Correlated Fading in Broadcast MIMO Channels: Curse or Blessing?

Bruno Clerckx, Gil Kim and Sungjin Kim
Communication Lab, Corporate Technology Operations
Samsung Electronics
Yongin-Si, Gyeonggi-Do, Korea 446-712
Email: bruno.clerckx@samsung.com

***Abstract*—The impact of transmit correlation on the performance of limited feedback Multi-User MIMO is investigated. Correlated fading is shown to highly influence the probability of scheduling orthogonal users and the ability to achieve small quantization errors. Correlated fading is indeed beneficial to MU-MIMO performance as it can potentially reduce the quantization error, if an appropriate codebook is used. Grassmanian Line Packing is very much inappropriate for MU-MIMO in correlated fading environments, while DFT-based codebooks and channel statistics-based codebooks are promising candidates. Additionally correlated fading is very beneficial to the scheduler. User correlation is shown to decrease more quickly in transmit correlated channels than in i.i.d. channels as the number of active users increases. However, correlated fading is also shown to be detrimental to the scheduler performance when the number of active users is rather small and cell sectorization is performed.**

I. Introduction

Multi-User (MU) MIMO with limited feedback has drawn a considerable attention very recently. It is well known that multiple antennas at the base station provide a sum rate capacity increase that grows linearly with the minimum of the number of transmit antennas and users [1], [2]. A suboptimum but practical scheme, called Zero-Forcing Beamforming (ZFBF), has been shown to exhibit the optimum scaling as the number of user increases [4]. However such gain is only possible if channel knowledge at the transmitter is available. Contrary to the SU-MIMO, limited feedback in MU-MIMO considerably limits the spatial multiplexing gain by inducing a ceiling effect due to the quantization error [3], [5].

The very common assumption when analyzing SU and MU-MIMO schemes is to consider i.i.d. Rayleigh fading channels. However limited feedback-based SU and MU-MIMO for correlated channels has been less addressed in the literature (for SU-MIMO [6]–[11] and for MU-MIMO [12]–[16]).

In [6], [7], low complexity and high performance solutions are proposed by rotating and scaling a vector codebook designed for i.i.d. channels as a function of the transmit correlation matrix. Recently, an upper bound on the distortion function is derived for correlated fading case, showing that transmit correlation may help reducing quantization errors [8], [9]. Transmit correlation-based codebook design is also proposed. Recently in [10], [11], some quantities have been introduced to address the robustness of codebooks in correlated fading when the channel statistics are not known by the transmitter and the codebook cannot be adapted depending on the channel statistics.

In [12], line-of-sight channels are shown to be beneficial to MU-MIMO since it can reduce the amount of feedback overhead. Other papers [13]–[16] have tried to evaluate the impact of transmit correlation on the MU-MIMO sum-rate. Those papers however rely on the very strong and unrealistic assumption that all users experience the same transmit correlation matrix. This assumption naturally leads to the conclusion that transmit correlation is detrimental to MU-MIMO. Indeed this implies that all users should be scheduled in the same beam as the correlation increases, which is totally contradictory to the basic of MU-MIMO (e.g. SDMA).

In this paper, contrary to what is claimed by [13]–[16], it is shown that correlated fading can be very beneficial to the quantizer and the scheduler. Firstly, correlated fading can be beneficial from the distortion point of view as the quantization error can be reduced if an appropriate codebook is chosen. Channel statistics-based codebooks and DFT-based codebooks are shown to lead to small quantization errors while codebooks designed for i.i.d. channels or based on the commonly-used Grassmanian Line Packing (GLP) approach are very sensitive to correlated fading. Secondly, for a small number of active users, correlated fading decreases multi-user diversity as it reduces the probability of finding orthogonal users. However, as the number of active users increases, the probability of finding orthogonal users decreases more quickly in correlated channels than in i.i.d. channels. Cell sectorization is also shown to be particularly detrimental to MU-MIMO sheduler in the presence of correlated fading.

