Climate Policies under Dynamic Factor Adjustment *

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Abstract

This paper develops and estimates a dynamic general equilibrium model of the global economy to investigate how imperfect and gradual adjustments of capital and labor across sectors affect the effectiveness of climate policies in reducing global emissions. The model's estimates indicate that both capital assets and workers exhibit low responsiveness to changes in climate policies, limiting the policies' ability to permanently and swiftly shift capital and labor out of the sector that extracts and processes raw fossil fuels. I demonstrate that a subsidy implemented by China on non-fossil fuel energy sources would boost consumption and investment in the targeted sector but may inadvertently raise global emissions due to increased aggregate output and investment. Imperfect factor mobility amplifies this effect by limiting the reallocation of capital and labor away from the sector that produces fossil fuels. My findings underscore the need to (i) combine a subsidy on non-fossil fuel energy sources with a carbon tax and (ii) improve capital and labor mobility to both reduce global emissions and enhance aggregate consumption.

JEL Classification: F18, Q56, Q58

Keywords: Climate Policy, Capital Investment, Labor Mobility, International Trade

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1 Introduction

Failure to meet the Paris Agreement's target of limiting global warming to 1.5°C above pre-industrial levels is projected to significantly increase the frequency and severity of extreme weather events, such as heat waves, heavy rainfall, and droughts.[1] A major obstacle to transitioning to a low-carbon economy is the challenge of replacing entrenched infrastructure and institutions, which slows the shift to renewable energy and keeps economies dependent on existing structures (Lee et al., 2023). Despite this concern in policy discussions, existing frameworks appear limited in assessing the effectiveness of climate policies in reducing global emissions when capital and labor adjust gradually and imperfectly across sectors in response to policy changes.[2]

This paper develops a quantitative, dynamic framework of the global economy to examine how imperfect and gradual adjustments of capital and labor across sectors influence the effectiveness of climate policies in reducing global emissions. The framework emphasizes two key margins of adjustment: (i) the extent of adjustment, which determines how capital and labor are reallocated across sectors in the long run in response to changes in climate policies, and (ii) the speed of adjustment, which governs the transition process.

The imperfect adjustment arises because capital and labor supply do not solely respond to sectoral differences in average rental rates of capital and wages. Each capital investment opportunity faces sector-specific idiosyncratic shocks, resulting in substantial differences in returns. For example, returns can vary widely across sectors due to differences in sector maturity and, within sectors, due to unique risks or characteristics tied to producers managing sector-specific assets (e.g., drilling rigs, hydroelectric dams). Similarly, workers decide which sector to enter based not only on wages but also on non-pecuniary factors, such as family considerations, personal tastes, and search frictions.

The gradual adjustment arises because the profitability of investment opportunities in each sector (i.e., capital efficiency) depends on past investments, and workers face mobility costs. Sector-specific capital efficiency builds on prior capital allocations, leading to persistent capital allocation and gradual capital adjustment (Aghion et al., 2016). Additionally, workers face costs (e.g., reloca-

[1] If emissions remain at 2023 levels, global temperatures could reach 1.5°C above pre-industrial levels within a decade and exceed 2°C in 27 years (Lee et al., 2023). The remaining carbon budget is estimated at 500 $GtCO_2$ for a 50% chance of limiting warming to 1.5°C and 1150 $GtCO_2$ for a 67% chance of staying below 2°C (Lee et al., 2023). Global emissions in recent years were approximately 35-37 $GtCO_2$.

[2] Imperfect factor mobility is an empirical regularity extensively documented in the international trade literature. For examples, see Artuç et al. (2010), Dix-Carneiro (2014), Dix-Carneiro and Kovak (2017).

tion and training costs) when moving between sectors, decelerating the speed of adjustment. Since capital and labor are complements in production, the gradual adjustment of either factor slows the mobility of the other, amplifying sluggishness in sectoral reallocation. I estimate the structural parameters governing these two margins of adjustment and show that both capital and labor exhibit low responsiveness to changes in factor prices and would adjust gradually to climate policy changes.

I begin by presenting motivational evidence on the imperfect and dynamic responses of capital and employment in the brown sector (i.e., the sector that extracts and processes raw fossil fuels) to environmental policy changes. Using data from the 2016 release of the World Input-Output Database (WIOD), I use the local projection method of Jordà (2005) to estimate the impulse response of capital and employment in the brown sector to an increase in the Environmental Policy Stringency (EPS) index.[3] I find that long-run adjustments in the brown sector to stricter environmental policies differ between low- and high-income countries: in high-income countries, capital in the brown sector declines by approximately 16 percentage points more than in low-income countries 8 years after policy implementation. Moreover, I show that short-run responses to stricter environmental policies differ from long-run responses, with capital in the brown sector in high-income countries declining by more than 2.8 times after 8 years compared to the initial drop. These findings suggest the presence of frictions and barriers that drive cross-country differences in both the short- and long-run responses of capital and labor to environmental policy changes.

To examine how imperfect factor mobility affects the impact of climate policies on global output and emissions, I develop a dynamic general equilibrium model with sectoral and international production linkages. The model features multiple sectors, divided into three groups: (i) a brown sector that directly uses raw fossil fuels for energy production, (ii) a non-brown energy sector that produces an alternative to fossil fuel-based energy using intermediate inputs such as solar panels and wind turbines, and (iii) non-energy sectors reliant on intermediate inputs from the brown and non-brown energy sectors. The use of raw fossil fuels by the brown sector generates carbon emissions, which reduce economy-specific environmental quality and, in turn, aggregate productivity (Barrage and Nordhaus, 2024).

The decisions of capitalists and workers to supply capital and labor across sectors depend on

[3] The Environmental Policy Stringency (EPS) index is a country-specific, internationally comparable measure constructed by Kruse et al. (2022) that assesses how strongly countries impose explicit costs (e.g., carbon taxes) or implicit costs (e.g., emissions limits) on environmentally harmful activities.

both factor prices and non-price considerations, which vary across sectors. Capitalists decide to accumulate aggregate capital and allocate each unit to the sector that (i) offers the highest rental rate and (ii) has the highest capital efficiency (i.e., provides the most profitable investment opportunity). Workers choose which sector to work in for the next period by selecting the sector that offers (i) the highest wage, (ii) the lowest moving cost, and (iii) the greatest idiosyncratic non-pecuniary utility.

I demonstrate that optimal sectoral capital allocation decisions depend on sector-specific rental rates, sector-specific capital efficiencies, and aggregate capital. The immediate response to climate policy changes hinges on the dispersion of capital efficiencies, reflecting differences in investment opportunities across and within sectors. Additionally, the dynamic and long-run response depends on the evolution of sector-specific capital efficiencies, driven by a dynamic agglomeration externality from past capital allocations (e.g., within-sector knowledge spillovers, learning-by-doing) that capitalists do not internalize.

Furthermore, I show that labor flows across sectors depend on future wage differentials, sector-specific non-pecuniary preferences, and bilateral mobility costs. On the one hand, the extent of labor adjustment to climate policy changes depends on the dispersion of workers' idiosyncratic preferences across sectors, which influences the strength of non-pecuniary motives for mobility. On the other hand, the speed of adjustment— or the magnitude of labor flows— depends on bilateral mobility costs. The optimal capital and labor supply conditions can be mapped to data to estimate the key structural parameters governing the extent and speed of adjustment: (i) within-sector dispersion of capital efficiencies, (ii) dynamic agglomeration externality, (iii) dispersion of non-pecuniary preferences, and (iv) bilateral mobility costs.

To illustrate the mechanisms behind the impact of climate policies, I focus on two types of policy instruments for reducing carbon emissions that are widely used in practice: a carbon tax and a subsidy on non-brown energy. A carbon tax, implemented either as a consumption tax on brown sector goods or on raw fossil fuels, reduces emissions by decreasing demand for the brown sector. In contrast, a consumption subsidy on non-brown energy reduces emissions by encouraging the reallocation of demand, capital, and labor from the brown sector to the non-brown energy sector.

The overall impact of climate policies on global emissions depends on both substitution and scale effects.[4] A carbon tax, which raises the price of brown sector goods, reduces emissions in two ways. First, through the substitution effect, demand, capital, and labor shift from the brown

[4] Refer to Stern (2004) for a detailed discussion of substitution and scale effects.

sector to the non-brown energy sector. Second, the scale effect further lowers emissions as higher intermediate energy costs reduce production in non-energy sectors. Similarly, a subsidy on non-brown energy, which lowers its price relative to brown sector goods, also reduces emissions through the substitution effect by shifting demand away from the brown sector. However, unlike the carbon tax, a subsidy can increase emissions through the scale effect, as lower energy input costs encourage higher production and energy demand in non-energy sectors.

I employ the Environmental Policy Stringency (EPS) index, which varies at the country-year level, as a demand shifter to estimate the capital and labor supply parameters. I leverage the notion that changes in the EPS have a heterogeneous impact across sectors due to their differing reliance on intermediate inputs from the brown sector. To estimate capital supply parameters, I utilize data from the World Input-Output Database 2016 Release, which includes information on capital stocks and capital compensation for 43 countries and 56 industries over the period 2000 to 2014. The dataset includes industry-level capital stock measures, constructed as a weighted average of eight asset types, reflecting the heterogeneity of capital use across sectors. For labor supply parameters, I merge individual-level longitudinal labor surveys from eight countries—the United States, China, India, France, the United Kingdom, Australia, South Korea, and Argentina. This merged dataset includes information on labor income, industry affiliation, and employment status over time, enabling the estimation of country-specific intersectoral mobility costs and the dispersion of non-pecuniary motives for mobility at a global level.

The estimates reveal (i) significant variation in capital efficiencies within and across sectors and (ii) strong non-pecuniary motives for worker mobility. Specifically, controlling for lagged sectoral capital allocation and economy-sector fixed effects, I find that a 1 percent increase in the sectoral rental rate of capital leads to only a 0.02 percent increase in capital allocation within the same period. Additionally, controlling for economy-origin and economy-destination sector fixed effects, I find that a 1 percent increase in sectoral wages increases annual labor inflows by only 0.55 percent. These findings underscore the significant role of non-price mechanisms in constraining the responsiveness of factor flows to changes in sectoral factor prices.

The estimates further show that workers in the brown sector face significant mobility costs equivalent to sacrificing at least 97.2 percent of their current real wage, indicating extreme difficulty in moving across sectors. Additionally, a 1 percent increase in capital allocation to a sector in one period is associated with a 1 percent increase in capital allocation to the same sector in the next period, highlighting the persistence of capital allocations across sectors.

In the last part of the paper, I use the estimated models to evaluate the impact of counterfactual policy scenarios on emissions and output. First, I compare the impact of unilateral climate policies on global emissions across models with varying degrees of capital and labor mobility but identical initial conditions. I calibrate the initial conditions of the models using data for the year 2022, or the most recent available year, on production, international trade, fossil fuel production, emissions, and global temperature. Then, I assess three counterfactual policies that China—the largest emitter of carbon dioxide—could implement.[5] The policies I consider are: (i) a subsidy for non-brown energy that removes its value-added tax, (ii) a carbon tax set to the level currently implemented by the EU, and (iii) both policies applied simultaneously. In each scenario, the policy intervention is unexpected and permanent.

The key result is that imperfect factor mobility significantly dampens the substitution effect of climate policies on global emissions. A subsidy for non-brown energy reallocates capital and employment to the non-brown energy sector, and this substitution effect is amplified with greater factor mobility. However, the subsidy lowers energy prices, stimulating consumption and increasing production in sectors that use energy inputs, which ultimately boosts aggregate output and investment. Part of this increase in aggregate capital spills over to the brown sector, leading to an increase in capital in the brown sector—a mechanism I refer to as the scale effect. Consequently, a non-brown energy subsidy leads to a net rise in global emissions due to increased energy demand and capital spillover to the brown sector. This emissions increase is greater in the baseline model with low factor mobility, as limited reallocation between sectors weakens the substitution effect. Specifically, the baseline model predicts at least a 118 percent higher cumulative emissions increase by 2030 and a 213 percent increase by 2100 compared to a model with perfect factor mobility.

I further examine the effects of unilateral climate policies on consumption at both the country and global levels. A unilateral carbon tax implemented by China results in a significant long-term decline in domestic consumption, as the tax depresses aggregate investment. Although the carbon tax initially boosts consumption through tax revenues, it leads to a long-term reduction due to increased intermediate input costs and lower investment. In contrast, global consumption rises as improved environmental quality follows China's substantial emission reductions. This outcome suggests the possibility of a Kaldor-Hicks improvement, where economies benefiting from these environmental gains could compensate China for its economic losses through international

[5] China accounted for 30.7 percent of global emissions in 2022 (Ritchie and Roser, 2020).

transfers.

On the other hand, the impact of a unilateral non-brown energy subsidy implemented by China on aggregate consumption depends on the strength of the dynamic agglomeration externality, which influences the evolution of capital efficiency. When the scale effect dominates the substitution effect, the non-brown energy subsidy increases global emissions and worsens environmental quality across all economies. However, if the dynamic agglomeration externality is strong, the consumption gains from internalizing this externality can offset the losses from deteriorating environmental quality. In this case, the government could reduce emissions and boost consumption by implementing both a non-brown energy subsidy and a carbon tax. Conversely, if the agglomeration externality is weak, a unilateral non-brown energy subsidy results in a decline in global consumption both in the short and in the long run.

Related Literature This paper contributes to the literature on the economics of climate change (Nordhaus, 1992; Golosov et al., 2014; Nordhaus, 2015; Hémous, 2016; Shapiro, 2021; Kortum and Weisbach, 2022; Cruz, 2023; Känzig, 2023; Cruz and Rossi-Hansberg, 2023; Arkolakis and Walsh, 2023; Xiang, 2023; Bilal and Känzig, 2024; Barrage and Nordhaus, 2024; Farrokhi and Lashkaripour, 2024). It closely relates to the work of Cruz and Rossi-Hansberg (2023), who examine how labor mobility frictions and agents' idiosyncratic non-pecuniary preferences for different regions influence the welfare impact of climate change. In contrast to the existing literature, which primarily focuses on labor mobility across regions, this paper incorporates and estimates the imperfect and gradual adjustments of both capital and labor across sectors, particularly within the energy sectors. These adjustments across sectors are crucial for understanding the impact of climate policies, which influence emissions through their effects on the energy sectors.

This paper also relates to a large body of literature on imperfect capital mobility, which has documented that capital adjustments are costly and occur gradually over time (Dixit, 1989; Dixit and Pindyck, 1994; Ramey and Shapiro, 2001; Dix-Carneiro and Kovak, 2017; Lanteri, 2018; Artuc et al., 2022; Lanteri et al., 2023). The key contribution of the paper is the tractable incorporation of imperfect and dynamic capital adjustment into the analysis of climate policies. This is achieved by extending the capital allocation framework of Kleinman et al. (2023) to a multi-sector setting and incorporating dynamic capital adjustments driven by the dynamic agglomeration externality, in the spirit of the directed technical change literature (Acemoglu et al., 2012, 2016; Aghion et al., 2016; Acemoglu et al., 2023). This approach enables the estimation of key parameters that govern the extent and speed of capital adjustment in response to climate policies. The extent and speed of adjustment are especially important for understanding the impact of climate policies: the low responsiveness of capital and labor to policy changes affects the long-run allocation of capital and employment across sectors, while the speed of adjustment influences long-term environmental quality, given that a significant portion of emissions remains permanently in the atmosphere.

Moreover, this paper relates to the quantitative international trade literature that analyzes the global impact of policies (Armington, 1969; Anderson, 1979; Eaton and Kortum, 2002; Arkolakis et al., 2012; Head and Mayer, 2014; Caliendo and Parro, 2015). Specifically, it contributes to the trade literature examining the impact of policies when workers gradually and imperfectly adjust to policy changes (Artuç et al., 2010; Dix-Carneiro, 2014; Artuç and McLaren, 2015; Caliendo et al., 2019; Traiberman, 2019; Caliendo et al., 2021; Dix-Carneiro et al., 2023). Understanding the extent and speed of labor adjustment in major economies responsible for global emissions is central to this paper. Hence, compared to previous work that estimated labor supply parameters for a single country, I merge longitudinal labor surveys from eight major economies to estimate country-level bilateral labor mobility costs and non-pecuniary motives for worker mobility.

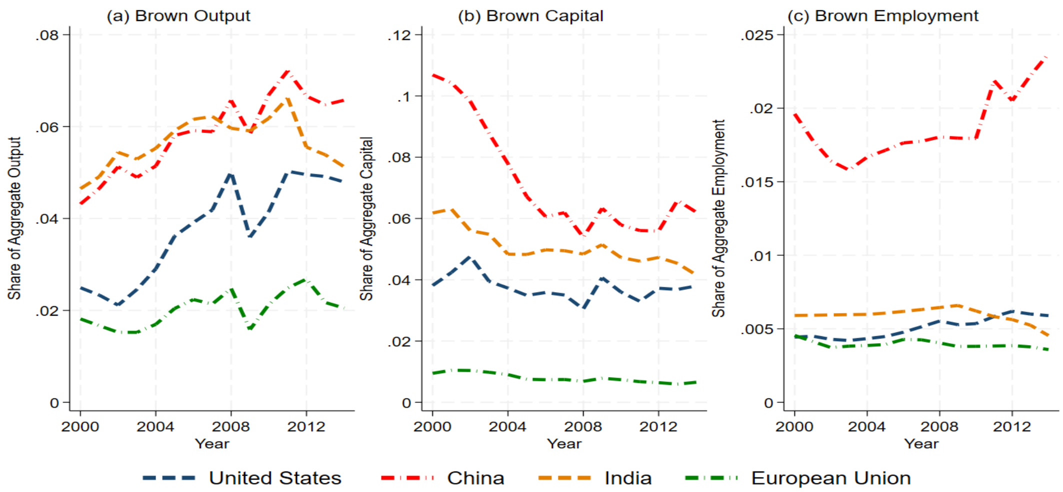
The remainder of the paper is organized as follows. Section 2 documents the motivating facts. Section 3 introduces a dynamic general equilibrium model of a global economy that features imperfect capital and labor mobility, production linkages, and environmental damage due to carbon emissions. In Section 4, I describe the data, the estimation procedure, and the estimation results. Section 5 presents the quantitative results. Section 6 concludes the paper.

2 Motivating Facts

In this section, I present descriptive facts about fossil fuel-producing industries in major economies and provide motivational evidence for two key margins of imperfect capital and labor adjustment in response to changes in environmental policies: (i) the extent of adjustment and (ii) the speed of adjustment.

Brown Sector The primary data source for this analysis is the World Input-Output Database (WIOD) 2016 Release (Timmer et al., 2015), covering 43 countries and 56 industries from 2000 to 2014. The dataset provides industry-level data on gross output, capital stock, employment, capital compensation, and labor compensation.

Figure 1: Output, Capital, and Employment in the Brown Sector



Notes: Subplot (a) shows the gross output in the brown sector as a share of aggregate gross output in the United States, China, India, and the European Union from 2000 to 2014. Subplot (b) shows the capital stock in the brown sector as a share of aggregate capital stock in the same regions from 2000 to 2014. Subplot (c) shows employment in the brown sector as a share of aggregate employment in these regions over the same period.
Source: World Input-Output Database 2016 Release.

In this paper, I define the brown sector as industries that extract and process raw fossil fuels, such as crude oil, natural gas, and coal. As a producer of fossil fuel energy, the brown sector is directly responsible for global emissions. Following O'Mahony and Timmer (2009), it is classified as a subset of the energy sector, including the mining and quarrying industry and the industry that manufactures coke and refined petroleum products.

As shown in subplot (a) of Figure 1, the gross output of the brown sector accounted for 2 to 6 percent of aggregate output in four major economies responsible for approximately 60 percent of global emissions in 2022: the United States (US), China, India, and the European Union (EU). The subplot also indicates that these output shares remained stable or even increased from 2000 to 2014, suggesting that reliance on fossil fuel products persisted before the Paris Agreement, despite the adoption of the Kyoto Protocol in 1997.

Subplots (a) and (b) demonstrate that the share of aggregate capital allocated to the brown sector is at least double that of employment in all economies, with this pattern remaining stable post-2010. This highlights that the brown sector is capital-intensive and downsizing it will require reallocating more capital than labor.

Table 1: Summary Statistics for Annual Changes in EPS ($EPS_{n,t} - EPS_{n,t-1}$)

Statistic	**EPS**
25th Percentile	0.000
Median (50th)	0.000
75th Percentile	0.133
Minimum	-1.400
Maximum	1.112
Mean	0.079
Variance	0.066
Observations	771

Notes: Summary statistics for annual changes in the environmental policy stringency index (EPS) for 40 countries from 1990 to 2020.
Source: OECD (Kruse et al., 2022).

Environmental Policy To understand the impact of changes in environmental policies on capital and employment in the brown sector, I employ the Environmental Policy Stringency index (EPS) constructed by the OECD (Kruse et al., 2022), which measures the degree to which environmental policies impose an explicit or implicit price on polluting activities. It covers 40 countries from 1990 to 2020 and is constructed by assessing 13 types of environmental policies divided into three categories: market-based, non-market-based, and technology support policies. Each policy is assigned a score ranging from 0 to 6, where 0 indicates the weakest implementation and 6 indicates the most stringent implementation.[6] The index $EPS_{n,t}$ for country n in year t is computed as the average of the scores assigned to each environmental policy.

Table 1 presents the summary statistics for the annual changes in $EPS_{n,t}$, as well as those for the market-based EPS and non-market-based EPS. The table shows that changes in $EPS_{n,t}$ are rare, with the median being zero. Motivated by this observation, I define the environmental policy

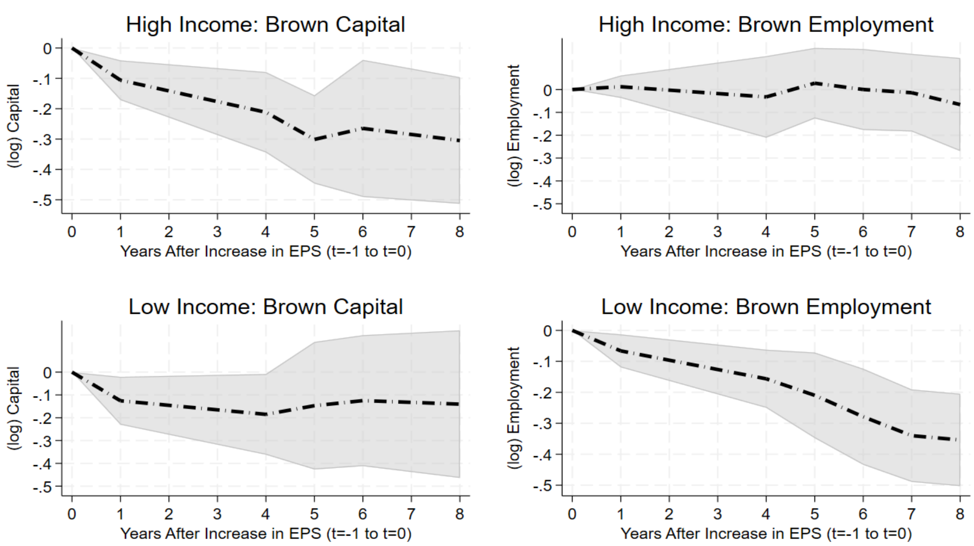
[6]Refer to the Appendix A.7 for details on thresholds for each policy.

shock $x_{n,t}$ as follows:

$$x_{n,t} = \mathbb{1}[EPS_{n,t} - EPS_{n,t-1} > 0, EPS_{n,t-1} - EPS_{n,t-2} = 0, EPS_{n,t-2} - EPS_{n,t-3} = 0].$$

In words, a country n experiences an environmental policy shock in year t if $EPS_{n,t}$ increases relative to the previous year. To focus on sudden and likely unanticipated policy changes, I restrict the analysis to the set of environmental policy changes where $EPS_{n,t}$ remained unchanged for the two years preceding the increase.

Figure 2: The Impact of the Environmental Policy Shock on the Brown Sector



Notes: The top plots show the impulse responses of capital and employment in the brown sector following an increase in EPS for countries with above-average gross output per capita. The bottom plots display the impulse responses of capital and employment in the brown sector for countries with below-average gross output per capita. Standard errors are clustered at the country-industry level. The shaded area represents the 95% confidence interval.
Source: WIOD 2016 Release, OECD (Kruse et al., 2022).

Empirical Specification I employ the local projection method of Jordà (2005) to estimate the impulse response of capital and employment in industry j to the environmental policy shock. I specify the regression specification as follows:

$$\underbrace{y^j_{n,t+h}}_{\text{Outcome}} = \alpha + \underbrace{\beta_h}_{\text{Average Effect}} \cdot \underbrace{x_{n,t}}_{\text{Policy Change}} + \underbrace{\gamma_h}_{\text{Differential Effect}} \cdot x_{n,t} \times \mathbb{1}[j = \text{Brown Sector}]$$

$$+ \underbrace{\delta_{n,t}}_{\text{Country-Year FE}} + \sum_{k=1}^{2} \lambda_k \underbrace{y_{n,t-k}}_{\text{Lagged Outcome}} + \underbrace{\Psi^j_{n,t}}_{\text{Vector of Controls}} + \varepsilon^j_{n,t}$$

where $y^j_{n,t+h}$ is the outcome variable of interest (e.g., log capital, log employment) h years after the shock; β_h captures the average effect of the shock on the outcome variable in year $t + h$ across all industries; γ_h captures the differential impact of the shock on the outcome variable in the brown sector in year $t + h$; $\delta_{n,t}$ are country-year fixed effects; $\{y_{n-k}\}$ are lagged outcomes, and $\Psi^j_{n,t}$ is a vector of controls such as k-lagged annual changes in $EPS_{n,t}$, k-lagged log industry output, k-lagged log wage, capital input share, k-lagged capital input share where k ranges from 1 to 2; $\varepsilon^j_{n,t}$ is a residual term.

Results Figure 2 displays the coefficients γ_h for capital and employment in the brown sector, where h ranges from 0 to 8. The top plots show the impulse responses of capital and employment in the brown sector following an increase in EPS for countries with above-average gross output per capita (high-income countries). The bottom plots show the corresponding responses for countries with below-average gross output per capita (low-income countries).

The figure highlights notable differences in the extent of adjustment between the two groups. In low-income countries, capital in the brown sector declines by approximately 15 percent eight years after the shock, while in high-income countries, the decline reaches nearly 35 percent. Looking at employment responses in the brown sector, high-income countries experience a drop of about 7 percent, whereas employment in low-income countries falls by approximately 42 percent.

Additionally, the figure demonstrates that the full impact of environmental policy on capital and employment takes time to materialize. For example, in high-income countries, the decline in capital after 8 years is more than 2.8 times larger than the decline after 1 year. Similarly, in low-income countries, the decline in employment after 8 years is over 5.3 times the initial drop.

These empirical findings indicate that long-run adjustments in both capital and labor differ across countries, suggesting country-specific barriers and frictions that prevent these factors from perfectly responding to environmental policy changes. Additionally, capital and labor respond gradually to such policy changes. Motivated by these findings, the next section presents a quantitative framework to assess the impact of environmental policies on global emissions when factor adjustments are imperfect and gradual.

3 Model

In this section, I develop a dynamic general equilibrium model of the global economy to examine the effects of climate policies on emissions, accounting for the imperfect and gradual adjustments of capital and labor to environmental policy changes. Subsection 3.1 outlines the setting. Subsection 3.2 describes consumers' demand for sector-specific differentiated products as in Armington (1969) and the aggregation of these products into sectoral, energy, and final goods. Subsection 3.3 then extends the production framework of Caliendo and Parro (2015) to explicitly include energy production. Subsection 3.4 links production to carbon emissions and environmental quality, following the work of Cruz and Rossi-Hansberg (2023) and Barrage and Nordhaus (2024). Subsection 3.5 extends the capital allocation framework developed in Kleinman et al. (2023) to a multi-sector setting, where capitalists accumulate aggregate capital and allocate capital assets across sectors; furthermore, the efficiency of using capital in each sector evolves based on past capital allocation, motivated by the empirical findings of Aghion et al. (2016) and Bradt (2024). Subsection 3.6 describes the workers' dynamic problem which follows Artuç et al. (2010). Finally, Subsection 3.7 describes the role of governments in regulating emissions and implementing environmental policies. The theoretical derivations for all the equations that appear in this section are provided in Appendix A.10.

3.1 Setting

Time is discrete and is indexed by $t \in \{0, 1, ...\} \equiv \mathcal{T}$. The world economy consists of N economies, indexed by $n, m \in \mathcal{N}$, where $\mathcal{N}$ represents the set of economies. Each economy contains J sectors, indexed by $j, k \in \mathcal{J}$, where $\mathcal{J}$ denotes the set of sectors. I categorize these sectors into three categories: non-employment (NE) sector, which does not produce output; energy sectors, denoted by $\mathcal{J}_E$, which produce energy goods; and non-energy sectors, denoted by $\mathcal{J}_N$, which use energy inputs for production. The set $\mathcal{J}_E$ includes both the brown (B) sector, which extracts and processes raw fossil fuels such as crude oil, natural gas, and coal, and the non-brown energy (NB) sector, which produces energy without directly extracting and processing raw fossil fuels. Motivated by the policies commonly implemented in practice, I limit the government's policy instruments to the following: (i) a consumption tax on the brown sector ($\tau^{B}_{n,t}$), (ii) consumption taxes on raw fossil fuels ($\tau^{f,h}_{n,t}$), (iii) a consumption subsidy on the non-brown energy sector ($\tau^{NB}_{n,t}$), and (iv) import tariffs ($\rho^{j}_{nm,t}$).

