References

1 CHOI, S., and SHIN, K.G.: 'An uplink CDMA system architecture with diverse QoS guarantees for heterogeneous traffic', *IEEE/ACM Trans. Netw.*, 1999, **17**, (5), pp. 616–628

2 LAM, A.W., and OZLUTURK, F.M.: 'Performance bound for DS/SSMA communications with complex signature sequences', *IEEE Trans. Commun.*, 1992, **40**, (10), pp. 1607–1614

Deficiencies of 'Kronecker' MIMO radio channel model

H. Özcelik, M. Herdin, W. Weichselberger, J. Wallace and E. Bonek

Evidence that the 'Kronecker' model underestimates the channel capacity of an 8×8 multiple input, multiple output (MIMO) system in indoor non-line-of-sight (NLOS) scenarios is presented. Moreover, the model does not render the multipath structure correctly, which is the cause for the systematic capacity mismatch.

Introduction: MIMO channels with independent Rayleigh fading channel matrix elements exhibit very high capacity [1]. Correlation at either the receive or transmit side reduces MIMO capacity significantly [2]. As a consequence, a stochastic MIMO radio channel model has been developed to account for this spatial correlation in a realistic way [3]. McNamara *et al.* [4] have found the performance of the model [3] to be satisfactory under non-line-of-sight (NLOS) propagation, but not under line-of-sight (LOS).

The idea of [3], in slightly different notation [4], is to generate a correlated Rayleigh fading channel matrix $\tilde{\mathbf{H}} \in \mathbf{C}^{m \times n}$ (for m receive and n transmit antennas) according to

$$\mathrm{vec}(\tilde{\mathbf{H}}) = \mathbf{R}_{\mathbf{H}}^{1/2} \mathrm{vec}(\mathbf{G}) \tag{1}$$

where $\mathrm{vec}(\cdot)$ is the vector operator, $(\cdot)^{1/2}$ denotes any matrix square root fulfilling $\mathbf{R}_{\mathbf{H}}^{1/2} \cdot (\mathbf{R}_{\mathbf{H}}^{1/2})^H = \mathbf{R}_{\mathbf{H}}$ and

$$\mathbf{R}_{\mathbf{H}} = E\{\mathrm{vec}(\mathbf{H})\mathrm{vec}(\mathbf{H})^H\} \tag{2}$$

is the $(m \cdot n) \times (m \cdot n)$ channel correlation matrix containing the complex correlation between all $(m \cdot n)$ elements of the $m \times n$ channel matrix $\mathbf{H}$. Here, $(\cdot)^H$ is the complex conjugate transpose and $E\{\cdot\}$ the expectation operator. $\mathbf{G}$ is an $m \times n$ random matrix with zero-mean i.i.d. complex Gaussian entries. Under the assumption of [3], the channel correlation matrix is given by the Kronecker product of the transmitter correlation matrix $R_{Tx} = E\{(\mathbf{H}^H \ \mathbf{H})^T\}$ and the receiver correlation matrix $\mathbf{R}_{Rx} = E\{\mathbf{H}\mathbf{H}^H\}$,

$$\mathbf{R}_{\mathbf{H}} = \frac{1}{\mathrm{tr}\{\mathbf{R}_{Rx}\}} \mathbf{R}_{Tx} \otimes \mathbf{R}_{Rx} \tag{3}$$

where $\mathrm{tr}\{\cdot\}$ is the trace of a matrix, $(\cdot)^T$ is the transpose and $\otimes$ denotes the Kronecker product. Using the identity $[\mathbf{B}^T \otimes \mathbf{A}]\mathrm{vec}(\mathbf{D}) = \mathrm{vec}(\mathbf{ADB})$, (1) simplifies to the so-called 'Kronecker' model

$$\tilde{\mathbf{H}} = \frac{1}{\sqrt{\mathrm{tr}\{\mathbf{R}_{Rx}\}}} \mathbf{R}_{Rx}^{1/2} \mathbf{G} (\mathbf{R}_{Tx}^{1/2})^T \tag{4}$$

Besides simplifying analytical treatment or simulation of MIMO systems, (4) allows for independent array optimisation at Tx and Rx. Hence, the 'Kronecker' model has become popular.