II. System Model

Assume a broadcast channel with n_t transmit antennas and K single antenna active users. Assuming that M users are simultaneously scheduled, the broadcast channel is written as[1]

$$y_i = \mathbf{h}_i^H \mathbf{x} + n_i, \quad i = 1, \ldots, K \tag{1}$$

where y_i is the signal received by user i, $\mathbf{h}_i$ $[n_t \times 1]$ is user i channel, n_i is complex Gaussian noise with unit variance and $\mathbf{x}$ is the transmitted signal. We denote the concatenation of

[1] $\mathcal{E}$ stands for expectation, ^{T} for transposition, * for elementwise conjugation, ^{H} for conjugate transpose, $|.|$ is the absolute value.

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the channels by $\mathbf{H} = [\mathbf{h}_1, \ldots, \mathbf{h}_M]$. In this paper, we assume uniform power allocation among M users and linear transmit beamforming, such that the transmitted signal $\mathbf{x}$ can be given as

$$\mathbf{x} = \sqrt{\frac{P}{M}} \sum_{i=1}^{M} \mathbf{v}_i s_i \tag{2}$$

where s_i is user i data symbol and $\mathbf{v}_i$ is user i unitary beamforming vector. A popular MU-MIMO beamforming strategy is Zero-Forcing Beamforming (ZFBF) [4], where vectors $\mathbf{v}_i$ are obtained as the normalized columns of matrix $\mathbf{V}_{\bar{\mathbf{H}}} = \bar{\mathbf{H}}\left(\bar{\mathbf{H}}^H\bar{\mathbf{H}}\right)^{-1}$ with $\bar{\mathbf{H}} = \left[\bar{\mathbf{h}}_1, \ldots, \bar{\mathbf{h}}_M\right]$, i.e. $\mathbf{v}_i = \mathbf{V}_{\bar{\mathbf{H}}}(:,i)/\|\mathbf{V}_{\bar{\mathbf{H}}}(:,i)\|$. The channel directions are defined as $\bar{\mathbf{h}}_i = \mathbf{h}_i / \|\mathbf{h}_i\|$.

In a limited feedback scheme, channel directions $\bar{\mathbf{h}}_i$ are unknown at the transmitter. Only quantized versions of those vectors, denoted as $\hat{\mathbf{h}}_i$, are fedback to the transmitter. In such case, vectors $\mathbf{v}_i$ are obtained as the normalized columns of matrix $\mathbf{V}_{\hat{\mathbf{H}}} = \hat{\mathbf{H}}\left(\hat{\mathbf{H}}^H\hat{\mathbf{H}}\right)^{-1}$ where $\hat{\mathbf{H}} = \left[\hat{\mathbf{h}}_1, \ldots, \hat{\mathbf{h}}_M\right]$, i.e. $\mathbf{v}_i = \mathbf{V}_{\hat{\mathbf{H}}}(:,i)/\|\mathbf{V}_{\hat{\mathbf{H}}}(:,i)\|$. Every user i quantizes its channel using a B_i-bits codebook $\mathcal{W}_i$ of codevectors $\mathbf{w}_{i,k}$, $k = 1, \ldots, n_{p,i} = 2^{B_i}$. The best codevector $\hat{\mathbf{h}}_i$ for user i is therefore selected as

$$\hat{\mathbf{h}}_i = \arg \max_{1 \leq k \leq n_{p,i}} \left|\mathbf{h}_i^H \mathbf{w}_{i,k}\right|^2. \tag{3}$$