Within each sector, a continuum of perfectly competitive producers (i.e., firms) produce economy-sector-specific varieties.[7] The numeraire of the model is one unit of final good in economy n_0, i.e., $P_{n_0,t} = 1$ for all $t \in \mathcal{T}$. Within each economy, a domestic market for riskless assets exists where only domestic agents can participate. Finally, there is no aggregate uncertainty; thus, all agents possess perfect foresight over aggregate variables.

3.2 Demand

Sectoral Goods Sectoral goods aggregate sector-specific products (or varieties) differentiated by the economy of origin (Armington, 1969; Dixit and Stiglitz, 1977). These varieties are either (i) consumed by final goods consumers or (ii) used as intermediate inputs for the production of varieties. Formally, the sectoral good $Y^{j}_{n,t}$ in sector j of economy n at time t is a Constant Elasticity of Substitution (CES) aggregate of sector-specific varieties $\{y^{j}_{nm,t}\}$ produced in economy m and consumed in economy n:

$$Y^{j}_{n,t} = \left(\sum_{m \in \mathcal{N}} \left(y^{j}_{nm,t} \right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \tag{1}$$

where $\sigma > 1$ dictates the elasticity of substitution across varieties.

International Trade The share $\pi^{j}_{nm,t}$ of sector j expenditures of economy n spent on varieties produced in sector j of economy m is given by:

$$\pi^{j}_{nm,t} \equiv \frac{p^{j}_{nm,t} y^{j}_{nm,t}}{P^{j}_{n,t} Y^{j}_{n,t}} = \left(\frac{d^{j}_{nm,t} p^{j}_{nm,t}}{P^{j}_{n,t}} \right)^{1-\sigma} \tag{2}$$

[7] I assume perfect competition to avoid distortions due to market power. In a model with market power, government intervention through production subsidies would be necessary to boost production and restore efficiency. Since this paper focuses on environmental externalities, I abstract from market power considerations.

where $p^j_{nm,t}$ denotes the price of variety produced in sector j of economy m and consumed in economy n at time t; $P^j_{n,t}$ is the sector-level price index in economy n at time t given by $P^j_{n,t} \equiv \Big(\sum_{m\in\mathcal{N}}(p^j_{nm,t})^{1-\sigma}\Big)^{\frac{1}{1-\sigma}}$.

The term $d^j_{nm,t} \geq 1$ captures the sector-level bilateral iceberg trade cost of shipping goods from economy m to economy n, which can be further decomposed into the bilateral import tax $e^j_{nm,t}$ and an exogenous bilateral trade cost $\varrho^j_{nm,t}$, i.e., $d^j_{nm,t} \equiv e^j_{nm,t}\varrho^j_{nm,t}$ (Krugman, 1980).[8] The elasticity of trade flows with respect to changes in trade costs (i.e., trade elasticity) is $1-\sigma$. Equation (2) is commonly referred to as the gravity equation in the international trade literature (Armington, 1969; Anderson, 1979).

Energy Composite Goods Energy composite goods are bundles that represent consumers' demand for energy goods produced by brown (B) and non-brown energy (NB) sectors. The energy composite goods are either (i) consumed by final goods consumers, or (ii) used as a factor of production in sectors other than the brown sector. Formally, the energy composite good $Y^E_{n,t}$ in economy n at time t is a Constant Elasticity of Substitution (CES) aggregate of the sectoral goods $Y^{NB}_{n,t}$ and $Y^B_{n,t}$:

$$Y^E_{n,t} = \Big((\alpha^{NB}_n)^{\frac{1}{\eta_1}}(Y^{NB}_{n,t})^{\frac{\eta_1-1}{\eta_1}} + (\alpha^B_n)^{\frac{1}{\eta_1}}(Y^B_{n,t})^{\frac{\eta_1-1}{\eta_1}}\Big)^{\frac{\eta_1}{\eta_1-1}} \tag{3}$$

where the weights α^{NB}_n and α^B_n represent the reliance of economy n on non-brown energy and brown sectors, respectively, with $\alpha^{NB}_n + \alpha^B_n = 1$; the parameter η_1 governs the elasticity of substitution across these energy goods. Equation (3) highlights that an economy can reduce emissions through substitution if brown and non-brown energy goods are substitutes, i.e., $\eta_1 > 1$.

The energy price index $P^E_{n,t}$ is given by:

$$P^E_{n,t} = \left(\alpha^{NB}_n\Big((1+\tau^{NB}_{n,t})P^{NB}_{n,t}\Big)^{1-\eta_1} + \alpha^B_n\Big((1+\tau^B_{n,t})P^B_{n,t}\Big)^{1-\eta_1}\right)^{\frac{1}{1-\eta_1}} \tag{4}$$

where $\tau^{NB}_{n,t}$ and $\tau^B_{n,t}$ are consumption taxes imposed on the non-brown energy sector and the brown sector, respectively. Equation (4) shows that the government can shift demand away from the brown sector, which is referred to as the *substitution effect*, by either increasing the consumption

[8] Also, the triangle inequality holds, i.e., $d^j_{nm,t} \leq d^j_{nh,t} \cdot d^j_{hm,t}$ for all $m, n, h \in \mathcal{N}$.

tax on brown goods, $\tau_{n,t}^{B}$ (hereafter referred to as a carbon tax), or reducing the consumption tax on non-brown energy goods, $\tau_{n,t}^{NB}$ (hereafter referred to as a non-brown energy subsidy).

Raising the carbon tax not only shifts demand away from the brown sector but also reduces overall demand for the energy composite good. This effect, which I refer to as the negative *scale effect*, further reduces demand for the brown sector. Conversely, increasing the subsidy for non-brown energy raises demand for the energy composite good. This effect, which I refer to as the positive *scale effect*, increases demand for the brown sector.

Final Goods Final goods are bundles of sectoral and energy composite goods consumed by workers and capitalists, and used by capitalists for investment. Formally, the final good $Y_{n,t}$ in economy n at time t is a Cobb-Douglas aggregate of sectoral goods $\{C_{n,t}^{j}\}$ produced in non-energy sectors and the energy composite good $C_{n,t}^{E}$:

$$Y_{n,t} = \mathcal{D}_n(\mathcal{C}_t)\Big(\prod_{j\in\mathcal{J}_N}(C_{n,t}^{j})^{\alpha_n^j}\Big)^{\alpha_n}(C_{n,t}^{E})^{1-\alpha_n}. \tag{5}$$

where α_n^j governs the share of final goods expenditure spent on non-energy sector j, with $\sum_{j\in\mathcal{J}_N}\alpha_n^j = 1$; $\alpha_n \in [0,1]$ represents the share of total final goods expenditure allocated to non-energy goods.

Following Barrage and Nordhaus (2024), environmental quality $\mathcal{D}_n(\mathcal{C}_t)$, which depends on the global stock of carbon dioxide $\mathcal{C}_t$, influences the total factor productivity (TFP) of an economy. Consequently, it affects consumption by both workers and capitalists, as well as capitalists' investment decisions. This term will be explained in detail in Section 3.4. Since consumers and producers do not account for the impact of their activities on the global carbon stock, a *global environmental externality* arises, opening the door for government intervention to reduce global emissions and prevent the deterioration of environmental quality.

Materials Materials are bundles of sectoral goods reflecting non-energy intermediate inputs used for production.[9] Formally, the material $M_{n,t}^{j}$ in sector j of economy n is a Cobb-Douglas aggregate

[9] Materials are included in the model for two main reasons. First, intermediate input expenditures account for 20 to 70 percent of gross revenue across all sectors in all economies, making it a quantitatively significant feature necessary to match the data. Second, materials capture the interconnections between sectors through production linkages. They illustrate how changes in the output price of one sector affect the economy by altering the input prices of other sectors that use these outputs for production. This is crucial for analyzing the impact of policies targeting specific sectors.

of sectoral goods $\{M_{n,t}^{j,k}\}_{k\in\mathcal{J}}$: $M_{n,t}^{j} = \prod_{k\in\mathcal{J}} \left(M_{n,t}^{j,k}\right)^{\gamma_n^{j,k}}$ where intermediate input shares satisfy $\sum_{k\in\mathcal{J}} \gamma_n^{j,k} = 1$.

3.3 Production

Variety Production The output $y_{n,t}^{B}$ of brown varieties produced in economy n at time t is produced using a Cobb-Douglas function that takes fossil fuel $F_{n,t}$ as an input for production in addition to labor $L_{n,t}^{B}$, capital $K_{n,t}^{B}$, and materials $M_{n,t}^{B}$:

$$y_{n,t}^{B} = z_{n,t}^{B}\Big((F_{n,t}^{B})^{\psi_n^{B,F}}(L_{n,t}^{B})^{\psi_n^{B,L}}(K_{n,t}^{B})^{\psi_n^{B,K}}\Big)^{\gamma_n^{B,V}}\Big(M_{n,t}^{B}\Big)^{1-\gamma_n^{B,V}} \tag{6}$$

where $z_{n,t}^{B}$ is the total factor productivity of the variety producer in sector B of economy n at time t, and input shares satisfy $(1-\gamma_n^{B,V}) + \sum_{h\in\{L,K,F\}} \gamma_n^{B,V}\psi_n^{B,h} = 1$.

In contrast to producers in the brown sector, the output $y_{n,t}^{j}$ produced in sector $j \in \mathcal{J}\setminus\{B\}$ of economy n at time t requires energy inputs— as shown in Equation (3)— rather than raw fossil fuel inputs.[10] In particular, the production function is expressed as a Cobb-Douglas function that takes energy composite $Q_{n,t}^{j,E}$, labor $L_{n,t}^{j}$, capital $K_{n,t}^{j}$, and materials $M_{n,t}^{j}$ as inputs:

$$y_{n,t}^{j} = z_{n,t}^{j}\Big((Q_{n,t}^{j,E})^{\psi_n^{j,E}}(L_{n,t}^{j})^{\psi_n^{j,L}}(K_{n,t}^{j})^{\psi_n^{j,K}}\Big)^{\gamma_n^{j,V}}\Big(M_{n,t}^{j}\Big)^{1-\gamma_n^{j,V}} \tag{7}$$

where $z_{n,t}^{j}$ is the total factor productivity of the variety producer in sector j of economy n at time t, and input shares satisfy $(1-\gamma_n^{j,V}) + \sum_{h\in\{L,K,E\}} \gamma_n^{j,V}\psi_n^{j,h} = 1$.

Fossil Fuel Extraction Variety producers in the brown sector of economy n use fossil fuel inputs $F_{n,t}$ at time t, which are a Constant Elasticity of Substitution (CES) composite of crude oil $f_{n,t}^{o}$, natural gas $f_{n,t}^{g}$, and coal $f_{n,t}^{c}$:[11] $F_{n,t} = \left(\sum_{h\in\{o,g,c\}}(\omega_n^{h})^{\frac{1}{\eta_2}}\left(f_{n,t}^{h}\right)^{\frac{\eta_2-1}{\eta_2}}\right)^{\frac{\eta_2}{\eta_2-1}}$ where the weights

[10] The model features roundabout production where the non-brown energy producers can substitute between intermediate energy inputs produced by the brown sector and the non-brown energy sector. As the price of non-brown energy goods decreases relative to the price of brown goods, the non-brown energy producers rely less on the inputs produced by the brown sector through substitution.

[11] Including different types of fossil fuels is necessary to capture differing carbon emission trends across countries due to the changing composition of emissions. In 2022, more than 65% of the emissions in China and India were from coal, while less than 25% of the emissions in the United States and the European Union were from coal, driven by a shift to oil and natural gas.

$\{\omega_n^h\}_{h\in\{o,g,c\}}$ represent the reliance of economy n on different types of fossil fuels for production with $\sum_{h\in\{o,g,c\}}\omega_n^h = 1$. I assume that crude oil, natural gas, and coal are non-tradable in their raw form and become tradable only after being refined by producers in the brown sector.

The parameter η_2 governs the elasticity of substitution across these fossil fuels. When fossil fuels are substitutes (i.e., $\eta_2 > 1$), fuel-specific taxes $\{\tau_{n,t}^{f,h}\}_{h\in\{o,g,c\}}$ can discourage the use of carbon-intensive fossil fuels (e.g., coal) and reduce carbon emissions.

To make the model tractable, I assume that either state-owned enterprises or perfectly competitive and atomistic firms extract each type of fossil fuel.[12] These firms solve static extraction problems. Consequently, the before-tax price $s_{n,t}^h$ of fossil fuel type $h \in \{o, g, c\}$ in economy n at time t equals the marginal cost of extraction φ_n^h, measured in units of the final good: $s_{n,t}^h = \varphi_n^h P_{n,t}$ where $P_{n,t}$ denotes the price of the final good in economy n at time t. Hence, the price of a fossil fuel input $s_{n,t}$ in economy n at time t is given by:

$$s_{n,t} = \left(\sum_{h\in\{o,g,c\}} \omega_n^h \Big((1+\tau_{n,t}^{f,h})s_{n,t}^h\Big)^{1-\eta_2}\right)^{\frac{1}{1-\eta_2}} \tag{8}$$

where $\tau_{n,t}^{f,h}$ represents the consumption tax specific to fossil fuel type h. Equation (8) shows that the government can reduce carbon emissions through the use of fuel-specific consumption taxes that discourage the use of fuels emitting more carbon dioxide per unit (e.g., coal).

Finally, the law of motion for the reserve $D_{n,t}^h$ of fossil fuel type $h \in \{o, g, c\}$ is given by: $D_{n,t+1}^h = D_{n,t}^h - f_{n,t}^h$ with the constraint $D_{n,t+1}^h \geq 0$. The existence of exhaustible fossil fuel resources implies that the steady state of the global economy can only be reached either when (i) fossil fuel reserves in all economies are depleted, or (ii) the non-brown energy sector becomes extremely productive, allowing the non-brown energy sector to completely, or asymptotically, dominate the brown sector in all economies. In both scenarios, global emissions will be zero or close to zero, and climate policies will no longer be necessary. This highlights the importance of understanding the impact of climate policies during the transition period, not just in the long-run steady state.

[12] A more realistic model would involve fossil fuel extracting firms with market power solving dynamic extraction problems, considering the future path of fossil fuel reserves. However, this is beyond the scope of this paper, as such an inclusion would introduce additional externalities and inefficiencies (e.g., resource depletion externalities, price distortion). For quantification, I account for any observed differences between retail prices and supply costs of fossil fuel using the fuel-specific consumption tax, which may also reflect firm markups.

3.4 Carbon Emissions and Environmental Quality

Carbon dioxide (CO_2) emissions arise from the consumption of fossil fuels $\{f^h_{n,t}\}$ used in the brown sector. Specifically, consuming one unit of fossil fuel of type h emits ϕ^h units of CO_2. Hence, the total carbon emissions $E_{n,t}$ due to fossil fuel production in economy n at time t is given by: $E_{n,t} = \sum_{h\in\{o,g,c\}} \phi^h \cdot f^h_{n,t}$. The measure $E_{n,t}$ represents territorial carbon emissions based on fossil fuel production within the economy.[13] The global CO_2 emissions at time t, denoted by E_t, are given by: $E_t = \sum_{n\in\mathcal{N}} E_{n,t}$.

Following Cruz and Rossi-Hansberg (2023), I assume the global stock of carbon dioxide $\mathcal{C}_t$ in the atmosphere evolves as follows: $\mathcal{C}_t = \mathcal{S} + \sum_{\tau=T_{min}}^{t}(1-\delta^C_{t-\tau})\Big(E_\tau + E^L\Big)$ where $\mathcal{S}$ denotes the pre-industrial stock of carbon dioxide; T_{min} denotes the start period for measuring carbon emissions; $(1-\delta^C_{t-\tau})$ represents the fraction of carbon emissions at time τ that remains in the atmosphere at time t, and E^L denotes exogenous carbon emissions due to land-use change (e.g., deforestation, urbanization, desertification).

Furthermore, following Golosov et al. (2014), the global average temperature $\mathbb{T}(\mathcal{C}_t)$ at time t relative to the global average temperature at time T_{min} is approximated as follows: $\mathbb{T}(\mathcal{C}_t) = \lambda_T \log_2\left(\frac{\mathcal{C}_t}{\mathcal{S}}\right)$ where λ_T governs the increase in the global average temperature due to the increase in the global stock of carbon.

Following Barrage and Nordhaus (2024), I assume the economy-specific environmental quality function $\mathcal{D}_n(\cdot)$ decreases non-linearly with the global average temperature $\mathbb{T}(\mathcal{C}_t)$: $\mathcal{D}_n(\mathcal{C}_t) = 1 - \xi_n\Big(\mathbb{T}(\mathcal{C}_t)\Big)^2$ where $\xi_n > 0$ captures the environmental damage specific to economy n resulting from changes in global average temperature. The convexity of this function reflects the non-linear impact of rising global carbon stock on the climate system and the global economy, encompassing effects such as natural disasters and tipping points.

3.5 Capitalists

Now, I describe the capitalists' problem and the determinants of the extent and the speed of capital adjustment across sectors. Within each economy, there are infinitely-lived capitalists of measure one who are assumed to be homogeneous in their initial aggregate capital $W_{n,0}$. Given this homogeneity, I simplify the analysis by considering an infinitely-lived representative capitalist.

[13] In Appendix A.12.6, I present an alternative measure of carbon emissions based on the economy's total consumption of brown products, which includes both domestically produced and imported products. However, since countries typically report territorial carbon emissions, I use the production-based measure for the analysis.

The representative capitalist in economy n aggregates all capital income $R_{n,t}W_{n,t}$, and government transfers $\Omega_{n,t}$ and makes collective decisions on aggregate consumption $C^K_{n,t}$, aggregate capital accumulation $W_{n,t+1}$, and capital allocation across sectors $\{K^j_{n,t}\}$. For a given aggregate capital $W_{n,t}$ at time t, the capitalist solves a static discrete choice problem, similar to Kleinman et al. (2023), to allocate each unit of capital to the sector that generates the largest return.[14] Then, the representative capitalist makes a dynamic decision to accumulate aggregate capital.

Capital Allocation At each period t, the representative capitalist in economy n holds $W_{n,t}$ units of aggregate capital, which evolves according to the following law of motion:

$$W_{n,t+1} = (1 - \delta^k_n)W_{n,t} + I^k_{n,t} \tag{9}$$

where $I^k_{n,t}$ represents aggregate investment in units of the final good at time t, and $\delta^k_n \in [0, 1]$ denotes the economy-specific capital depreciation rate.

Aggregate capital in economy n at time t, $W_{n,t}$, consists of capital assets with heterogeneous returns across sectors, within sectors, and over time (Gonçalves et al., 2020; Smith et al., 2023). This assumption captures the notion that returns may differ across sectors due to variations in sector maturity and within sectors depending on how efficiently investments can be utilized. For instance, returns on investment might be high in the fossil fuel sector if infrastructure like oil rigs and refineries is already in place, while returns in sectors manufacturing solar panels or wind turbines could be low if essential infrastructure, such as wafer or turbine production facilities, is not yet in place.

Even within the same sector, capital returns can differ due to variations among producers in their ability to use capital efficiently. Additionally, capital returns fluctuate randomly over time, driven by idiosyncratic factors such as changes in firm-investor match quality and the stochastic costs of searching for the most efficient producers to invest in (Hortaçsu and Syverson, 2004).

To tractably capture this notion, I assume that the return on each unit of capital in sector j depends on (i) the sector-specific rental rate of capital $r^j_{n,t}$ and (ii) the sector-specific capital efficiency $\zeta^j_{n,t}$. Formally, each infinitesimal unit of aggregate capital $W_{n,t}$ is characterized by its profile $\vec{\zeta}_{n,t} = \{\zeta^j_{n,t}\}_{j\in\mathcal{J}}$, which contains the idiosyncratic capital efficiency for each sector, independently

[14] Kleinman et al. (2023) utilize this modeling approach in a one-sector model of international trade and capital holdings to match key facts about international capital holdings. I adapt this approach in a multi-sector model to match the allocation of capital stock across sectors and the heterogeneous returns on capital observed in the data.

drawn from a Fréchet distribution $F^j_{n,t}(\zeta^j_{n,t})$ at each period:

$$F^j_{n,t}(\zeta^j_{n,t}) = e^{-(\zeta^j_{n,t}/a^j_{n,t})^{-\rho_K}} \tag{10}$$

where $a^j_{n,t}$ represents the average capital efficiency in sector j at time t, while the dispersion parameter ρ_K captures the variation in investment opportunities within sectors due to idiosyncratic factors. When ρ_K is low, indicating high dispersion, investment opportunities associated with capital assets vary greatly across sectors, even when sector-specific capital efficiencies $\{a^j_{n,t}\}$ are equal. On the other hand, when ρ_K is high, indicating low dispersion, investment opportunities are more similar within sectors, and, hence, sector-specific capital efficiencies $\{a^j_{n,t}\}$ primarily determine differences in returns.

If a capitalist invests an infinitesimal unit of aggregate capital in sector j, the return before depreciation is $\zeta^j_{n,t}P_{n,t}r^j_{n,t}$. The unit of aggregate capital translates to $\zeta^j_{n,t}$ units of capital available for production in sector j. The capitalist allocates a given unit of aggregate capital to sector j if its return is the highest among all sectors; that is, $\zeta^j_{n,t}P_{n,t}r^j_{n,t} = \max_h\{\zeta^h_{n,t}P_{n,t}r^h_{n,t}\}$. As shown in Appendix A.12.5, the probability that one unit of aggregate capital is allocated to sector j is given by:

$$\Phi^j_{n,t} = \frac{(r^j_{n,t}a^j_{n,t})^{\rho_K}}{\sum_{k\in\mathcal{J}}(r^k_{n,t}a^k_{n,t})^{\rho_K}}. \tag{11}$$

By the law of large numbers, the share of aggregate capital allocated to sector j coincides with $\Phi^j_{n,t}$ shown in Equation (11). This equation shows that the sector-level capital allocation depends not only on the sector-specific rental rate $r^j_{n,t}$ but also on the sector-specific capital efficiency $a^j_{n,t}$ and the dispersion of capital efficiencies captured by the parameter ρ_K.

Given that each unit of aggregate capital allocated to a given sector differs in terms of its efficiency, the efficiency units of capital allocated to sector j differ from the total aggregate capital allocated to sector j, i.e., $\Phi^j_{n,t}W_{n,t}$. Specifically, the efficiency units of capital allocated to sector j, $K^j_{n,t}$, is given by $K^j_{n,t} = \Gamma\left(\frac{\rho_K-1}{\rho_K}\right)\left(a^j_{n,t}\right)\left(\Phi^j_{n,t}\right)^{\frac{\rho_K-1}{\rho_K}} W_{n,t}$, where $\Gamma(\cdot)$ is the gamma function.

Furthermore, let $v_{n,t}$ denote the average return per unit of capital investment after accounting idiosyncratic capital efficiencies, i.e., $v_{n,t} \equiv \mathbb{E}\big[\zeta^j_{n,t}P_{n,t}r^j_{n,t}|\zeta^j_{n,t}P_{n,t}r^j_{n,t} = \max_h\{\zeta^h_{n,t}P_{n,t}r^h_{n,t}\}\big]$. As shown in Appendix A.12.5, the average return per unit of capital investment, $v_{n,t}$, is given by

$\frac{v_{n,t}}{P_{n,t}} = \Gamma\left(\frac{\rho_K - 1}{\rho_K}\right)\left(\sum_{j \in \mathcal{J}} (a^j_{n,t} r^j_{n,t})^{\rho_K}\right)^{\frac{1}{\rho_K}}$ which is a power mean of sector-specific rental rates with exponent ρ_K weighted by the sector-specific capital efficiencies $\{a^j_{n,t}\}$. By the law of large numbers, $v_{n,t}$ is also the average return on investing one unit of aggregate capital.

Lastly, as shown in Equation (11), the capital allocation share would gradually change over time if the rental rate of capital, $r^j_{n,t}$, or sector-specific capital efficiency, $a^j_{n,t}$, gradually evolve. However, if sector-specific efficiencies $\{a^j_{n,t}\}$ are time-invariant, then, given that capitalists possess perfect foresight and there are no physical adjustment costs, they would swiftly reallocate capital across sectors in response to climate policy changes. This reallocation would be reflected in the rapid adjustment of sector-specific rental rates, $\{r^j_{n,t}\}$, which could potentially conflict with the gradual capital adjustment observed in Figure 2 in Section 2.

Hence, to rationalize the observed gradual capital adjustment, I introduce a dynamic agglomeration externality, where past capital allocations to a given sector influence current sector-specific capital efficiency. This externality reflects external learning-by-doing and within-sector knowledge spillovers, generating gradual and persistent capital reallocation in response to policy changes (Aghion et al., 2016; Bradt, 2024). It is considered an externality because individual capitalists do not internalize the broader impact of their past capital allocations on current sector-specific capital efficiency.

Formally, the law of motion for sector-specific efficiency, $a^j_{n,t}$, which captures this idea, is given by: $\ln(a^j_{n,t}) = \ln(a^j_n) + \rho_\Phi \ln\left(\frac{K^j_{n,t-1}}{W_{n,t-1}}\right)$. In this equation, a^j_n represents the time-invariant economy-sector-specific efficiency, while the parameter ρ_Φ governs the strength of the dynamic agglomeration externality. Specifically, ρ_Φ indicates the degree to which capital efficiency improves as a result of allocating aggregate capital to sector j in the previous period. A high value of ρ_Φ suggests that past capital allocation decisions significantly influence current efficiency. Conversely, the externality disappears when $\rho_\Phi = 0$. The inclusion of past aggregate capital, $W_{n,t-1}$, captures the diminishing gains in capital efficiency across all sectors from aggregate investment, similar to the approach of Hémous (2016).

As derived in Appendix A.12.5, the optimal sector-level capital allocation condition is given by

$$\underbrace{\ln(K^j_{n,t})}_{\text{Capital Allocation}} = \underbrace{(\rho_K - 1)}_{\text{Responsiveness}} \cdot \underbrace{\ln\Big(\frac{P_{n,t} r^j_{n,t}}{v_{n,t}}\Big)}_{\text{Rental Rate of Capital}} + \underbrace{\ln(W_{n,t})}_{\text{Aggregate Capital}} - \underbrace{\rho_K \cdot \rho_\Phi \cdot \ln(W_{n,t-1})}_{\text{Lagged Wealth}} \tag{12}$$
$$+ \underbrace{\rho_K \cdot \rho_\Phi \cdot \ln(K^j_{n,t-1})}_{\text{Lagged Capital Allocation}} + \underbrace{\rho_K \cdot \ln(a^j_n)}_{\text{Sector-specific Capital Efficiency}} + \rho_K \ln\Big(\Gamma\big(\frac{\rho_K - 1}{\rho_K}\big)\Big).$$

Equation (12) serves as the key moment condition used to estimate the capital supply parameters ρ_K and ρ_Φ. This condition indicates that sector-level capital allocation depends on (i) the sector-specific rental rate of capital ($r^j_{n,t}$), (ii) aggregate capital ($W_{n,t}$), (iii) lagged capital allocation ($K^j_{n,t-1}$), and (iv) time-invariant economy-sector-specific capital efficiency (a^j_n). It suggests that the extent of capital adjustment depends on the dispersion parameter ρ_K. If ρ_K is low, capital allocation responds inelastically to changes in the rental rate of capital induced by environmental policies, as allocation decisions are primarily driven by idiosyncratic investment opportunities, which vary significantly even within sectors. Additionally, the speed of capital adjustment depends on the magnitude of ρ_Φ. A high ρ_Φ implies that past capital allocations across sectors would strongly persist over time in the absence of policy intervention, while also leading to persistent dynamic responses to temporary policy changes.

Aggregate Capital Accumulation The representative capitalist at time 0 with initial aggregate capital $W_{n,0}$ chooses a sequence of future aggregate capital $\{W_{n,t+1}\}$, in units of final good, to maximize her life-time utility $V^K_{n,0}$:

$$V^K_{n,0} = \max_{W_{n,t+1}} \sum_{t=0}^{\infty} \beta^t \mathbb{E}_0 \Big[\ln(C^K_{n,t})\Big] \tag{13}$$

s.t.

$$C^K_{n,t} = \Big(\frac{v_{n,t}}{P_{n,t}} + (1 - \delta^k_n)\Big) W_{n,t} - W_{n,t+1} + \frac{\Omega_{n,t}}{P_{n,t}}, \tag{14}$$

$$0 = \lim_{t\to\infty} \Big(\prod_{s=0}^{t} \Big(\frac{v_{n,s}}{P_{n,s}} + 1 - \delta^k_n\Big)^{-1}\Big) W_{n,t+1} \tag{15}$$

where $C^K_{n,t}$ denotes consumption of final good by the capitalist; β denotes the discount factor; $\Omega_{n,t}$ denotes the lump-sum transfer from the government in economy n at time t; Equation (15) represents the transversality condition of the representative capitalist.

Although not explicitly shown in Equation (14), there exists a domestic financial asset market where capitalists can trade risk-free bonds, $A_{n,t+1}$, at a gross interest rate $R_{n,t+1}$. The no-arbitrage condition implies the gross interest rate on bonds equals the net return on wealth: $R_{n,t+1} = \frac{v_{n,t+1}}{P_{n,t+1}} + (1 - \delta^k_n)$.