Now, does this model predict MIMO capacity correctly? We present evidence that this is not the case in general. When considerable correlation exists at either or both ends of the MIMO link, then the 'Kronecker' model predicts a lower capacity than that of the directly measured channel. Also, the 'Kronecker' model does not render the multipath structure correctly.

Measurement setup and scenario: Channel matrices were measured at the Institut für Nachrichtentechnik und Hochfrequenztechnik, Technische Universität Wien, at 5.2 GHz. The transmitter (Tx) consisted of a positionable monopole antenna on a 20×10 grid with an inter-element spacing of $\lambda/2$, where only a 10×5 submatrix was used to avoid large-scale fading effects. The receiver (Rx) was a directional 8-element linear patch array with 0.4λ inter-element spacing and 120° 3 dB beamwidth. The channel was probed at 193 equi-spaced frequency bins over 120 MHz of bandwidth. The (virtual) transmit array was positioned in a hallway and the receiver assumed 24 different positions each looking into three different directions (rotated by 120°) in several offices connected to this hallway without LOS (except one position/direction), leading to 72 different 'scenarios'.

Evaluation: For each Rx position and direction we created 15 spatial realisations of the 8×8 MIMO channel matrix by moving a virtual 8-element uniform linear array (ULA) over the 10×5 grid. Taking the 193 frequency bins, the total set of measured channel samples is formed by $193 \times 15 = 2895$ different spatial and frequency realisations. Considering a channel unknown at Tx, the MIMO capacity for each realisation was calculated by [1]

$$C = \log_2 \det\left(\mathbf{I}_m + \frac{\rho}{n} \mathbf{H}\mathbf{H}^H\right) \tag{5}$$

where $\mathbf{I}_m$ denotes the $m \times m$ identity matrix, ρ the mean receive SNR, n the number of Tx antennas and $\mathbf{H}$ the normalised $m \times n$ MIMO channel matrix. Here, $m = n = 8$. Channel matrices were normalised such that the equivalent average SISO pathloss (over all frequency and spatial realisations) for each scenario was 0 dB [5], and an average receive SNR of 20 dB was assumed. Additionally, we synthesised $193 \times 15 = 2895$ different realisations of $\tilde{\mathbf{H}}$ according to the 'Kronecker' model (4), normalised them and compared the resulting capacities for both measured and 'Kronecker' model channel matrices.

Results: The comparison of *average* capacities is shown in Fig. 1. Each point in this plot corresponds to one scenario. As can be seen, the 'Kronecker' model always underestimates the true capacity, i.e. the points lie below the identity (dashed) line. The dash-dotted and dashed lines correspond to 10 and 20% relative error, respectively. Moreover, the capacity mismatch gets worse with decreasing 'measured' capacity. Since decreasing 'measured' capacity is related to increasing correlation at either or both receive and transmit sides [2, 5], it shows that the 'Kronecker' model fails in correlated environments.

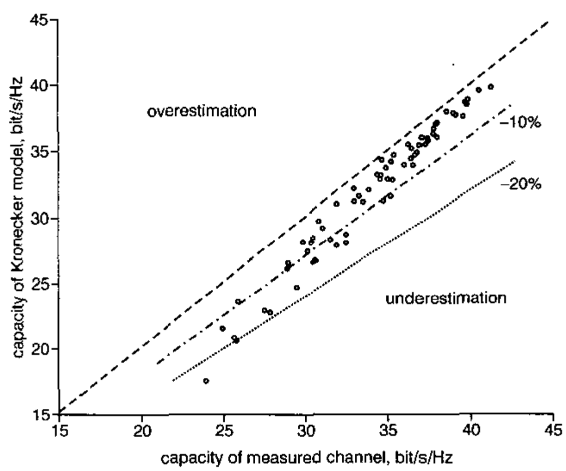

Fig. 1 *Scatter plot of average 'Kronecker' model capacity against average capacity of measured channel for 8 × 8 MIMO*
-------- identity line

