, in order to calculate the total amount of exported goods, we multiply the tonnage of each boat with the number of times an itinerary is taken. For 9L, that is $70 \times 428.5714 \simeq 30000$ tons of exported wheat and imported iron. For 12L, the value is $70 \times 285.714286 \simeq 20000$ tons. We can thus conclude that both the wheat export and iron import necessities of Bom are satisfied.

Taking into account the calculations shown above, we can conclude that all constraints with respect to exports ands imports have been maximized. Every port exported everything they had available, and imported the full quantity that was needed. With some extra calculations we can also infer that the total available time for small vessels and large vessels has been maximized, therefore utilizing all resources available.

One important aspect of the solution still needs to be considered, and that is the total yearly cost of the fleet. As previously stated, the cost of each vessel can be calculated using its purchase cost, the crew cost, the maintenance cost and the fuel cost. We can perform the needed calculations with the help of the values in Figure 4.

Regarding the small vessels, we know that the full purchase cost of one unit is 1000×10^3, however given that the purchase is done over 25 years, in yearly installments, the yearly purchase cost is $\frac{1}{25}$ of that, equalling $40000. Crew costs for small vessels are $70000, and finally maintenance costs are $100000, 10% of the initial purchase value. For large vessels, the calculations are similar. The yearly purchase cost is $1000 \times 10^3 \times \frac{1}{25} = \$60000$, the crew cost is $75000 and the maintenance cost $150000.

In relation to the fuel costs, having into account the itineraries chosen from each type of vessel, their fuel consumption when the boats are loaded and empty, and the price of a liter of fuel ($0.8), the full fuel consumption of the full fleet is $285714.2857 when the boats are empty, and $18600000 when the boats are full.

With this information, we can conclude that the full cost of the fleet, including fuel, maintenance and crew costs, is **$44,678,502.42**.

All the values that were calculated and are relevant for the linear programming model solution can be seen in the left-hand side of the *Solve* page, in the supplemental Excel file. It includes all the decision variables, objective function values, and constraints.

7 Question 4: Sensitivity analysis - Labor costs

The goal of question 4 is to find the range of labor cost of each vessel that would keep the solution unchanged. To do this we resorted to the sensitivity report (see *Sensitivity Report 1* page), in particular the value of the allowable increase and decrease of the coefficient associated with the number of vessels. This coefficient is related to fixed costs of each type of vessel, as seen in Equation 3 and Equation 4, and is composed of vessel, crew and maintenance costs. Since we are only interested in the crew cost term we can attribute the whole allowable increase and decrease to this portion.

For vessel type S, the value of the fixed cost coefficient is 210000 and the crew costs is 70000. Since we have an allowable increase and decrease for the coefficient variable n_S of 7526250 and 135375 we can change the value of the crew cost by these amounts which leaves us with a acceptable range for the crew cost that would leave the optimal solution unchanged of:

$$\begin{aligned} &crewCost_S \in [70000 - 135375, 70000 + 7526250] \leftrightarrow \\ &crewCost_S \in [0^*, 7596250] \\ &\text{*because crew costs can't be negative} \end{aligned} \tag{15}$$

For vessel type L, the value of the fixed cost coefficient is 285000 and the crew costs is 75000. Since we have an allowable increase and decrease for the coefficient variable n_L of 216600 and

285000 we can change the value of the crew cost by these amounts which leaves us with a acceptable range for the crew cost that would leave the optimal solution unchanged of:

$$
\begin{gathered}
crewCost_L \in [75000 - 285000, 75000 + 216600] \leftrightarrow \\
crewCost_L \in [0^*, 291600] \\
\text{*because crew costs can't be negative}
\end{gathered} \tag{16}
$$

8 Question 5: Sensitivity analysis - Maintenance costs

To find the impact on the annual cost of the fleet if the maintenance of vessels of type 1 (S) changed from the current value of 20 days to 15 days per year, we can use once again the sensitivity report. We will focus on the time constraint defined in Equation 12.