3.6 Workers

Now, I describe the workers' dynamic problem of moving across sectors, which highlights the extent and speed of labor adjustment across sectors. Within each economy, there are infinitely-lived hand-to-mouth workers of measure $L_{n,t}$ who are assumed to be homogeneous in their ability but heterogeneous in their sector of employment.[15] At each period, workers make dynamic discrete choice decisions regarding which sector to work in during the subsequent period. The decision to move from sector j (origin) to sector k (destination) depends on several factors, including (i) the destination-specific wage ($w^k_{n,t}$), (ii) the origin-destination-specific mobility costs ($\kappa^k_{n,t}$), (iii) the discounted expected life-time utility from working in sector k ($\beta \mathbb{E}_t[\mathcal{U}^k_{n,t+1}]$), (iv) the sector-specific non-pecuniary benefit (μ^j_n), and (v) the destination-specific idiosyncratic preference ($\varepsilon^k_{n,t}$).[16]

Formally, the lifetime utility $\mathcal{U}^j_{n,t}$ of a worker in sector j at time t is given by:

$$\mathcal{U}^j_{n,t} = \ln(c^j_{n,t}) + \ln(\mu^j_n) + \max_{k \in \mathcal{J}} \{\beta \mathbb{E}_t[\mathcal{U}^k_{n,t+1}] + \ln(1 - \kappa^{j,k}_n) + \rho_L \varepsilon^k_{n,t}\} \tag{16}$$

where $c^j_{n,t}$ denotes the consumption of final goods; The parameter $\kappa^{j,k}_n$ captures costs including the psychological costs, imperfect inter-sectoral transferability of experience, and training costs (Kambourov, 2009; Artuç et al., 2010; Dix-Carneiro, 2014). The cost of staying in the origin sector is normalized to 0, i.e., $\kappa^{j,j}_n = 0$ for all $j \in \mathcal{J}$. As $\kappa^{j,k}_n$ approaches 1, the utility of moving from sector j to sector k approaches negative infinity, reflecting extreme difficulty in moving to

[15] The assumption that workers are hand-to-mouth is a standard assumption made in the labor market dynamics literature for tractability (Artuç et al., 2010). While allowing workers to make saving decisions is both desirable and realistic, it complicates the analysis because it requires keeping track of the asset distribution of workers, where each worker's asset would depend on the full path of her past income and employment history.

[16] The inclusion of the sector-specific non-pecuniary benefit primarily serves a quantitative purpose. Specifically, it captures factors like better work-life balance and job security that rationalize the inter-sectoral wage differentials that cannot be explained by estimated mobility frictions.

sector k.[17] Since future idiosyncratic preference shocks $\{\varepsilon^k_{n,t+h}|h>0\}$ are not realized at time t, workers make moving decisions based on the expected life-time utility from working at sector k at time $t+1$, $\mathbb{E}_t[\mathcal{U}^k_{n,t+1}]$.

At each period, a worker draws an i.i.d. idiosyncratic preference shock $\varepsilon^k_{n,t}$ for each sector from a standard Gumbel distribution with mean zero, i.e., $\varepsilon^k_{n,t} \overset{\text{i.i.d.}}{\sim} \text{Gumbel}(\gamma,1)$, where γ is the Euler-Mascheroni constant.[18] The parameter ρ_L governs the dispersion of idiosyncratic non-pecuniary preferences and affects how responsive labor flows are to changes in future wages. A larger ρ_L indicates a greater influence of non-pecuniary reasons on labor mobility.

Since workers are assumed to be hand-to-mouth, the consumption of a worker in sector j in economy n at time t only depends on the sector-specific wage $w^j_{n,t}$ and the price of final goods $P_{n,t}$, i.e., $c^j_{n,t}=\frac{w^j_{n,t}}{P_{n,t}}$. Moreover, I assume workers in the non-employment (NE) sector receive income $w^{NE}_{n,t}$ from the government. In particular, $w^{NE}_{n,t}$ is assumed to depend on the economy-specific component w^{NE}_n, which will be estimated in Section 4, and the average wage of the economy: $w^{NE}_{n,t}=w^{NE}_n\times\left(\frac{\sum_{j\in\mathcal{J}\setminus\{NE\}}w^j_{n,t}L^j_{n,t}}{L_{n,t}}\right)$.

Denote $V^j_{n,t}$ as the expected lifetime utility of a worker in sector j of economy n at time t, i.e., $V^j_{n,t}\equiv\mathbb{E}_t[\mathcal{U}^j_{n,t}]$. As derived in Appendix A.10, the expected lifetime utility $V^j_{n,t}$ can be expressed as follows:

$$V^j_{n,t}=\ln\left(\frac{w^j_{n,t}}{P_{n,t}}\right)+\ln(\mu^j_n)+\rho_L\ln\left(\sum_{k\in\mathcal{J}}\exp\left((\frac{\beta}{\rho_L})V^k_{n,t+1}+(\frac{1}{\rho_L})\ln(1-\kappa^{j,k}_n)\right)\right). \tag{17}$$

Furthermore, the probability of a worker in sector j at time t moving to sector k at time $t+1$ is given by:

$$m^{j,k}_{n,t}=\frac{\exp\left((\frac{\beta}{\rho_L})V^k_{n,t+1}+(\frac{1}{\rho_L})\ln(1-\kappa^{j,k}_n)\right)}{\sum_{h\in\mathcal{J}}\exp\left((\frac{\beta}{\rho_L})V^h_{n,t+1}+(\frac{1}{\rho_L})\ln(1-\kappa^{j,h}_n)\right)}. \tag{18}$$

[17] Utility from consumption and the utility loss from moving can be combined as follows: $\ln\left((1-\kappa^{j,k}_n)c^j_{n,t}\right)=\ln(1-\kappa^{j,k}_n)+\ln(c^j_{n,t})$. Therefore, for a worker in sector j, moving from sector j to k is equivalent to sacrificing $\kappa^{j,k}_n$ fraction of her current consumption.

[18] The functional form is assumed for the tractability of the model. For examples of papers that use similar methodology, see Artuç et al. (2010), Dix-Carneiro (2014), Artuç and McLaren (2015), Caliendo et al. (2019), Dix-Carneiro et al. (2023). The idiosyncratic preference reflects various reasons associated with changing sectors, including but not limited to pursuing passion, relocation due to family reasons, and tedium.

By the law of large numbers, $m_{n,t}^{j,k}$ is also the share of workers in sector j at time t moving to sector k at time $t+1$. Therefore, the law of motion for labor supply in sector j is given by $L_{n,t+1}^{j} = \sum_{k\in\mathcal{J}} L_{n,t}^{k,j}$ where $L_{n,t+1}^{j}$ denotes the employment in sector j at time $t+1$ and $L_{n,t}^{k,j}$ denotes the number of workers moving from sector k at time t to sector j at time $t+1$, i.e., $L_{n,t}^{k,j} \equiv m_{n,t}^{k,j} L_{n,t}^{k}$.

Combining equations (17) and (18) leads to the following inter-sectoral labor flow condition:

$$\underbrace{\ln\left(\frac{L_{n,t}^{j,k}}{L_{n,t}^{j,j}}\right)}_{\text{Labor Flows}} = \underbrace{\left(\frac{\beta}{\rho_L}\right)}_{\text{Responsiveness}} \cdot \underbrace{\ln\left(\frac{w_{n,t+1}^{k}}{w_{n,t+1}^{j}}\right)}_{\text{Future Wage Differential}} + \underbrace{\left(\frac{1-\beta}{\rho_L}\right)\ln(1-\kappa_n^{j,k})}_{\text{Bilateral Mobility Cost}} \tag{19}$$

$$+ \underbrace{\beta\ln\left(\frac{\mu_n^{k}}{\mu_n^{j}}\right)}_{\text{Non-pecuniary Preferences}} + \underbrace{\beta\ln\left(\frac{L_{n,t+1}^{j,k} L_{n,t+1}^{k}}{L_{n,t+1}^{j} L_{n,t+1}^{k,k}}\right)}_{\text{Option Value}}.$$

Equation (19) serves as the key moment condition describing the relationship between labor flows and key labor supply parameters. The extent of labor adjustment is governed by the dispersion of non-pecuniary preferences ρ_L. When ρ_L is high, workers will reallocate less to the destination sector because economic incentives matter less for labor mobility decisions. Furthermore, the speed of adjustment depends on the bilateral mobility costs. In particular, higher bilateral mobility costs, $\kappa_n^{j,k}$, reduce the size of labor flows from the origin to the destination, causing labor adjustments in response to sector-level policy changes to occur more slowly over time.

3.7 Governments

Since this paper does not aim to solve for optimal climate policies, I assume that governments do not make endogenous decisions to maximize domestic social welfare. Instead, I model three types of pre-implemented sector-specific policy instruments that reflect climate policies used in practice: (i) sector-specific taxes/subsidies $\{\tau_{n,t}^{j}\}$, (ii) fuel-specific taxes $\{\tau_{n,t}^{f,h}\}$, and (iii) trade taxes $\{e_{nm,t}^{j}\}$. I assume the government maintains a balanced budget each period through a lump-sum transfer $\Omega_{n,t}$ to the representative capitalist conditional on the set of policy instruments

$\{\tau^{f,h}_{n,t}, \tau^{j}_{n,t}, e^{j}_{nm,t}, \}$:

$$\Omega_{n,t} = \underbrace{\sum_{j\in\mathcal{J}_E} \tau^{j}_{n,t} P^{j}_{n,t} Y^{j}_{n,t}}_{\text{Consumption Tax Revenue}} + \underbrace{\sum_{h\in\{o,g,c\}} \tau^{f,h}_{n,t} s^{h}_{n,t} f^{h}_{n,t}}_{\text{Fuel Tax Revenue}} \tag{20}$$
$$+ \underbrace{\sum_{j\in\mathcal{J}} \sum_{m\in\mathcal{N}} \left(\frac{e^{j}_{nm,t} - 1}{e^{j}_{nm,t}}\right) p^{j}_{nm,t} y^{j}_{nm,t}}_{\text{Import Tariff Revenue}} - \underbrace{w^{NE}_{n,t} L^{\text{NE}}_{n,t}}_{\text{Non-employment Expenditure}}, \quad \forall t.$$

As shown in Equation (20), the lump-sum government transfer increases with tax revenues from consumption taxes and import tariffs and decreases with expenditure towards workers in the non-employment sector.

3.8 General Equilibrium

In equilibrium, all markets in every economy must clear at every period. The final goods market clears in economy n at time t if the quantity of final goods equals the sum of aggregate consumption, aggregate investment, and extraction costs:

$$Y_{n,t} = \sum_{j\in\mathcal{J}} c^{j}_{n,t} L^{j}_{n,t} + C^{K}_{n,t} + W_{n,t+1} - (1-\delta^{k}_{n}) W_{n,t} + \sum_{h\in\{o,g,c\}} \varphi^{h}_{n} f^{h}_{n,t}. \tag{21}$$

Denote by $X_{n,t}$ the aggregate expenditure of final goods consumers which equals the aggregate value, i.e., $X_{n,t} = P_{n,t} Y_{n,t}$.

The sectoral goods market clears in sector $j \in \mathcal{J}$ of economy n at time t if the aggregate quantity of sectoral goods equals the sum of the demand by final goods consumers and variety producers for sector j goods:

$$Y^{j}_{n,t} = \begin{cases} C^{j}_{n,t} + \sum_{k\in\mathcal{J}} M^{k,j}_{n,t}, & j \in \mathcal{J}_N, \\ \alpha^{j}_{n} \left(\frac{(1+\tau^{j}_{n,t}) P^{j}_{n,t}}{P^{E}_{n,t}}\right)^{-\eta_1} C^{E}_{n,t} + \sum_{k\in\mathcal{J}} M^{k,j}_{n,t}, & j \in \mathcal{J}_E. \end{cases} \tag{22}$$

The market for varieties produced in sector j of economy n at time clears if the gross revenue equals the sum of demand from all economies:

$$\sum_{m\in\mathcal{N}} p^j_{mn,t} y^j_{n,t} = \sum_{m\in\mathcal{N}} \left(\frac{p^j_{mn,t}}{P^j_{m,t}}\right)^{1-\sigma} P^j_{m,t} Y^j_{m,t}. \tag{23}$$

The markets for labor, capital, and fossil fuel clear in sector j of economy n at time t if the aggregate factor income equals the total expenditure of variety producers on given inputs:

$$w^j_{n,t} L^j_{n,t} = \left(\psi^{j,V}_n \psi^{j,L}_n\right) \sum_{m\in\mathcal{N}} p^j_{mn,t} y^j_{n,t}, \tag{24}$$

$$r^j_{n,t} P_{n,t} K^j_{n,t} = \left(\psi^{j,V}_n \psi^{j,K}_n\right) \sum_{m\in\mathcal{N}} p^j_{mn,t} y^j_{n,t}, \tag{25}$$

$$s_{n,t} F_{n,t} = \left(\psi^{J,V}_n \psi^{B,F}_n\right) \sum_{m\in\mathcal{N}} p^B_{mn,t} y^B_{n,t}. \tag{26}$$

Finally, given the net supply of risk-free financial assets is zero, the domestic financial asset market at time t clears if the financial asset position decision of the representative capitalist equals zero:

$$A_{n,t+1} = 0, \quad \forall n \in \mathcal{N}, t \in \mathcal{T}. \tag{27}$$

Now, I define the sequential equilibrium of the model. For readability, detailed descriptions of the sets of parameters Θ, prices $\mathbb{P}$, and endogenous variables $\mathbb{Y}$ are relegated to Appendix A.1.

Definition 1. ***Sequential Equilibrium.***
Given an initial distribution of aggregate capital and labor allocations $\{W_{n,0}, L^j_{n,0}\}_{n\in\mathcal{N},j\in\mathcal{J}}$, fossil fuel reserves $\{D^h_{n,0}\}_{h\in\{o,g,c\},n\in\mathcal{N}}$, and the set of parameters Θ, the sequential equilibrium of the model is characterized by a sequence of factor prices, interest rates, goods prices, and taxes $\mathbb{P}$, and the sequence of consumption, investment, saving, output, supply and demand for labor, capital, and fossil fuel, trade shares, and transfers $\mathbb{Y}$, that (i) maximize the profits of producers (6), (7), (ii) maximize the life-time utilities of capitalists (13) and workers (16) (iii) satisfy governments' balanced budget conditions (20), and (iv) satisfy the conditions for bilateral trade shares (2), output market clearing (21), (22), (23), factor market clearing (24), (25), (27) as well as the evolution of capital and labor allocation across sectors, and the law of motion for the stock of global carbon dioxide.

4 Quantification

The model is calibrated to a global economy consisting of three countries and two country aggregates: the United States (US), China, India, the European Union (EU), and the Rest of the World (ROW).[19] I employ the industrial classification from the World Input-Output Database (WIOD) 2016 Release (Timmer et al., 2015) to group 56 industries into five sectors, excluding the non-employment sector: Agriculture (AG), Manufacturing (MN), Services (SR), Non-Brown Energy (NB), and Brown (B).[20] Following O'Mahony and Timmer (2009), the non-brown energy sector includes the industry supplying electricity, and the brown sector includes the mining and quarrying industry as well as the industry manufacturing coke and refined petroleum products. See Appendix A for a detailed description of how economies and sectors are categorized.

I categorize the model parameters into three categories: (i) parameters that are imposed based on values previously used in the literature (summarized in Table 2), (ii) parameters estimated outside of the model (summarized in Table 3), and (iii) parameters estimated using the method of simulated moments (MSM) (summarized in Table 6). The parameters are calibrated using various datasets from the year 2022 or the most recent year available.

4.1 Imposed Parameters

Table 2: Summary of Imposed Parameters

Parameter	Value	Description	Source
β	0.98	Discount factor	Rennert et al.
σ	6.03	Trade elasticity	HM
δ_{τ}^{C}	See Text	Carbon Stock Depreciation	CR
$\mathcal{S}$	2200	Pre-industrial carbon stock (Gt)	GHKT

Notes: HM: Head and Mayer (2014), Rennert et al.: Rennert et al. (2022), CR: Cruz and Rossi-Hansberg (2023), GHKT: Golosov et al. (2014).

The imposed parameters are derived from established values in the literature. These parameters,

[19] The selection of these economies is motivated by the fact that the US, China, India, and the EU accounted for approximately 60% of global carbon emissions in 2022.

[20] In principle, the model can be extended to an arbitrary number of economies and sectors. In this paper, I consider a limited number of economies and sectors due to data limitations. For instance, the individual-level data is not available for all countries in the World Input-Output Database. Furthermore, the longitudinal individual-level labor survey in China, called the China Family Panel Studies, aggregates all manufacturing industries into one manufacturing sector.

detailed in Table 2, include the discount factor (β) set at 0.98, following Rennert et al. (2022), and the trade elasticity (σ) set at 6.03, following Head and Mayer (2014).[21]

Additionally, the carbon stock depreciation rate (δ_τ^C) is calibrated following the approach of Cruz and Rossi-Hansberg (2023). In particular, I parameterize δ_τ^C as follows: $\delta_\tau^C = 1 - a_0 - \sum_{s=1}^{3} a_s e^{-\tau/b_s}$ with $a_0 = 0.2173$, $a_1 = 0.2240$, $a_2 = 0.2824$, $a_3 = 0.2763$, $b_1 = 394.4$, $b_2 = 36.54$, and $b_3 = 4.304$. This approximation reflects the finding of Stocker et al. (2013) that more than 20% of carbon emissions will remain in the atmosphere for more than 1000 years. Following Golosov et al. (2014), the pre-industrial carbon stock ($\mathcal{S}$) is set to 2200 Gt of CO_2, with the baseline year for the measurement being 1850 ($T_{min} = 1850$).

4.2 Parameters Estimated Outside of the Model

In this section, I describe the estimation procedure and results for the key parameters of the model: labor mobility costs $\kappa_n^{j,k}$, the dispersion of non-pecuniary preferences ρ_L, the dispersion of Capital efficiencies ρ_K, and the dynamic agglomeration externality ρ_Φ. The estimation procedure and results for the other parameters listed in Table 3 are described in Appendix A.3.

Labor Mobility Costs To estimate the inter-sectoral labor mobility costs $\kappa_n^{j,k}$, I merge longitudinal labor force surveys from the following eight countries: the United States (Current Population Survey, 2000-2023), China (China Family Panel Studies, 2010-2020), India (Periodic Labour Force Survey, 2018-2020), France (Continuous Labour Force Survey, 2014-2020), Argentina (Permanent Household Survey, 2003-2019), Australia (The Household, Income and Labour Dynamics in Australia, 2002-2022), South Korea (Korean Labor & Income Panel Study, 1999-2022), and the United Kingdom (Labour Force Survey, 2016-2022). These datasets track the flows $L_{n,t}^{j,k}$ of individuals across sectors and employment status over time and report their income as well as individual characteristics such as gender, age, and education. Details on the data construction procedure are provided in Appendix A.2.

Following Artuç and McLaren (2015), I estimate the following regression specification (see

[21] Head and Mayer (2014) study 744 estimates of trade elasticity from a sample of 32 papers. The median of estimates is -5.03, i.e., $1 - \sigma = -5.03$.

Table 3: Summary of Parameters Estimated Outside of the Model

Parameter	Description	Source
$\kappa_n^{j,k}$	Labor mobility costs	See Text
ρ_L	Dispersion of non-pecuniary preferences	See Text
ρ_K	Dispersion of Capital efficiencies	WIOD
ρ_Φ	Dynamic agglomeration externality	WIOD
η_1	Energy demand elasticity	See Appendix
η_2	Fossil fuel elasticity	See Appendix
w_n^{NE}	Non-employment income	See Appendix
$\varrho_{nm,0}^j$	Exogenous trade costs	See Appendix
δ_n^K	Capital depreciation	PWT
φ_n^h	Supply cost of fossil fuel by type	IMF
ϕ^h	Carbon emission by fossil fuel	RR2020
E_L	Land-change emissions	RR2020
λ_T	Carbon-temperature conversion	MET
α_n	Non-energy expenditure share	WIOD
α_n^j	Non-energy expenditure share by sector	WIOD
$\gamma_n^{j,V}$	Value added share	WIOD
$\gamma_n^{j,k}$	Material input share	WIOD
$\psi_n^{j,L}$	Labor input share	WIOD
$\psi_n^{j,K}$	Capital input share	WIOD
$\psi_n^{j,E}$	Energy demand share	WIOD
$\psi_n^{B,F}$	Fossil fuel input share	WIOD

Notes: RR2020: Ritchie and Roser (2020), MET: Met Office Hadley Centre (2024), PWT: Penn World Table, WIOD: World Input-Output Database 2016 Release, WIOD 2013: World Input-Output Database 2013 Release, IMF: Black et al. (2023), MacMap: Guimbard et al. (2012).

Appendix A.13 for the derivation):

$$L_{n,t}^{j,k} = \exp\left(C_n^{j,k} + \lambda_{n,t}^k + \lambda_{n,t}^j\right) + \xi_{n,t}^{j,k}. \tag{28}$$

where $C_n^{j,k}$, $\lambda_{n,t}^j$, $\lambda_{n,t}^k$ are, respectively, economy-origin-destination fixed effects, economy-origin-year fixed effects, and economy-destination-year fixed effects defined as $C_n^{j,k} \equiv \frac{\ln(1-\kappa_n^{j,k})}{\rho_L}$, $\lambda_{n,t}^k \equiv \frac{\beta}{\rho_L} V_{n,t+1}^k$, and $\lambda_{n,t}^j \equiv \ln(L_{n,t}^j) - \ln\left(\sum_{h\in\mathcal{J}} \exp\left(\frac{\beta}{\rho_L} V_{n,t+1}^h + \frac{1}{\rho_L}\ln(1-\kappa_n^{j,h})\right)\right)$, and $\xi_{n,t}^{j,k}$ is the residual term.

Given data on the labor flows across sectors over time $L_{n,t}^{j,k}$, I use the Poisson Pseudo Maximum

Likelihood (PPML) method of Silva and Tenreyro (2010) to estimate $C_n^{j,k}$ using the regression specification (28). For estimates of $C_n^{j,k}$ which are not statistically significant at the 5% level, I re-estimate these mobility costs using the following specification (see Appendix A.13 for the derivation):

$$\frac{L_{n,t}^{j,k} L_{n,t}^{k,j}}{L_{n,t}^{j,j} L_{n,t}^{k,k}} = \exp(\tilde{C}_n^{j,k}) + \tilde{\xi}_{n,t}^{j,k} \tag{29}$$

where $\tilde{C}_n^{j,k}$ are economy-origin-destination fixed effects defined as $\tilde{C}_n^{j,k} \equiv \frac{1}{\rho_L} \ln(1 - \kappa_{n,t}^{j,k}) + \frac{1}{\rho_L} \ln(1 - \kappa_{n,t}^{k,j})$, and $\tilde{\xi}_{n,t}^{j,k}$ is a residual term. The regression specification (29) cannot separately identify $\frac{1}{\rho_L} \ln(1 - \kappa_{n,t}^{j,k})$ and $\frac{1}{\rho_L} \ln(1 - \kappa_{n,t}^{k,j})$. Hence, for the estimates re-estimated using the second method, I assume that bilateral mobility costs are symmetric, i.e., $\kappa_n^{j,k} = \kappa_n^{k,j}$. Refer to the Appendix A for the estimates of $\frac{\ln(1-\kappa_n^{j,k})}{\rho_L}$.

Dispersion of Non-pecuniary Preferences To estimate the dispersion of non-pecuniary preferences ρ_L, I use the following specification (see Appendix A.13 for the derivation):

$$y_{n,t}^{j,k} = \omega_n^k + \omega_n^j + \left(\frac{\beta}{\rho_L}\right) \ln\left(\frac{w_{n,t+1}^k}{w_{n,t+1}^j}\right) + \eta_{n,t+1}^{j,k} \tag{30}$$

where $y_{n,t}^{j,k} \equiv \ln\left(\frac{L_{n,t}^{j,k}}{L_{n,t}^{j,j}}\right) - (1-\beta)C_n^{j,k} - \beta \ln\left(\frac{L_{n,t+1}^{k} L_{n,t+1}^{j,k}}{L_{n,t+1}^{j} L_{n,t+1}^{k,k}}\right)$; ω_n^j and ω_n^k are, respectively, economy-origin and economy-destination fixed effects defined as $\omega_n^j \equiv -\beta \ln(\mu_n^j)$ and $\omega_n^k \equiv \beta \ln(\mu_n^k)$; $\ln(\frac{w_{n,t+1}^k}{w_{n,t+1}^j})$ captures the (log) residualized wage of the destination sector k relative to the residualized wage of the origin sector j at time $t+1$; and, $\eta_{n,t+1}^{j,k}$ is a forecast error. For the estimation, I assume the discount factor β equals 0.98.

To address the potential endogeneity concern that forecast errors may be correlated with the regressors, I estimate the regression specification (30) using an Instrumental Variable Generalized Method of Moments (IV-GMM) approach. Two instruments are employed: (i) two-period lagged residualized wages $\ln(\frac{w_{n,t-1}^k}{w_{n,t-1}^j})$, as suggested by Artuç et al. (2010), and (ii) sector-level exposure to economy-level environmental policies, $\text{EPS}_{n,t+1}^j$.

The sector-level exposure to environmental policies, $\text{EPS}_{n,t+1}^j$, is calculated as the product of the country-level Environmental Policy Stringency (EPS) index, constructed by the OECD (Kruse et al., 2022), and the country-sector-level carbon emissions per output in the year 2000, based on data from the World Input-Output Database Environmental Accounts (Corsatea et al., 2019). As discussed in Section 2, the EPS index, available for 40 countries from 1990 to 2023, evaluates the stringency of 13 policy instruments—including market-based policies, non-market-based policies, and technology support policies—on a scale from 0 to 6, with higher scores indicating greater stringency. These scores are averaged to form the EPS index. To address the endogeneity concern that sector-level carbon emissions per output might respond to the EPS index over time, I use sector-level carbon emissions per output from the year 2000. The parameter ρ_L is then identified under the assumption that the environmental policy $\text{EPS}^j_{n,t+1}$ influences future labor flows solely by affecting producers' demand for labor.

Table 4: Estimation Result for Dispersion of Non-pecuniary Preferences $\left(\frac{\beta}{\rho_L}\right)$ with $\beta = 0.98$

Dependent: $y^{j,k}_{n,t}$	(1) OLS	(2) IV: $\ln\left(w^k_{n,t-1}/w^j_{n,t-1}\right)$	(3) IV: $\text{EPS}^j_{n,t+1}$
$\ln\left(\frac{w^k_{n,t+1}}{w^j_{n,t+1}}\right)$	0.270***	0.424***	0.547*
	(0.048)	(0.068)	(0.323)
Country-Origin FE	YES	YES	YES
Country-Destination FE	YES	YES	YES
Observations	3,466	2,906	2,356
First Stage F-Statistic		111.93	59.42
R-squared	0.067		

Notes: $y^{j,k}_{n,t}$ denotes the (log) difference of labor flows adjusted for mobility costs and option values. $\ln\left(w^k_{n,t+1}/w^j_{n,t+1}\right)$ denotes the (log) residualized wage ratio between sectors k and j at time $t+1$. Clustered standard errors (country-origin-destination) in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. Sources: Panel labor force surveys of the US, China, India, France, Argentina, Australia, South Korea, and the UK. Column 2 uses two-period lagged wages as an instrument. Column 3 uses the sector-level exposure to environmental policies as an instrument.

The results are presented in Table 4. Column 3 provides the preferred IV estimate for $\frac{\beta}{\rho_L}$, yielding $\rho_L = 1.791$. This estimate suggests that a 1 percent increase in the next-period wage of the destination sector relative to the next-period wage of the origin sector increases labor flows to the destination sector by 0.547 percent. Hence, labor flows are not highly responsive to changes in future wage differentials and the dispersion of non-pecuniary motives for worker mobility is significant. The labor mobility costs $\kappa_n^{j,k}$ can be determined using the estimates of ρ_L and $\frac{\ln(1-\kappa_n^{j,k})}{\rho_L}$. Table A30 in the appendix presents the average costs faced by workers to move out of the brown sector, in 2014 USD. The costs are substantial (e.g., 130,344 USD for workers in the US), indicating that labor adjustment in response to climate policy changes would be small and gradual.

Capital Supply Elasticities I use the following specification to estimate the capital supply elasticities ρ_K and ρ_Φ (see Appendix A.13 for the derivation):

$$\ln\left(\frac{K_{n,t}^j}{K_{n,t}^o}\right) = \tilde{\omega}_n^j + \left(\rho_K - 1\right)\ln\left(\frac{r_{n,t}^j}{r_{n,t}^o}\right) + \left(\rho_K \cdot \rho_\Phi\right)\ln\left(\frac{K_{n,t-1}^j}{K_{n,t-1}^o}\right) + \tilde{\eta}_{n,t}^j \tag{31}$$

where $\tilde{\omega}_n^j$ are economy-sector fixed effects defined as $\tilde{\omega}_n^j \equiv \rho_K \ln(a_n^j/a_n^o)$ for some base sector o, and $\tilde{\eta}_{n,t}^j$ is the residual term. To run the regression, I obtain data on sector-level capital stock $K_{n,t}^j$ and capital compensation $r_{n,t}^j P_{n,t} K_{n,t}^j$ from the Socio-Economic Accounts (SEA) of the WIOD 2016 Release for 56 sectors in 43 countries from years 2000 to 2014. Rental rates of capital $r_{n,t}^j$ are computed by dividing sector-level capital compensation by sector-level capital stocks. I consider the petroleum and coke manufacturing (P) sector as the base sector for the estimation.