Aside the feedback of the index of the preferred codevector in the codebook, the Base Station requires an estimate of the SINR for every user in order to perform scheduling and rate adaptation. An accurate estimate of the SINR is complicated in the limited feedback case as the interference cannot be evaluated. In [20], a lower bound on an expected value of the SINR where the expectation is performed with respect to the interference term was shown to be given by

$$\mathcal{E}_I \left\{SINR_i\right\} \geq \frac{p_i}{P/n_t} CQI_i \tag{4}$$

where

$$CQI_i = \frac{\frac{P}{n_t}\|h_i\|^2 \cos^2\left(\theta_i\right)}{1 + \frac{P}{n_t}\|h_i\|^2 \sin^2\left(\theta_i\right)} \tag{5}$$

and $p_i = \frac{P/M}{\|\mathbf{V}_{\hat{\mathbf{H}}}(:,i)\|^2}$. The angle θ_i is defined as $\cos^2(\theta_i) = \left|\bar{\mathbf{h}}_i^H \hat{\mathbf{h}}_i\right|^2$. The CQI (Channel Quality Indicator) is evaluated at the mobile station and fed back to the base station. A similar expression for the CQI was proposed in [5].

For simulation, the channel for user i is assumed Rayleigh distributed $\mathbf{h}_i = \mathbf{R}_{t,i}^{1/2}\mathbf{h}_{w,i}$ with the correlation matrix $\mathbf{R}_{t,i}$ given by an exponential model, e.g.

$$\mathbf{R}_{t,i} = \begin{bmatrix} 1 & t_i & t_i^2 & t_i^3 \\ t_i^* & 1 & t_i & t_i^2 \\ t_i^{*2} & t_i^* & 1 & t_i \\ t_i^{*3} & t_i^{*2} & t_i^* & 1 \end{bmatrix} \tag{6}$$

for 4 transmit antennas. Parameter t_i is the transmit correlation coefficient for user i. In order to evaluate the impact of correlation on the performance of MU-MIMO, all users are assumed to experience the same value of $|t_i| = |t|$. This is motivated by the fact that $|t|$ is directly function of the base station inter-element spacing. While not exactly realistic, this assumption gives some good insight into how performance of ZFBF is affected by transmit correlation. Contrary to [13]–[16], we however assume that the phase of t_i is randomly generated and is independent from one user to another. Hence the eigenvalues of the users correlation matrices may be the same, but their eigenvectors are independent. When the label $360°$ is used, we refer to the case where the phase of t_i is uniformly distributed over 2π. Similarly, $70°$ refers to the case where the phase of t_i is uniformly generated between $-35°$ and $35°$. This last scenario is used to investigate the effect of cell sectorization. Indeed as $|t|$ is large, the phase of t_i is strongly related to the physical location of users. In a 3-sectors cell, the -3dB aperture is usually fixed to $70°$. In a 6-sectors cell, it would be even smaller.

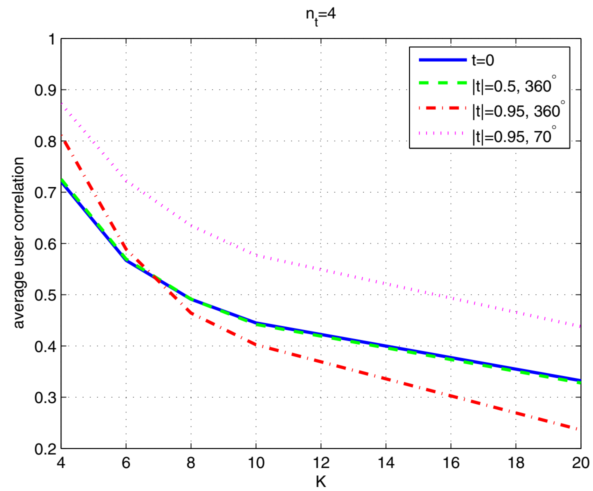

Fig. 1. Average user correlation as a function of the number of active users in the sector K and the magnitude of the transmit correlation coefficient $|t|$.

A multi-user MIMO scheme with limited feedback is affected by two dominant factors. The first is the quantization error induced by the limited feedback scheme, which leads to a residual interference term that does not vanish with the SNR [3] and affects the calculation of the CQI (5). The second factor is the probability of finding orthogonal users. Indeed all scheduling algorithms [4], [5], [21] try to find some semi-orthogonal or orthogonal users to be scheduled, as orthogonal users provide larger sum-rate, by removing naturally the multi-user interference. In the following sections, we discuss and investigate the impact of transmit correlation on the user correlation and on the quantization error .