$$
\begin{aligned}
&\textbf{Original constraint} \\
&\sum_{id}^{itinerariesS} totalTime_{idS} \times x_{idS} - n_S \times 345 \times 24 \leq 0 \\
&\textbf{New constraint} \\
&\sum_{id}^{itinerariesS} totalTime_{idS} \times x_{idS} - n_S \times 350 \times 24 \leq 0 \leftrightarrow \\
&\sum_{id}^{itinerariesS} totalTime_{idS} \times x_{idS} - n_S \times (345 + 5) \times 24 \leq 0 \leftrightarrow \\
&\sum_{id}^{itinerariesS} totalTime_{idS} \times x_{idS} - n_S \times 345 \times 24 - n_S \times 5 \times 24 \leq 0 \leftrightarrow \\
&\sum_{id}^{itinerariesS} totalTime_{idS} \times x_{idS} - n_S \times 345 \times 24 \leq n_S \times 5 \times 24
\end{aligned} \tag{17}
$$

The problem is then reduced to a variation of the original constraint with a change in the right hand side. Since the value of n_S for the obtained solution is 45.3186105360019, this represents an **increase** in the right hand side of $45.3186105360019 \times 5 \times 24 = 5438.23$.

Using the shadow price for the constraint (-25.3623), which is present in the *Sensitivity Report 1* page of the spreadsheet, we can estimate that the decrease in maintenance hours will lead to a decrease in the final cost of $25.3623 \times 5438.23 = 137926.02$ dollars.

This result makes sense because having shorter maintenance times on vessels makes them more available to complete trips which in turn may reduce the amount of vessels needed. There also isn't a guarantee that the values of the decision variables, in particular n_S and n_L remain unchanged in the new solution.

9 Question 6: Model reformulation

Most of the constraints and decision variables we're kept from the previous formulation. As the question specifies, the demand of corn and wheat of Fatland and Sealand doubles, so the restrictions with respect to the imported products to these countries were changed, doubling the minimum amount of wheat and corn necessary to satisfy the constraints.

However, if we tried to solve the linear programming model as is, no solution would be available, because due to the increase in import demands from Fatland and Sealand, Sweetland does not have all the needed resources to satisfy them. We need to consider that there may be a part of the import demand that is not satisfied. We solved this by changing the sign of some of our constraints.

Since we know that the amount that needs to be imported is larger than the amount available to export, we know that everything will be exported. As such, we changed the export constraints to:

$$\begin{aligned} totalExported(\overline{x}) = exportThreshold \\ \text{For each export material of each port} \end{aligned} \tag{18}$$

In addition, since some imports can't be fulfilled, particularly the ones from Sealand and Fatland, we changed the import constraints from these countries to use "less-than".

$$\begin{aligned} totalImported(\overline{x}) \leq importThreshold \\ \text{For each import material of each port on Sealand and Fatland} \end{aligned} \tag{19}$$

10 Question 7: Model solution 2

Decision Variable	Value	Quantity
n_S	29.56061652	—
n_L	68.26547044	—
x_{2S}	285.7142857	10,000
x_{3S}	285.7142857	10,000
x_{5L}	142.8571429	10,000
x_{6L}	142.8571429	10,000
x_{9L}	285.7142857	20,000
x_{12L}	428.5714286	30,000
Objective function (Cost)	43,206,245.69	—

Table 3: Values of the decision variables obtained using the simplex method to solve the reformulated problem. For brevity, we omitted the zero valued ones. The right-most column represents the quantity (tons) imported/exported by all the trips of the itinerary.

In Table 3, we present the values of the decision variables for the optimal solution, which we can compare with the values obtained for the first formulation present in Table 2. In this case, the total cost of the ship fleet is **$43,206,245.69**, which is less than the cost obtained for the first formulation. The reason for the lower cost is that despite the quantity of exported products remaining the same, there is greater freedom when choosing the destination of the materials. Because of this, products can be exported to places accessible by cheaper trips instead of forcefully having to fulfill the necessities of a port that requires a more expensive trip to get there. We can see this comparing the export flows between this and the previous solution.

Regarding the exports of corn from Doce, the number of trips of itineraries 2_S and 3_S (**285.7142857** trips, each itinerary satisfying 10,000 tons of corn) remain unchanged when compared to the first solution. Itineraries 1_S and 8_L — which satisfied 10,000 tons each in the first solution — are now unused, and the exports of corn satisfied by those itineraries are now satisfied by itineraries 5_L and 6_L, each travelled **142.8571429** times and satisfying 10,000 tons.