The specification in Equation (31) may suffer from simultaneity bias since capital demand and capital supply jointly determine the equilibrium capital allocation and the rental rate. To address this issue, I estimate the regression using two instruments: (i) lagged wages $\ln(\frac{w_{n,t-1}^j}{w_{n,t-1}^o})$ and (ii) sector-level exposure to environmental policies $EPS_{n,t}^j$. The coefficient $(\rho_K - 1)$ is identified under the assumption that lagged wages correlate with the persistent sectoral productivity process, which influences producers' demand for capital but does not directly affect the current supply of capital. Additionally, sectoral exposure to environmental policies $EPS_{n,t}^j$—discussed in the estimation of the parameter governing the dispersion of non-pecuniary preferences—is assumed to impact the capitalists' supply decisions only through its effect on producers' demand for capital.

The results are presented in Table 5. Column 3 provides the preferred IV estimates for $\rho_K - 1$ and $\rho_K \cdot \rho_\Phi$, yielding $\rho_K = 1.021$ and $\rho_\Phi = 0.981$. These results indicate that a 1 percent increase in the rate of return for a given sector, relative to the petroleum and coke manufacturing sector, is associated with a 0.021 percent increase in the allocation of capital to that sector, implying that returns on capital investment vary significantly even within sectors. Moreover, the estimate for ρ_Φ suggests that a 1 percent increase in past capital allocation to a given sector, relative to the petroleum and coke manufacturing sector, is associated with a 1.001 percent increase in current capital allocation. This implies that the evolution of sector-specific Capital efficiency is highly dependent on past capital investment in that sector.

Table 5: Estimation Result for Capital Supply Elasticities ($\rho_K - 1$, $\rho_K \cdot \rho_\Phi$)

Dependent: $\ln\left(\frac{K^j_{n,t}}{K^P_{n,t}}\right)$	(1)	(2)	(3)
	OLS	IV: $\ln(w^j_{n,t-1}/w^P_{n,t-1})$	IV: EPS$^j_{n,t}$
$\ln\left(\frac{r^j_{n,t}}{r^P_{n,t}}\right)$	-0.002**	0.123***	0.0209*
	(0.0011)	(0.0435)	(0.0113)
$\ln\left(\frac{K^j_{n,t-1}}{K^P_{n,t-1}}\right)$	0.992***	1.032***	1.001***
	(0.0008)	(0.0134)	(0.0041)
Country-Sector FE	YES	YES	YES
Observations	24,183	22,740	20,616
First Stage F-Statistic		50.10	56.84
R-squared	0.987		

Notes: $\ln(K^j_{n,t}/K^P_{n,t})$ denotes the (log) ratio of sector-level capital stocks relative to the petroleum and coke manufacturing sector (P). $\ln(r^j_{n,t}/r^P_{n,t})$ denotes the (log) ratio of rental rates of capital relative to the petroleum and coke manufacturing sector (P). Clustered standard errors (country-sector) in parentheses. *** p < 0.01, ** p < 0.05, * p < 0.1.
Source: WIOD 2016 Release.

4.3 Parameters Estimated by Method of Simulated Moments

The parameters listed in Table 6 are estimated using the Method of Simulated Moments (MSM), which is particularly suitable for estimating model parameters where traditional estimation techniques may be infeasible. This method involves simulating the full dynamic general equilibrium model and matching the moment generated by the model with the moments observed in the data. The MSM procedure involves the following steps: First, I specify the moments from the data that the model should replicate, such as sectoral output, sectoral factor prices, trade shares, fossil fuel production, and energy consumption. Next, I simulate the model using initial parameter guesses and compute the corresponding model moments. The parameters are then adjusted iteratively to minimize the distance between the simulated model moments and the observed data moments. The

Table 6: Summary of Parameters Estimated by Method of Simulated Moments

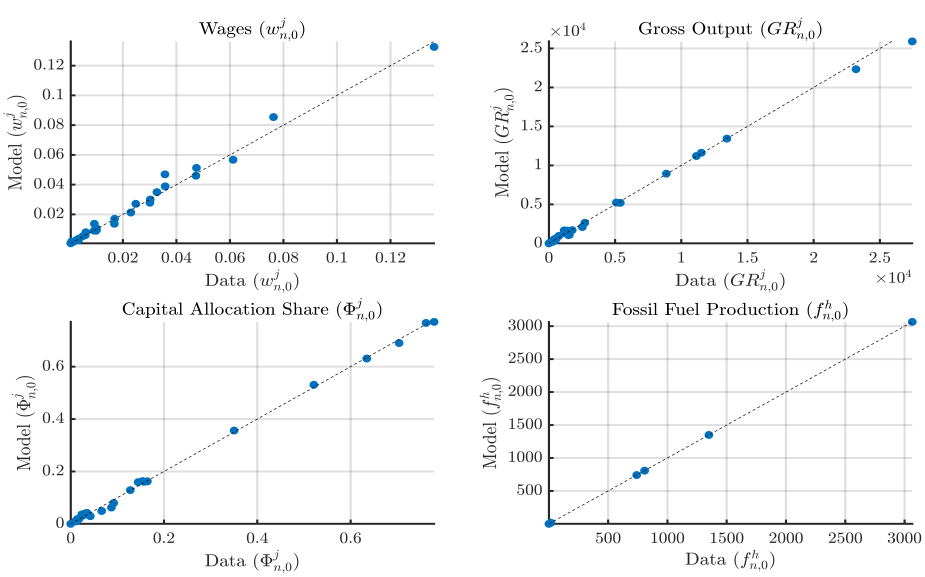
Parameter	Description	Targeted Moments
a_n^j	Capital efficiency	Real interest, capital allocation, capital compensation
μ_n^j	Non-pecuniary benefits	Sector-level wages
$z_{n,0}^j$	Sector-specific TFP	Bilateral trade shares, sector-level gross output
ω_n^h	Weights on fossil fuels	Fossil fuel production by type
α_n^{NB}	Weight on non-brown energy	Expenditure on the Non-Brown Energy
ξ_n	Environmental damage	GDP response to 1°C increase in global temperature

Notes: Parameters are estimated by matching model moments to observed data moments using the Method of Simulated Moments.

parameters are jointly estimated conditional on the parameters imposed or estimated outside of the model. For a detailed description of the targeted moments and the MSM procedure, refer to the Appendix A.4.

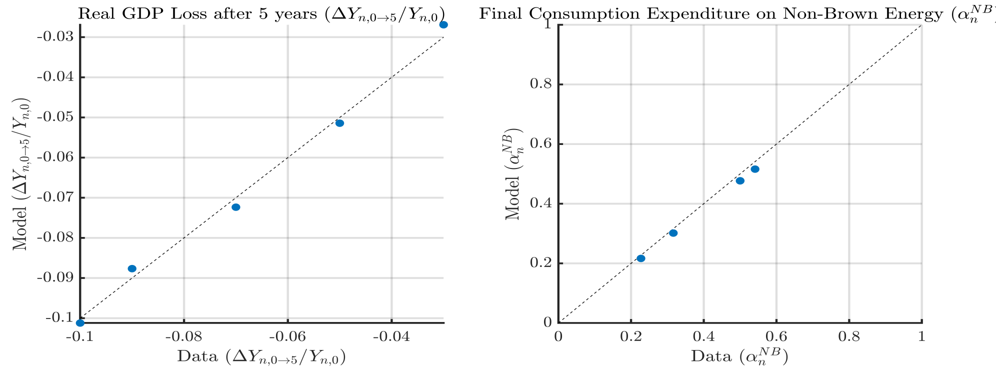
4.4 Model Fit

Figure 3: Model Fit: Model vs. Data Moments



Notes: Comparison between the model-generated moments and the observed data moments for wages, gross output, capital allocation share, and fossil fuel production.

Figure 4: Model Fit: Model vs. Data Moments



Notes: Comparison between the model-generated moments and the observed data moments for real GDP loss after 5 years in response to a 1 °C increase in the global temperature and final consumption expenditure on the non-brown energy sector.

Figures 3 and 4 compare the data moments targeted during the MSM estimation procedure with the model-generated moments. Overall, the calibrated model provides a good fit to the data. The parameter estimates obtained through the MSM procedure are presented in Appendix **??**.

4.5 Initial Conditions

Table 7: Summary of Initial Conditions

Variable	Description	Source
$W_{n,0}$	Aggregate capital	WIOD
$L^j_{n,0}$	Labor allocation	WIOD
$\mathcal{C}_0$	Carbon stock	RR2020
$\mathcal{D}_{n,0}$	Environmental quality	BK, MET
$A_{n,0}$	Financial asset position	–
$D^h_{n,0}$	Fossil fuel reserves	RR2017
$\tau^j_{n,0}$	Sector-specific consumption taxes	–
$\tau^{f,h}_{n,0}$	Fuel-specific taxes	IMF
φ^h_n	Fossil fuel extraction costs	IMF
$e^j_{nm,0}$	Trade taxes	MacMap

Notes: RR2020: Ritchie and Roser (2020), BK: Bilal and Känzig (2024), MET: Met Office Hadley Centre (2024), WIOD: World Input-Output Database 2016 Release, IMF: Black et al. (2023), RR2017: Ritchie and Rosado (2017) MacMap: Guimbard et al. (2012).

Table 7 summarizes the variables that characterize the initial conditions of the model. The initial conditions are computed for the year 2022 or the most recent available year. For a detailed description of the variables and data sets used to compute the initial conditions, refer to the Appendix **??**.

5 Quantitative Analysis

In this section, I present the key quantitative findings from the calibrated model. Given that China is the largest contributor to global emissions, accounting for 30.7 percent of global emissions in 2022, I simulate potential policies that China could implement to reduce its emissions. First, I examine how a unilateral subsidy to the non-brown energy sector in China impacts emissions, considering different degrees of factor mobility. Second, I evaluate how implementing a carbon tax in addition to the non-brown energy subsidy affects emissions and consumption in China.

The rich structure of the model can be used to evaluate various policy scenarios. In Appendix **??**, I also address the following policy-relevant questions: (i) How does a unilateral carbon tax in China affect emissions across models with varying levels of factor mobility? (ii) How large does the global carbon tax need to be to limit global warming to 1.5°C above pre-industrial levels by 2100, and how does this result change when imperfect mobility is relaxed? (iii) How large should temporary subsidies to the non-brown energy sector be to generate a lasting shift of capital into the non-brown energy sector?

Counterfactual Models To understand the influence of factor mobility, I consider three counterfactual models in addition to the baseline: (i) a model with high labor responsiveness, (ii) a model with high capital responsiveness, and (iii) a model combining both modifications. For each counterfactual model, I re-calibrate the parameters estimated using the MSM to match the targeted moments and initial conditions presented in Section 4.3.

First, to assess the impact of labor mobility, I modify the baseline model by significantly reducing the dispersion of non-pecuniary preferences, ρ_L, by a factor of five, from 1.79 to 0.358. This reduction weakens the influence of non-pecuniary motives for worker mobility and increases the extent of labor mobility by making labor flows more responsive to wage changes, as shown in the labor flow condition (19). Since the mobility costs $\{\kappa_n^{j,k}\}$ are estimated conditional on the value of ρ_L, I re-calibrate the mobility costs $\{\kappa_n^{j,k}\}$ so that the estimated values of $\left(\frac{1-\beta}{\rho_L}\right)\ln(1-\kappa_n^{j,k})$ in the labor flow condition (19) remain unchanged.

Second, to evaluate the impact of capital mobility, I consider a counterfactual model that significantly reduces differences in capital efficiencies across sectors by increasing ρ_K five-fold, from $\rho_K = 1.02$ to 5.10. The reduction in the dispersion of capital efficiencies increases the extent of capital mobility, as investment opportunities become more similar both within and across sectors, as shown in the capital allocation condition (12).

Since ρ_Φ is estimated conditional on the value of ρ_K, I also re-calibrate the dynamic agglomeration externality parameter ρ_Φ by reducing the value by a factor of five so that the estimated value of $\rho_K \cdot \rho_\Phi$ in the capital allocation condition (12) remains unchanged.

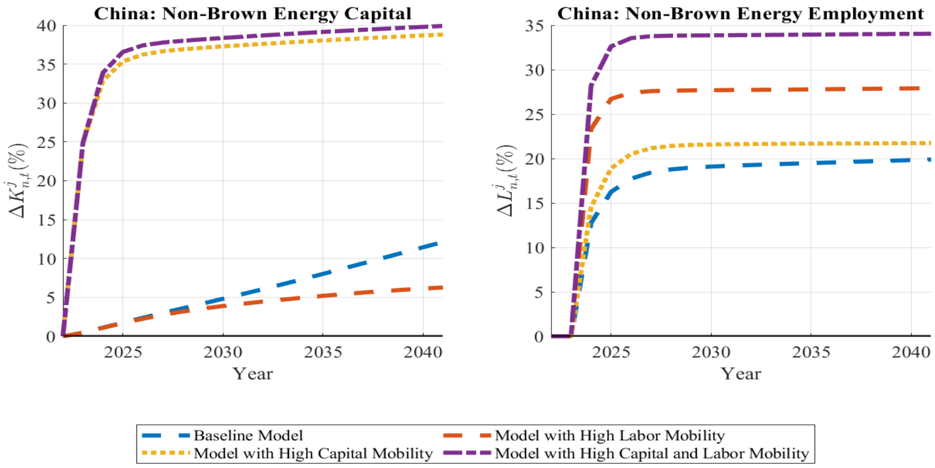
Finally, I combine the modifications from the first two models to create a third counterfactual model, which simultaneously increases the responsiveness of capital and labor supply by reducing the dispersion of capital efficiencies and the dispersion of non-pecuniary preferences.

5.1 Non-Brown Energy Subsidy in China

To understand how subsidies on the non-brown energy sector influence emissions, I consider a unilateral and permanent 11.5 percentage point increase in the consumption subsidy on the non-brown energy sector implemented by China. This policy exercise examines the impact of subsidizing the non-brown energy sector, equivalent to removing China's current value-added tax applied to the sector (Black et al., 2023). The policy scenario is compared to the business-as-usual (BAU) scenario where this policy is not implemented.

Capital and Employment in the Non-Brown Energy Sector The impact of China's policy on capital and employment in the non-brown energy sector across models is illustrated in Figure 5. In the baseline model, in response to the subsidy, capital and employment in the non-brown energy sector are higher than the business-as-usual (BAU) level in both the short and long run. This is because the subsidy on the non-brown energy sector incentivizes a shift in demand from the brown sector to the non-brown energy sector (*substitution effect*) and increases energy consumption by both producers and consumers, which leads to an overall rise in aggregate output and demand for both energy sectors (*scale effect*). Models with greater factor mobility result in higher levels of capital and employment in the non-brown energy sector compared to the baseline model. This suggests that greater factor mobility amplifies the substitution effect by significantly reallocating capital and labor to the non-brown energy sector.

Figure 5: Impact of a Non-Brown Energy Subsidy in China
on Non-Brown Energy Capital and Employment in China

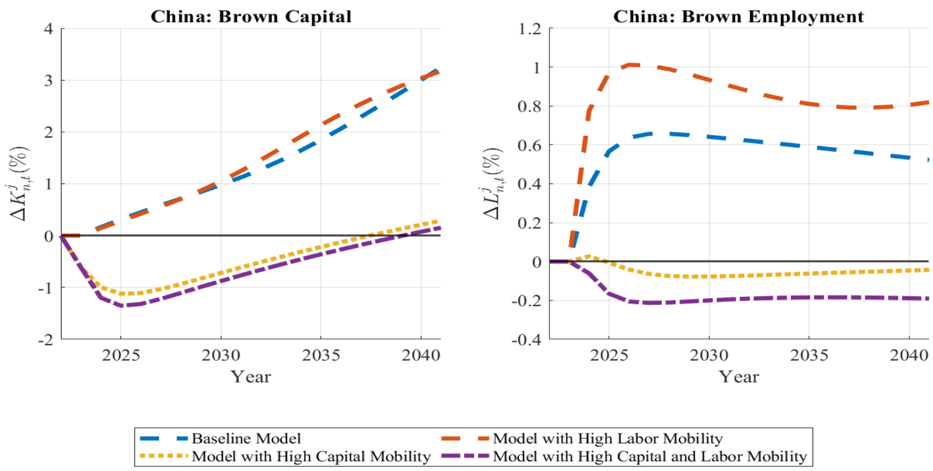


Notes: The impact of China's policy of unexpectedly and permanently reducing its consumption tax on the non-brown energy sector by 11.5 percentage points on capital and employment in China's non-brown energy sector. The blue line is the baseline model, and the red line represents the model with high labor mobility. The yellow line represents the model with high capital mobility, and the purple line indicates the model with high capital and labor mobility. The lines represent their values relative to the business-as-usual (BAU) case where the policy is not implemented.

Capital and Employment in the Brown Sector The impact of China's subsidy on the non-brown energy sector on capital and employment in the brown sector across models is illustrated in Figure 6. The figure shows that the effect of the subsidy on changes in capital and employment in the brown sector critically depends on the degree of factor mobility. In the baseline model, in response to the subsidy, capital and employment in the brown sector are higher than the business-as-usual (BAU) level in both the short and long run. This happens because the subsidy encourages energy consumption by both producers and consumers, increasing aggregate output and raising demand for both brown and non-brown energy sectors.

In contrast, in the model with high capital and labor mobility, capital and employment in the non-brown energy sector are lower than the BAU level in both the short and long run. Greater factor mobility amplifies the shift in demand from the brown sector to the non-brown energy sector by significantly reallocating capital and labor away from the brown sector. In the model with

Figure 6: Impact of a Non-Brown Energy Subsidy in China
on Brown Capital and Employment in China



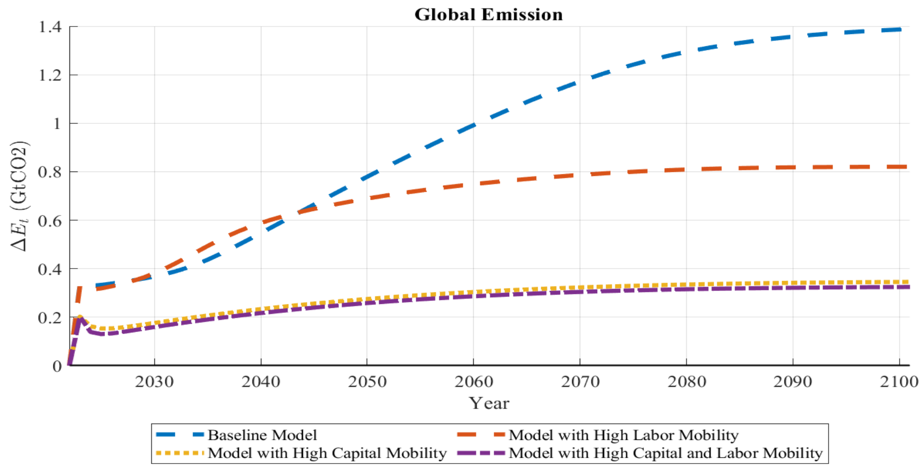
Notes: The impact of China's policy of unexpectedly and permanently decreasing its consumption tax on the brown sector by 11.5 percentage points on capital and employment in China's brown sector. The blue line is the baseline model, and the red line represents the model with high labor mobility. The yellow line represents the model with high capital mobility, and the purple line indicates the model with high capital and labor mobility. The lines represent their values relative to the business-as-usual (BAU) case where the policy is not implemented.

capital and labor mobility, the substitution effect outweighs the scale effect from increased energy consumption, resulting in lower capital and employment in the brown sector compared to the BAU level.

Figure 6 also shows that this amplification of the substitution effect is primarily driven by greater capital mobility rather than greater labor mobility. A model with high labor mobility amplifies the scale effect by allowing workers to more easily respond to rising wages in the non-brown energy sector, which helps equalize wages across sectors. This boosts aggregate output, consumption and investment, leading to increased accumulation of aggregate capital, and consequently, more capital in the brown sector. Due to the complementarity between capital and labor, employment in the brown sector also increases.

Global Emissions Figure 7 shows that global emissions rise across all models in response to China's subsidy on the non-brown energy sector, both in the short and long run. Although models

Figure 7: Impact of a Non-Brown Energy Subsidy in China on Global Emissions

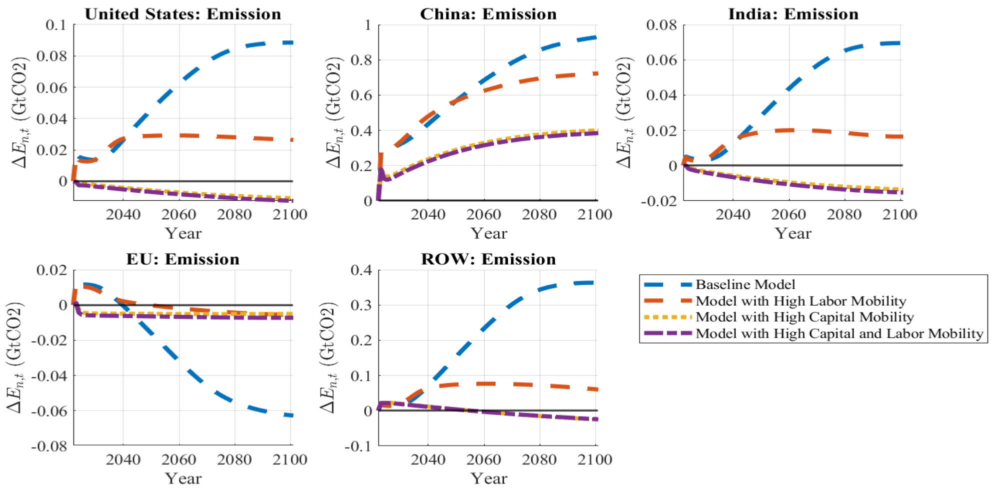


Notes: The impact of China's implementation of a permanent 11.5 percentage point decrease in its consumption tax on the non-brown energy sector on global emissions. The lines represent economy-level contributions to the changes in global emissions compared to the global emissions in the business-as-usual (BAU) case where the policy is not implemented.

with greater capital mobility result in lower capital and employment in the brown sector than the BAU level, as shown in Figure 6, the decline in material prices stimulates brown sector production. However, models with higher capital mobility still show a smaller overall increase in emissions, as greater mobility strengthens the substitution effect. Specifically, in the baseline model, global emissions in response to the subsidy are 0.39 $GtCO_2$ higher in 2023 and 1.53 $GtCO_2$ higher in 2100 compared to the BAU level. In contrast, in the model with high capital and labor mobility, global emissions are 0.22 $GtCO_2$ higher in 2023 and 0.43 $GtCO_2$ higher in 2100 than the BAU level.

International Spillovers Figure 8 shows the impact of China's implementation of the non-brown energy subsidy on emissions across economies. In the baseline model, emissions increase in all economies except the EU in the long run due to the scale effect. However, in models with high capital mobility, emissions in economies outside of China decrease in the long run due to the decline in the price of the non-brown energy good imported from China which encourages a shift in demand from the brown sector to the non-brown energy sector. The figure highlights that such

Figure 8: Impact of a Non-Brown Energy Subsidy in China on Economy-level Emissions



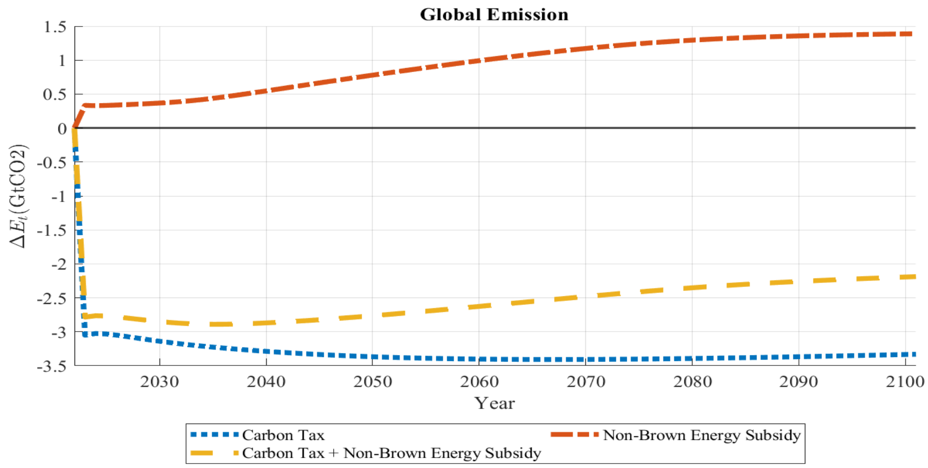
Notes: The impact of China's policy of unexpectedly and permanently decreasing its consumption tax on the brown sector by 11.5 percentage points on economy-level emissions. The blue line is the baseline model, and the red line represents the model with high labor mobility. The yellow line represents the model with high capital mobility, and the purple line indicates the model with high capital and labor mobility. The lines represent their values relative to the business-as-usual (BAU) case where the policy is not implemented.

an effect exists only when capital is highly mobile across sectors.

5.2 Non-Brown Energy Subsidy and Carbon Tax in China

In this section, I use the baseline model to compare how the following potential policy scenarios affect global emissions and consumption: (i) China implements a subsidy on the non-brown energy sector, as discussed in the previous section; (ii) China permanently increases consumption taxes on raw fossil fuels (hereafter referred to as a carbon tax) to the level currently implemented by the EU (Black et al., 2023); and (iii) China implements both of these policies together. The combined implementation of the subsidy on the non-brown energy sector and a carbon tax in China represents a scenario in which China raises tax revenues by levying taxes on raw fossil fuels to subsidize non-brown energy sources.

Figure 9: Climate Policies in China on Global Emissions

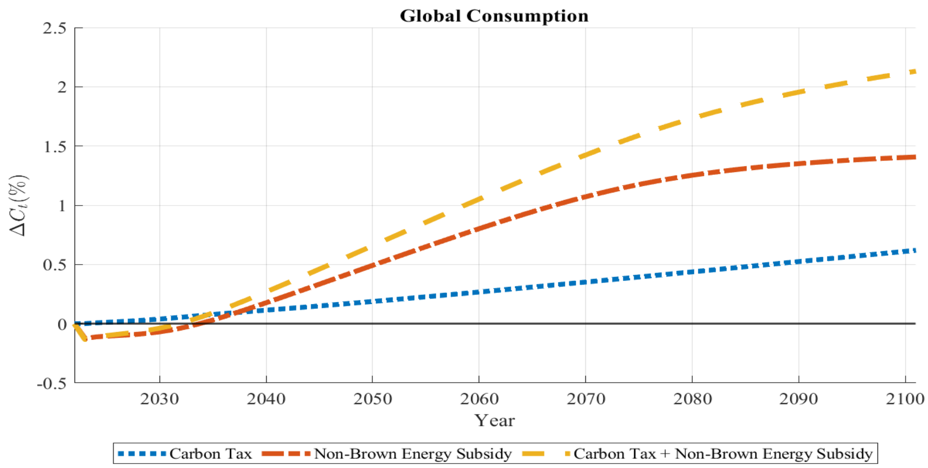


Notes: The impact of China's climates policies on global emissions. The blue line represents the response when China only implements a carbon tax; the red line represents the response when China implements a consumption subsidy on the non-brown energy sector. The yellow line represents the response when both the carbon tax and the subsidy on the non-brown energy sector are implemented. The lines represent their values relative to the business-as-usual (BAU) case where the policy is not implemented.

Global Emissions Figure 9 shows that the carbon tax reduces global emissions while the subsidy on the non-brown energy sector increases them relative to the BAU level, as discussed earlier. When both policies are implemented, global emissions fall below the BAU level, but the reduction is smaller than with the carbon tax alone. The combination of the carbon tax and the subsidy on the non-brown energy sector amplifies the substitution effect, shifting demand and production from the brown sector to the non-brown energy sector. However, the positive scale effect from the subsidy and the negative scale effect from the tax offset each other, resulting in an overall emissions reduction compared to the subsidy-only case. Nonetheless, due to the strong scale effect of the subsidy, the emissions reduction remains smaller than in the carbon tax-only scenario, both in the short and long run.

Global Consumption Figure 10 shows the response of global consumption when climate policies are implemented relative to the BAU level. Global consumption is the sum of all consumption

Figure 10: Baseline Model: Climate Policies in China on Global Consumption



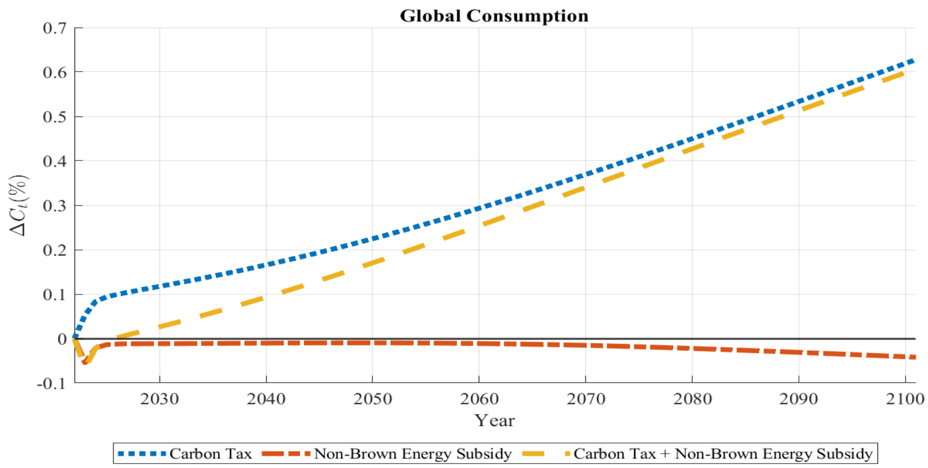
Notes: The impact of China's climates policies on global consumption. The blue line represents the response when China only implements a carbon tax; the red line represents the response when China implements a consumption subsidy on the non-brown energy sector. The yellow line represents the response when both the carbon tax and the subsidy on the non-brown energy sector are implemented. The lines represent their values relative to the business-as-usual (BAU) case where the policy is not implemented.

by workers and capitalists across sectors and economies. When subsidies are implemented, global consumption initially decreases for two reasons. First, as shown in Figure 9, global emissions rise, worsening environmental quality. Second, the subsidies are financed by lump-sum taxes on capitalists in China, causing a short-term drop in consumption.