III. User Correlation vs. Transmit Correlation

Among K active users, the probability of finding n_t (semi-)orthogonal users is investigated. Denoting as $\mathcal{T}$ all sets of n_t users among the K active users, and as τ one of those sets, we investigate by simulation the pdf of the user correlation $\rho_u = \mathcal{E}\left\{\min_{\tau \in \mathcal{T}} \max_{k,l \in \tau} \left|\bar{\mathbf{h}}_k^H \bar{\mathbf{h}}_l\right|\right\}$.

Figure 1 displays the average value of ρ_u as a function of $|t|$ and K, where the average value here refers to the averaging over several sets of links (i.e. for different sets of transmit correlation matrices $\mathbf{R}_{t,i}$). The exponential model is used for generating correlation matrices and it is assumed that $|t_i| = |t|\ \forall i$. As it is well known, increasing the number of users provides more opportunity to schedule orthogonal users [4]. But more importantly, we see that the transmit correlation coefficient has a non-negligible impact on the probability of scheduling orthogonal users. User correlation decreases more rapidly in correlated environments than in i.i.d. scenarios, as K increases. Indeed, for a moderate number of active users, the transmit correlation decreases the probability of finding semi-orthogonal users. On the other hand, if the number of active users is large, transmit correlation lowers the user correlation, as long as the phase of the transmit correlation coefficient is able to vary over a large range. If the phase is limited (due to e.g. sectorized cells), the user correlation remains however quite large. Transmit correlation is thus expected to be detrimental for a small number of users and beneficial for a large number of users, as long as the number of sectors in the cell is not too large.

The reason for such behavior is pretty intuitive. Correlation tends to decrease the size of the space channel directions may evolve in. As correlation increases, channel directions mainly lie in the space spanned by the transmit array responses. In sectorized cells, such space is reduced because the directions of departure (DOD) are constrained by the sectors. Therefore, for a small number of users, finding orthogonal users is rather complicated because it would come to find directions of departure such that their corresponding transmit array responses are orthogonal. In an i.i.d. space, the situation is much easier since channel directions are uniformly distributed. However, as K increases, since the correlated space is reduced, a smaller number of users is enough to cover the correlated space compared to the i.i.d. space. Hence the probability of finding orthogonal users drops more rapidly in the correlated case than in the i.i.d. case. This is only valid as long as the correlated space is not too much reduced because of the sectorization, i.e. as long as the range of DODs is wide enough to find sets of orthogonal transmit array responses.

IV. Average Distortion vs. Transmit Correlation

The previous explanation would also suggest another effect. If the space is reduced, for a fixed number of feedback bits, the quantization error should be smaller too. The average distortion induced by the quantization process is investigated as a function of the correlation. For single user i beamforming, the average distortion function [18] is a measure of the average array gain loss induced by quantization (3) with a codebook $\mathcal{W}_i$. In order to normalize w.r.t. to the channel gain, a normalized distortion function $(0 \leq G_n(\mathcal{W}_i) \leq 1)$ is introduced as follows $G_n(\mathcal{W}_i) = \mathcal{E}_{\mathbf{h}_i}\left\{|\mathbf{h}_i|^2\left[1 - |\bar{\mathbf{h}}_i^H \hat{\mathbf{h}}_i|^2\right]\right\} / \mathcal{E}_{\mathbf{h}_i}\left\{|\mathbf{h}_i|^2\right\}$.

The Generalized Lloyd Algorithm, well known in vector quantization [17], is used to derive the optimal codebook given the channel statistics (denoted as “CDIT-based” codebook) and to evaluate the distortion created by such optimal codebooks. The distortion induced by other codebooks (DFT codebook [19], Samsung 3GPP LTE codebook [10], Grassmanian Line Packing GLP codebook [18]) is also evaluated. Comparison of distortion $G_n(\mathcal{W})$ achieved by all those codebooks is presented in Figure 2.

