Reducing the Price Risks of Electric Power Plant Operation using Dynamic Delta Hedging

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Executive Summary

The profitability of natural gas fueled electric power plants is highly sensitive to the price of natural gas and the price that consumers are willing to pay for generated electricity. Both of these prices are extremely volatile, fluctuating with consumer habits, weather, and geopolitical events. This large uncertainty in future profitability is problematic for the producers that own and operate the plants. This paper shows that by buying and selling forward contracts in the electricity and natural gas markets, the economic value of a portfolio of power plants can be preserved, eliminating the risk of extreme loss due to commodity price fluctuations.

1. Introduction

This paper analyzes the price risks of operating natural gas-fired electric power plants in an environment of volatile electricity and natural gas prices and how these risks can be reduced through dynamic delta hedging with forward contracts. A financial model of power plant profits is created and simulated for a hypothetical portfolio of power plants under a variety of hedging strategies and assumptions. Based on the simulation results, a hedging strategy is recommended that strikes a balance between the reduction in risks and reduction in rewards due to transaction costs. Finally, a business opportunity for facilitating the implementation of such a hedging strategy is outlined.

2. Methodology: Monte-Carlo Simulation

2.1. Power Plants as Real Options

Natural-gas fired electric power plants burn natural gas as fuel and produce electricity which is ultimately sold and delivered to consumers. Profitability for a single plant for a given month, denoted by $V(T)$ (measured in dollars) can be approximated as the net profit of generating one unit of electricity (mmbtu) times the capacity of a plant to generate electricity per unit time (mmbtu/hour), denoted C, times the number of hours in the month the electricity is to be delivered, R. The cost of generating a single unit of electricity is the difference between the spot electricity price on the first day of the delivery month $L(T)$ (dollars per mmbtu) and the cost of natural gas to produce one mmbtu. The quantity of natural gas needed to produce one unit of electricity per unit time is the product of the physical heat rate of the plant, denoted by H (mmbtu/MWh) and the spot price of natural gas on the first day of the delivery month, $G(T)$ (dollars per mmbtu). More succinctly this is represented by equation 1.

$$V(T) = CR\{L(T) - HG(T)\} \tag{1}$$

At delivery if the cost to burn the natural gas exceeds the cost of electricity, a plant operator will choose not to operate the plant rather than lose money, in which case the profit of the plant will be zero as represented in equation 2.

$$V(T) = CRmax\{L(T) - HG(T), 0\} \tag{2}$$

Equation 2 is formally the same as the value of a financial spread option contract at expiration and we can leverage the same techniques used to value and hedge a portfolio of financial options to value and hedge a portfolio of power plants, an approach called real options valuation (ROV), pioneered by Brennan and Schwartz (1985).

To investigate the effects of hedging, we apply this model by simulating the future value of a hypothetical portfolio of thirty power plants over a time period of two hundred and eight days under a variety of scenario parameters.

A consequence of modelling power plants as real options is that the Black Scholes Merton pricing equations (Black and Scholes (1973)) may be used to analytically determine the price of an unhedged portfolio of plants. This is used to verify the accuracy of the simulation.

2.2. Modelling and Simulating Electricity and Natural Gas Prices

To use equation 2 to calculate or simulate the value of operating a power plant over a future time period, it is necessary to have a predictive model of natural gas and electricity prices. Future prices are uncertain due to unforeseen market conditions and may be modelled as a stochastic process. Maribu et al. (2007) present a review of several different models. The simulation implements a geometric Brownian motion process with a constant correlation coefficient as shown in equation 3, where ρ is the correlation coefficient and σ_L and σ_G, represent the electricity and natural gas annualized volatilities, respectively. Deng (199) showed that over a long time period geometric Brownian motion does not account for the mean-reverting tendency and sporadic jump behavior of commodity prices, so our results are more applicable to time horizons less than one year.

