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ECEN 422 - A9

David Dobbie

```
clc
clear
```

Q1 a

simple_portfolio_data

```
rand('state', 5);
randn('state', 5);
n=20;
pbar = ones(n,1)*.03+[rand(n-1,1); 0]*.12;
S = randn(n,n);
S = S'*S;
S = S/max(abs(diag(S)))*.2;
S(:,n) = zeros(n,1);
S(n,:) = zeros(n,1)';
x_unif = ones(n,1)/n;


A = randn(n);


% cvx_begin sdp
%     variable P(n,n) symmetric
%     minimize(trace(P))
%     A'*P + P*A <= -eye(n)
%     P >= eye(n)
% cvx_end

% -------no additional constraints


cvx_begin sdp quiet
variable x(n,1);
    minimize(x' * S * x); % minimise risk
    sum(x) <= 1; % all investments must sum to 1
    -pbar'*x <= -pbar'*x_unif ;% bound current invest with uniform
 invest strat
```

```
cvx_end;

disp('No additional constraints')

risk_uniform_invest = x_unif' * S * x_unif;
risk_optimal_invest = x' * S * x;



% -------long only

cvx_begin sdp quiet
variable x(n,1);
    minimize(x' * S * x); % minimise risk
    sum(x) <= 1; % all investments must sum to 1
    -pbar'*x == -pbar'*x_unif ;% bound current invest with uniform
 invest strat
    x >= 0 % long only
cvx_end;

disp(['Long only'])

risk_uniform_invest = x_unif' * S * x_unif;
risk_optimal_long_only_invest = x' * S * x;




% ------ limit on total short position

cvx_begin sdp quiet
variable x(n,1);
    minimize(x' * S * x); % minimise risk
    sum(x) <= 1; % all investments must sum to 1
    -pbar'*x <= -pbar'*x_unif ;% bound current invest with uniform
 invest strat
    sum(max(-x,0)) <= 0.5 % limit short to up to half of investment
cvx_end;
sum(max(-x,0));
disp(['Limit Short to half of investment'])

risk_uniform_invest = x_unif' * S * x_unif;
risk_optimal_limit_short_invest = x' * S * x;

risk_variance= [risk_uniform_invest; risk_optimal_invest; ...
    risk_optimal_long_only_invest; risk_optimal_limit_short_invest];
rows = {'Uniform Investment', 'Unconstrained Investment',...
    'Long-only investment', 'Short-limited investment'};


table(risk_variance, 'RowNames', rows)
```

