Heterogeneous Climate Game: A Dynamic Model for Renewables Adoption

Our model treats renewable energy adoption as a finite horizon, multi-round threshold public goods game with complete observability between players. Each round the players (nations) simultaneously decide to adopt renewable energy or continue to rely on fossil fuels. This binary strategic choice is shaped by heterogeneous cost and benefit functions that are influenced by feedback mechanisms from the previous rounds, the game unfolds over N rounds, this is known to the players. Player choices are not equal in influence, some players are more impactful.

Design Justification

Each modeling choice reflects a key aspect of real world climate dynamics.

1. A finite-horizon is chosen to capture the reality of impending climate deadlines, while still allowing countries strategies to adjust over time.

2. Complete-observability mirrors real world transparency in climate policy, where commitments and agreements are closely monitored.

3. The use of a threshold public goods framework represents the nonlinear outcomes of climate outcomes, where benefits/penalties are only gained/avoided once global collaboration crosses some key critical mass.

4. Limiting strategies to a binary choice serves to simplify the problem whilst capturing the policy dichotomy nations face. The simultaneous nature of the game reflects that nations have to make decisions without knowing the immediate choices of other nations

5. Heterogeneous costs and benefits, reflect real economic asymmetries, e.g Norway can adopt renewables more cheaply than Afghanistan, yet face less of an immediate climate threat.

6. The dynamic nature of these costs and benefits across N rounds, allows us to model strategic adaptation, changing policy influences and tipping points overtime.

7. Player choices are not given an equal weighting towards the game, instead each player is assigned a weighted contribution metric based on economic size and energy usage, quantifying each player's impact on global transition. These scores are then normalised by the sum of the weights such that each player's impact can be measured as a % of global influence.

Contents

Game Structure

Players and Strategies

Each country can choose between two strategies:

1. Adopt Green Energy (G) – Pay an upfront cost Ci to transition to renewables.
2. Free-Ride (F) – Continue using fossil fuels and benefit if enough others invest

Formally, $Gi \in \{0,1\}$, *where 1 = adopt green energy, 0 = free-ride*

Weighted Threshold Contributions

A successful Green transition depends on the total adoption effort (W), not just the number of adopters. A country's contribution is weighted by economic size and energy impact. (e.g the USA's decision has a larger impact than Malawi's on global adoption and the climate)

$$W = \Sigma (w_i)$$

$$w_i = S_i * G_i$$

Gi is a binary decision: Gi = 1 if the country adopts, Gi = 0 if it free-rides.
Si represents a country's influence on the transition, determined by its GDP and fossil fuel consumption.
A country's impact factor Si is given by:

$$S_i = \alpha * GDP_i + \beta * Energy\ Usage_i$$

per country influence scores can be normalised to sum to 1 for easier interpretability through $\boldsymbol{s_i} / \Sigma S_i$ *(each* s_i *now represents a countries % impact on the global push towards the Threshold)*

where alpha and beta control the relative importance of economic power versus fossil fuel dependence in determining a country's role in the transition.

Threshold Scenarios

T = the minimum effort for a successful green transition
Mathematically, $T = \theta * \Sigma S_i$ where $\theta \in (0,1)$
Intuitively T is the % of global influence needed for a successful green transition.

If total adopters W ≥ T (Threshold Met, Green Transition Succeeds):
Adopters (G) receive full benefits (potential economic gains + climate damage avoidance) but incur adoption costs. Free-riders (F) gain economic benefits without costs, as they also avoid climate damages.

If total adopters W < T (Threshold Not Met, Partial Transition):
Adopters (G) receive some potential economic benefits but face climate damages, making payoffs lower. Free-riders (F) gain no potential adoption benefits and bear the full climate disaster cost.

Defining the Costs and Benefits of Adoption

Defining the Cost of Adoption

A country's cost of transitioning to green energy depends on two key factors:

1. Economic Strength – Wealthier countries have more resources to fund the transition.
2. Energy Dependence – Countries reliant on fossil fuels face higher transition costs.

A simplified initial cost function is:

$C\_i\_0 = \alpha * GDP\_i * (1 + (Fossil\_share\_i / 100))$

GDP_i: represents a country's economic size and ability to invest in renewables.

α, where $\alpha \in (0,1)$

cost per unit of GDP is lower for nations with more established renewable industries

$Fossil\_share\_i$ where $Fossil\_share\_i \in (0,100)$

represents % fossil fuel dependency, higher costs for fossil fuel dependant nations

Evolving Cost of Adoption

In a dynamic setting, the cost of adoption evolves over time, as technological improvements and economies of scale lower costs. As more countries adopt, transition becomes cheaper.