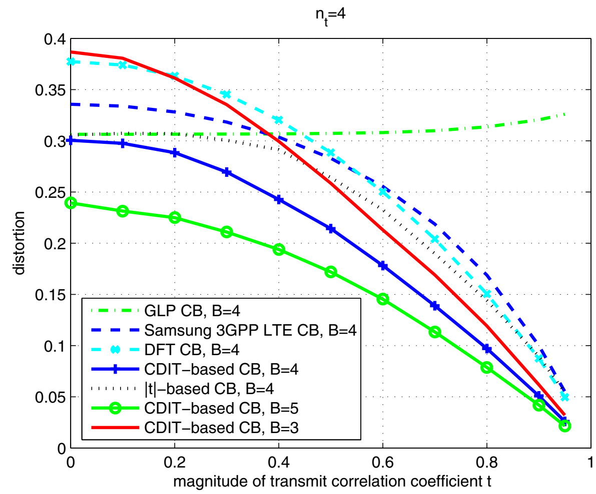

Fig. 2. Distortion $G_n(\mathcal{W}_i)$ as a function of the codebook size $n_p = 2^B$ and the transmit correlation in correlated scenarios with four transmit antennas.

The distortion achieved by CDIT-based codebooks is displayed for 3,4 and 5-bits codebook sizes (i.e. $n_p = 8, 16, 32$ respectively). It is seen that the distortion $G_n(\mathcal{W}_i)$ considerably decreases as the $|t|$ increases. Additionaly the gain of having larger codebook sizes is only beneficial at low correlation. For large correlation, small size CDIT-based codebook achieve very small quantization errors for a very small feedback overhead. In that sense, correlation is highly beneficial to MU-MIMO as it decreases simultaneously the feedback overhead and the quantization error. Note however that this requires to use a codebook that adapts itself to the correlation structure of the channel. In a MU-MIMO scheme, every user i would therefore have a different codebook $\mathcal{W}_i$ that is optimized to match to their own individual statistics (i.e. to their own $\mathbf{R}_{t,i}$ eigenvectors and eigenvalues).

In Figure 2 performance of other codebooks (DFT, Samsung 3GPP LTE and GLP) are also evaluated as a function of $|t|$ for $B = 4$. The distortion is averaged over the phase of t. Also we compare with a codebook designed only based on the information of $|t|$ and for which the phase of t is not known but can vary from 0 to 2π. Hence for such codebook, the training sequence in the Lloyd algorithm is generated in such a way that it spans all phases of t for a given magnitude $|t|$. This can be seen as a lower bound on the performance of a fixed codebook.

Clearly the behavior highly depends on the codebook used. GLP codebook, originally designed for i.i.d. channels is extremely sensitive to transmit correlations. On the other hand the DFT codebook and Samsung codebook (optimized DFT codebook) proposed in 3GPP LTE [10] are definitely more

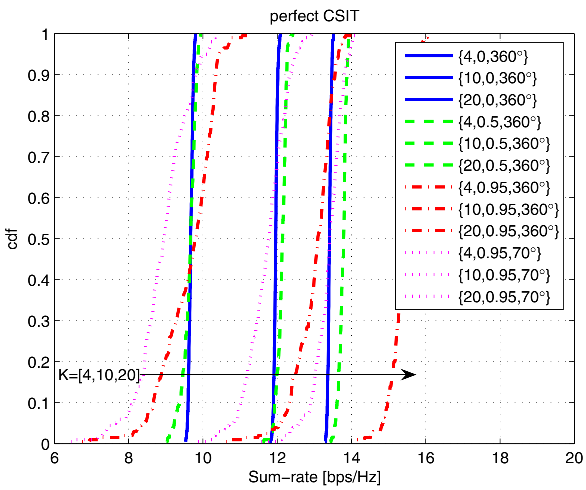

Fig. 3. CDF of the performance of ZFBF with perfect CSIT and Greedy user selection as a function of K and the transmit correlation (SNR=10 dB). $\{K, |t|, t/|t|\}$ refers to the set of parameters.