No additional constraints
Long only

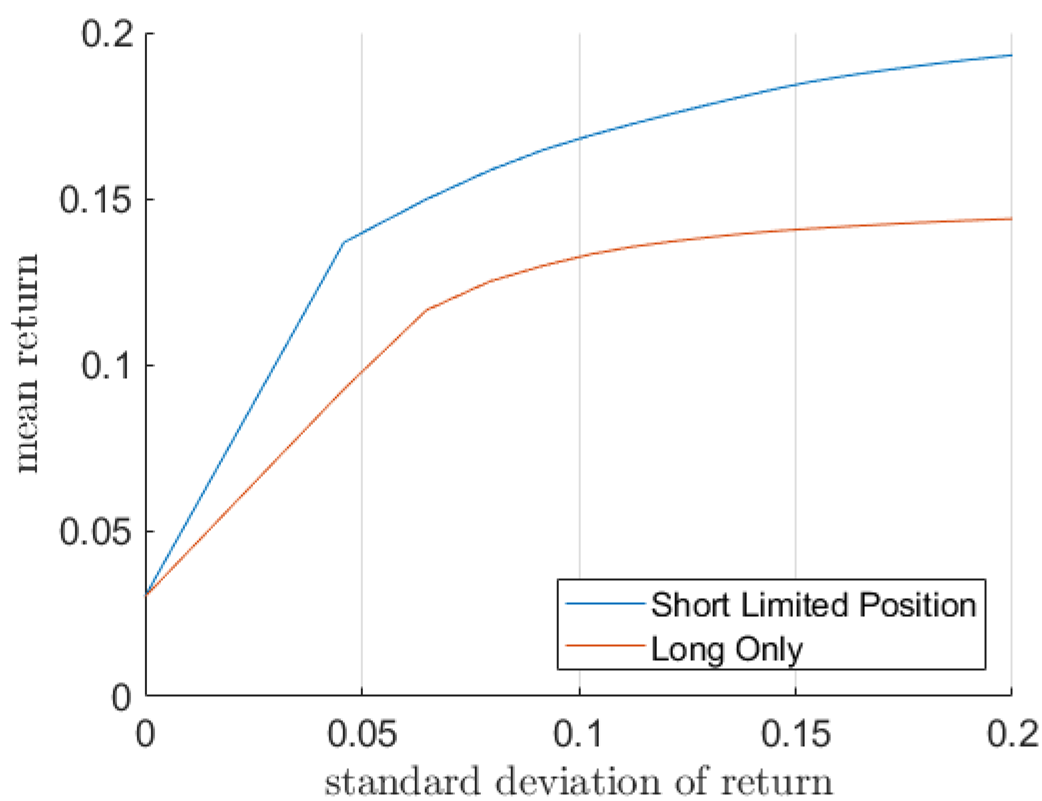
```
Limit Short to half of investment

ans =

  4×1 table

                                 risk_variance
                                 _____________

    Uniform Investment             0.0075753
    Unconstrained Investment      0.00034623
    Long-only investment           0.0025663
    Short-limited investment      0.00044078
```

We see that the no constraints of investment type achieves the lowest risk. Long-only has the highest risk, and short limit has a compromise between the two. All of these results are still better than a uniform investment however even though the return is the constant.

Q1 b

```
rand('state', 5);
randn('state', 5);
n=20;
pbar = ones(n,1)*.03+[rand(n-1,1); 0]*.12;
S = randn(n,n);
S = S'*S;
S = S/max(abs(diag(S)))*.2;
S(:,n) = zeros(n,1);
S(n,:) = zeros(n,1)';
x_unif = ones(n,1)/n;

length_res = 20;
var_limit_half_short = linspace(0,0.20^2,length_res);
mean_return_half_short = zeros(1,length_res);
var_limit_long_only = linspace(0,0.20^2,length_res);
mean_return_long_only = zeros(1,length_res);



% calculate wrt to a set variance
for indx = 1:length_res
    cvx_begin sdp quiet
    variable x(n,1);
        minimize(-pbar'*x); % minimise risk
        sum(x) <= 1; % all investments must sum to 1
        x' * S * x <= var_limit_half_short(indx)
        sum(max(-x,0)) <= 0.5 % limit short to up to half of
 investment
    cvx_end;
    mean_return_half_short(indx) = pbar'*x;

    cvx_begin sdp quiet
```

```
    variable x(n,1);
        minimize(-pbar'*x); % minimise risk
        sum(x) <= 1; % all investments must sum to 1
        x' * S * x <= var_limit_long_only(indx)
        x >= 0 % long only
    cvx_end;
    mean_return_long_only(indx) = pbar'*x;

end



std_dev_limit_half_short = sqrt(var_limit_half_short);
std_dev_limit_long_only = sqrt(var_limit_long_only);

figure(1)
clf
xlabel('standard deviation of return')
ylabel('mean return')
hold on
plot(std_dev_limit_half_short, mean_return_half_short)
plot(std_dev_limit_long_only, mean_return_long_only)
hold off
legend('Short Limited Position','Long Only','Location',"SouthEast")
```