Mathematically cost at time t is,

$\boldsymbol{C\_i\_t = C\_i\_0 * (1 - Z * W\_t)}$

C_i_t

is the cost for country i at time t. Where $C\_i\_t > 0$ at all t ensuring no negative cost scenarios, this condition is satisfied when $Z*W\_t < 1$

C_i_0

is the initial cost for country i, defined previously

W_t

is the total adoption effort at time t, defined previously

Z where $Z \in (0,1)$

is a scaling parameter that determines how much adoption reduces costs, the Z term includes factors like economies of scale, technology improvements, etc anything that would make switching cheaper

Defining the Benefits of Adoption

A country's benefit from adopting green energy depends on two key factors:

1. Economic Gains – Job creation, reduced energy costs, and new industries.
2. Climate Stability- If W > T then there is a realised benefit of avoiding the climate disaster.

We define the initial benefit function as:
*B_i = EconomicGains_i + T_d * (ClimatePayoff_i)*

EconomicGains_i *domestic economic benefits*

Climate_Payoff_i *The real benefit a country gets from avoiding climate disaster*

T_d $\in \{0,1\}$ *A threshold dummy variable, 1 if global adoption meets or passes the threshold, else 0*

Evolving Benefits and Trade Penalties

As more countries adopt, free-riders face increasing economic penalties from trade restrictions and carbon tariffs or conversely they may receive support to aid in adoption. These penalties/subsidies evolve based on global adoption, hence the benefit function will too.

The benefit function at a given time t is mathematically,

B_i_t = B_i_0 + T_t(W_t)
B_i_t : benefit function at time t | *B_i_0: initial benefit function, defined previously*

T_t(W_t)
represents increasing subsidies/tariffs for free-friders as adoption grows, both are modeled in the benefits section for simplicity, avoiding a tariff is treated as a pro of adopting, as is receiving a subsidy for adopting

*T_t = T_0 + γ * W_t*
T_0 shows any initial pressure free-riders may face

γ, γ∈(0,1)
parameter tied to how aggressively adopters try to push free-riders to adopt via subsidies/tariffs/taxes

Intuitively:
If W_t is low (few adopters), penalties are minimal, making free-riding attractive.

If W_t is moderate (some major economies adopt it), trade restrictions begin increasing, reducing free-rider incentives.

If W_t is high (most major economies adopt), free-riders face severe economic disadvantages, making adoption a better option)

Multi-Round Decision Process

At the start of each round $t \in \{1,...,N\}$, countries observe global adoption trends, reassess costs and benefits, and select their strategy $Gi \in \{0,1\}$. Global weighted contribution W_t is then updated, affecting the payoff structure for subsequent rounds.

<u>Each round consists of the following steps:</u>

1. Cost and Benefit Updates

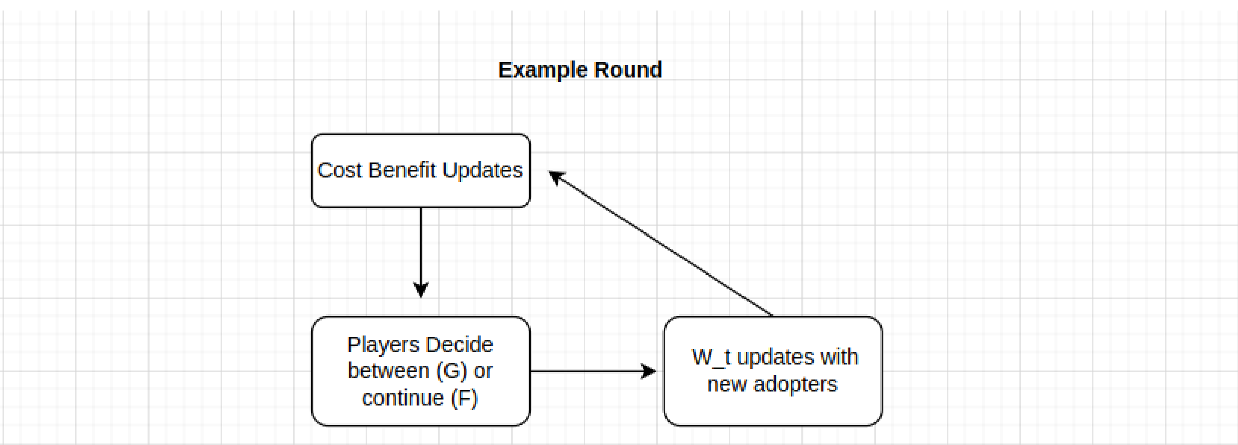
1. When more players Adopt (G), the cost decreases for all other players.
2. When more players Adopt (G), political pressure on Free-Riders (F) increases. Increasing the benefit of switching and joining the adopters.

2. Adoption Decisions

1. Based on updated costs, policy changes, and global adoption levels, each country decides whether to adopt green energy or continue free-riding for another round.
2. Countries that delay adoption may benefit from cheaper adoption in the future, but they also face increasing pressure to adopt.