$$\begin{aligned} dL(t) &= \sigma_L L(t) dW_L(t) \\ dG(t) &= \sigma_G G(t) dW_G(t) \\ dW_L(t) dW_G(t) &= \rho dt \end{aligned} \tag{3}$$

The time evolution of these asset prices are simulated via a Monte Carlo method. The time period of 208 days is divided into 209 time slices (one additional slice for the initial day). Electricity and natural gas prices on the first day are set to their initial forward prices. On each successive day, the electricity and natural gas prices increase or decrease by random amounts following simulated correlated geometric Brownian motion represented by equation 4, where $Z1$ and $Z2$ are normally distributed pseudo-random numbers generated using the MATLAB implementation of the Ziggurat algorithm described in Marsaglia and Tsang (2007).

$$\begin{aligned} L(t+1) &= L(t) \exp\{-\frac{1}{2}\sigma_L^2 \delta t + \sigma_L \sqrt{\delta t} \epsilon_L\} \\ G(t+1) &= G(t) \exp\{-\frac{1}{2}\sigma_G^2 \delta t + \sigma_G \sqrt{\delta t} \epsilon_G\} \\ \epsilon_L &\sim Z_1 \\ \epsilon_G &\sim \rho \epsilon_L + \sqrt{1-\rho^2} Z_2 \end{aligned} \tag{4}$$

Using 4, a single set of one hundred thousand random price paths is generated and reused in a series of simulations. For each plant in the portfolio, equation 2 is evaluated using the terminal asset price to achieve the simulated profits on the future delivery date. The profits are averaged across all paths to obtain a simulated distribution of profits. The profit of the portfolio is then calculated as the sum of the profits of each constituent plant.

2.3. Hedging

Similar to hedging financial options, the power plant portfolio may be hedged by maintaining a portfolio of assets that are negatively correlated with the value of the spot price of the underlying asset. Practically, this is best achieved by hedging with electricity and natural gas forward contracts. A power plant is naturally long electricity and short natural gas, so the position is hedged by short electricity and long natural gas contracts.

Forward contracts have the advantage that they may be purchased in the over-the-counter (OTC) market, whereas it is more difficult to purchase physical electricity and natural gas commodities in the spot market and store them.

Hedging with forwards also simplifies the simulation. It avoids the murky task of accounting for the storage costs and convenience yield of the physical commodities. Additionally, since forward contracts are settled on the delivery date, it requires no cost to enter into a forward position and it is not necessary to discount the value of the cash flows at each point in time that a hedge is bought or sold.

The ideal hedge position for a plant is one that directly offsets changes in the valuation of the plant due to changes in underlying prices. Approximating the the sensitivity of the plant valuation for small changes in asset price as a linear relationship (the first term in the Taylor expansion) yields the expression for an ideal hedge in equation 5.

$$\begin{aligned} \{\text{Notional Value of Ideal Electricity Hedge}\} &= -\frac{\partial f}{\partial L} L(t) \\ \{\text{Notional Value of Ideal Natural Gas Hedge}\} &= -\frac{\partial f}{\partial G} G(t) \end{aligned} \tag{5}$$

As the value of the plant changes it is necessary to readjust the hedge based on equation 5. Theoretically, if it were possible to trade the hedge position continuously and there were no jumps in the process, this would offset the plant value perfectly.

By extension, delta hedging can be applied to a portfolio of plants where the notional value of the plant in equation 5 is replaced by the notional value of a portfolio of plants. This is easier to implement. It is not possible to maintain a perfect hedge and an imbalance in the hedge of one plant can offset an opposing imbalance in another plant making it more efficient to hedge a portfolio of plants.

