# Bayesian Model (Analytical Expression)

### Goal

To form an expression that may be directly used to estimate the density function

### Motivation

The main deliverable of the project is to test the viability of a Bayesian framework in estimation of bound fluid fraction. To do this, an analytical tractable equation must be presented

### Constraints

We have been given:

* Measurement data (m)
* Prior data Gaussian (mu\_f and C\_f)
* Measurement noise
* Exponential decay kernel

### Candidate Solutions

Only the proof that is made, we justify each point

### Experimental Work (Proof)

### Final Decision

Get the final f|m expression

# Construction of the Prior

## Forms of Interpolation

### Goal

* To interpolate the experimental data into the dimensionality required for the Bayesian framework to be used and compared

### Motivation

* We cannot compare between different techniques if we have explicit difference in the sizes of the measurement vectors and density function vectors. This compatibility levels the playing field

### Constraints

* The density function must be non-negative at all points
* Where the true density function is zero, the interpolated density should be zero
* The absolute error between the interpolated and true density functions should be minimised

### Candidate Solutions

1. Linear
2. Nearest
3. Next
4. Previous
5. Shape-perserving Piecewise Cubic Interpolation (PCHIP)
6. Modified Akima cubic Hermite interpolation
7. Spline

### Experimental Work (Proof)

Try the different techniques provided in the MATLAB function for a sample density function

### Final Decision

Should be PCHIP

## Estimation of the prior mean

### Goal

* To form the mean prior for each of the f(T2) points as an accurate estimator

### Constraints

* We only have the previous experimental measurements. We may have one density function that is a significant outlier

### Candidate Solutions

* Use the median
* Use the mean

### Experimental Work (Proof)

Using the Bayesian estimator expression – create a CDF of the rocks of the error

### Final Decision

## Estimation of the prior covariance

### Goal

To form an accurate model of the covariance between each of the density functions

### Constraints

### Candidate Solutions

1. Uniform Independent T2 density
2. Non-uniform independent T2 density
3. Dependent T2 density

### Experimental Work (Proof)

Using Bayesian model, create a CDF of the error given the equation

See where it is minimised

Create plot of heat map for covariance estimation – illuminates the estimated dependence

### Final Decision

# Integral Transform

## BFV

### Goal

To create the best integral transform for the bound fluid volume to deal with inter bin ‘leaking’

### Constraints

Each will have the same cost in terms of computation.

### Candidate Solutions

* Sharp BFV
* Tapered BFV

### Experimental Work (Proof)

* Plot the RMS of the error in BFV for different scaling of C\_f with alpha – its equivalent in the Bayesian context

### Final Decision

## Porosity

### Goal

To estimate the porosity in a closed form expression to compute the BFF

### Constraints

We know only measurement data and the density function we have made

### Candidate Solutions

* With estimate density function only integral transform
* Slope of the measurement data
* Combination of the two

### Experimental Work (Proof)

* Plot the error in porosity for a sample density function

### Final Decision

# Evaluation Metrics

## Accurate Form of Error

### Goal

We want to fairly evaluate each of the different estimators.

### Constraints

We have all of this experimental rock data but we want to prove the generality of the situation.

### Candidate Solutions

* Absolute error
* Squared error
* Normalised Root Mean Square Error

### Experimental Work (Proof)

### Final Decision

## Computation Time

### Goal

To assess how long each of the techniques take to compute

### Constraints

We cannot directly compute the O complexity unless we unpack all of the functions

* This is a secondary goal compared to estimation accuracy as long as its not too long

### Candidate Solutions

* Seconds taken
* O complexity

### Justification (Proof)

* We can make a useful comparative analysis with the seconds taken as long as it is on the same computer
* Must keep in mind that as MATLAB is dynamic, it will behave differently to a static embedded implementation.

### Final Decision