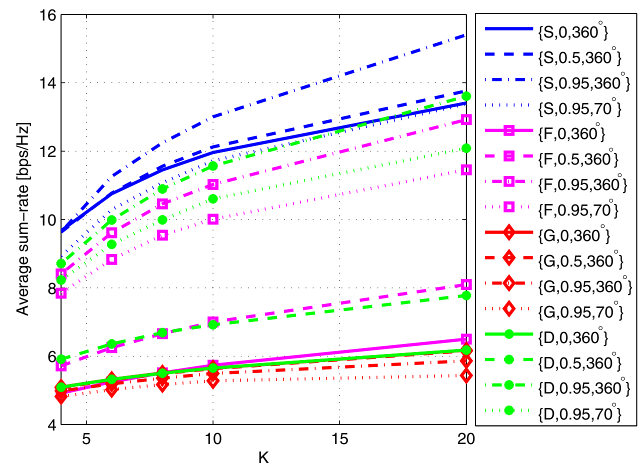

Fig. 4. Performance of ZFBF with Greedy user selection and various codebook strategies as a function of K for various transmit correlation coefficient $t = 0$, $|t| = 0.5$, and $|t| = 0.95$ (SNR=10 dB, $B = 4$). "D" stands for CDIT-based codebook, "F" for DFT, "G" for GLP and "S" for perfect CSIT. $\{``X", |t|, t/|t|\}$ refers to the set of parameters.

robust. Also they come quite close to the $|t|$-based codebook, even though the $|t|$-based codebook requires to be adapted as $|t|$ varies. Note that this good behavior of DFT codebooks is only guaranteed in uniform linear array scenarios [11].

V. Performance of Zero-Forcing Beamforming

We investigate by simulations the performance of ZFBF with various codebook strategies: perfect CSIT, Grassmanian Line Packing (GLP) [18], DFT-based codebooks [10], [11], [19], channel-statistics based codebooks (CDIT CB). The CDIT CB is generated using the rotation and scaling approach proposed in [6]. The i.i.d. component codebook used in the rotation and scaling process is optimized using Lloyd algorithm. Such codebook construction for correlated channels has been shown to come very close to the optimality [7]. In all simulations, the CQI is estimated as in (5) and fedback to the base station in order to perform scheduling. It is assumed that perfect link adaptation is performed. No time delay and feedback errors are considered. The scheduling is performed using a greedy user selection (US) algorithm [21].

In Figures 3 and 4, we investigate by simulation the impacts of the phenomena explained in Section III and IV on the performance of ZFBF with 4 transmit antennas in various correlated environment. The exponential model is used to generate the correlation matrices. We assume that all users experience the same value of $|t_i| = |t|$.

Simulation results in Figure 3 suggest the following observations. In the case of perfect CSIT and no sectorization (i.e. 360°), the CDF of the achievable throughput flattens as the transmit correlation increases. Additionally the transmit correlation increases the average throughput as the number of active users increases. Therefore for a small number of active users ($K < 10$), correlated fading may be beneficial and detrimental to the performance. However for a larger number of active users ($10 \leq K \leq 20$), correlated fading is always beneficial to the performance. This is perfectly inline with the behaviors observed in Section III. Figure 3 also shows that a sectorized cell is always detrimental to the performance in the presence of high transmit correlation, as it becomes more complicated to schedule semi-orthogonal users in such scenarios. Indeed, simulations suggested that in transmit correlated scenarios only 3 users are most of the time simultaneously scheduled when 20 users are active in the sector, while 4 users would be scheduled in lower correlation scenarios or non-sectorized cells.

In Figure 4, several conclusions can be drawn. Firstly it is observed that the slope of the sum-rate vs. K increases as the correlation increases. Hence in highly correlated channels, the sum rate increases very quickly with K while in i.i.d. channels the increase is rather slow. In the limited feedback case, the same behavior is observed for DFT and channel statistics-based codebooks, but not for GLP codebooks.