To simulate hedging a plant portfolio, it is necessary to calculate the derivatives in equation 5 (deltas) at each point in time that the hedge is adjusted. Deltas are calculated analytically based on the Black Scholes Merton pricing equations as shown in equation 6 where f is the value of the power plant portfolio and Φ is the cumulative normal distribution function.

$$
\begin{aligned}
\frac{\partial f}{\partial L} &= CR\{\Phi(d1)\} \\
\frac{\partial f}{\partial G} &= CRH\{-\Phi(d2)\} \\
d1 &= \frac{log(\frac{L_0}{HG_0}) + \frac{\sigma^2 T}{2}}{\sigma\sqrt{T}} \\
d2 &= d1 - \sigma\sqrt{T} \\
\sigma &= \sqrt{\sigma_L^2 + \sigma_G^2 - 2\rho\sigma_L\sigma_G}
\end{aligned}
\tag{6}
$$

In the hedging simulations the hedging frequency is always represented as a multiple of days (once a day, once every two days, etc.). This ensures alignment with the generated asset price paths.

To better represent the real-world constraints of hedging, an estimate of transaction costs are calculated for each change in hedge position as a percentage of the notional value of the contracts bought or sold. Additionally, some of the simulations restrict the hedged position to an integral number of forward contracts.

2.4. Outages

Plant outages are modelled as a probability of failure g_i characterisic of each plant. This yields a value of a plant at the end of the time horizon given by equation 8, where R is a uniformly distributed pseudo-random number.

$$V(T) = \begin{cases} CRmax\{L(T) - HG(T), 0\}, & \text{if } R \geq g \\ 0, & \text{otherwise} \end{cases} \tag{7}$$

For the scenarios that account for potential outages, the above valuation is applied for each simulated pair of asset prices and for each plant in the portfolio.

3. Numerical Results and Discussion

Refer to table 4 for a summary of the parameters used to configure each scenario.

3.1. Scenario A: An unhedged portfolio

To validate the correctness of the asset price path generation code, the unhedged value of each plant was simulated and compared to the theoretical value given by the Black Scholes pricing equation. The results are shown in tables 1 and 2. The simulated values are all slightly larger than the theoretical values, most less than 1% with the largest outlier at 2.6% of the theoretical. The percentage accuracy varies directly with how deep in-the-money the plant is.

Some experiments using antithetic variables were done to improve the accuracy without increasing the number of simulated paths, but these did not provide significant improvements and were abandoned.

The unhedged portfolio profitability is shown in table 3 with a mean profit of \$3.7 million and a large standard deviation of \$3 million. This will be used as the baseline for comparison with the other scenarios.

3.2. Scenario B: Hedging only electricity exposure

Scenario B is disastrous. Only the electricity delta of the portfolio is hedged resulting in a negligible gain over the unhedged scenario while introducing a much larger standard deviation of \$5.9 million.

3.3. Scenario C: Hedging both electricity and natural gas

In scenario C both electricity and natural gas are hedged daily assuming no transaction costs or outages. In this case, hedging works almost perfectly. The mean profit is approximately the same as scenario A but the standard deviation is reduced by two orders of magnitude to \$38,878.

3.4. Scenario D1: Hedging with transaction costs and outages

Scenario D1 adds transaction costs for buying and selling forwards as well as the probability for outage discussed in the Methodology section. The mean profit dropped to \$2.9 million, \$629 thousand of which are due to transaction costs and the most of the remaining losses due to outages.

This scenario also introduces the constraint that only an integral number of forward contracts can be bought resulting in some differences from the ideal hedge which proved to be small.

3.5. Scenario D2: An unhedged portfolio with transaction costs and outages

Scenario D2 is the same as scenario A with the addition of outages. Compared to A the mean profit dropped by \$200 thousand to \$3.5 million, all attributable to outages.