We can notice that all the trips following path 1_S were replaced with trips using path 5_L. Both of these trips are used to export corn from Doce and import copper from Sky, however 1_S exports to Moon, and 5_L exports to Mars. Given that Moon's corn import needs have doubled from 30k to 60k, and Mars' from 10k to 20k, it becomes more advantageous to choose path 5_L to export the corn (since we are obligated to export all of it), due to the use of large vessels instead of small ones. Even though the distance from Doce to Mars (5500) is bigger than the distance from Doce to Moon (5000), the change from 1_S to 5_L is still favorable as a consequence of the vessel type that is used. The difference can be seen in the Excel file, on the tab *Itineraries*, if we look at the *FuelCostPerTon* column: for 1_S, the value is **$13.05142857**, and for 5_L it is only **$5.942857143**.

We can also see that the 10,000 tons that were exported using itinerary 8_L are now exported using itinerary 6_L. Since 8_L imports iron from Mars, but all of the available iron from Mars is already imported using 12_L, the itinerary is replaced by itinerary 6_L, which also exports corn to Mars, but imports iron from Sky. 8_L is replaced by a more expensive itinerary, however this is a trade-off since having 12_L importing all iron from Mars as described below is more advantageous.

The itineraries used to export wheat from Bom remain the same (x_{9L} and x_{12L}, which import iron from Sky and Mars, respectively), with changes in the number of trips of each itinerary. The number of trips of x_{9L} is **285.7142857** instead of **428.5714286**, exporting 20,000 tons of wheat instead of 30,000. The number of trips of x_{12L} is now **428.5714286** instead of **285.7142857**, exporting 30,000 tons instead of 20,000. The reason for this change is that while in the first formulation Mars only requires 20,000 wheat (which it receives with itinerary x_{12L}), it requires 40,000 wheat in this formulation. Since trip x_{12L} (which exports wheat to Mars) is shorter (and cheaper) than trip x_{9L} (which exports wheat to Sky), it is better to export more wheat to Mars, even though the maximum is 30,000 since it is the amount of iron that Mars has available.

11 Question 8: Sensitivity analysis - Importation changes

This question wants to know what would be the change in cost if corn imports from Mars increased to 25000 tons.

This question can be answered using the sensitivity report for the updated model (see *Sensitivity Report 2* page of the spreadsheet). We'll focus on row *46* which pertains the Mars corn import constraint. As we can see, increasing the corn imports to 25,000 equates to an increase of 5000 tons. Since this value is within the allowable increase, we can use the shadow price (-142.917529330572) to determine the change in cost caused by this increase: $-142.917529330572 \times 5000 = -714587$\$.

This means that the cost will decrease as a result of Mars importing more corn, which might seem counter-intuitive at first, but is explained by the fact that in the current situation, since it's impossible to satisfy all import constraints, the goal is to find the itinerary configuration that minimizes the cost, while trying to deliver the maximum amount of corn possible. By increasing the amount of corn imported by Mars we are allowing the choosing of more itineraries that satisfy this import constraint, reducing the amount satisfied for other corn constraints, such as Moon. This is also corroborated by the fact that the itineraries used to satisfy the Mars constraints use large vessels, which are more fuel efficient than small vessels, and can't be used in travels to Moon. The fact that we were able to get a decrease in cost despite the contradictory nature of increasing the imported amount serves as a testament to the usefulness of sensitivity analysis.

12 Conclusion

In general, we can conclude we achieved all the desired goals in this case study, applying various concepts of operational research to solve the different questions that were presented. Regarding the initial problem, we were able to interpret it and formulate an initial linear programming model, composed of decision variables, an objective function, and constraints, that correspond to the actual restrictions of the situation. This linear programming model was then solved, using the Excel Solver, in order to generate the optimal solution for the number of vessels of each type that should be bought, and other metrics like the total cost of the fleet.

In order to get real-life, meaningful results for the number of vessels, we would need to perform integer programming, so as to avoid getting solutions that are not integers. However, solving the problem using the Excel Solver turned out to be unfeasible, considering it would either take a large amount of time to solve, or it would not even find a feasible result. The group also attempted to estimate the integer results based on the results got using linear programming, but that also proved to not be possible, since the estimated solutions would also not be feasible. Nevertheless, given that the focus of this case study was to explore and use linear programming, we focused on that goal and used the solutions generated from it, considering that the non-integer results already give some insight to Sweetland's government.

Some other operational research and linear programming concepts were used, like sensitivity reports, allowable ranges, and shadow prices, that allow us to arrive to insights regarding the sensitivity of the solution, such as how would the result change if the maintenance time of some vessels was lower.

All in all, we believe the case study was a success, as the group leaves this project with a better knowledge in the field of operational research and linear programming.