In the long run, all policies lead to higher global consumption compared to the BAU level, with the combination of the carbon tax and the subsidy on the non-brown energy sector generating the greatest increase. However, the channels through which global consumption rises differ across policies. The carbon tax boosts global consumption by reducing emissions and improving environmental quality, while the subsidy on the non-brown energy sector allows capitalists to internalize the dynamic agglomeration externality, increasing production and consumption.

Dynamic Agglomeration Externality The long-term consumption gains from the subsidy depend largely on the strength of the dynamic agglomeration externality. Figure 11 illustrates the

Figure 11: Model with High Capital Mobility: Climate Policies in China on Global Consumption



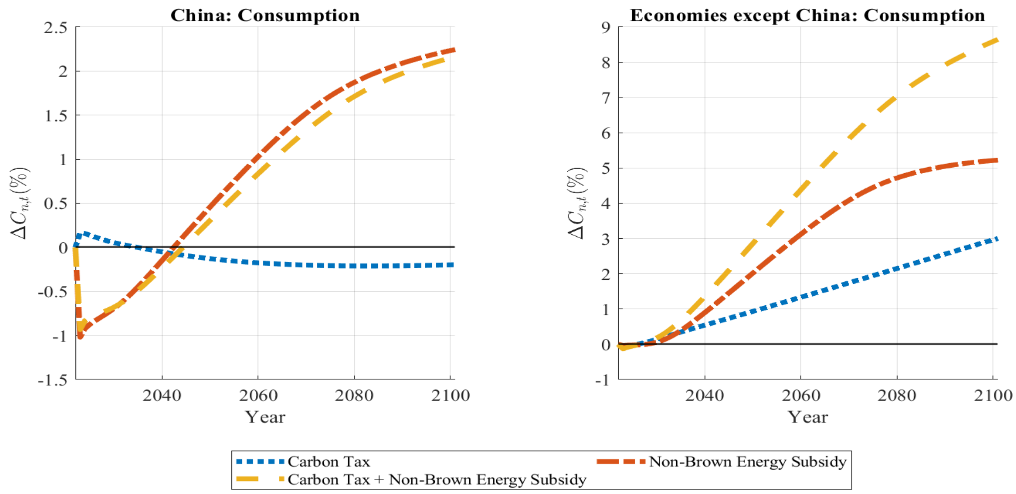
Notes: The impact of China's climates policies on global consumption in a model with high capital mobility and low dynamic agglomeration externality. The blue line represents the response when China only implements a carbon tax; the red line represents the response when China implements a consumption subsidy on the non-brown energy sector. The yellow line represents the response when both the carbon tax and the subsidy on the non-brown energy sector are implemented. The lines represent their values relative to the business-as-usual (BAU) case where the policy is not implemented.

response of global consumption to climate policies in a model with high capital mobility. In this model, the dynamic agglomeration externality ρ_Φ is reduced by a factor of five to match the observed persistence of capital allocation in the data. As shown in the figure, global consumption in response to the subsidy on the non-brown energy sector falls below the BAU level in both the short and long run, as the negative impact of environmental deterioration from increased emissions outweighs the benefits of internalizing the dynamic agglomeration externality.

Economy-level Consumption Figure 12 shows the consumption responses to China's climate policies in China and other economies. In China, the carbon tax initially raises consumption as capitalists benefit from the tax revenue. However, over time, the permanent carbon tax reduces aggregate output and investment by increasing energy costs faced by the non-energy sector, leading to a long-term decline in consumption.

In contrast, the subsidy on the non-brown energy sector initially reduces consumption in China

Figure 12: Climate Policies in China on Economy-level Consumption



Notes: The impact of China's climates policies on consumption in China and economies other than China. The blue line represents the response when China only implements a carbon tax; the red line represents the response when China implements a consumption subsidy on the non-brown energy sector. The yellow line represents the response when both the carbon tax and the subsidy on the non-brown energy sector are implemented. The lines represent their values relative to the business-as-usual (BAU) case where the policy is not implemented.

due to the lump-sum taxes imposed on capitalists to finance the subsidy. However, in the long run, China's aggregate consumption rises above the BAU level as the country's productivity improves from internalizing the dynamic agglomeration externality. When both policies are implemented together, the short-term drop in consumption is smaller, and long-term gains are similar to those from the subsidy alone. These results, along with those in Figure 9, suggest that China could achieve both emissions reductions and consumption gains by implementing the carbon tax and the subsidy on the non-brown energy sector together.

In economies outside China, all policies lead to an increase in aggregate consumption. When China implements the carbon tax, the rise in consumption elsewhere results from improved environmental quality. In the case of the subsidy on the non-brown energy sector, other economies benefit from lower energy prices and productivity spillovers from China through international trade.

6 Conclusion

This paper develops a dynamic general equilibrium model to evaluate the effectiveness of climate policies in reducing global emissions under imperfect capital and labor mobility. The findings reveal that both capital and labor exhibit limited responsiveness to changes in climate policies, constraining the large-scale resource reallocation needed to meet the Paris Agreement's emission reduction target. The results show that subsidies for non-fossil fuel energy sources can inadvertently increase emissions by lowering energy input prices and boosting aggregate production and investment, particularly when capital and labor mobility are limited.

The paper also explores the economic and environmental impacts of China's unilateral carbon tax. While the tax leads to a significant decline in China's domestic consumption due to its negative impact on the fossil fuel sector, global consumption rises as substantial emission reductions improve environmental quality. These findings suggest that international cooperation and compensation mechanisms may be necessary to enhance the effectiveness of carbon taxes and mitigate the economic costs for countries implementing such policies. Additionally, I show that if the productivity gains from reallocating capital are substantial, a country can reduce emissions and increase aggregate consumption by combining a carbon tax with a subsidy for the non-fossil fuel energy sector.

There are several fruitful avenues that future research could explore. While this paper provides a framework for understanding the dynamic effects of environmental policies, it abstracts from several critical aspects that merit further investigation. In particular, future research could focus on developing methods for solving global optimal environmental policies in a dynamic setting, as this remains a crucial yet under-explored area in the literature. Additionally, the strategic interactions between governments will play a pivotal role in designing globally implementable policies. The significant economic burden of unilateral carbon taxes, as highlighted in this paper, suggests that such policies may not be feasible without international cooperation and compensation mechanisms.

Another important direction for future research involves exploring the distributional consequences of environmental policies. This paper assumes homogeneous capitalists, abstracting from the ownership differences in heterogeneous capital assets across households. In practice, ownership of sector-specific assets can lead to substantial distributional effects, particularly between households with assets tied to the fossil fuel sector and those linked to the renewable energy sector. Furthermore, the assumption of infinitely lived agents overlooks potential generational conflicts.

Addressing these distributional issues will be crucial for designing equitable and effective environmental policies.

Lastly, future research could explore policies specifically designed to increase factor mobility alongside climate policies. For example, subsidies to reduce worker mobility costs or capital subsidies for the non-brown energy sector could facilitate the reallocation of resources. The model presented in this paper is flexible enough to simulate and evaluate these complementary policies, making this an important extension to consider.

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A Appendix

A.1 Description of Equilibrium Variables

In this section, I provide detailed descriptions of the sets of parameters Θ, prices $\mathbb{P}$, and endogenous variables $\mathbb{Y}$ shown in Section **??**. The variables in the set of prices $\mathbb{P}$ are the shown in Table A1. The variables in the set of parameters Θ are shown in Table A2. The variables in the set of endogenous variables $\mathbb{Y}$ are shown in Table A3.

Table A1: List of Prices and Taxes

Variable	**Description**
$p^j_{mn,t}$	Variety price in economy m for variety produced in sector j of economy n at time t
$P^{M,j}_{n,t}$	Price of material in sector j of economy n at time t
$P^E_{n,t}$	Energy price index in economy n at time t
$s^h_{n,t}$	Before-tax price of fossil fuel type h in economy n at time t
$s_{n,t}$	Price of a fossil fuel input in economy n at time t
$P_{n,t}$	Price of the final good in economy n at time t
$P^j_{n,t}$	Sector-level price index for sector j in economy n at time t
$w^j_{n,t}$	Wage in sector j of economy n at time t
$r^j_{n,t}$	Rental rate of capital in sector j of economy n at time t
$R_{n,t+1}$	Gross interest rate in economy n at time $t+1$
$\tau^B_{n,t}$	Consumption tax on the brown sector in economy n at time t
$\tau^{NB}_{n,t}$	Consumption tax on the non-brown energy sector in economy n at time t
$\tau^{f,h}_{n,t}$	Consumption tax specific to fossil fuel type h in economy n at time t
$e^j_{nm,t}$	Bilateral import tax in sector j between economies n and m at time t

Table A2: List of Model Parameters

Variable	**Description**
$\kappa_n^{j,k}$	Bilateral labor mobility costs
ρ_L	Dispersion of idiosyncratic non-pecuniary preferences
ρ_K	Dispersion of capital efficiencies
ρ_Φ	Dynamic capital allocation externality
η_1 (η_2)	Elasticity of substitution between energy goods (fossil fuels)
β (σ)	Discount factor (Elasticity of substitution across varieties)
δ_n^K (δ_t^C)	Economy-specific capital depreciation (Carbon stock depreciation rate)
ξ_n	Environmental damage in economy n
λ_T	Carbon-temperature conversion
α_n	Non-energy consumption share in economy n
α_n^j	Non-energy share by sector j in economy n
$\gamma_n^{j,V}$	Value added share for sector j in economy n
$\gamma_n^{j,k}$	Material input share for sector j in economy n
$\psi_n^{j,L}$	Labor input share for sector j in economy n
$\psi_n^{j,K}$	Capital input share for sector j in economy n
$\psi_n^{j,E}$	Energy input share for sector j in economy n
$\psi_n^{B,F}$	Fossil fuel input share in economy n
$\varrho_{nm,t}^j$	Exogenous trade costs between economies n and m for sector j at time t
z_n^j	Sector-specific TFP for sector j in economy n
μ_n^j	Non-pecuniary benefits for sector j in economy n
$a_{n,t}^j$	Capital efficiency for sector j in economy n
φ_n^h	Supply cost of fossil fuel by type h
ϕ^h	Carbon emission by fossil fuel type h
E_L	Land-change emissions
ω_n^h	Weight on fossil fuels for fuel type h in economy n
$\alpha_n^B(\alpha_n^{NB})$	Weights on brown (non-brown energy) in economy n
w_n^{NE}	Non-employment income in economy n

Table A3: List of Endogenous Variables

Variable	Description
$C^K_{n,t}$	Consumption of the representative capitalist in economy n at time t
$c^j_{n,t}$	Consumption of a worker in sector j of economy n at time t
$W_{n,t+1}$	Aggregate capital of the representative capitalist in economy n at time $t+1$
$A_{n,t+1}$	Risk-free assets held by capitalists in economy n at time $t+1$
$Y_{n,t}$	Output of final goods in economy n at time t
$L_{n,t}$	Aggregate supply of labor in economy n at time t
$L^j_{n,t}$	Supply of labor in sector j of economy n at time t
$K_{n,t}$	Aggregate stock of capital in economy n at time t
$K^j_{n,t}$	Stock of capital in sector j of economy n at time t
$F_{n,t}$	Aggregate supply of fossil fuels in economy n at time t
$F^h_{n,t}$	Supply of fossil fuel type h in economy n at time t
$y^j_{mn,t}$	Output produced in sector j of economy n and consumed in economy m at time t
$Y^j_{n,t}$	Sectoral output in sector j of economy n at time t
$C^j_{n,t}$	Consumption of sectoral goods from sector j in economy n at time t
$Q^{j,E}_{n,t}$	Energy input in sector j of economy n at time t
$M^j_{n,t}$	Material input in sector j of economy n at time t
$M^{j,k}_{n,t}$	Material input from sector k used in sector j of economy n at time t
$\pi^j_{nm,t}$	Share of sector j expenditure of economy n spent on sector j of economy m at time t
$E_{n,t}$	Total carbon emissions due to fossil fuel production in economy n at time t
E_t	Global CO_2 emissions at time t
$\mathcal{C}_t$	Global stock of carbon dioxide in the atmosphere at time t
$\mathbb{T}(\mathcal{C}_t)$	Global temperature at time t the global temperature at time T_{min}
$\mathcal{D}_n(\mathcal{C}_t)$	Environmental quality in economy n at time t
$\Phi^j_{n,t}$	Probability of allocating one unit of aggregate capital to sector j in economy n at time t
$V^j_{n,t}$	Expected lifetime utility of a worker in sector j of economy n at time t
$V^K_{n,t}$	Lifetime utility of the representative capitalist in economy n at time t
$m^{j,k}_{n,t}$	Share of workers moving from sector j to sector k in economy n at time t
$\Omega_{n,t}$	Lump-sum government transfer in economy n at time t

A.2 Description of Labor Force Surveys and Variable Construction

In this section, I describe the labor force surveys used in the analysis which track the movement of individuals across sectors and employment status over time and report individuals' income as well as individual characteristics such as gender, age, and education.

United States The data for the United States is sourced from the IPUMS Current Population Survey (CPS) Annual Social and Economic Supplement (ASEC) for the years 2000 to 2023. The dataset is annual and includes variables such as individual panel ID, year, sex, age, marital status, individual weight, employment status, sector of employment, sector of employment last year, education, income, and unemployment benefits. The sample includes individuals aged 25 to 65, excluding retirees and individuals unable to work. Income values exceeding 99990 or less than zero are treated as missing. The dataset tracks employment status (unemployed, employed) and movements across sectors.

China The data for China comes from the China Family Panel Studies (CFPS), covering the years 2010 to 2020. The survey is biennial, and the data includes variables such as sector of origin (sector of employment last year) and sector of destination (sector of employment), with sectors categorized into Agriculture, Brown, Manufacturing, Non-Brown Energy, and Services. The labor mobility shares are computed by annualizing the labor flows using the method of Artuç et al. (2010). The dataset also includes individual-level data such as income and employment status (unemployed, employed).

India The data for India is sourced from the Periodic Labour Force Survey (PLFS) for the years 2018 to 2020. The survey is quarterly, and the data includes variables such as quarter, visit, stratum, subsample, individual panel ID, years of education, sector, income, individual weight, employment status, age, hours worked per week, wage, and sex. The sample includes individuals aged 25 to 65. Employment status is derived based on a combination of variables indicating self-employment, wage employment, unemployment, and inactivity.

France The data for France is sourced from the Continuous Labour Force Survey (EEEC) for the years 2014 to 2020. The survey is quarterly, and the data includes variables such as individual panel ID, year, quarter, age, individual weight, birth year, birth month, start year, sector, employment status, sex, education, income, and unemployment benefits. The sample includes individuals aged 25 to 65 who are either employed or actively seeking employment. Sector classification follows NAF Rev. 2, which aligns with ISIC Rev. 4. The labor mobility shares are computed by annualizing the labor flows using the method of Artuç et al. (2010).

Argentina The data for Argentina is derived from the Permanent Household Survey (EPH) for the years 2003 to 2019. The survey is quarterly, and the data includes variables such as year, quarter, age, individual weight, income, wage, sector, education, employment status inactivity status, sex, job search, and individual panel ID. The sample includes individuals aged 25 to 65 who are either employed or actively seeking employment. Sector classification follows CAES-Mercosur, which aligns with ISIC Rev. 4. The labor mobility shares are computed by annualizing the labor flows using the method of Artuç et al. (2010).

Australia The dataset for Australia is the the Household, Income and Labour Dynamics in Australia (HILDA) Survey, covering 2002 to 2022. The survey is annual and provides longitudinal information on individual movements across sectors and employment status, including income and personal characteristics. Variables include individual panel ID, age, sex, marital status, individual weight, sector, education, income, employment status, school years, and annual work hours. The sample includes individuals aged 25 to 65 who are either employed or actively seeking employment.

South Korea The data for South Korea is from the Korean Labor & Income Panel Study (KLIPS) for the years 1999 to 2022. The survey is annual, and the data includes variables such as individual panel ID, year, sector, employment status, income, education, and weekly work hours. The sample includes individuals aged 25 to 65 who are either employed or actively seeking employment. Sector classification follows the 3-digit Korean Standard Industry Classification (KSIC), which aligns with the 3-digit ISIC Rev. 4.

United Kingdom The data for the United Kingdom comes from the Labour Force Survey (LFS) for the years 2016 to 2022. The survey is both quarterly and annual, and the data includes variables such as individual panel ID, year, quarter, age, individual weight, employment status, sector, and income. The sample includes individuals aged 25 to 65 who are either employed or actively seeking employment. Sector classification follows the 2-digit UK Standard Industry Classification (SIC), which aligns with the 2-digit ISIC Rev. 4.

A.3 Detailed Estimation Procedure for Parameters

This section discusses the estimation procedure for parameters not discussed in the main text.

Energy Demand Elasticity I estimate the energy demand elasticity using the following specification (see Appendix A.13.3 for the derivation):

$$\ln\left(\frac{Y_{n,t}^{NB}}{Y_{n,t-1}^{NB}}\right) - \ln\left(\frac{Y_{n,t}^{B}}{Y_{n,t-1}^{B}}\right) = -\eta_1\left(\ln\left(\frac{P_{n,t}^{NB}}{P_{n,t-1}^{NB}}\right) - \ln\left(\frac{P_{n,t}^{B}}{P_{n,t-1}^{B}}\right)\right) + \delta_t + \iota_{n,t} \tag{32}$$

where δ_t is the year fixed effects, and $\iota_{n,t}$ is the residual term.[22] In words, I regress the difference in output growth between the brown and non-brown energy sectors on the difference in price growth between these two sectors.

Table A4: Estimation Result for Energy Demand Elasticity ($-\eta_1$)

Dependent: $\ln\left(\frac{Y_{n,t}^{NB}}{Y_{n,t-1}^{NB}}\right) - \ln\left(\frac{Y_{n,t}^{B}}{Y_{n,t-1}^{B}}\right)$	(1) OLS	(2) IV: $\ln\left(\frac{r_{n,t-1}^{NB}}{r_{n,t-2}^{NB}}\right) - \ln\left(\frac{r_{n,t-1}^{B}}{r_{n,t-2}^{B}}\right)$
$\ln\left(\frac{P_{n,t}^{NB}}{P_{n,t-1}^{NB}}\right) - \ln\left(\frac{P_{n,t}^{B}}{P_{n,t-1}^{B}}\right)$	-0.37**	-2.28***
	(0.19)	(0.77)
Year FE	YES	YES
Observations	630	574
First Stage F-Statistic		9.89
R-squared	0.262	

Notes: The dependent variable is the one-period lagged (log) difference in the growth rates of non-brown energy sector output and brown sector output. Column 2 uses the difference in the rental rates of capital as the instrument. Robust standard errors are in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

[22] In particular, $\delta_t \equiv \frac{\eta_1}{|\mathcal{N}|}\sum_{m\in\mathcal{N}} \ln\left(\frac{(1+\tau_{m,t}^{NB})(1+\tau_{m,t-1}^{B})}{(1+\tau_{m,s}^{B})(1+\tau_{m,t-1}^{NB})}\right)$ and
$\iota_{n,t} \equiv -\eta_1 \ln\left(\frac{(1+\tau_{m,t}^{NB})(1+\tau_{m,t-1}^{B})}{(1+\tau_{m,s}^{B})(1+\tau_{m,t-1}^{NB})}\right) - \frac{\eta_1}{|\mathcal{N}|}\sum_{m\in\mathcal{N}} \ln\left(\frac{(1+\tau_{m,t}^{NB})(1+\tau_{m,t-1}^{B})}{(1+\tau_{m,s}^{B})(1+\tau_{m,t-1}^{NB})}\right)$.

To estimate the specification (32), I use the Socio-Economic Accounts (SEA) of the WIOD 2016 release, which provides sector-level output and output price indices normalized to 2010 levels. To address the potential endogeneity issue where prices may be correlated with the demand shifters in the residuals, I use the one-period lagged (log) difference between the growth rates of rental rates of capital in the non-brown energy sector and the brown sector, $\ln\left(\frac{r^{NB}_{n,t-1}}{r^{NB}_{n,t-2}}\right) - \ln\left(\frac{r^{B}_{n,t-1}}{r^{B}_{n,t-2}}\right)$, as an instrument. The parameter η_1 is identified under the assumption that variations in growth rates of rental rates of capital influence the relative output of the energy sectors only by affecting the production costs across these sectors and not the demand for energy. The results are presented in Table A4. Column 2 displays the preferred estimate for $-\eta_1$, yielding $\eta_1 = 2.28$, which suggests that brown and non-brown energy goods are substitutes.

Fossil Fuel Elasticity I estimate the elasticity of substitution across fossil fuels using the following specification (see Appendix A.13.4 for the derivation):

$$\ln\left(\frac{f^k_{n,t}}{f^h_{n,t}}\right) = -\eta_2 \ln\left(\frac{s^k_{n,t}}{s^h_{n,t}}\right) - \eta_2 \ln\left(\frac{1+\tau^k_{n,t}}{1+\tau^h_{n,t}}\right) + \delta^k_n + \tilde{\iota}^k_{n,t} \tag{33}$$

where δ^k_n are country-fuel fixed effects defined as $\delta^k_n \equiv \ln\left(\frac{\omega^k_n}{\omega^h_n}\right)$ for some base fuel type h; and, $\tilde{\iota}^k_{n,t}$ is an error term. In words, I regress the relative fossil fuel production on the relative supply costs and fuel-specific taxes. For the estimation, I consider crude oil (o) as the base fuel type.

Table A5: Estimation Result for Fossil Fuel Elasticity ($-\eta_2$)

Dependent: $\ln\left(\frac{f^k_{n,t}}{f^o_{n,t}}\right)$	(1)	(2)
$\ln\left(\frac{s^k_{n,t}}{s^o_{n,t}}\right)$	-2.56***	-2.67***
	(0.41)	(0.31)
Country-Fuel FE	YES	YES
Year FE	NO	YES
Observations	1,219	1,219
R-squared	0.985	0.985

Notes: The dependent variable is the (log) difference in the production of fuel type h relative to the production of crude oil (o). Clustered standard errors (country-fuel) are in parentheses.
*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

To estimate the specification (33), I use the IMF Fossil Fuel Subsidies Data (Black et al., 2023) which provides information on supply costs ($s^h_{n,t}$) and retail prices ($(1+\tau^h_{n,t})s^h_{n,t}$) for 168 countries from 2015 to 2019. The extraction costs ($\{s^h_{n,t}\}$) for oil, natural gas, and coal are set equal to the supply costs of gasoline (bil. USD/bil. m^3), natural gas (bil. USD/bil. m^3), and coal (bil. USD/Gt). The fuel-specific taxes ($\{1+\tau^h_{n,t}\}$) are computed by dividing the retail prices by the supply costs. Production levels of crude oil, natural gas, and coal ($\{f^h_{n,t}\}$) are sourced from Ritchie and Rosado (2017) for 169 countries from 2015 to 2019, with units of production for crude oil, natural gas, and coal provided in billion m^3, billion m^3, and billion tons, respectively.

The results are presented in Table A5. Column 1 shows the baseline estimate for $-\eta_2$, yielding $\eta_2 = 2.56$. Column 2 presents the preferred estimate for $-\eta_2$ when I additionally control for year fixed effects. The estimate is 2.67, showing minimal change from the estimate in Column 1. These estimates suggest that fossil fuel inputs are substitutes.

Non-Employment Income To estimate the non-employment income w^{NE}_n as a share of average labor income, I employ the panel labor force surveys described above which contain information on income of non-employed individuals. I begin by estimating the Mincerian regression of (log) wages on sector-year fixed effects, along with education, age, and gender fixed effects, weighted by individual weights to obtain residualized sector-year-level wages (Mincer, 1974):

$$\underbrace{\omega^{j,i}_{n,t}\Big(\log(w^{j,i}_{n,t})\Big)}_{\text{Weighted (log) wage in sector } j \text{ of individual } i} = \underbrace{\omega^{j,i}_{n,t}}_{\text{Weight}}\Bigg(\underbrace{\delta_{jt}}_{\text{Sector-Year FE}} + \underbrace{\delta_e}_{\text{Education FE}} + \underbrace{\delta_a}_{\text{Age FE}} + \underbrace{\delta_g}_{\text{Gender FE}} + \underbrace{\varepsilon^{j,i}_{n,t}}_{\text{Residual}}\Bigg)$$

Next, I compute the average residualized wage of employed workers, weighted by employment:

$$\underbrace{L^j_{n,t}\cdot\delta_{jt}}_{\text{Weighted (log) residualized wage}} = \underbrace{L^j_{n,t}}_{\text{Weight}}\Bigg(\underbrace{\delta_t}_{\text{Year FE}}\,\mathbb{1}[j\neq \text{NE}] + \underbrace{\delta_t}_{\text{Year FE}}\,\mathbb{1}[j=\text{NE}] + \underbrace{\varepsilon^j_{n,t}}_{\text{Residual}}\Bigg)$$

Finally, I estimate the non-employment income parameter w^{NE}_n by dividing the residualized non-employment income by the average residualized wage of employed workers and averaging over time.

Exogenous Trade Costs The gravity equation leads to the following regression specification:

$$\ln\left((\pi^j_{nm,t})^{\frac{1}{1-\sigma}}(e^j_{nm,t})^{-1}\right) = \ln(\varrho^j_{nm,t}) + \delta^j_{m,t} + \delta^j_{n,t} + \varepsilon^j_{nm,t}$$

where $\delta^j_{m,t}$ and $\delta^j_{n,t}$ are, respectively, exporter-sector-year fixed effects and importer-sector-year fixed effects defined as $\delta^j_{m,t} \equiv \ln(mc^j_{m,t}) - \ln(z^j_{m,t})$ and $\delta^j_{n,t} \equiv \frac{1}{\sigma-1}\ln\left(\sum_{i\in\mathcal{N}}(z^j_{i,t})^{\sigma-1}(d^j_{ni,t}mc^j_{i,t})^{1-\sigma}\right)$, and $\varepsilon^j_{nm,t+1}$ is the residual term.

Although the goal is to estimate trade costs for 2022 or the most recent year available, the WIOD only provides data up to 2014. Therefore, I assume the initial exogenous trade costs $\varrho^j_{nm,0}$ are equal to the time-averaged trade costs, calculated as $\rho^j_{nm,0} = \frac{1}{15}\sum_{s=0}^{14}\rho^j_{nm,-s}$ for the period 2000 - 2014. Under this assumption, the trade costs are estimated as the coefficients of the importer-exporter-sector fixed effects in the regression specification (34). The bilateral trade shares $(\pi^j_{nm,t})$ are sourced from the WIOD 2016 Release, while trade taxes $(e^j_{nm,t})$ are obtained from Guimbard et al. (2012). Refer to the Appendix A.8 for the detailed estimates.

Other Parameters The capital depreciation rates (δ^K_n) are taken from the Penn World Table 10.01 (Feenstra et al., 2015) for the year 2019, which is the most recent year available. For country aggregates, I take the GDP-weighted average; the values range from 0.042 to 0.058. For detailed estimates, refer to the Appendix A.8.

Supply costs of fossil fuels (φ^h_n) are estimated based on the IMF Fossil Fuel Subsidies Data (Black et al., 2023) for the year 2022. The extraction costs for oil, natural gas, and coal are set equal to the supply costs of gasoline (bil. USD/bil. m^3), natural gas (bil. USD/bil. m^3), and coal (bil. USD/Gt). See Appendix A.8 for detailed estimates.

Carbon emissions data from years 1850 to 2022 are sourced from Ritchie and Roser (2020), with land-change emissions (E_L) computed as the average global land-change emissions over this period. The estimate for E_L is 4.9617 $GtCO_2$. The carbon-temperature conversion parameter (λ_T) is obtained by estimating Equation (**??**) using data on global temperature relative to the year 1850 obtained from Met Office Hadley Centre (2024) and global carbon stock computed based on Equation (**??**). The estimated value for λ_T is 1.671 with a robust standard error of 0.032 for the period 1980 - 2022. The model fit and the regression results are presented in Figure A1 and Table A16, respectively.

Consumption expenditure shares (α_n, α^j_n) and input shares (value added $(\gamma^{j,V}_n)$, material $(\gamma^{j,k}_n)$, labor ($\psi_n^{j,L}$), capital ($\psi_n^{j,K}$), energy ($\psi_n^{j,E}$), fossil fuel ($\psi_n^{B,F}$)) are directly computed using the World Input-Output Table (WIOT) and Socio-Economic Accounts (SEA) of the WIOD 2016 Release for the year 2014, which is the most recent year available. Refer to the Appendix A.8 for detailed estimates.

A.4 Method of Simulated Moments (MSM) Procedure

This section discusses the targeted moments and estimation procedure for parameters estimated using the method of simulated moments.

A.4.1 Targeted Moments

Capital Efficiencies and Non-pecuniary Benefits Capital efficiencies (a_n^j) target real interest rates, capital allocation shares, and economy-level capital compensation in the year 2014. Real interest rates are sourced from International Financial Statistics (IFS), and the capital allocation shares and economy-level capital compensation are calculated using the WIOD 2016 Release. Non-pecuniary benefits (μ_n^j) are matched to sector-level wages in the year 2014 computed using the WIOD 2016 Release.