3.6. Scenario D3: Adjusting delta for outages

To compensate for outages the simulation uses an outage-adjusted delta. The delta of each plant is scaled down by the probability that it will not experience an outage. The effective portfolio delta is then the computed as the sum of the individual plant deltas and hedging is applied against the outage-adjusted portfolio delta.

$$\Delta^{OutageAdjusted} = \sum_i (1 - g_i)\Delta_i \tag{8}$$

This results in virtually no increase in mean profit (a increase of less than 1% compared to the same scenario with non-adjusted deltas), but provides a significant reduction in standard deviation of 15% from \$318 thousand to \$273 thousand. This is not surprising when considered in light of scenario B. Scenario B showed that an imbalanced hedge does not result in a significant change in profitability, but does have a higher standard deviation. In D2, the non-adjusted hedge was effectively imbalanced when outages are included. Scenario D3, including the outage-adjusted deltas removes the imbalances, reducing the standard deviation.

3.7. Scenario X1-X17: Varying the hedge frequency

In scenario D1, although the standard deviation is significantly reduced compared to the unhedged scenario A, the transaction costs from hedging were \$629 thousand dollars, 22% of the total profits. The reduction in volatility was not cheap.

This series of scenarios explore reducing the transaction by hedging less frequently (one every two days, once every three days, etc.) As discussed in the Methodology section, hedging less frequently will result in a larger standard deviation, but how much? And can we find some compromise between the expensive daily hedging of scenario D1 and the extreme volatility of scenario A?

The results are shown in table 5. Note that scenario X1 is identical to scenario D1. The results are also graphed in 1 where the hedging frequency is allowed to vary parametrically to trace all possible combinations of profits and standard deviation (the risk-reward spectrum).

4. Recommendation for a Hedging Strategy

The risk-reward spectrum 1 demonstrates that by decreasing the hedging frequency to span multiple days transaction costs can be significantly reduced for only a marginal increase in standard deviation, but it raises an interesting general question as to how to how a rational investor, given a continuum of risk-reward possibilities, should choose the best one.

The answer could depend on a number of factors - what other investments opportunities are available, at what rate money can be borrowed, the ability and cost of maintaining operational capital to cover potential losses, and the operational costs of having uncertain future profits.

That said, a reasonable recommendation is to choose the strategy that maximizes the ratio of risk to reward, equivalent to the Sharpe Ratio (Sharpe (1994)). This point occurs when the hedge is rebalanced every 5 days (scenario X4).

In summary, given the specified plant portfolio, including transactions costs, non-fractional hedge positions, and outages, the recommended strategy is:

- Hedge both electricity and natural gas using the outage-adjusted deltas defined in 8.
- Start the hedge on the first day and rebalance the hedge every 5 days until delivery.
- The expected distribution of profits should correspond to scenario X4 : a mean profit of $3.2 million and standard deviation of $275 thousand, pay-

ing $322 thousand in transactions costs as the ”price” of reduced volatility.

5. Proposal of a Business Opportunity: A Venue for Exchange-Traded Electricity-Natural Gas Spreads

The hedging strategy described in the paper is complex and costly, requiring constant periodic trading activity and monitoring the risk of the hedged forward positions. For an electric company to implement this strategy they would require a full-time commodities trading team to manage their portfolio. The electric company would be investing outside of their core competency of producing electricity.

An alternative possibility is that an investment bank could offer services to sell a electricity-natural gas spreads to electric companies over-the-counter (OTC). This investment bank takes the responsibility of hedging the spread contract with its own holdings, leveraging economies of scale and experience in trade execution and risk management to manage the portfolio. While this is a valuable service it suffers from lack of price transparency to ensure competitive prices for the purchaser of the contracts and can expose the client to credit risk if the writer of the contract defaults.

What is proposed instead is to create a public electronic trading venue (an electronic exchange) that provides standardized electricity-natural gas spread contracts as well as standardized electricity and natural gas futures. This would provide price transparency and protection from counter-party credit risk (since the exchange assumes ensures that all trades are cleared even if participants default). There is a strong precedent for exchange-traded energy derivatives. The Chicago Mercantile Exchange (CME) offers crack spread contracts (CME (2015)) which are generally purchased by oil refiners to hedge the spread between crude and refined oil, essentially the same real option energy model described in this paper. Why not do the same thing for electricity?

