Proposed Bayesian Estimation Method for T2 Relaxation Times from measurement data.

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Given a set of measured data - $\mathbf{x}$, we seek to determine the T2 relaxation times that are present in it. With this we have $\theta(T_2)$, our distribution function of T2 values. This means we can use Bayes' theorem (eq. 1) to find the likely distribution of x's T2 relaxation times given that we know the measurement we would likely get for a speicifc T2 distribution $\theta(T_2)$.

$$P(\theta|\mathbf{x}) = \frac{P(\theta)P(\mathbf{x}|\theta)}{P(\mathbf{x})} \tag{1}$$

The individual parts of this crucial rule are as follows:

- $P(\theta)$ is the likeliness of what distribution we will get i.e. what rock we get.
- $P(\mathbf{x}|\theta(T_2))$ is the chance that given a specific T2 relaxation time distirbution, we would get the measured data, $\mathbf{x}$
- $P(\theta(T_2)|\mathbf{x})$ is the distirbution of our T2 relaxation times given what we've measured with $\mathbf{x}$. This is the genuine result we want.

The important part to construct is the priori, $P(\mathbf{x}|\theta(T_2))$. To do this we need to provide the a high SNR example of x to construct this and a proper $\theta(T_2)$. To construct this we can use eq. 2. This equation uses a expontial kernel K for the shape of our sample and multiplies it with a density function ($f(T_2)$) of T2 relaxation times. In a high SNR environment, we have a very accurate density function that we can use to construct the estimated $\hat{\mathbf{x}}$. We would form a series of these estimate measured data in a controlled environment outside of the use case.

$$\hat{\mathbf{x}} = Kf(T_2) \tag{2}$$

Now we want a numerical value to give us the closeness of our data with the set of hypothesies given to us. In order to do that, we require construct $P(\mathbf{x}|\theta(T_2))$ as a Gaussian distribution since we are looking for the probability that our actual $\mathbf{x}$ has a certain T2 relaxation distriubution generated from $\hat{\mathbf{x}}$. This can be done in the form of a multivariate Gaussian with $\mathbf{x} - \hat{\mathbf{x}} = \mathbf{x} - Kf(T_2)$ as the mean. This means that the more closer our hypothesis is to our measured data, the less bias there is and the higher our symbol is.

With our weighting formed, we can create our $P(\theta|\mathbf{x})$ density T2 distribution for the specific measured data. This can be done by forming a series of delta functions in the continuous space offset by the T2 times available 3 (where $\mathcal{N}(0, \sigma_\epsilon$ with noise standard deviation) is our non shifted Gaussian function. The set of weighted dirac functions can be convoluted with the Gaussian to give us a continuous function we can interpolate from. This allows us to create a distribution of T2 times specially for our current measured data using previous information in a high SNR environment. This would allow for a low SNR measurement to use high SNR data to construct the T2 distribution.

$$\hat{\theta}(T_2) = \sum_{n=0}^{N} w_n \delta(\tau_2 - T2_n) * \mathcal{N}(0, \sigma_\epsilon) \tag{3}$$