Productivity Sector-specific total factor productivity ($z_{n,0}^j$) is estimated by targeting bilateral trade shares and sector-level output in 2014, which is the most recent year available in the WIOD 2016 Release. Bilateral trade shares pin down relative sector-level productivities across economies for a given sector. To pin down the relative productivities across sectors within each economy, I exploit the positive and monotonic relationship between sector-level output and sector-level productivity. In the estimation, I assume that relative productivity levels across sectors and economies remain constant over time, although I do not assume the global economy is initially in steady state.

Weights on Fossil Fuels and Non-Brown Energy Weights on fossil fuels (ω_n^h) target the production of fossil fuels at the economy level by fuel type sourced from Ritchie and Rosado (2017). The relative fossil fuel demand is given by: $\frac{\omega_n^h}{\omega_n^k} = \left(\frac{\varphi_n^h}{\varphi_n^k}\right)^{\eta_2}\left(\frac{f_{n,t}^h}{f_{n,t}^k}\right)$ which shows that weights are a function of fossil fuel production ($f_{n,t}^k$,$f_{n,t}^h$), elasticity η_2, and the efficiencies (φ_n^k, φ_n^h). Similarly, the energy demand can be re-arranged as follows: $\frac{\alpha_n^{NB}}{\alpha_n^B} = \left(\frac{(1+\tau_{n,t}^{NB})P_{n,t}^{NB}}{(1+\tau_{n,t}^{B})P_{n,t}^{B}}\right)^{\eta_1-1}\left(\frac{P_{n,t}^{NB}Y_{n,t}^{NB}}{P_{n,t}^{NB}Y_{n,t}^{B}}\right)$. Th equation implies that the weight on the non-brown energy sector (α_n^{NB}) can be identified by the share of final energy expenditure spent on the non-brown energy relative to the brown sector calculated using the WIOD 2016 Release.

Environmental Damage The environmental damage parameter (ξ_n) is estimated to match the impulse response of real GDP per capital of an economy to a 1 °C increase in global temperature, as estimated by Bilal and Känzig (2024). Specifically, I simulate an unexpected increase in exogenous carbon emissions E_0^X in year 2023, which results in an increase in global temperature by 1 °C in 2023. The shock can be computed as follows: $E_0^X = \mathcal{S} \cdot 2^{\frac{\mathbb{T}(\mathcal{C}_0)}{\lambda_T}} \left(2^{\frac{1}{\lambda_T}} - 1\right)$. I then match the drop in real GDP per capita of each economy, relative to the baseline simulation without the shock, for each economy after 5 years with the estimates of Bilal and Känzig (2024).

A.4.2 Moments and Parameter Identification

The MSM procedure involves estimating parameters such that the moments generated by the model match the observed moments in the data. The set of parameters Θ^{MSM} includes:

- **Sector-Specific Total Factor Productivity** ($z_{n,0}^j$):
  - **Identified by**: Bilateral trade shares and sectoral output.
  - **Moments**: Bilateral trade shares $\pi_{nm,0}^j$, sectoral output $P_{n,0}^j Y_{n,0}^j$.
- **Non-Pecuniary Benefit** (μ_n^j):
  - **Identified by**: Inter-sectoral wage differences.
  - **Moments**: Wages relative to non-employment income $\frac{w_{n,0}^j}{w_{n,0}^{NE}}$.
- **Capital Efficiency** ($a_{n,t}^j$):
  - **Identified by**: Real interest rates, capital allocation shares, and capital compensation.
  - **Moments**: Real Interest Rate ($R_{n,t}$), Capital stock $K_{n,0}^j$, capital allocation $\Phi_{n,0}^j$, and capital compensation $P_{n,0} \sum_{j \in \mathcal{J}} r_{n,0}^j K_{n,0}^j$.
- **Weight on Fossil Fuel** (ω_n^h):
  - **Identified by**: Fossil fuel production by type.

- **Moments**: Production shares of different fossil fuels $f^h_{n,0}$.

- **Weight on Brown** (α^B_n):

  - **Identified by**: Brown consumption.
  - **Moments**: Consumption shares of brown and non-brown energy α^B_n.

- **Environmental Damage** (ξ_n):

  - **Identified by**: GDP response to a 1°C increase in global temperature.
  - **Moments**: GDP changes ΔGDP_n after 5 years following global temperature shocks.

A.4.3 Computation of Moments

The moments $\mathbb{M}^{\text{Data}}$ are computed from observed data as follows:

- **Trade Share** ($\pi^j_{nm,0}$): The share of total expenditure spent on products produced in a given sector of a given country.
  Source: WIOD 2016 Release.

- **Sectoral output and Wage** ($P^j_{n,0}Y^j_{n,0}$, $w^j_{n,0}$): Gross output and wage by sector.
  Source: Panel labor force surveys from eight countries (US, China, India, France, Argentina, Australia, South Korea, UK).

- **Real Interest Rate, Capital Allocation, and Compensation** ($R_{n,t}$, $K^j_{n,0}$, $\Phi^j_{n,0}$, $P_{n,0}\sum_{j\in\mathcal{J}} r^j_{n,0}K^j_{n,0}$): Real interest rate and the distribution of capital stock, allocation, and compensation across sectors.
  Source: International Financial Statistics (IFS) and WIOD 2016 Release.

- **Fossil Fuel Production Share** ($f^h_{n,0}$): The proportion of each type of fossil fuel produced in different countries.
  Source: Ritchie and Rosado (2017).

- **Energy Consumption** (α^B_n): The ratio of brown to non-brown energy consumption.
  Source: WIOD 2016 Release.

- **GDP Response to Temperature Increase** (ΔGDP_n): The percentage change in GDP for each country due to a $1°$C rise in global temperature.
  Source: Bilal and Känzig (2024).

A.4.4 Calibration Procedure

The MSM calibration procedure involves the following steps:

1. **Initialize Parameters**: Start with initial guesses for the parameters Θ^{MSM}.

2. **Simulate Model**: Run the dynamic general equilibrium model with the initial parameter guesses to generate simulated moments $\mathbb{M}^{\text{Model}}$.

3. **Compute Model Moments**: Calculate the model moments corresponding to the observed data moments.

4. **Minimize Error Function**: Adjust the parameters to minimize the distance between the simulated model moments and the observed data moments using the error function:

$$Q_T(\Theta) = \left[\frac{\mathbb{M}_T^{\text{Data}} - \mathbb{M}_T^{\text{Model}}(\Theta)}{\mathbb{M}_T^{\text{Data}}}\right]' I \left[\frac{\mathbb{M}_T^{\text{Data}} - \mathbb{M}_T^{\text{Model}}(\Theta)}{\mathbb{M}_T^{\text{Data}}}\right],$$

   where I is the identity matrix.

5. **Iterate**: Repeat the simulation and adjustment steps until the parameters converge to values that minimize the error function $Q_T(\Theta)$.

6. **Sequential Procedure**: The estimation of environmental damage parameters (ξ_n) is performed after the estimation of other parameters. The estimated environmental damage parameter is then fed back into the first step, and the entire procedure is repeated until convergence.

A.4.5 Environmental Damage Estimation

The environmental damage parameter (ξ_n) is estimated to match the impulse response of real GDP per capita of an economy to a $1°$C increase in global temperature, as estimated by Bilal and Känzig (2024). Specifically, I simulate an unexpected increase in exogenous carbon emissions E_0^X

at time 0, which results in an increase in global temperature by 1°C at time 1. This can be shown as follows, based on Equation (**??**):

$$E_0^X = \mathcal{S} \cdot 2^{\frac{\mathbb{T}(\mathcal{C}_0)}{\lambda_T}} \left(2^{\frac{1}{\lambda_T}} - 1\right). \tag{34}$$

I then match the drop in real GDP per capita of each economy, relative to the baseline simulation without the shock, for each economy after 5 years with the estimates of Bilal and Känzig (2024).

A.5 Initial Conditions

This section describes the initial conditions of the models. The initial conditions of the model are computed for the year 2022 or the most recent year available. The initial distribution of aggregate capital ($W_{n,0}$) and labor allocations ($L^j_{n,0}$) across sectors and economies are derived from the Socio-Economic Accounts (SEA) of the World Input-Output Database (WIOD) 2016 Release. I assume the labor force ($L_{n,t}$) in each economy remains constant over time.

The global carbon stock ($\mathcal{C}_0$) in the year 2022 is computed based on pre-industrial stock ($\mathcal{S}$), global carbon emissions from years 1850 to 2022 sourced from Ritchie and Roser (2020). The initial environmental quality ($\mathcal{D}_{n,0}$) for each economy is determined by the initial carbon stock and the estimated environmental damage parameter (ξ_n). Given the assumption that only domestic financial asset markets exist in each economy, the initial financial asset position of the representative capitalist in each economy is set to zero, i.e., $A_{n,0} = 0$ for all $n \in \mathcal{N}$.

Fossil fuel reserves ($D^h_{n,0}$) are estimated based on Ritchie and Rosado (2017), with units of reserves for crude oil, natural gas, and coal provided in billion m^3, billion m^3, and billion tons, respectively, for the year 2019. Using the same dataset, I compute the average annual discovery from 2010 to 2019 and calculate the reserves assuming new reserves will continue to be discovered at this average annual rate for the next 500 years. This assumption is primarily made to focus on the impacts of environmental policies rather than resource exhaustion; this assumption amplifies the environmental benefits of environmental policies, as larger reserves imply that emissions will continue unless the global economy achieves a clean transition. Estimates are presented in Appendix A.

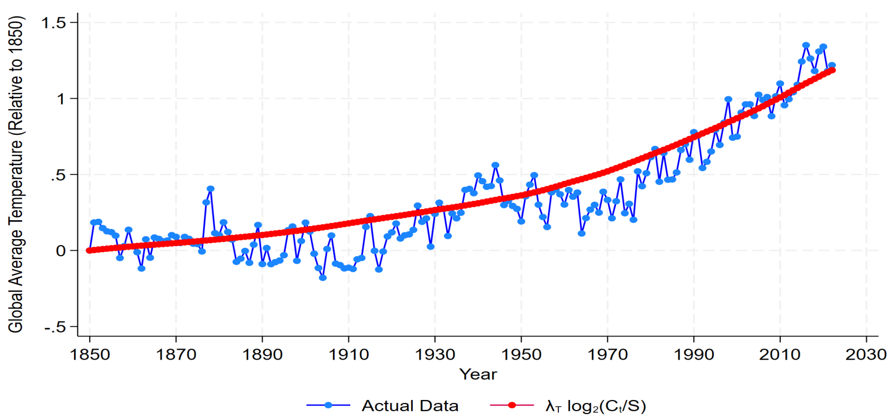
In the baseline calibration, sector-specific consumption taxes ($\tau^j_{n,0}$) are set to zero for two main reasons. First, countries primarily report value-added taxes (VAT) or sales taxes, which are applied uniformly across all or most products. Therefore, incorporating these taxes into the model would not meaningfully alter the distribution of economic activities across sectors. Second, taxes introduce distortions in the model. Removing uniform consumption taxes and redistributing through government transfers would enhance welfare by reducing these distortions, which is not the primary focus of this paper.

Fuel-specific tax rates are estimated based on the measures of supply costs of fossil fuels (φ_n^h) and retail prices of fossil fuels sourced from IMF Fossil Fuel Subsidies Data (Black et al., 2023) for the year 2022. Fuel-specific tax rates are computed as the difference between the retail price after deducting value-added tax (VAT) of fossil fuels (gasoline, natural gas, coal) and the supply costs of fossil fuels.

Trade taxes ($e_{nm,0}^j$) are estimated based on the MAcMap-HS6 dataset (Guimbard et al., 2012) for the year 2019. The MAcMap-HS6 (Market Access Map HS6) provides a detailed, comprehensive, and bilateral measurement of applied tariff duties for goods, considering regional agreements and trade preferences. Sector-specific trade taxes are computed as the average of the tariffs on products within each sector. See Appendix A.8 for detailed estimates.

A.6 Figures

Figure A1: Global Average Temperature (°C) relative to Year 1850



Notes: The blue line represents the actual temperature (°C) data, while the red line represents the predicted temperature (°C) based on the carbon-temperature conversion parameter (λ_T). The prediction is based on the equation $\mathbb{T}(\mathcal{C}_t) = \lambda_T \log_2(\mathcal{C}_t/\mathcal{S})$, where $\mathbb{T}(\mathcal{C}_t)$ is the global temperature, $\mathcal{C}_t$ is the carbon stock and $\mathcal{S}$ is the pre-industrial carbon stock. The estimate for the parameter λ_T is 1.671 with a robust standard error of 0.032 for the period 1980 - 2022.
Sources: Ritchie and Roser (2020) (carbon emissions), Met Office Hadley Centre (2024) (temperature).

A.7 Tables

Table A6: Threshold for Market-Based EPS

Score	Market based policies				
	CO_2 certificate	Renewable energy certificates (%)	CO_2 taxes (USD/tonne CO_2)	NO_x taxes (USD/tonne NO_x)	SO_x taxes (USD/tonne SO_x)
0	0	0	0	0	0
1	$0 < x \leq 10$	$0 < x \leq 0.05$	$0 < x \leq 10$	$0 < x \leq 90$	$0 < x \leq 116$
2	$10 < x \leq 20$	$0.05 < x \leq 0.08$	$10 < x \leq 20$	$90 < x \leq 137$	$116 < x \leq 180$
3	$20 < x \leq 30$	$0.08 < x \leq 0.11$	$20 < x \leq 30$	$137 < x \leq 184$	$180 < x \leq 244$
4	$30 < x \leq 40$	$0.11 < x \leq 0.14$	$30 < x \leq 40$	$184 < x \leq 231$	$244 < x \leq 308$
5	$40 < x \leq 50$	$0.14 < x \leq 0.17$	$40 < x \leq 50$	$231 < x \leq 278$	$308 < x \leq 372$
6	> 50	> 0.17	> 50	> 278	> 372

Source: Kruse et al. (2022).

Table A7: Threshold for Non-Market-Based EPS

Score	Non-market based policies			
	Emission Limit NO_x (mg/m^3)	Emission Limit SO_x (mg/m^3)	Emission Limit PM (mg/m^3)	Emission Limit Sulphur (mg/m^3)
0	No limit	No limit	No limit	No limit
1	>563	>643	>44	>1602
2	$458 \leq x \leq 563$	$518 \leq x \leq 643$	$39 \leq x \leq 44$	$1204 \leq x \leq 1602$
3	$353 \leq x \leq 458$	$393 \leq x \leq 518$	$32 \leq x \leq 39$	$806 \leq x \leq 1204$
4	$248 \leq x \leq 353$	$268 \leq x \leq 393$	$26 \leq x \leq 32$	$408 \leq x \leq 806$
5	$143 \leq x \leq 248$	$143 \leq x \leq 268$	$20 \leq x \leq 26$	$10 \leq x \leq 408$
6	$0 \leq x \leq 143$	$0 \leq x \leq 143$	$0 \leq x \leq 20$	$0 \leq x \leq 10$

Source: Kruse et al. (2022).

Table A8: Threshold for Technology Support EPS

Score	**Technology support policies**		
	R&D expenditure (1000 USD/GDP)	**FIT solar (USD/kWh)**	**FIT wind (USD/kWh)**
0	0	0	0
1	$0 < x \leq 0.14$	$0 < x \leq 0.41$	$0 < x \leq 0.95$
2	$0.14 < x \leq 0.27$	$0.41 < x \leq 0.81$	$0.95 < x \leq 1.27$
3	$0.27 < x \leq 0.4$	$0.81 < x \leq 1.21$	$1.27 < x \leq 1.59$
4	$0.4 < x \leq 0.53$	$1.21 < x \leq 1.61$	$1.59 < x \leq 1.91$
5	$0.53 < x \leq 0.66$	$1.61 < x \leq 2.01$	$1.91 < x \leq 2.23$
6	> 0.66	> 2.01	> 2.23

Source: Kruse et al. (2022).

Table A9: List of Economies and Countries in Country Aggregates

Economy	**Country**
United States (US)	United States
China (CHN)	China
India (IND)	India
European Union (EU)	Austria, Belgium, Bulgaria, Croatia, Czech Republic, Denmark, Germany, Estonia, Greece, Spain, France, Ireland, Italy, Cyprus, Latvia, Lithuania, Luxembourg, Hungary, Malta, Netherlands, Poland, Portugal, Romania, Slovenia, Slovakia, Finland, Sweden
Rest of the World (ROW)	Australia, Brazil, Canada, Switzerland, United Kingdom, Indonesia, Mexico, Malta, Norway, Russia, Turkey, Japan, South Korea, Taiwan

Notes: The European Union includes 27 member countries, and the Rest of the World (ROW) includes selected major economies and a composite of other countries.
Source: World Input-Output Database (WIOD) 2016 Release.

Table A10: Industry Classification - Non-Brown Energy, Brown

Sector	**Industry**
Non-Brown Energy (NB)	Electricity, gas supply
Brown (B)	Mining and quarrying Manufacture of coke, petroleum

Notes: Industries are categorized into sectors based on the World Input-Output Database 2016 Release classification. The non-brown energy sector includes industries related to renewable energy and technology, while the Brown sector includes industries related to fossil fuels.

Table A11: Industry Classification - Agriculture, Manufacturing, Services

Sector	**Industries**
Agriculture (AG)	Crop and animal production
	Forestry and logging
	Fishing and aquaculture
Manufacturing (MN)	Manufacture of food, beverages, tobacco
	Manufacture of textiles, apparel, leather
	Manufacture of wood and paper products
	Printing and recorded media
	Manufacture of chemicals, pharmaceuticals
	Manufacture of rubber, plastic products
	Manufacture of non-metallic, basic, and fabricated minerals
	Manufacture of computer, electronics
	Manufacture of electrical equipment
	Manufacture of motor vehicles
Services (SR)	Motor vehicle trade and repair
	Wholesale trade, excl. motor vehicles
	Retail trade, excl. motor vehicles
	Land, Water, Air transport, pipelines
	Warehousing, support for transportation
	Postal and courier activities
	Accommodation, food services
	Publishing activities, Motion picture, video production
	Telecommunications
	Computer programming, consultancy
	Financial services, Insurance, pension funding
	Auxiliary financial services
	Real estate activities
	Legal, accounting, management consultancy
	Architectural, engineering activities
	Scientific research and development
	Advertising, market research
	Other professional, technical activities
	Administrative and support services
	Public administration, defence
	Education, Human health, social work
	Other service activities, Household activities as employers
	Activities of extraterritorial organizations

Notes: Industries are categorized into sectors based on the World Input-Output Database (WIOD) 2016 Release classification.

A.8 Parameter Estimates

In this section, I provide the estimates of parameters described in Section 4.

Table A12: Capital Depreciation Rates (δ_n^K) for 2019

Country/Aggregate	**Depreciation Rate** (δ_n^K)
United States (US)	0.0460
China	0.0523
India	0.0579
European Union (EU)	0.0424
Rest of the World (ROW)	0.0463

Source: Penn World Table 10.01 (Feenstra et al., 2015).

Table A13: Supply Costs of Fossil Fuels (2022)

Economy	**Crude Oil** (φ_n^o)	**Natural Gas** (φ_n^{NB})	**Coal** (φ_n^c)
United States (US)	1626.3	0.3	99.2
China	1741.5	0.9	156.1
India	1741.5	0.8	248.5
European Union (EU)	1771.1	1.3	265.8
Rest of the World (ROW)	1730.0	0.8	243.4

Notes: The supply costs for crude oil, natural gas, and coal are in units of billion USD per billion m^3 for gasoline, billion USD per billion m^3 for natural gas, and billion USD per gigaton for coal, respectively.
Source: IMF Fossil Fuel Subsidies Data (Black et al., 2023).

Table A14: Fuel-Specific Taxes for Fossil Fuels (2022)

Economy	**Crude Oil** ($\tau^{f,o}_{n,0}$)	**Natural Gas** ($\tau^{f,g}_{n,0}$)	**Coal** ($\tau^{f,c}_{n,0}$)
United States (US)	0.3117	0.0146	-0.0411
China	0.2843	-0.5373	0.0121
India	0.3200	-0.0998	-0.0896
European Union (EU)	0.8667	-0.2865	0.3040
Rest of the World (ROW)	0.2889	-0.2547	-0.0139

Notes: Fuel-specific tax rates are computed as the difference between the retail price after deducting value-added tax (VAT) of fossil fuels (gasoline, natural gas, coal) and the supply costs of fossil fuels.
Source: IMF Fossil Fuel Subsidies Data (Black et al., 2023).

Table A15: Fossil Fuel Reserves ($D^h_{n,0}$) for 2019

Economy	**Crude Oil** ($D^o_{n,0}$)	**Natural Gas** ($D^g_{n,0}$)	**Coal** ($D^c_{n,0}$)
United States (US)	471.19	750388.90	279.22
China	170.92	235184.33	3500.82
India	17.90	42669.44	1867.59
European Union (EU)	9.91	18646.31	716.86
Rest of the World (ROW)	4561.01	1836887.27	6446.29

Notes: The units of reserves for crude oil, natural gas, and coal are in billion m^3, billion m^3, and billion tons, respectively. Estimates are based on Ritchie and Rosado (2017) for the baseline year 2019. I compute the average annual discovery from 2010 to 2019; then, I calculate the reserves assuming new reserves will continue to be discovered at this average annual rate for the next 500 years.

Table A16: Regression Results: Carbon-Temperature Conversion Parameter (λ_T)

Dependent: Global Average Temperature $\left(\mathbb{T}(\mathcal{C}_t)\right)$	(1) 1850 - 2022	(2) 1980 - 2022
$\log_2(\mathcal{C}_t)$	1.556*** (0.037)	1.671*** (0.032)
Observations	173	43
R-squared	0.919	0.984

Notes: Data on global temperature $\mathbb{T}(\mathcal{C}_t)$ in celsius ($^\circ$C) relative to the year 1850 obtained from Met Office Hadley Centre (2024) and global carbon emissions from Ritchie and Roser (2020). Global carbon stock ($\mathcal{C}_t$) computed based on Equation (**??**). Column 1 presents the regression result for the period 1850 - 2022. Column 2 presents the regression result for the period 1980 - 2022. Robust standard errors in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Table A17: Non-energy Consumption Share (α_n)

	US	**China**	**India**	**EU**	**ROW**
α_n	0.9163	0.8941	0.9215	0.9214	0.9112

Notes: α_n represents the Non-energy consumption share in economy n.
Source: WIOD 2016 Release. US: United States, EU: European Union, ROW: Rest of the World.

Table A18: Non-energy Share by Sector (α_n^j)

Sector	**US**	**China**	**India**	**EU**	**ROW**
AG	0.0051	0.0462	0.1157	0.0130	0.0337
MN	0.1534	0.2741	0.2796	0.1842	0.1978
SR	0.8416	0.6797	0.6047	0.8027	0.7685

Notes: α_n^j represents the Non-energy share of sector j in economy n.
Source: WIOD 2016 Release. AG: Agriculture, MN: Manufacturing, SR: Services, NB: Non-Brown Energy, B: Brown. US: United States, EU: European Union, ROW: Rest of the World.

Table A19: Value Added Share ($\gamma_n^{j,V}$)

Sector	US	China	India	EU	ROW
AG	0.4978	0.6156	0.7844	0.4758	0.6458
MN	0.3907	0.2849	0.2827	0.3461	0.3814
SR	0.6405	0.4960	0.7043	0.5861	0.6264
NB	0.8279	0.7805	0.7434	0.7212	0.8414
B	0.8155	0.7001	0.8502	0.5879	0.7399

Notes: $\gamma_n^{j,V}$ represents the value added share for sector j in economy n.
Source: WIOD 2016 Release. AG: Agriculture, MN: Manufacturing, SR: Services, NB: Non-Brown Energy, B: Brown. US: United States, EU: European Union, ROW: Rest of the World.

Table A20: Material Input Share ($\gamma_n^{j,k}$)

From (k) $\rightarrow$ To (j)	US	China	India	EU	ROW
AG $\rightarrow$ **AG**	0.4146	0.3585	0.4965	0.2744	0.3416
MN $\rightarrow$ **AG**	0.2652	0.5019	0.1755	0.3834	0.3462
SR $\rightarrow$ **AG**	0.3202	0.1396	0.3280	0.3422	0.3122
AG $\rightarrow$ **MN**	0.0854	0.0875	0.1062	0.0650	0.0886
MN $\rightarrow$ **MN**	0.5878	0.7389	0.5618	0.5785	0.6269
SR $\rightarrow$ **MN**	0.3268	0.1736	0.3320	0.3565	0.2844
AG $\rightarrow$ **SR**	0.0023	0.0205	0.0499	0.0052	0.0129
MN $\rightarrow$ **SR**	0.1634	0.4865	0.3603	0.1699	0.2716
SR $\rightarrow$ **SR**	0.8342	0.4930	0.5898	0.8249	0.7154
AG $\rightarrow$ **NB**	0.0001	0.0005	0.0003	0.0086	0.0008
MN $\rightarrow$ **NB**	0.0814	0.4928	0.3150	0.2351	0.3098
SR $\rightarrow$ **NB**	0.9184	0.5067	0.6847	0.7563	0.6894
AG $\rightarrow$ **B**	0.0006	0.0014	0.0005	0.0021	0.0015
MN $\rightarrow$ **B**	0.2391	0.4784	0.3199	0.2701	0.2951
SR $\rightarrow$ **B**	0.7300	0.3685	0.6043	0.6651	0.5010
NB $\rightarrow$ **B**	0.0303	0.1517	0.0753	0.0627	0.2023

Notes: $\gamma_n^{j,k}$ represents the material input share from sector k to sector j in economy n.
Source: WIOD 2016 Release. AG: Agriculture, MN: Manufacturing, SR: Services, NB: Non-Brown Energy, B: Brown. US: United States, EU: European Union, ROW: Rest of the World.

Table A21: Labor Input Share ($\psi_n^{j,L}$)

Sector	US	China	India	EU	ROW
AG	0.3327	0.9044	0.5378	0.5741	0.6599
MN	0.4656	0.3075	0.2641	0.5428	0.4162
SR	0.5732	0.5199	0.4919	0.5901	0.5443
NB	0.2152	0.0826	0.2546	0.1355	0.1120
B	0.0993	0.1862	0.0749	0.0834	0.1153

Notes: $\psi_n^{j,L}$ represents the labor input share for sector j in economy n.
Source: WIOD 2016 Release. AG: Agriculture, MN: Manufacturing, SR: Services, NB: Non-Brown Energy, B: Brown. US: United States, EU: European Union, ROW: Rest of the World.

Table A22: Capital Input Share ($\psi_n^{j,K}$)

Sector	US	China	India	EU	ROW
AG	0.5526	0.0477	0.4456	0.3306	0.2792
MN	0.4590	0.3782	0.4508	0.3387	0.3473
SR	0.3885	0.4059	0.4532	0.3746	0.4012
NB	0.5967	0.1781	0.2698	0.3356	0.2422
B	0.4160	0.2758	0.2434	0.1707	0.4852

Notes: $\psi_n^{j,K}$ represents the capital input share for sector j in economy n.
Source: WIOD 2016 Release. AG: Agriculture, MN: Manufacturing, SR: Services, NB: Non-Brown Energy, B: Brown. US: United States, EU: European Union, ROW: Rest of the World.

Table A23: Energy Input Share ($\psi_n^{j,E}$)

Sector	US	China	India	EU	ROW
AG	0.1147	0.0478	0.0166	0.0952	0.0608
MN	0.0754	0.3143	0.2851	0.1185	0.2364
SR	0.0383	0.0743	0.0549	0.0352	0.0546
NB	0.1882	0.7393	0.4755	0.5289	0.6457

Notes: $\psi_n^{j,E}$ represents the energy input share for sector j in economy n.
Source: WIOD 2016 Release. AG: Agriculture, MN: Manufacturing, SR: Services, NB: Non-Brown Energy, B: Brown. US: United States, EU: European Union, ROW: Rest of the World.

Table A24: Fossil Fuel Input Share ($\psi_n^{B,F}$)

	US	**China**	**India**	**EU**	**ROW**
$\psi_n^{B,F}$	0.4847	0.5380	0.6816	0.7459	0.3995

Notes: $\psi_n^{B,F}$ represents the fossil fuel input share in economy n. **Source**: WIOD 2016 Release. US: United States, EU: European Union, ROW: Rest of the World.

