The Bayesian estimator proposed in this report has a comprehensive component of technical implementation to provide a strong conclusion of its effectiveness. These components include:

1. Other techniques proposed by previous studies for this problem domain,
2. The validation of the recreation of those competing techniques such that each of their distinct features can be compared and analysed,
3. A fair test procedure and algorithm that is used to compare and contrast the different estimators, and
4. The proposed estimator itself with consideration for its different input requirements compared to the previous techniques.

These several components are developed, prototyped, and analysed in MATLAB such that analysis is general and compatible with potential embedded implementation.

# Existing Techniques:

Reproducing the pre-existing techniques allows for analysis beyond using the density functions discussed within their publications. This means that the implementation and the validation of it accompany each other. Figure XX demonstrates the validation topology used by this project. This component provides additional potential perspective of estimation performance.

## Validation of Implementation:

The validation framework compares an extraction of the original estimate versus the reproduced estimate. This allows for an accurate perspective of any analysis made between the benchmark estimators and the proposed estimator.

The components of the validation framework process as follows:

* The image extractor uses an image of the model in the publication to acquire actual estimate of the density function and the actual density function. An off the shelf image extractor was used [].
* The interpolator discussed in Section XXX is utilised to allow for direct compatibility and comparability between the different frameworks.
* The estimator being tested develops an estimate of the density function
* The comparison allows for analysis of the difference in the true estimate and the recreated estimate. This takes the form of visual display and the numerical indicators.

## Inverse Laplace Transform Approximation (ILT)

The canonical benchmark for density function estimation is the ILT approximation detailed in [] and Section XXX. It was implemented as a self-contained function (in []) and returns a bound fluid fraction estimation for that methodology.

The validation of this function used the experimental conditions detailed in [] for models 1 to 4. This project explores one-dimensional density functions rather than two dimensional as detailed in Venkataramanan et al however. This meant that the re-implementation tended towards the one-dimensional case proposed by Butler et al instead.

### Time Axis

The time axis was assumed set to limits of 2\*echo spacing to 3 seconds as the simulation results in [] is bounded within these values. Interestingly, the 3 seconds maximum is beyond the space of the simulation, as it does not enough data points to depict up to that point. The justification for this was that we should not predict a density function outside the frequencies that we can detect. This sensitivity issue is apparent in the nature of this inversion problem, such as in [].

### Regularisation

The starting alpha value used before the iterative process has a large effect on the resulting function. For example, if the initial alpha were a very low number such as 0.1 to 1 (less than 10% the published regularisation average), the result would involve near inversion of an un-invertible matrix. For a very large initial value such as 1000 (over 1000% the published regularisation value), this was not the case. This led to the assumption that a large value initial regularisation alpha was sufficient to achieve convergence towards the typically smaller optimal alpha.

[figure demonstrating the difference of regularisation start for the same seed]

### Inconsistent SNR

The simulation implementation from Gruber et al is complicated by the inclusion of additional short pulses as well as the main pulse result. There are a series of thirty pulses repeated 10 times that increase the SNR of short relaxation times. This means that to reproduce the technique exactly, this component had to be reproduced as well. The algorithm used to implement this detail was:

1. Simulate the data for only 30 samples in the time domain 10 times
2. Average these all together with the first 30 samples of the ‘long’ pulse sequence
3. Concatenate this onto the measurement vector.
4. Multiply the kernel rows for the 30 sample points by the SNR increase given by the short pulse sequence.

### Scaling the Noise

The published noise standard deviation of 0.1 and SNR of 10 requires that the signal power be unity. However, the density functions benchmarked in publications do not have unity power. The porosity is not unity. This means to recreate these appropriately, the SNR held constant while noise standard deviation is reduced such that the SNR is requirement is satisfied. This implies that the SNR is the primary known value rather than the noise standard deviation.

## ILT+

This technique has a significant amount of complexity as it combines moment estimation in Venkataramanan et al [], tapered area estimation in Gruber et al [], and the initial ILT methods in Venkataramanan et al [] and Butler et al []. The implementation follows many of the same points as the ILT method. Additionally it also requires testing for the tapered areas and moment estimators that are used to augment the ILT approximation.

## Tapered Area Estimator

The tapered area estimator has a far simpler set up compared to the previous approximation techniques. This is due to it being fully self-contained and analytically tractable. It contains the exponential Haar transform detailed in [].

# Bayesian Technique

The implementation of the Bayesian technique is comparable to the tapered area estimator rather than the regularised least squares methods of the ILT and ILT+. This is due to the Gaussian assumption of the prior density function and the measurement data providing an analytical expression of the estimate density function (as discussed in Section XXX).

## Estimator Architecture

We adopt a computational sequence for the Bayesian to minimise unnecessary computation. The sequence is as follows:

1. We take the high quality known density functions and construct our prior mean and covariance before time sensitive procedures. The prior can be independently constructed before estimation
2. Create the estimate density function from the Bayesian prior in equation XXX
3. Feed this estimate into the integral transform that will provide over goal value

This can be very adaptive to lower computations power as the time consuming complex computation of the prior function does not have to be outside of time-constrained use.

# Test Environment

The test environment’s implementation required attention into providing immediate comparability and fairness. The implementation of the test environment is required to be such that it provides an illuminating and fair comparison of the different techniques.

## Test Architecture

The test architecture that compares each of estimators aims to demonstrate the uncertainty and typical performance of all of the techniques. In order to keep the Bayesian estimator blind from the exact rock it is testing, leave one-out cross validation is utilised for evaluation. Figure XXX details the architecture of the technique.

An extension of the test architecture can allow for an analysis of how different cut-off time lead to different results. This proves a perspective of where one variable is better suited than another value. This comes at the cost of computational time, as going over 30 rock samples over 30 different cut off times is a non-trivial amount of computation.

In addition to the test environment, utilising the estimators re-implemented there will also be visual comparison of different case study density functions. There are published results by Gruber et al [] that form a typical benchmark of typical density functions used for estimation. Even with a potential flaw in the re-implemented estimators, the Bayesian technique can still be plot over definitive previous results.

|  |  |
| --- | --- |
| Variable | Values |
| N2 (Number of time samples) | 5000 |
| Ny (number of porosity bins) | 30 |
| t\_e (sample period) | 200 mu sec |
| T2 axis bounds | 400mu - 3 |
| SNR\_linear | 10 |
| T\_c | 33 ms |