Secondly, the performance of CSIT, DFT and CDIT codebooks is also shown to increase as the correlation increases while the performance of GLP decreases as the correlation increases. The number of scheduled users was found to be actually always larger for CDIT-based codebooks and DFT codebooks than with GLP codebooks as the correlation increases. This is in agreement with the behavior of the distortion function[2]. Hence, GLP is concluded to be totally inappropriate to ZFBF in correlated channels while DFT and CDIT CB are appropriate candidates.

Thirdly, interestingly it is observed that DFT is better than GLP and CDIT CB for low correlation and a large number of users (when small and realistic codebook sizes are used) while it is the opposite for small K, even though the distortion

[2]Note that simulations are done for a given structure of the correlation matrix. Conclusions might be slightly different for other structures.

function is smaller for GLP than for DFT codebooks in i.i.d. channels. It was observed by simulations that the number of scheduled users is larger for DFT codebooks than for GLP and CDIT codebooks in the presence of low correlation and large K. This shows that the optimal vector codebook for SU beamforming may not be optimal anymore in ZFBF. Indeed the scheduling aspects are never taken into account when designing GLP codebooks. We explain the better performance of DFT coming from the fact that some vectors of a DFT codebook are orthogonal to each other while it is not the case in GLP codebook. Hence when the probability of finding orthogonal users increases (large K), users should be quantized with orthogonal vectors in order not to loose their orthogonality. Small (e.g. 4bits for $n_t = 4$) GLP codebooks do not allow to do so. Finally, note that sectorization hurts full CSIT and limited feedback ZFBF performance.

Those simulations show that correlated fading is beneficial to MU-MIMO performance as it reduces the quantization error even for very small codebook sizes (small feedback) and increases the probability of finding orthogonal users. A third reason to justify the use of MU-MIMO in correlated scenarios is that in OFDM transmissions, a single codevector could be used over the whole band, therefore significantly reducing the feedback overhead in the frequency domain. This is due to the fact that the channel statistics (e.g. the transmit correlation matrix) remain the same over the whole bandwidth as long as the ratio bandwidth/carrier frequency is small (e.g. as in 3GPP LTE or IEEE 802.16m). In uncorrelated channels, on the other hand, the channel directions may vary significantly from one resource block to another.

It is worth noting the following drawbacks of the deployment of a small antenna spacing transmit array. Firstly, it may affect the performance of the control channels by reducing the achievable spatial diversity. It should be carefully investigated to make sure that the achievable frequency diversity order and the small transmit diversity order are sufficient to guarantee enough reliability and the required coverage. Secondly, single user spatial multiplexing cannot be deployed anymore in such scenarios. Hence the cell would be optimized for boosting the cell throughput only, without consideration of the peak throughput. Finally, it is important to mention that some level of fairness have not been considered in the Greedy scheduler used in the previous simulations. It may happen that in some correlated scenarios, the use of a Greedy scheduler may select most of the time the same users.

VI. Conclusions

The impact of transmit correlation on the performance of MU-MIMO schemes (e.g. ZFBF) with limited feedback is investigated. It is shown that correlated fading leads to two mechanisms that originate from the reduction of quantization space. Firstly, the probability of finding (semi-)orthogonal users decreases for a small number of users and increases for a large number of users as the correlation increases. Secondly, correlated fading helps in reducing the quantization error and therefore partly helps removing the ceiling effect experienced by limited feedback MU-MIMO. This however requires the use of an appropriate codebook. Channel statistics and DFT based codebooks are shown to particularly adapted to correlated fading, while GLP is totally inappropriate. Finally, it is shown that cell sectorization hurts MU-MIMO by reducing the scheduling flexibility and limiting the number of simultaneously scheduled users when transmit correlation is large. Discussions suggest that MU-MIMO should be primarily optimized for small antenna spacing scenarios.

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