Fig. 2 provides a compelling answer by comparing the joint DoD-DoA Fourier spectrum of a typical scenario for measured channels (Fig. 2*a*) and channels synthesised with the 'Kronecker' model (Fig. 2*b*). In the measured channel, specific DoDs are clearly linked to specific DoAs, such that the joint spectra is non-separable (not described by a product of separate DoD and DoA spectra). The Kronecker factorisation, however, forces the joint spectrum to be separable, thus introducing artifact paths lying at the intersections of DoD and DoA spectral peaks. As explained in [6], these artifacts lower average capacity but increase diversity order. We note that these effects will likely be more pronounced for large arrays with high angular resolution.

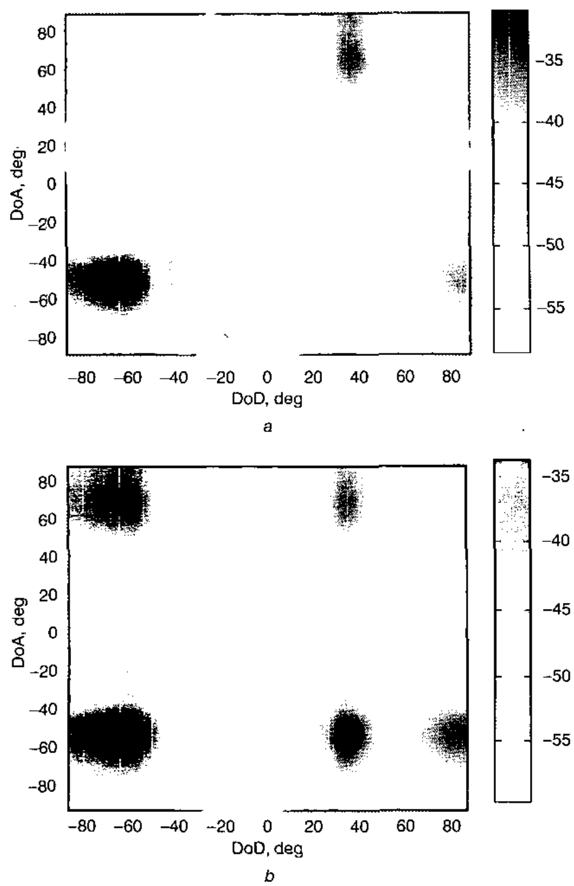

Fig. 2 *Joint DoD–DoA Fourier spectrum (in dB) of measured channel matrix and 'Kronecker' model—example scenario*

a Measured channel matrix *b* 'Kronecker' model

Conclusion: Using a large set of broadband NLOS indoor measurements, we have shown that the Kronecker factorisation of the full channel correlation matrix into independent receive and transmit correlation matrices leads to systematic prediction errors in capacity and multipath structure. The 8×8 MIMO capacity predicted by the 'Kronecker' model is always below the capacity extracted directly from the measurement. We found a capacity mismatch of up to 27%, increasing with increasing correlation, which is an alarming indicator of inadequate modelling. We have shown also that the Kronecker factorisation, by assuming non-existent separability of the spectra, introduces artifact paths in the joint DoD-DoA spectrum, necessarily decreasing capacity but increasing diversity order [6]. Our result is not in contradiction to Kai Yu *et al.* [7], who claimed to have validated Kronecker factorisation and modelling, because of the reduced spatial resolution of their 2×2 and 3×3 MIMO systems.

Acknowledgments: We wish to thank H. Hofstetter (ftw.) for help with the measurements and T-Systems Nova GmbH for providing an 8-element linear patch array.