Despite the current lack of such an infrastructure, it is not a new idea. The history of the electricity derivatives market is detailed in Hale et al. (2002). As early as 1988 the U.S. Power Marketing Association predicted the electricity industry would eventually support more than a trillion dollars of trading in standardized electricity futures contracts. In the beginning of 2000, several public commodities exchanges started offering electricity future contracts, but by the end of the year the market had collapsed. Enron traded electricity natural gas swaps until their ultimately imploded due to scandalous practices (Henriques (2001)).

It is widely believed that the collapse of the electricity derivatives market is due to the market inefficiencies of the spot market (Hale et al. (2002)), specifically balkanization of the market in the U.S. based on state-specific laws and regulations and monopolisitc practices carried forward from the era of public utilities. Efforts to deregulate the market backfired resulting in the near collapse of the energy market in California.

A well-functioning spot market is the main challenge to provide electricity derivatives including the electricity-natural gas spreads proposed in this paper. The Federal Energy Regulatory Commission (FERC) is taking active steps to reform the industry. Once this hurdle is overcome, the opportunities for a publicly trade electricity derivatives market are vast.

6. Conclusion

Possibilities for further lines of investigation include:

- Re-running the simulations with a more sophisticated model for electricity and natural gas such as Maribu et al. (2007).
- Adding stochastic volatility to the model
- Exploring other real options which follow a similar pattern

Plant Number	Theoretical Mean Profit($)	Simulated Mean Profit($)
1	158,440	157,650
2	446,550	445,500
3	360,970	359,980
4	346,370	345,380
5	7,688	7,527
6	291,660	290,700
7	197,110	196,230
8	3,570	3,486
9	93,821	93,300
10	92,112	91,596
11	182,180	181,340
12	138,410	137,690
13	4,243	4,136
14	308,770	307,810
15	48,830	48,409

Table 1: Unhedged Analytical versus Theoretical Profits for Power Plants 1 - 15

A Tables and Figures

Plant Number	Theoretical Mean Profit($)	Simulated Mean Profit($)
16	199,790	199,070
17	235,880	235,160
18	624	608
19	11,239	11,050
20	675	658
21	21,038	20,784
22	149,730	149,070
23	2,167	2,125
24	72,974	72,575
25	223,270	222,750
26	97,293	96,859
27	717	702
28	690	676
29	548	534
30	15,139	14,962
Total (for all 30 plants)	3,712,497	3,698,318

Table 2: Unhedged Analytical versus Theoretical Profits for Power Plants 16 - 30

Simulation	Mean Profit($)	Median Profit($)	Std Dev Profit($)	Transaction Costs($)
A	3,698,322	2,985,769	3,016,885	0
B	3,731,702	4,542,200	5,859,767	0
C	3,712,333	3,711,275	38,878	0
D1	2,899,236	3,007,538	318,846	629,356
D2	3,514,048	2,818,260	2,887,585	0
D3	2,928,098	2,965,500	273,818	601,387

Table 3: Simulated Profits for Daily Hedging Scenarios

Simulation	Hedge Electricity	Hedge Nat. Gas	Fractional Positions	Transaction Costs	Plant Outages	Adjusted Delta
A			✓			
B	✓		✓			
C	✓	✓	✓			
D1	✓	✓		✓	✓	
D2				✓	✓	
D3	✓	✓		✓	✓	✓