Table A25: Import Taxes ($e^{j}_{nm,0}$)

From \ To	US	China	India	EU	ROW
			Agriculture (AG)		
US	1.0000	1.0476	1.0375	1.0460	1.0315
China	1.1267	1.0000	1.1186	1.1267	1.1415
India	1.3798	1.3174	1.0000	1.3798	1.3013
EU	1.1422	1.1467	1.1332	1.0000	1.0541
ROW	1.1519	1.1578	1.1597	1.1436	1.1545
			Manufacturing (MN)		
US	1.0000	1.0445	1.0313	1.0443	1.0314
China	1.0692	1.0000	1.0638	1.0692	1.1014
India	1.1179	1.1113	1.0000	1.1179	1.0864
EU	1.0480	1.0482	1.0329	1.0000	1.0121
ROW	1.0860	1.0873	1.0859	1.0740	1.0870
			Services (SR)		
US	1.0000	1.0000	1.0000	1.0000	1.0000
China	1.0000	1.0000	1.0000	1.0000	1.0000
India	1.0000	1.0000	1.0000	1.0000	1.0000
EU	1.0000	1.0000	1.0000	1.0000	1.0000
ROW	1.0000	1.0000	1.0000	1.0000	1.0000
			Non-Brown Energy (NB)		
US	1.0000	1.0159	1.0033	1.0159	1.0089
China	1.0678	1.0000	1.0616	1.0678	1.0822
India	1.1025	1.0988	1.0000	1.1025	1.0754
EU	1.0210	1.0210	1.0059	1.0000	1.0048
ROW	1.0594	1.0590	1.0581	1.0506	1.0596
			Brown (B)		
US	1.0000	1.0135	1.0020	1.0135	1.0069
China	1.0542	1.0000	1.0513	1.0542	1.0644
India	1.0820	1.0727	1.0000	1.0820	1.0598
EU	1.0166	1.0166	1.0166	1.0000	1.0039
ROW	1.0669	1.0672	1.0660	1.0576	1.0670

Notes: Trade taxes ($e^{j}_{nm,0}$) are estimated based on the MAcMap-HS6 dataset (Guimbard et al., 2012) for the year 2019. Due to the unavailability of information for the services sector, trade taxes for this sector are imputed as 1.
Source: MAcMap-HS6 (Market Access Map HS6).

Table A26: Exogenous Trade Costs ($\varrho^j_{nm,0}$) - Agriculture, Manufacturing, Services

From \ To	**US**	**China**	**India**	**EU**	**ROW**
			Agriculture (AG)		
US	1.0000	3.5076	2.8975	2.6888	1.4766
China	2.4547	1.0000	3.6859	3.6420	1.6869
India	3.1896	3.8452	1.0000	3.8841	1.8247
EU	2.3837	2.8882	2.7970	1.0000	1.4708
ROW	2.2668	2.8426	3.0167	2.3440	1.0000
			Manufacturing (MN)		
US	1.0000	1.5562	2.6075	1.6877	1.2952
China	2.6936	1.0000	3.8920	2.2264	1.5460
India	2.1878	1.7732	1.0000	1.7996	1.4172
EU	2.0853	1.7659	2.8148	1.0000	1.4741
ROW	1.7958	1.5828	2.6152	1.5765	1.0000
			Services (SR)		
US	1.0000	4.4265	5.4879	3.0378	2.3623
China	3.7658	1.0000	4.7236	2.9896	2.4606
India	2.7947	3.8830	1.0000	2.6310	2.2056
EU	2.4649	3.4372	4.7347	1.0000	2.0299
ROW	2.3324	2.6026	3.2605	2.1337	1.0000

Notes: Exogenous trade costs ($\varrho^j_{nm,0}$) are estimated using a fixed effects regression specification (34), which is conditional on the trade elasticity σ, the bilateral trade shares ($\pi^j_{nm,t}$), and trade taxes ($e^j_{nm,t}$). Bilateral trade shares for the years 2000 to 2014 are computed using the WIOD 2016 Release. Tri-annual trade taxes ($e^j_{nm,t}$) for the years 2001 to 2014 are computed using Guimbard et al. (2012). Missing values for years between two observed years are imputed based on the assumption that trade taxes did not vary during the intervening period. The estimation assumes that exogenous trade costs did not vary from 2000 to 2014.

Table A27: Exogenous Trade Costs ($\varrho^j_{nm,0}$) - Non-Brown Energy, Brown

From \ To	**US**	**China**	**India**	**EU**	**ROW**
			Non-Brown Energy (NB)		
US	1.0000	3.8274	62.3328	3.0329	2.3645
China	8.3960	1.0000	118.3053	4.3355	3.3871
India	9.4558	3.3788	1.0000	3.2306	2.2408
EU	3.8112	3.8161	43.0767	1.0000	1.8855
ROW	5.2842	4.3795	41.5787	2.9739	1.0000
			Brown (B)		
US	1.0000	3.4274	4.0656	2.4224	1.3113
China	3.0898	1.0000	2.5712	3.5095	1.3407
India	2.5363	2.4576	1.0000	2.5766	1.0333
EU	1.9383	2.7517	2.8199	1.0000	1.0466
ROW	2.0465	2.6036	2.7203	2.1445	1.0000

Notes: Exogenous trade costs ($\varrho^j_{nm,0}$) are estimated using a fixed effects regression specification (34), which is conditional on the trade elasticity σ, the bilateral trade shares ($\pi^j_{nm,t}$), and trade taxes ($e^j_{nm,t}$). Bilateral trade shares for the years 2000 to 2014 are computed using the WIOD 2016 Release. Tri-annual trade taxes ($e^j_{nm,t}$) for the years 2001 to 2014 are computed using Guimbard et al. (2012). Missing values for years between two observed years are imputed based on the assumption that trade taxes did not vary during the intervening period. The estimation assumes that exogenous trade costs did not vary from 2000 to 2014.

Table A28: Inter-Sectoral Labor Mobility Costs $\left(\frac{\ln(1-\kappa_n^{j,k})}{\rho_L}\right)$ for US, China, and India

From \ To	**NE**	**AG**	**MN**	**SR**	**NB**	**B**
			United States (US)			
NE	0	-2.46	-2.12	-1.46	-2.84	-4.32
AG	-2.79	0	-4.46	-2.13	-5.50	-6.04
MN	-3.22	-5.57	0	-1.82	-3.49	-5.55
SR	-3.81	-6.28	-4.89	0	-5.29	-6.64
NB	-3.73	-6.45	-3.44	-2.26	0	-5.97
B	-3.51	-5.97	-4.00	-2.14	-4.07	0
			China			
NE	0	-1.02	-1.95	-1.13	-3.84	-2.74
AG	-1.02	0	-2.68	-2.32	-4.19	-3.57
MN	-1.95	-2.68	0	-2.09	-3.21	-3.35
SR	-1.13	-2.32	-2.09	0	-3.07	-3.30
NB	-3.84	-4.19	-3.21	-3.07	0	-5.96
B	-2.74	-3.57	-3.35	-3.30	-5.96	0
			India			
NE	0	-0.49	-3.19	-2.41	-4.76	-5.25
AG	-5.35	0	-5.02	-4.28	-8.90	-8.90
MN	-3.19	-5.02	0	-3.58	-4.61	-5.56
SR	-2.41	-4.28	-3.58	0	-3.45	-4.97
NB	-4.76	-8.90	-4.61	-5.55	0	-5.55
B	-5.25	-8.90	-5.56	-4.97	-5.55	0

Notes: Inter-sectoral labor mobility costs ($\frac{\ln(1-\kappa_n^{j,k})}{\rho_L}$) are estimated using the Poisson Pseudo Maximum Likelihood (PPML) method.
Sources: Current Population Survey, China Family Panel Studies, Periodic Labour Force Survey, Continuous Labour Force Survey, Permanent Household Survey, The Household, Income and Labour Dynamics in Australia, Korean Labor & Income Panel Study, Labour Force Survey.

Table A29: Inter-Sectoral Labor Mobility Costs $\left(\frac{\ln(1-\kappa_n^{j,k})}{\rho_L}\right)$ for EU and ROW

From \ To	**NE**	**AG**	**MN**	**SR**	**NB**	**B**
European Union (EU)						
NE	0	-4.87	-5.60	-2.80	-6.61	-9.28
AG	-4.24	0	-7.32	-5.20	-9.28	-9.28
MN	-3.55	-7.32	0	-3.68	-7.26	-8.90
SR	-2.80	-8.67	-8.28	0	-9.28	-8.90
NB	-5.34	-9.28	-8.51	-5.94	0	-9.28
B	-4.94	-9.28	-8.90	-4.45	-9.28	0
Rest of the World (ROW)						
NE	0	-3.38	-2.07	-1.75	-3.98	-4.81
AG	-4.65	0	-4.52	-3.08	-6.94	-7.10
MN	-4.25	-5.65	0	-2.38	-4.59	-5.84
SR	-1.75	-6.09	-4.38	0	-6.04	-7.05
NB	-4.52	-6.55	-3.37	-2.75	0	-5.65
B	-3.92	-4.88	-2.88	-2.00	-4.13	0

Notes: Inter-sectoral labor mobility costs ($\frac{\ln(1-\kappa_n^{j,k})}{\rho_L}$) are estimated using the Poisson Pseudo Maximum Likelihood (PPML) method.
Sources: Current Population Survey, China Family Panel Studies, Periodic Labour Force Survey, Continuous Labour Force Survey, Permanent Household Survey, The Household, Income and Labour Dynamics in Australia, Korean Labor & Income Panel Study, Labour Force Survey.

Table A30: Average Cost of Moving Out of the Brown Sector

Economy	Average Mobility Cost (2014 USD)
United States	130,344
China	9,897
India	5,900
European Union	30,060
Rest of the World	46,953

Notes: Average mobility costs faced by workers to move out of the brown sector, displayed in 2014 USD. Sources: Panel labor force surveys of the US, China, India, France, Argentina, Australia, South Korea, and the UK.

Table A31: Non-Employment Income Parameter (w_n^{NE})

Economy	w_n^{NE}
United States (US)	0.16
China	0.55
India	0.24
European Union (EU)	0.57
Rest of the World (ROW)	0.45

Notes: Non-employment income (w_n^{NE}) as a share of average labor income is estimated using panel labor force surveys. The estimation involves Mincerian regression of (log) wages on sector-year fixed effects, along with education, age, and gender fixed effects, weighted by individual weights to obtain residualized sector-year-level wages.
Sources: Current Population Survey, China Family Panel Studies, Periodic Labour Force Survey, Continuous Labour Force Survey, Permanent Household Survey, The Household, Income and Labour Dynamics in Australia, Korean Labor & Income Panel Study, Labour Force Survey.

Table A32: Sector-Specific Total Factor Productivity (z_n^j)

Economy	**AG**	**MN**	**SR**	**NB**	**B**
United States (US)	3.9533	7.2295	1.4806	0.2408	12.0749
China	0.2903	9.2016	1.4455	0.5143	13.3632
India	0.4670	6.4430	0.9737	0.2955	12.1159
European Union (EU)	2.1120	6.9764	1.2983	0.3245	12.2564
Rest of the World (ROW)	0.8215	8.7546	1.5253	0.4251	12.4059

Notes: Sector-specific total factor productivity (z_n^j) is estimated by targeting bilateral trade shares and wages across sectors and countries. Sectors: AG (Agriculture), MN (Manufacturing), SR (Services), NB (Non-Brown Energy), B (Brown).
Source: WIOD 2016 Release.

Table A33: Non-Pecuniary Benefits (μ_n^j)

Economy	**NE**	**AG**	**MN**	**SR**	**NB**	**B**
United States (US)	1.0000	2.1216	1.0103	3.4218	0.5545	0.2389
China	1.0000	11.4915	5.7615	8.2755	1.5491	2.2657
India	1.0000	0.0836	0.3391	0.3032	0.0265	0.0455
European Union (EU)	1.0000	7.3734	2.0734	2.7443	1.9517	1.3907
Rest of the World (ROW)	1.0000	2.1176	0.6950	1.2364	0.3972	0.3041

Notes: Non-pecuniary benefits (μ_n^j) are matched to inter-sectoral labor mobility shares. Sectors: NE (Non-Employment), AG (Agriculture), MN (Manufacturing), SR (Services), NB (Non-Brown Energy), B (Brown).
Source: Panel labor force surveys.

Table A34: Weights on Fossil Fuels $\left((\omega_n^h)^{\frac{1}{\eta_2}}\right)$ with $\eta_2 = 2.67$

Economy	**Crude Oil**	**Natural Gas**	**Coal**
United States (US)	0.9999	0.0021	0.0429
China	0.9945	0.0022	0.2046
India	0.9865	0.0039	0.2861
European Union (EU)	0.9854	0.0057	0.2950
Rest of the World (ROW)	0.9994	0.0033	0.0918

Notes: Weights on fossil fuels (ω_n^h) target economy-level fossil fuel production by fuel type. Fossil fuels: Crude Oil, Natural Gas, Coal.
Source: Ritchie and Rosado (2017).

Table A35: Weights on Non-Brown Energy and Brown $\left((\alpha_n^{NB})^{\frac{1}{\eta_1}}, (\alpha_n^B)^{\frac{1}{\eta_1}}\right)$ with $\eta_1 = 2.28$

Economy	$(\alpha_n^{NB})^{\frac{1}{\eta_1}}$	$(\alpha_n^B)^{\frac{1}{\eta_1}}$
United States (US)	0.9977	0.0999
China	0.9998	0.0318
India	0.9544	0.3651
European Union (EU)	0.9995	0.0511
Rest of the World (ROW)	0.9996	0.0453

Notes: Weights on non-brown energy (α_n^{NB}) and brown (α_n^B) are matched to the final expenditure on the brown and non-brown energy sectors, respectively.
Source: WIOD 2016 Release.

Table A36: Environmental Damage Parameter (ξ_n)

Economy	ξ_n
United States (US)	0.015
China	0.009
India	0.025
European Union (EU)	0.021
Rest of the World (ROW)	0.029

Notes: Environmental damage parameter (ξ_n) is estimated to match the impulse response of real GDP per capita to a 1°C increase in global temperature.
Source: Bilal and Känzig (2024).

A.9 Additional Quantitative Exercises

In this section, I present and discuss the results of additional quantitative exercises.

A.9.1 Unilateral Carbon Tax in China

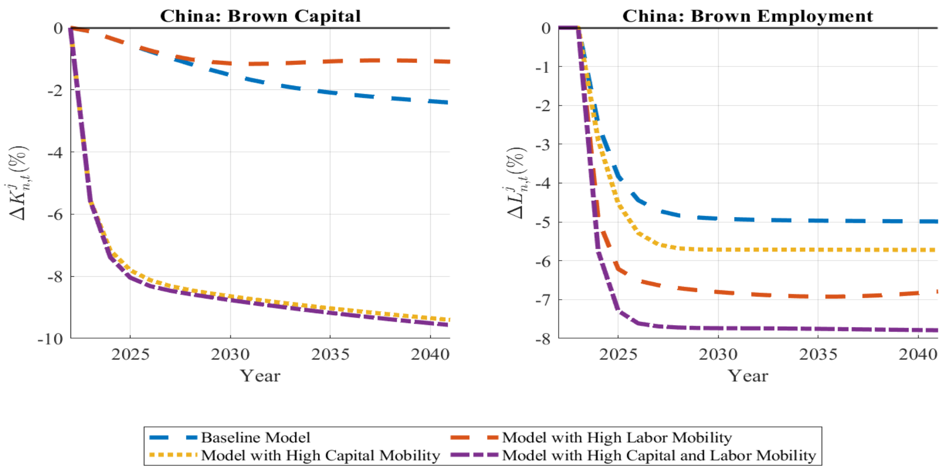
I discuss the change in global emissions under a policy scenario where China — which accounted for 30.7 percent of global emissions in 2022 — permanently increases consumption taxes on raw fossil fuels to the level currently implemented by the EU (Black et al., 2023). This scenario is compared to the business-as-usual (BAU) scenario where this policy is not implemented.

Capital and Employment in the Brown Sector The impact of China's policy on capital and employment in the brown sector across models is illustrated in Figure A2. The figure demonstrates that factor mobility significantly influences how much the carbon tax reduces capital and employment in the brown sector. In the baseline model, in response to the carbon tax, capital in the brown sector is 0.12 percent lower than the business-as-usual (BAU) level in 2023. In contrast, in the model with high capital mobility, capital in the brown sector is 5.57 percent below the BAU level in 2023. This reduction becomes more pronounced over time: the difference in capital between the two models increases from 5.45 percentage points of the BAU level in 2023 to 7.12 percentage points by 2040.

Making workers more responsive to wage changes leads to a greater reduction in employment in China's brown sector. For all models, the carbon tax does not affect employment in 2023, as workers cannot move in the short run. In the baseline model, employment in the brown sector is 2.50 percent lower than the business-as-usual (BAU) level in 2024. In contrast, in the model with high labor mobility, employment in the brown sector is 5.76 percent below the BAU level in 2024. The difference in employment reduction between the two models persists over time: the employment gap remains 1.60 percentage points below the BAU level in 2040.

Scale Effect The change in capital in the brown sector is influenced not only by the substitution effect but also by the scale effect. In the model with high labor mobility, capital in the brown sector drops less than in the baseline model. This is due to the scale effect, where rapid labor mobility allows workers in the brown sector to quickly adapt to the carbon tax, resulting in less consumption loss. A smaller drop in aggregate consumption leads to higher output and aggregate

Figure A2: Impact of a Carbon Tax in China on Brown Capital and Employment in China



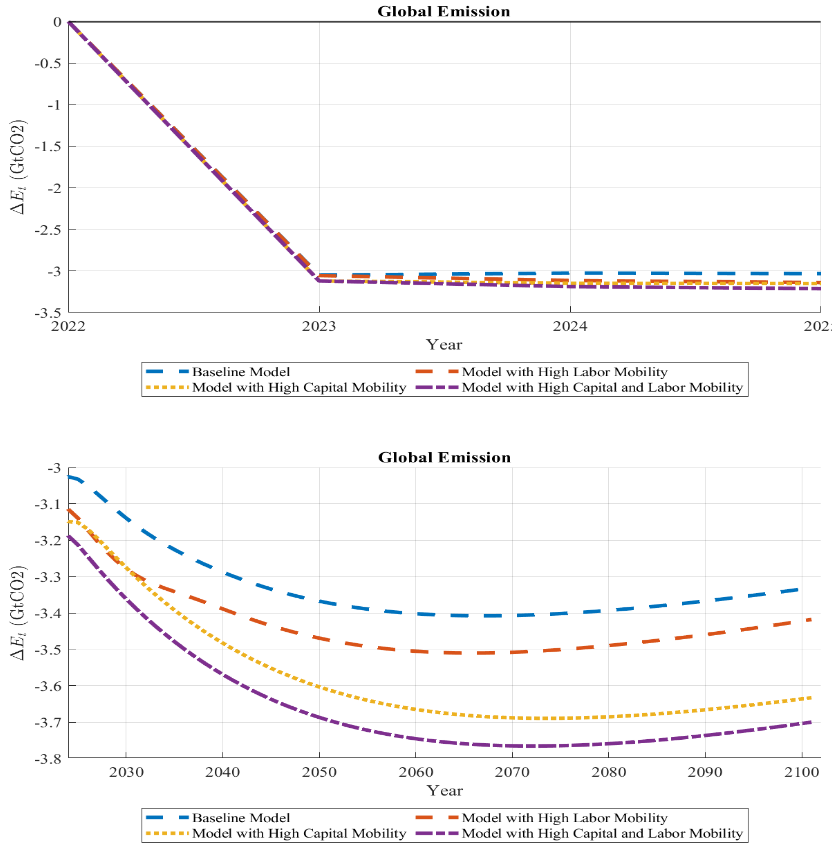
Notes: The impact of China's policy of unexpectedly and permanently increasing its consumption taxes on raw fossil fuels to the level currently implemented by the EU on capital and employment in China's brown sector. The blue line is the baseline model, and the red line represents the model with high labor mobility. The yellow line represents the model with high capital mobility, and the purple line indicates the model with high capital and labor mobility. The lines represent their values relative to the business-as-usual (BAU) case where the policy is not implemented.

capital. Consequently, despite a larger decline in employment in the brown sector, the scale effect mitigates the decline in capital in that sector.

Capital-Labor Complementarity Figure A2 also highlights the complementarity between capital and labor. The marginal product of labor in the brown sector decreases more in the rapid capital mobility model due to the larger decline in capital. Hence, China's carbon tax results in a greater reduction in employment in the model with rapid capital mobility compared to the baseline model.

Global Emissions The impact of China's unilateral carbon tax on global emissions is shown in Figure A3. In all models, the initial drop in global emissions is similar, with a reduction of 3.05 $GtCO_2$ (approximately 8.8 percent of global emissions in 2022) due to the increase in material prices and the subsequent decline in material usage and output. However, the figure demonstrates that the evolution of global emissions varies significantly across models. The model with greater la-

Figure A3: Impact of a Carbon Tax in China on Global Emissions

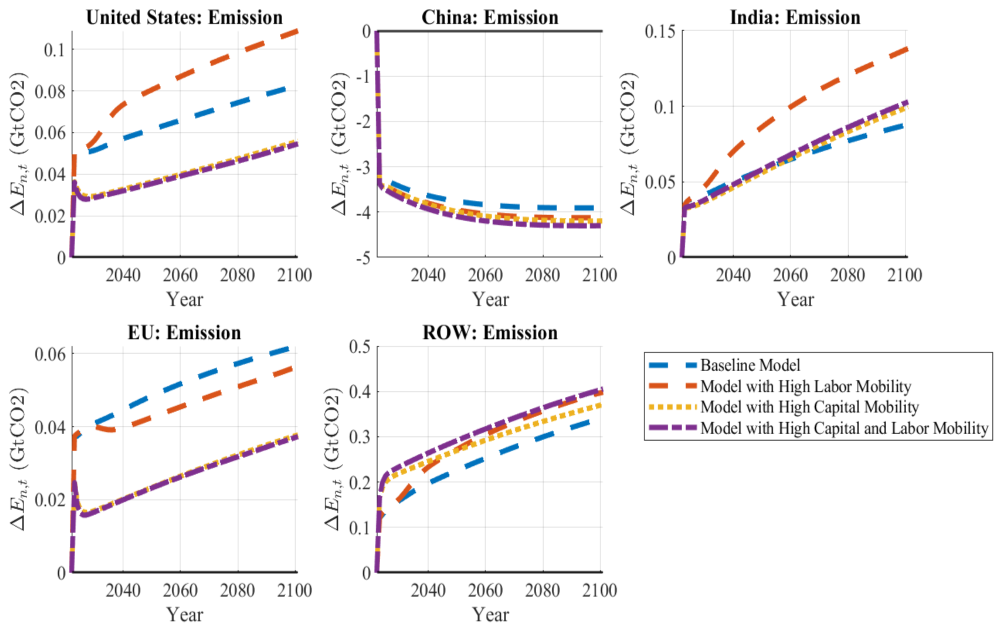


Notes: The impact of China's policy of unexpectedly and permanently increasing its consumption taxes on raw fossil fuels to the level currently implemented by the EU on global emissions. The top panel presents the results from 2022 to 2025. The bottm panel presents the results from 2023 to 2100. The blue line is the baseline model, and the red line represents the model with high labor mobility. The yellow line represents the model with high capital mobility, and the purple line indicates the model with high capital and labor mobility. The lines represent their values relative to the business-as-usual (BAU) case where the policy is not implemented.

bor mobility results in a 2.50 percent larger reduction in global emissions compared to the baseline by 2030 and a 2.63 percent larger reduction by 2100.

I find that even greater emission reductions occur with increased capital mobility, driven by a further decline in employment and capital in the brown sector, as shown in Figure A3. In the

Figure A4: Impact of a Carbon Tax in China on Economy-level Emissions



Notes: The impact of China's policy of unexpectedly and permanently increasing its consumption taxes on raw fossil fuels to the level currently implemented by the EU on economy-level emissions. The blue line is the baseline model, and the red line represents the model with high labor mobility. The yellow line represents the model with high capital mobility, and the purple line indicates the model with high capital and labor mobility. The lines represent their values relative to the business-as-usual (BAU) case where the policy is not implemented.

model with high capital mobility, the carbon tax achieves a 2.50 percent greater reduction in global emissions than the baseline by 2030, and a 9.05 percent greater reduction by 2100. Compared to the model with greater capital and labor mobility, the baseline achieves 6.61 percent less emission reduction by 2030 and 9.97 percent less emission reduction by 2100.

Carbon Leakage Figure A4 shows the impact of China's implementation of the carbon tax on emissions across economies. Emissions in economies outside of China increase as global demand for brown sector goods shifts from China to other economies, a phenomenon known as carbon leakage. The degree of factor mobility has an ambiguous effect on the magnitude of carbon leakage, as the size of the scale and substitution effects varies across economies.

A.9.2 Global Carbon Tax: Limiting Global Warming to 1.5 °C by 2100

I now compute the permanent global carbon tax required to meet the Paris Agreement's goal of limiting global warming to 1.5 °C above pre-industrial levels by 2100. Lee et al. (2023) estimates that, as of 2022, the remaining carbon budget is 428.17 $GtCO_2$ for a 50 percent chance of staying below the 1.5°C target. Using the baseline model, I calculate the minimum increase in uniform, permanent global consumption taxes on raw fossil fuels needed to keep cumulative emissions from 2022 to 2100 within this limit. The same policies are then simulated in counterfactual models with varying degrees of factor mobility and calculate how long it would take for cumulative emissions to reach 428.17 $GtCO_2$ in each model.

Table A37: Global Carbon Tax Required to Limit Warming to 1.5°C Above Pre-Industrial Levels

Model	**Δ Global Carbon Tax (p.p.)**	**Year 1.5°C Reached**
Baseline	0	2034
Baseline	410	2100
High Labor Mobility	410	2097
High Capital Mobility	410	2106
High Capital and Labor Mobility	410	2103

Notes: The table shows the permanent percentage points increase in the global fuel-specific taxes on raw fossil fuels in 2023 and the projected year in which global warming is expected to exceed 1.5 °C above pre-industrial levels, or when cumulative global emissions reach 428.17 $GtCO_2$, given the global carbon tax for each model.

As shown in Table A37, the baseline model predicts that global temperatures will exceed 1.5 °C above pre-industrial levels by 2034. To limit global warming to 1.5 °C by 2100, all economies must increase their consumption taxes on raw fossil fuels by at least 410 percentage points starting from 2023.

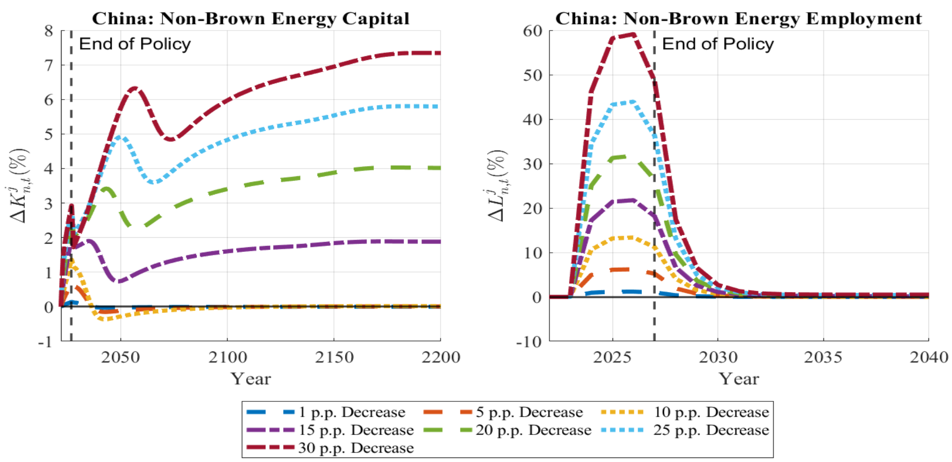
The table also shows that the same level of a global carbon tax in the model with high labor mobility does not delay the time to reach 1.5 °C; in fact, the target is reached three years earlier. This is because rapid labor mobility mitigates the scale effect that would otherwise push emissions downward, as workers in the brown sector adjust elastically to the shock.

In contrast, a model with high capital mobility significantly delays the point at which global temperatures exceed 1.5 °C. With a 410 percentage point increase in the global carbon tax, the 1.5 °C threshold is reached in 2106 in the high capital mobility model and in 2103 in the model with both rapid capital and labor mobility.

A.9.3 Big Push: Temporary Non-Brown Energy Subsidies

Temporary climate policies can have lasting effects due to high labor mobility costs and the dynamic agglomeration externality that governs the growth of sector-specific capital efficiencies. Thus, even temporary policies, if sufficiently large, can reshape the long-term distribution of capital and labor across sectors.

Figure A5: 5-year Non-Brown Energy Subsidies in China
on Non-Brown Energy Capital and Employment in China



Notes: Employment and capital allocations in the non-brown energy sector relative to the business-as-usual (BAU) allocations under various temporary the subsidy on the non-brown energy sector scenarios in China from 2023 to 2027. The lines represent scenarios with consumption tax reductions on the non-brown energy sector by 1, 5, 10, 15, 20, 25, and 30 percentage points.

To explore this channel, I simulate multiple scenarios where China introduces non-brown energy subsidies for five years, from 2023 to 2027, with consumption tax reductions on the non-brown energy sector ranging from 1 to 30 percentage points. I then compare how capital and labor allocations evolve under these policies against a business-as-usual (BAU) scenario.

Figure A5 illustrates the impact of these temporary subsidies on capital and labor allocations in China. The results show that reductions in the consumption tax by 1 to 10 percentage points have no lasting impact on capital allocation, as non-brown energy capital gradually reverts to BAU levels after the policies end. However, subsidies of 15 percentage points or more lead to a more persistent reallocation of capital toward the non-brown energy sector, even after the policy period. For instance, a 30 percentage point reduction results in over 7 percent more capital allocated to the non-brown energy sector by 2200 compared to the BAU scenario.

The long-term impact of these larger subsidies on capital allocation is driven by a strong dynamic agglomeration externality, governed by parameter ρ_Φ, which reinforces the growth of capital efficiency in the non-brown energy sector. In terms of employment, labor in the non-brown energy sector remains above BAU levels after the subsidies expire, though this effect diminishes over time, with employment converging to BAU levels within five years.