3. W_t updates

1. If there are new adopters W_t updates. Consequently players cost/benefits do for the next round



By round N, the game reaches a final equilibrium, where either:

1. **W >= T** : A successful transition to Green Energy occurs, and climate worst case scenarios are avoided.
2. **W < T** :A subset of countries continues to resist adoption, leading to long-term environmental consequences and corresponding economic effects.

Payoff Functions and Strategic Interactions

Payoff Functions

Each country can choose between the two strategies outlined previously (G) or (F)

Adopt Green Energy (G)

If total adoption is at or above the threshold T *(T_d = 1 in our benefit function)*:

Payoff = Full Benefit - Cost ("Full Benefit" is the potential economic and climate benefit)

If total adoption is below the threshold T *(T_d = 0 in our benefit function)*:

Payoff = Reduced Benefit - Cost ("Reduced Benefit" is just any potential economic benefit):

Free-Ride (F)

If total adoption is at or above the threshold T (*T_d = 1 in our benefit function)*:

Payoff = Climate Benefit (avoiding climate damage)

If total adoption is below the threshold T *(T_d = 0 in our benefit function):*

Payoff = 0 (no potential economic benefits, no climate benefit)

Adopters can gain potential economic benefits from being early adopters of green energy for instance Denmark's early investment into wind in the 1980s-1990s made Denmark a world leader in turbine manufacturing. But they also incur transition costs. Free-riders avoid costs but miss out on potential economic gain, however they will still benefit from avoiding climate disaster if W>T

Strategic Form Representation

Given the increasing complexity of payoff matrices as N grows this game is better represented in Strategic Form. In this representation countries aren't responding to individual players but are basing decisions on threshold functions, which relate to expected payoffs from each strategy at a given round.

A country switches from (G) to (F) when, U(G) > U(F)

1. Substituting in payoffs : $Bi + Td * Bi - Ci >= Td*Bi$

2. Best Response Condition : $Bi >= Ci$

3. Best Response function for a given round **:** $G\_i =\{\ 1,\ \text{if } B\_i \geq C\_i \quad 0,\ \text{otherwise}\}$

The best response function depends on the values of B and C, which update at the start of each round based on the total number of adopters from the previous round. As more countries adopt, costs decrease and trade pressures change, dynamically influencing payoffs.

Equilibrium Analysis

Case 1: Persistent Free-Riding Majority Equilibria

In earlier rounds, free-riding can dominate if adoption costs are too high, when costs are too high W_t is low, and the rate of which costs of adoption fall is lower, if it's below a certain level, costs don't fall fast enough to incentivise more players to adopt before the game ends. Additionally there isn't significant political and trade pressure to incentivise adoption. These combined create a free-riding equilibrium where the vast majority of players free-ride.

The best response condition requires $B_i \geq C_i$ for adoption to occur, but B_t and C_t remain near constant due to low initial W_t.

$C_{t+1} = C_t * (1 - Z*W_t) \approx C_t$ for all rounds following initial adoption:

$B_{t+1} = B_t + T_t(W_t) \approx B_t$ *for all rounds following initial adoption:*

Hence the initial strategies are fixed giving us our first Nash Equilibrium where no player has an incentive to deviate. A player initially free riding won't deviate as the inequality $C_i > B_i$ holds for all rounds, making F strictly dominant, intuitively the reverse is true for an initial adopter.

There are many strategy profiles here depending on country specific initial cost/benefits, but in general the profiles consist of a small subset of early adopters, and many more freeriders: $G1,G2,\ldots,Gm,Fm+1,\ldots,FN)$.

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Case 2: Crossed Threshold Equilibria

Once the threshold is crossed, adoption doesn't immediately halt or jump to 100%. Instead, adoption continues as long as the net payoff for new adopters remains positive, and stops only when the system reaches a point of strategic inertia — that is, no remaining country finds it worth switching.Why this is a stable equilibrium:

Threshold-Crossed + Stabilization

- $W_t \geq T$crossed at some round
- Continued adoption until $B_i \approx C_i$ for non-adopters
- Nash equilibrium where adoption plateaus with no further incentives to switch

Case 3: Mixed Outcome / Partial Momentum

- W_t rises due to early adopters but **fails to reach** T
- Creates a hybrid equilibrium with some adopters and some persistent free-riders
- Final state depends on heterogeneity of C_i and B_i

Part 2: Introduce the Tipping Point Concept

We'll formalize a key strategic insight:

- There exists a **minimum initial adoption level**, $W_{tipping}$, such that if: $W_0 \geq W_{tipping} \Rightarrow W_t \to T$ and beyond
- Below that value, the system stagnates or stalls

→ Use this to estimate how large W_0 must be to push C_i below B_i for some countries in the next round