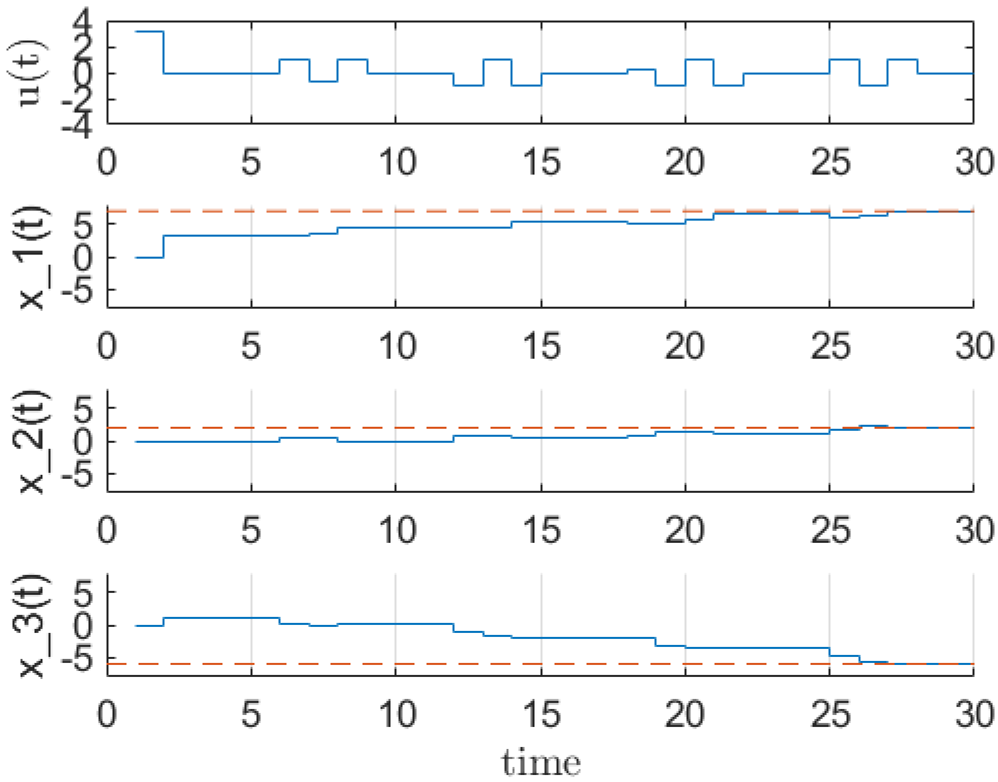


Q2

```
clear

A = [-1 0.4 0.8; 1 0 0; 0 1 0];
b = [1;0;0.3];
x_des = [7; 2; -6];
N = 30;
x_init = [0;0;0];

H = [];
for indx = 1:N
    H = [H  A^(indx - 1)*b];
end


cvx_begin sdp quiet
variable u(N,1)
variable t(N,1)
variable y(N,1)
    minimise(sum(t))
    %get with 1e-6 of target
    H*u <= x_des + 1e-6
    H*u >= x_des - 1e-6
    % input is element wise in absolute value by y
    -y <= u
    u <= y
    % relating cost of step to fuel function that we are minimising
    t >= y
    t >= 2*y -1
cvx_end;

H*u

x_out = zeros(3,N);
x_out(:,1) = x_init;
for indx = 2:N
    x_out(:,indx) =   H(:,1:indx)*u(1:indx);
end



figure(2)
clf

subplot(4,1,1)
stairs(u)
ylabel('u(t)')
ylim([-4 4])

subplot(4,1,2)
hold on
stairs(x_out(1,:)')
```

```
plot([0 N],[x_des(1) x_des(1)], '--')
ylabel('x_1(t)')
ylim([-8 8])
hold off

subplot(4,1,3)
hold on
stairs(x_out(2,:)')
plot([0 N],[x_des(2) x_des(2)], '--')
ylabel('x_2(t)')
ylim([-8 8])
hold off

subplot(4,1,4)
hold on
stairs(x_out(3,:)')
plot([0 N],[x_des(3) x_des(3)], '--')
ylabel('x_3(t)')
ylim([-8 8])
hold off

xlabel('time')
```

```
ans =

    7.0000
    2.0000
   -6.0000
```



The above figure generated shows the input required for x1,2,and 3 to acheive their goal state.