Table 4: Simulation Parameters for Daily Hedging Scenarios

Simulation	Hedge Freq (Days)	Profit Mean($)	Profit Median($)	Profit StdDev($)	Profit/ StdDev	Trans Costs($)
X1	1	2,926,984	2,965,099	275,491	10.62	601,387
X2	2	3,084,114	3,117,314	271,497	11.36	443,346
X3	3	3,144,221	3,178,046	273,590	11.49	383,305
X4	5	3,205,958	3,237,674	275,026	11.66	322,603
X5	6	3,223,814	3,254,998	277,457	11.62	304,354
X6	7	3,237,825	3,268,418	278,877	11.61	290,074
X7	8	3,243,556	3,273,642	281,360	11.53	283,339
X8	9	3,253,194	3,282,839	285,275	11.40	273,339
X9	10	3,267,949	3,294,811	285,157	11.46	260,676
X10	14	3,291,475	3,314,465	292,987	11.23	236,980
X11	21	3,316,122	3,331,795	307,160	10.80	212,729
X12	28	3,324,597	3,333,365	318,867	10.43	203,616
X13	35	3,340,625	3,341,615	335,041	9.97	187,279
X14	42	3,349,700	3,343,444	348,041	9.62	179,357
X15	140	3,379,113	3,303,102	466,508	7.24	148,441
X16	280	3,404,081	3,257,715	603,398	5.64	122,797
X17	365	3,403,896	3,256,252	603,144	5.64	122,797

Table 5: Simulated Profits for Multi-day Hedging Scenarios (All scenarios hedge both electricity and natural gas, include transactions costs, simulated outages, enforce non-fractional positions, and use outage-adjusted deltas. The only variable is the hedging frequency.)

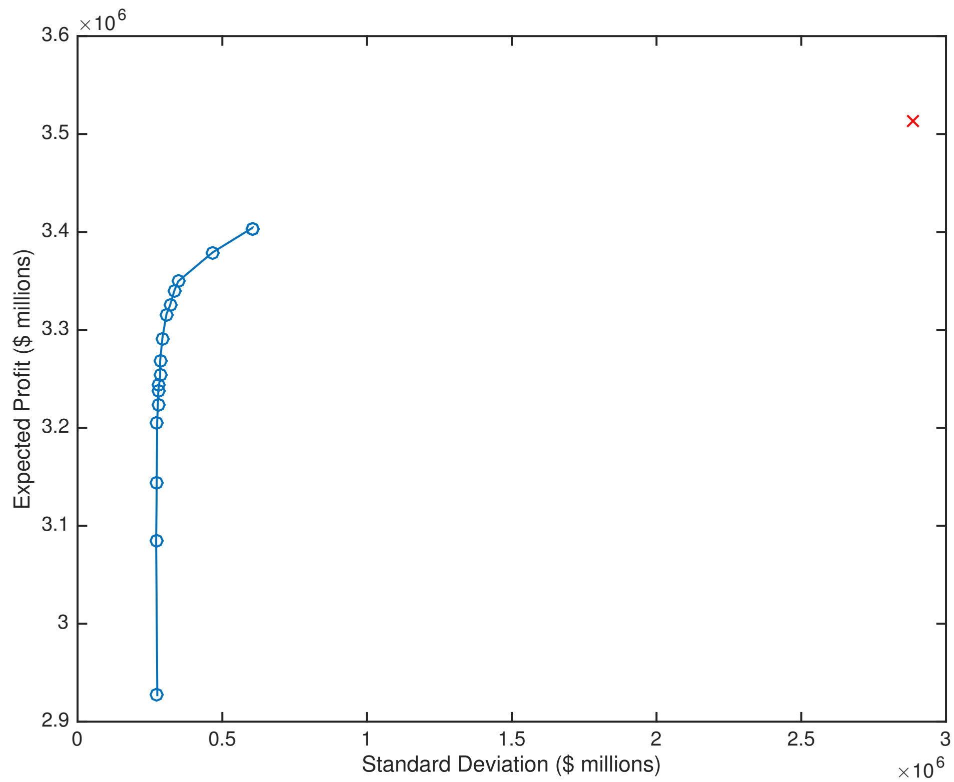

Figure 1: Profit versus Volatility as Hedge Frequency Varies. (The red x in the upper right is for a completely unhedged portfolio, which is provided for comparison. The blue curve on the left shows how expected profit and standard deviation change as the hedging frequency varies parametrically, corresponding to the data in table 5).

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