8 May 2003
Electronics Letters Online No: 20030785
DOI: 10.1049/el:20030785

H. Özcelik, M. Herdin, W. Weichselberger and E. Bonek (*Institut für Nachrichtentechnik und Hochfrequenztechnik, TU-Wien Gusshausstrasse 25/389, A-1040 Wien, Austria*)

J. Wallace (*Department of Electrical and Computer Engineering, Brigham Young University, 459 CB, Provo, UT, 84602, USA*)

References

1 TELATAR, I.E.: 'Capacity of multi-antenna Gaussian channels'. AT&T Bell Labs. Available online: http://mars.bell-labs.com/cm/ms/what/mars/papers/proof
2 SHUI, D.-S., FOSCHINI, G.J., GANS, M.J., and KAHN, J.M.: 'Fading correlation and its effect on the capacity of multielement antenna systems', *IEEE Trans. Commun.*, 2000, **48**, (3), pp. 502–513
3 KERMOAL, J.P., SCHUMACHER, L., PEDERSEN, K.I., MOGENSEN, P.E., and FREDERIKSEN, F.: 'A stochastic MIMO radio channel model with experimental validation', *IEEE J. Sel. Areas Commun.*, 2002, **20**, pp. 1211–1226
4 MCNAMARA, D.P., *et al.*: 'Spatial correlation in indoor MIMO channels'. Personal, Indoor and Mobile Radio Communications, Lisbon, Portugal, 2002, Vol. 1, pp. 290–294
5 ÖZCELIK, H., HERDIN, M., HOFSTETTER, H., and BONEK, E.: 'A comparison of measured 8×8 MIMO systems with a popular stochastic channel model at 5.2 GHz'. ICT 2003, Papeete, French Polynesia, Taniti, Vol. 2, pp. 1542–1546
6 SAYEED, A.M.: 'Deconstructing multiantenna fading channels', *IEEE Trans. Signal Process.*, 2002, **50**, pp. 2563–2579
7 YU, KAI, BENGTSSON, M., OTTERSTEN, B., MCNAMARA, D., KARLSSON, P., and BEACH, M.: 'Second order statistics of NLOS indoor MIMO channels based on 5.2 GHz measurements'. IEEE GLOBECOM'01, San Antonio, Texas, USA, 2001, Vol. 1, pp. 156–160

Methodology for performance analysis of randomly-spread CDMA systems over multipath fading channels via crosscorrelation matrix non-asymptotic average eigenvalue densities

Ö. Ertuğ, B. Baykal and B.S. Ünal

A new methodology for closed-form theoretical performance analysis of randomly-spread CDMA systems over multipath fading channels with multiuser receivers is presented. The basis of the analysis is the representation of the random signal to interference ratios at finite system parameters in terms of the eigenvalues of crosscorrelation matrices for which the non-asymptotic average densities are found or known. The methodology presented complements the asymptotic limiting theory used in similar previous work over Gaussian/single-path fading channels, but at finite system parameters and over multipath fading channels. One application of the method is demonstrated by spectral efficiency derivations with linear multiuser receivers.

Introduction: In this Letter, we present a new approach that is promising for closed-form theoretical performance analysis of randomly-spread CDMA systems over multipath fading channels in terms of system parameters with diversity-reception multiuser receivers via averaging the conditional performance metrics in terms of signal to interference ratios (SIRs) over the non-asymptotic average eigenvalue densities of random crosscorrelation matrices. The average eigenvalue density for a random matrix $\mathbf{A}$ stands for the average normalised histogram of the eigenvalues of a random matrix's samples, i.e. the normalised trace density $\bar{\lambda} = (1/N)\sum_{i=1}^{N}\lambda_i = (1/N)\mathrm{Tr}\{\mathbf{A}\}$, or the density for the probability of what percentage of the eigenvalues of $\mathbf{A}$ is below some specific value.

Analysis through random spreading sequences accurately models large CDMA systems with long spreading sequences, allows analysis independent of choice/assignment of spreading codes and provides a comparison baseline for systems with short sequences. Similar previous work on Gaussian and single-path channels [1, 2] based on asymptotic limiting theory considers asymptotic average eigenvalue densities as the number of users K and the processing gain L tends to infinity but with fixed load factor, $\beta \doteq K/L$, with the assumption that asymptotic distributions also model well systems with finite parameters.