Q3) a)

```
u1 = -2;
u2 = -3;

cvx_begin quiet
variable x1
variable x2
dual variable y1
dual variable y2
dual variable y3
    % equivalnet to min x1*x1 + 2*x2*x2 - x1*x2 - x1
    minimise( 0.5*quad_form(x1,1) + 1.5*quad_form(x2,1) +
 0.5*quad_form(x1-x2,1) +   -x1 )
    y1 : x1 + 2*x2 <= u1
    y2 : x1 - 4*x2 <= u2
    y3 : 5*x1 + 76*x2 <= 1
cvx_end;

% optimal variables
y1;
y2;
y3;
x1;
```

```
x2;

% check KKT - if true they hold
% primal
primal1 = x1 + 2*x2 <= u1;
primal2 = x1 - 4*x2 <= u2;
primal3 = 5*x1 + 76*x2 <= 1;
% dual
dual1 = y1 >= 0;
dual2 = y2 >= 0;
dual3 = y3 >= 0;
% complementary slackness
slack1 = abs(y1*(x1 + 2*x2 - u1) <= 1e-3);
slack2 = abs(y2*(x1 - 4*x2 - u2) <= 1e-3);
slack3 = abs(y3*(5*x1 + 76*x2 - 1) <= 1e-3);
% Lagrangian gradient is 0
lagan_x1 = abs(2*x1 - x2 - 1 + y1 + y2 + 5*y3) <= 1e-3;
lagan_x2 = abs(4*x2 - x1 + 2*y1 - 4*y2 + 76*y3) <= 1e-3;

% KKT conditions hold if all of these inequalities are true
KKT_cond_satisfied = (primal1 & primal2 & primal3 & dual1 & dual2 &
 dual3 ...
    & slack1 & slack2 & slack3 & lagan_x1 & lagan_x2)
```

```
KKT_cond_satisfied =

  logical

   1
```

Q3) b)

```
del = [0, 0;
        0, -0.1;
        0, 0.1;
        -0.1, 0;
        -0.1, -0.1;
        -0.1, 0.1;
        0.1, 0;
        0.1, -0.1;
        0.1, 0.1];

res = zeros(9,1);

for indx = 1:9

    u1 = -2 + del(indx,1);
    u2 = -3 + del(indx,2);

    cvx_begin quiet
    variable x1
```

```
    variable x2
    dual variable y1
    dual variable y2
    dual variable y3
        % equivalnet to min x1*x1 + 2*x2*x2 - x1*x2 - x1
        minimise( 0.5*quad_form(x1,1) + 1.5*quad_form(x2,1) +
 0.5*quad_form(x1-x2,1) +   -x1 )
        y1 : x1 + 2*x2 <= u1
        y2 : x1 - 4*x2 <= u2
        y3 : 5*x1 + 76*x2 <= 1
    cvx_end;
    p_exact = 0.5*quad_form(x1,1) + 1.5*quad_form(x2,1) +
 0.5*quad_form(x1-x2,1) +   -x1;
    res(indx) = p_exact;
end
var_name =
 {'del_1', 'del_2', 'p_opt_exact', 'clean_less_than_perturbed'};
table(del(:,1), del(:,2), res,  res<=res(1),'VariableNames',var_name)
```

```
ans =

  9×4 table
```

del_1	del_2	p_opt_exact	clean_less_than_perturbed
0	0	8.2222	true
0	-0.1	8.7064	false
0	0.1	7.98	true
-0.1	0	8.565	false
-0.1	-0.1	8.8156	false
-0.1	0.1	8.3189	false
0.1	0	8.2222	false
0.1	-0.1	8.7064	false
0.1	0.1	7.7515	true

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