A.10 Derivations

A.10.1 Sectoral Good and Bilateral Trade Share

- Given problem:

$$\min_{y^j_{nm,t}} \sum_{m \in \mathcal{N}} p^j_{nm,t} y^j_{nm,t}$$

- Subject to:

$$Y^j_{n,t} = \left(\sum_{m \in \mathcal{N}} \left(y^j_{nm,t} \right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

- Set up the Lagrangian:

$$\mathcal{L} = \sum_{m \in \mathcal{N}} p^j_{nm,t} y^j_{nm,t} + \lambda \left(Y^j_{n,t} - \left(\sum_{m \in \mathcal{N}} \left(y^j_{nm,t} \right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \right)$$

- First-order condition with respect to $y^j_{nm,t}$:

$$\frac{\partial \mathcal{L}}{\partial y^j_{nm,t}} = p^j_{nm,t} - \lambda \cdot \frac{\sigma}{\sigma-1} \left(\sum_{m \in \mathcal{N}} \left(y^j_{nm,t} \right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{1}{\sigma-1}} \cdot \frac{\sigma-1}{\sigma} \left(y^j_{nm,t} \right)^{-\frac{1}{\sigma}} = 0$$

- Simplify:

$$p^j_{nm,t} = \lambda \left(\sum_{m \in \mathcal{N}} \left(y^j_{nm,t} \right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{1}{\sigma-1}} \left(y^j_{nm,t} \right)^{-\frac{1}{\sigma}}$$

- Rearrange:

$$\left(y^j_{nm,t} \right)^{-\frac{1}{\sigma}} = \frac{p^j_{nm,t}}{\lambda} \left(\sum_{m \in \mathcal{N}} \left(y^j_{nm,t} \right)^{\frac{\sigma-1}{\sigma}} \right)^{-\frac{1}{\sigma-1}}$$

- Raise both sides to the power of $-\sigma$:

$$y^j_{nm,t} = \left(\frac{p^j_{nm,t}}{\lambda}\right)^{-\sigma} \left(\sum_{m \in \mathcal{N}} \left(y^j_{nm,t}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$$

- Substitute the constraint:

$$y^j_{nm,t} = \left(\frac{p^j_{nm,t}}{\lambda}\right)^{-\sigma} Y^j_{n,t}$$

- Sum over all m and use the constraint:

$$\begin{aligned} Y^j_{n,t} = \sum_{m \in \mathcal{N}} y^j_{nm,t} = Y^j_{n,t} \sum_{m \in \mathcal{N}} \left(\frac{p^j_{nm,t}}{\lambda}\right)^{-\sigma} \\ 1 = \sum_{m \in \mathcal{N}} \left(\frac{p^j_{nm,t}}{\lambda}\right)^{-\sigma} \\ \lambda = \left(\sum_{m \in \mathcal{N}} (p^j_{nm,t})^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \end{aligned}$$

- Substitute this back into the equation for $y^j_{nm,t}$:

$$y^j_{nm,t} = Y^j_{n,t} \frac{(p^j_{nm,t})^{-\sigma}}{\sum_{m \in \mathcal{N}} (p^j_{nm,t})^{1-\sigma}}$$

- Define the price index $P^j_{n,t}$:

$$P^j_{n,t} \equiv \left(\sum_{m \in \mathcal{N}} (p^j_{nm,t})^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$$

- Demand:

$$y^j_{nm,t} = \left(\frac{p^j_{nm,t}}{P^j_{n,t}}\right)^{-\sigma} Y^j_{n,t}$$

- Bilateral Trade Share:

$$\pi^j_{nm,t} \equiv \frac{p^j_{nm,t} y^j_{nm,t}}{P^j_{n,t} Y^j_{n,t}} = \left(\frac{p^j_{nm,t}}{P^j_{n,t}}\right)^{1-\sigma}$$

A.10.2 Energy Composite Good

- Problem: Minimize $(1+\tau^{NB}_{n,t})P^{NB}_{n,t} + (1+\tau^{B}_{n,t})P^{B}_{n,t}$ subject to the constraint:

$$Y^E_{n,t} = \left((\alpha^{NB}_n)^{\frac{1}{\eta_1}} (Y^{NB}_{n,t})^{\frac{\eta_1-1}{\eta_1}} + (\alpha^{B}_n)^{\frac{1}{\eta_1}} (Y^{B}_{n,t})^{\frac{\eta_1-1}{\eta_1}}\right)^{\frac{\eta_1}{\eta_1-1}}$$

- Form the Lagrangian:

$$\begin{aligned} L &= (1+\tau^{NB}_{n,t})P^{NB}_{n,t} + (1+\tau^{B}_{n,t})P^{B}_{n,t} \\ &\quad + \lambda\Big(Y^E_{n,t} - \Big((\alpha^{NB}_n)^{\frac{1}{\eta_1}} (Y^{NB}_{n,t})^{\frac{\eta_1-1}{\eta_1}} + (\alpha^{B}_n)^{\frac{1}{\eta_1}} (Y^{B}_{n,t})^{\frac{\eta_1-1}{\eta_1}}\Big)^{\frac{\eta_1}{\eta_1-1}}\Big) \end{aligned}$$

- Find the partial derivatives:

$$\begin{aligned} \frac{\partial L}{\partial Y^{NB}_{n,t}} &= (1+\tau^{NB}_{n,t})P^{NB}_{n,t} - \lambda \cdot (Y^E_{n,t})^{\frac{1}{\eta_1}} \cdot (\alpha^{NB}_n)^{\frac{1}{\eta_1}} \cdot (Y^{NB}_{n,t})^{-\frac{1}{\eta_1}} = 0 \\ \frac{\partial L}{\partial Y^{B}_{n,t}} &= (1+\tau^{B}_{n,t})P^{B}_{n,t} - \lambda \cdot (Y^E_{n,t})^{\frac{1}{\eta_1}} \cdot (\alpha^{B}_n)^{\frac{1}{\eta_1}} \cdot (Y^{B}_{n,t})^{-\frac{1}{\eta_1}} = 0 \\ \frac{\partial L}{\partial \lambda} &= Y^E_{n,t} - \Big((\alpha^{NB}_n)^{\frac{1}{\eta_1}} (Y^{NB}_{n,t})^{\frac{\eta_1-1}{\eta_1}} + (\alpha^{B}_n)^{\frac{1}{\eta_1}} (Y^{B}_{n,t})^{\frac{\eta_1-1}{\eta_1}}\Big)^{\frac{\eta_1}{\eta_1-1}} = 0 \end{aligned}$$

- Simplify the first two equations:

$$\begin{aligned} (1+\tau^{NB}_{n,t})P^{NB}_{n,t} &= \lambda \cdot (Y^E_{n,t})^{\frac{1}{\eta_1}} \cdot (\alpha^{NB}_n)^{\frac{1}{\eta_1}} \cdot (Y^{NB}_{n,t})^{-\frac{1}{\eta_1}} \\ (1+\tau^{B}_{n,t})P^{B}_{n,t} &= \lambda \cdot (Y^E_{n,t})^{\frac{1}{\eta_1}} \cdot (\alpha^{B}_n)^{\frac{1}{\eta_1}} \cdot (Y^{B}_{n,t})^{-\frac{1}{\eta_1}} \end{aligned}$$

- Divide these equations:

$$\frac{(1+\tau^{NB}_{n,t})P^{NB}_{n,t}}{(1+\tau^{B}_{n,t})P^{B}_{n,t}} = \frac{(\alpha^{NB}_n)^{\frac{1}{\eta_1}} \cdot (Y^{NB}_{n,t})^{-\frac{1}{\eta_1}}}{(\alpha^{B}_n)^{\frac{1}{\eta_1}} \cdot (Y^{B}_{n,t})^{-\frac{1}{\eta_1}}}$$

- Rearrange to get the ratio of $Y_{n,t}^{NB}$ to $Y_{n,t}^{B}$:

$$Y_{n,t}^{NB} = \Big(\frac{\alpha_n^{NB}}{\alpha_n^B}\Big) \cdot \Big(\frac{(1+\tau_{n,t}^{NB})P_{n,t}^{NB}}{(1+\tau_{n,t}^{B})P_{n,t}^{B}}\Big)^{-\eta_1} Y_{n,t}^{B}$$

- Substitute this ratio into the constraint equation:

$$\begin{aligned}
Y_{n,t}^{E} &= \Big((\alpha_n^{NB})^{\frac{1}{\eta_1}} (Y_{n,t}^{B}\Big(\frac{\alpha_n^{NB}}{\alpha_n^B}\Big) \cdot \Big(\frac{(1+\tau_{n,t}^{NB})P_{n,t}^{NB}}{(1+\tau_{n,t}^{B})P_{n,t}^{B}}\Big)^{-\eta_1})^{\frac{\eta_1-1}{\eta_1}} + (\alpha_n^{B})^{\frac{1}{\eta_1}} (Y_{n,t}^{B})^{\frac{\eta_1-1}{\eta_1}}\Big)^{\frac{\eta_1}{\eta_1-1}} \\
&= Y_{n,t}^{B}\Big((\alpha_n^{NB})^{\frac{1}{\eta_1}} \Big(\frac{\alpha_n^{NB}}{\alpha_n^B}\Big)^{\frac{\eta_1-1}{\eta_1}} \cdot \Big(\frac{(1+\tau_{n,t}^{NB})P_{n,t}^{NB}}{(1+\tau_{n,t}^{B})P_{n,t}^{B}}\Big)^{1-\eta_1} + (\alpha_n^{B})^{\frac{1}{\eta_1}}\Big)^{\frac{\eta_1}{\eta_1-1}} \\
&= (\alpha_n^{B})^{-1}((1+\tau_{n,t}^{B})P_{n,t}^{B})^{\eta_1} Y_{n,t}^{B}\Big(\alpha_n^{NB}\Big((1+\tau_{n,t}^{NB})P_{n,t}^{NB}\Big)^{1-\eta_1} + \alpha_n^{B}\Big((1+\tau_{n,t}^{B})P_{n,t}^{B}\Big)^{1-\eta_1}\Big)^{\frac{\eta_1}{\eta_1-1}}
\end{aligned}$$

- Define the price index $P_{n,t}^{E}$:

$$P_{n,t}^{E} = \left(\alpha_n^{NB}\Big((1+\tau_{n,t}^{NB})P_{n,t}^{NB}\Big)^{1-\eta_1} + \alpha_n^{B}\Big((1+\tau_{n,t}^{B})P_{n,t}^{B}\Big)^{1-\eta_1}\right)^{\frac{1}{1-\eta_1}}$$

- Demand for brown sector

$$Y_{n,t}^{B} = \alpha_n^{B}\Big(\frac{(1+\tau_{n,t}^{B})P_{n,t}^{B}}{P_{n,t}^{E}}\Big)^{-\eta_1} Y_{n,t}^{E}$$

A.10.3 Material

- Problem: Minimize $\sum_{k\in\mathcal{J}}(1+\tau_{n,t}^{k})P_{n,t}^{k}M_{n,t}^{j,k}$ subject to the constraint:

$$M_{n,t}^{j} = \prod_{k\in\mathcal{J}} \left(M_{n,t}^{j,k}\right)^{\gamma_n^{j,k}}$$

- Form the Lagrangian:

$$L = \sum_{k\in\mathcal{J}}(1+\tau_{n,t}^{k})P_{n,t}^{k}M_{n,t}^{j,k} + \lambda\left(M_{n,t}^{j} - \prod_{k\in\mathcal{J}} \left(M_{n,t}^{j,k}\right)^{\gamma_n^{j,k}}\right)$$

- Find the partial derivatives:

$$\frac{\partial L}{\partial M_{n,t}^{j,k}} = (1+\tau_{n,t}^{k})P_{n,t}^{k} - \lambda \cdot \gamma_n^{j,k} \cdot \prod_{k' \in \mathcal{J}} \left(M_{n,t}^{j,k'}\right)^{\gamma_n^{j,k'}} \cdot \left(M_{n,t}^{j,k}\right)^{-1} = 0$$
$$\frac{\partial L}{\partial \lambda} = M_{n,t}^{j} - \prod_{k \in \mathcal{J}} \left(M_{n,t}^{j,k}\right)^{\gamma_n^{j,k}} = 0$$

- Simplify the first equation:

$$(1+\tau_{n,t}^{k})P_{n,t}^{k} = \lambda \cdot \gamma_n^{j,k} \cdot \prod_{k' \in \mathcal{J}} \left(M_{n,t}^{j,k'}\right)^{\gamma_n^{j,k'}} \cdot \left(M_{n,t}^{j,k}\right)^{-1}$$
$$(1+\tau_{n,t}^{k})P_{n,t}^{k} = \lambda \cdot \gamma_n^{j,k} \cdot M_{n,t}^{j} \cdot \left(M_{n,t}^{j,k}\right)^{-1}$$

- Rearrange the equation to solve for $M_{n,t}^{j,k}$:

$$M_{n,t}^{j,k} = M_{n,t}^{j} \cdot \frac{\gamma_n^{j,k}}{(1+\tau_{n,t}^{k})P_{n,t}^{k}} \cdot \lambda$$

- Sum the demand equations and solve for λ:

$$M_{n,t}^{j} = \prod_{k \in \mathcal{J}} \left(M_{n,t}^{j,k}\right)^{\gamma_n^{j,k}}$$
$$M_{n,t}^{j} = \prod_{k \in \mathcal{J}} \left(M_{n,t}^{j} \cdot \frac{\gamma_n^{j,k}}{(1+\tau_{n,t}^{k})P_{n,t}^{k}} \cdot \lambda\right)^{\gamma_n^{j,k}}$$
$$M_{n,t}^{j} = M_{n,t}^{j} \cdot \left(\prod_{k \in \mathcal{J}} \left(\frac{\gamma_n^{j,k}}{(1+\tau_{n,t}^{k})P_{n,t}^{k}}\right)^{\gamma_n^{j,k}} \cdot \lambda^{\sum_{k \in \mathcal{J}} \gamma_n^{j,k}}\right)$$
$$1 = \prod_{k \in \mathcal{J}} \left(\frac{\gamma_n^{j,k}}{(1+\tau_{n,t}^{k})P_{n,t}^{k}}\right)^{\gamma_n^{j,k}} \cdot \lambda$$
$$\lambda = \prod_{k \in \mathcal{J}} \left(\frac{(1+\tau_{n,t}^{k})P_{n,t}^{k}}{\gamma_n^{j,k}}\right)^{\gamma_n^{j,k}}$$

- Define the price index:

$$P_{n,t}^{M,j} = \prod_{k \in \mathcal{J}} \left(\frac{(1+\tau_{n,t}^{k}) P_{n,t}^{k}}{\gamma_n^{j,k}} \right)^{\gamma_n^{j,k}}$$

- Substitute λ back into the demand equation:

$$M_{n,t}^{j,k} = M_{n,t}^{j} \cdot \frac{\gamma_n^{j,k} P_{n,t}^{M,j}}{(1+\tau_{n,t}^{k}) P_{n,t}^{k}}$$

A.10.4 Final Good

- Problem: Minimize $\sum_{k \in \mathcal{J}_D} (1+\tau_{n,t}^{k}) P_{n,t}^{j} C_{n,t}^{j} + P_{n,t}^{E} C_{n,t}^{E}$ subject to the constraint:

$$Y_{n,t} = \mathcal{D}_n(\mathcal{C}_t) \Big(\prod_{j \in \mathcal{J}_D} (C_{n,t}^{j})^{\alpha_n^j} \Big)^{\alpha_n} (C_{n,t}^{E})^{1-\alpha_n}$$

- Form the Lagrangian:

$$L = \sum_{j \in \mathcal{J}_D} (1+\tau_{n,t}^{j}) P_{n,t}^{j} C_{n,t}^{j} + P_{n,t}^{E} C_{n,t}^{E} + \lambda \left(Y_{n,t} - \mathcal{D}_n(\mathcal{C}_t) \Big(\prod_{j \in \mathcal{J}_D} (C_{n,t}^{j})^{\alpha_n^j} \Big)^{\alpha_n} (C_{n,t}^{E})^{1-\alpha_n} \right)$$

- Find the partial derivatives:

$$\begin{aligned}
\frac{\partial L}{\partial C_{n,t}^{j}} &= (1+\tau_{n,t}^{j}) P_{n,t}^{j} - \lambda \cdot \mathcal{D}_n(\mathcal{C}_t) \cdot \alpha_n \cdot \alpha_n^j \cdot \Big(\prod_{j' \in \mathcal{J}_D} (C_{n,t}^{j'})^{\alpha_n^{j'}} \Big)^{\alpha_n} \cdot (C_{n,t}^{j})^{-1} (C_{n,t}^{E})^{1-\alpha_n} = 0 \\
\frac{\partial L}{\partial C_{n,t}^{E}} &= P_{n,t}^{E} - \lambda \cdot \mathcal{D}_n(\mathcal{C}_t) \cdot (1-\alpha_n) \cdot \Big(\prod_{j \in \mathcal{J}_D} (C_{n,t}^{j})^{\alpha_n^j} \Big)^{\alpha_n} \cdot (C_{n,t}^{E})^{-\alpha_n} = 0 \\
\frac{\partial L}{\partial \lambda} &= Y_{n,t} - \mathcal{D}_n(\mathcal{C}_t) \Big(\prod_{j \in \mathcal{J}_D} (C_{n,t}^{j})^{\alpha_n^j} \Big)^{\alpha_n} (C_{n,t}^{E})^{1-\alpha_n} = 0
\end{aligned}$$

- Simplify the first equation:

$$(1+\tau_{n,t}^{j})P_{n,t}^{j} = \lambda \cdot \mathcal{D}_n(\mathcal{C}_t) \cdot \alpha_n \cdot \alpha_n^{j} \cdot \Big(\prod_{j' \in \mathcal{J}_D} (C_{n,t}^{j'})^{\alpha_n^{j'}} \Big)^{\alpha_n} \cdot (C_{n,t}^{j})^{-1} (C_{n,t}^{E})^{1-\alpha_n}$$
$$(1+\tau_{n,t}^{j})P_{n,t}^{j} = \lambda \cdot \mathcal{D}_n(\mathcal{C}_t) \cdot \alpha_n \cdot \alpha_n^{j} \cdot Y_{n,t} \cdot (C_{n,t}^{j})^{-1}$$

- Rearrange the equations to solve for $C_{n,t}^{j}$, $C_{n,t}^{E}$:

$$C_{n,t}^{j} = Y_{n,t} \cdot \frac{\alpha_n \cdot \alpha_n^{j}}{(1+\tau_{n,t}^{j})P_{n,t}^{j}} \cdot \lambda$$
$$C_{n,t}^{E} = Y_{n,t} \cdot \frac{(1-\alpha_n)}{P_{n,t}^{E}} \cdot \lambda$$

- Sum the demand equations and solve for λ:

$$Y_{n,t} = \mathcal{D}_n(\mathcal{C}_t)\Big(\prod_{j \in \mathcal{J}_D} (C_{n,t}^{j})^{\alpha_n^{j}} \Big)^{\alpha_n} (C_{n,t}^{E})^{1-\alpha_n}$$
$$Y_{n,t} = \lambda Y_{n,t} \mathcal{D}_n(\mathcal{C}_t)\Big(\prod_{j \in \mathcal{J}_D} \Big(\frac{\alpha_n \cdot \alpha_n^{j}}{(1+\tau_{n,t}^{j})P_{n,t}^{j}} \cdot \Big)^{\alpha_n^{j}} \Big)^{\alpha_n} \Big(\frac{(1-\alpha_n)}{P_{n,t}^{E}} \Big)^{1-\alpha_n}$$
$$1 = \lambda \mathcal{D}_n(\mathcal{C}_t)\Big(\prod_{j \in \mathcal{J}_D} \Big(\frac{\alpha_n \cdot \alpha_n^{j}}{(1+\tau_{n,t}^{j})P_{n,t}^{j}} \cdot \Big)^{\alpha_n^{j}} \Big)^{\alpha_n} \Big(\frac{(1-\alpha_n)}{P_{n,t}^{E}} \Big)^{1-\alpha_n}$$
$$\lambda = \Big(\mathcal{D}_n(\mathcal{C}_t) \Big)^{-1} \Big(\prod_{j \in \mathcal{J}_D} \Big(\frac{(1+\tau_{n,t}^{j})P_{n,t}^{j}}{\alpha_n \cdot \alpha_n^{j}} \cdot \Big)^{\alpha_n^{j}} \Big)^{\alpha_n} \Big(\frac{P_{n,t}^{E}}{(1-\alpha_n)} \Big)^{1-\alpha_n}$$

- Define the price index:

$$P_{n,t} = (\mathcal{D}_n(\mathcal{C}_t))^{-1} \left(\prod_{j \in \mathcal{J}_D} \Big((1+\tau_{n,t}^{j})P_{n,t}^{j} \Big)^{\alpha_n^{j}} \right)^{\alpha_n} \big(P_{n,t}^{E}\big)^{1-\alpha_n}$$

- Substitute λ back into the demand equation:

$$C_{n,t}^j = Y_{n,t} \cdot \frac{\alpha_n \cdot \alpha_n^j P_{n,t}}{(1+\tau_{n,t}^j)P_{n,t}^j}$$
$$C_{n,t}^E = Y_{n,t} \cdot \frac{(1-\alpha_n)P_{n,t}}{P_{n,t}^E}$$

A.11 Brown Variety

- Problem: Minimize

$$w_{n,t}^B L_{mn,t}^B + r_{n,t}^B K_{mn,t}^B + s_{n,t} F_{mn,t}^B + P_{n,t}^{M,B} M_{mn,t}^B$$

subject to the constraint:

$$y_{mn,t}^B = (\frac{z_{n,t}^B}{d_{mn,t}^B})((F_{mn,t}^B)^{\psi_n^{B,F}}(L_{mn,t}^B)^{\psi_n^{B,L}}(K_{mn,t}^B)^{\psi_n^{B,K}})^{\gamma_n^{B,V}}(M_{mn,t}^B)^{1-\gamma_n^{B,V}}$$

- First-order conditions:

$$L_{mn,t}^B = \lambda_{n,t} \cdot y_{mn,t}^B(\frac{\gamma_n^{B,V} \cdot \psi_n^{B,L}}{w_{n,t}^B})$$
$$K_{mn,t}^B = \lambda_{n,t} \cdot y_{mn,t}^B(\frac{\gamma_n^{B,V} \cdot \psi_n^{B,K}}{r_{n,t}^B})$$
$$F_{mn,t}^B = \lambda_{n,t} \cdot y_{mn,t}^B(\frac{\gamma_n^{B,V} \cdot \psi_n^{B,F}}{s_{n,t}})$$
$$M_{mn,t}^B = \lambda_{n,t} \cdot y_{mn,t}^B(\frac{1-\gamma_n^{B,V}}{P_{n,t}^{M,B}})$$

- Lagrangian multiplier equals marginal cost:

$$\lambda_{n,t} \cdot y_{mn,t}^B = w_{n,t}^B L_{mn,t}^B + r_{n,t}^B K_{mn,t}^B + s_{n,t} F_{mn,t}^B + P_{n,t}^{M,B} M_{mn,t}^B$$
$$\lambda_{n,t} = \frac{1}{y_{mn,t}^B}(w_{n,t}^B L_{mn,t}^B + r_{n,t}^B K_{mn,t}^B + s_{n,t} F_{mn,t}^B + P_{n,t}^{M,B} M_{mn,t}^B)$$

- Plugging into the constraint:

$$1 = \lambda_{n,t} \cdot (\frac{z^B_{n,t}}{d^B_{mn,t}})((\frac{\gamma_n^{B,V} \cdot \psi_n^{B,F}}{s_{n,t}})^{\psi_n^{B,F}} (\frac{\gamma_n^{B,V} \cdot \psi_n^{B,L}}{w^B_{n,t}})^{\psi_n^{B,L}} (\frac{\gamma_n^{B,V} \cdot \psi_n^{B,K}}{r^B_{n,t}})^{\psi_n^{B,K}})^{\gamma_n^{B,V}} (\frac{1-\gamma_n^{B,V}}{P^{M,B}_{n,t}})^{1-\gamma_n^{B,V}}$$

$$\lambda_{n,t} = (\frac{d^B_{mn,t}}{z^B_{n,t}})((\frac{s_{n,t}}{\gamma_n^{B,V} \cdot \psi_n^{B,F}})^{\psi_n^{B,F}} (\frac{w^B_{n,t}}{\gamma_n^{B,V} \cdot \psi_n^{B,L}})^{\psi_n^{B,L}} (\frac{r^B_{n,t}}{\gamma_n^{B,V} \cdot \psi_n^{B,K}})^{\psi_n^{B,K}})^{\gamma_n^{B,V}} (\frac{P^{M,B}_{n,t}}{1-\gamma_n^{B,V}})^{1-\gamma_n^{B,V}}$$

$$mc^B_{mn,t} = (\frac{d^B_{mn,t}}{z^B_{n,t}})((\frac{s_{n,t}}{\gamma_n^{B,V} \cdot \psi_n^{B,F}})^{\psi_n^{B,F}} (\frac{w^B_{n,t}}{\gamma_n^{B,V} \cdot \psi_n^{B,L}})^{\psi_n^{B,L}} (\frac{r^B_{n,t}}{\gamma_n^{B,V} \cdot \psi_n^{B,K}})^{\psi_n^{B,K}})^{\gamma_n^{B,V}} (\frac{P^{M,B}_{n,t}}{1-\gamma_n^{B,V}})^{1-\gamma_n^{B,V}}$$

A.12 Non-Brown Energy and Non-energy Varieties

- Problem: Minimize

$$w^j_{n,t} L^j_{mn,t} + r^j_{n,t} K^j_{mn,t} + P^E_{n,t} Q^j_{mn,t} + P^{M,j}_{n,t} M^j_{mn,t}$$

subject to the constraint:

$$y^j_{mn,t} = (\frac{z^j_{n,t}}{d^j_{mn,t}})[(Q^j_{mn,t})^{\psi_n^{j,E}} (L^j_{mn,t})^{\psi_n^{j,L}} (K^j_{mn,t})^{\psi_n^{j,K}}]^{\gamma_n^{j,V}} (M^j_{mn,t})^{1-\gamma_n^{j,V}}$$

- First-order conditions:

$$L^j_{mn,t} = \lambda_{n,t} \cdot y^j_{mn,t} (\frac{\gamma_n^{j,V} \cdot \psi_n^{j,L}}{w^j_{n,t}})$$

$$K^j_{mn,t} = \lambda_{n,t} \cdot y^j_{mn,t} (\frac{\gamma_n^{j,V} \cdot \psi_n^{j,K}}{r^j_{n,t}})$$

$$Q^j_{mn,t} = \lambda_{n,t} \cdot y^j_{mn,t} (\frac{\gamma_n^{j,V} \cdot \psi_n^{j,E}}{P^E_{n,t}})$$

$$M^j_{mn,t} = \lambda_{n,t} \cdot y^j_{mn,t} (\frac{1-\gamma_n^{j,V}}{P^{M,j}_{n,t}})$$

- Lagrangian multiplier equals marginal cost:

$$\lambda_{n,t} \cdot y^j_{mn,t} = w^j_{n,t} L^j_{mn,t} + r^j_{n,t} K^j_{mn,t} + P^E_{n,t} Q^j_{mn,t} + P^{M,j}_{n,t} M^j_{mn,t}$$
$$\lambda_{n,t} = \frac{1}{y^j_{mn,t}} (w^j_{n,t} L^j_{mn,t} + r^j_{n,t} K^j_{mn,t} + P^E_{n,t} Q^j_{mn,t} + P^{M,j}_{n,t} M^j_{mn,t})$$

- Plugging into the constraint:

$$1 = \lambda_{n,t} \cdot (\frac{z^j_{n,t}}{d^j_{mn,t}}) \cdot [(\frac{\gamma^{j,V}_n \cdot \psi^{j,E}_n}{P^E_{n,t}})^{\psi^{j,E}_n} (\frac{\gamma^{j,V}_n \cdot \psi^{j,L}_n}{w^j_{n,t}})^{\psi^{j,L}_n} (\frac{\gamma^{j,V}_n \cdot \psi^{j,K}_n}{r^j_{n,t}})^{\psi^{j,K}_n}]^{\gamma^{j,V}_n} (\frac{1-\gamma^{j,V}_n}{P^{M,j}_{n,t}})^{1-\gamma^{j,V}_n}$$
$$\lambda_{n,t} = (\frac{d^j_{mn,t}}{z^j_{n,t}}) \cdot [(\frac{P^E_{n,t}}{\gamma^{j,V}_n \cdot \psi^{j,E}_n})^{\psi^{j,E}_n} (\frac{w^j_{n,t}}{\gamma^{j,V}_n \cdot \psi^{j,L}_n})^{\psi^{j,L}_n} (\frac{r^j_{n,t}}{\gamma^{j,V}_n \cdot \psi^{j,K}_n})^{\psi^{j,K}_n}]^{\gamma^{j,V}_n} (\frac{P^{M,j}_{n,t}}{1-\gamma^{j,V}_n})^{1-\gamma^{j,V}_n}$$
$$mc^j_{mn,t} = (\frac{d^j_{mn,t}}{z^j_{n,t}}) \cdot [(\frac{P^E_{n,t}}{\gamma^{j,V}_n \cdot \psi^{j,E}_n})^{\psi^{j,E}_n} (\frac{w^j_{n,t}}{\gamma^{j,V}_n \cdot \psi^{j,L}_n})^{\psi^{j,L}_n} (\frac{r^j_{n,t}}{\gamma^{j,V}_n \cdot \psi^{j,K}_n})^{\psi^{j,K}_n}]^{\gamma^{j,V}_n} (\frac{P^{M,j}_{n,t}}{1-\gamma^{j,V}_n})^{1-\gamma^{j,V}_n}$$

A.12.1 Labor Mobility Share

- Expected lifetime utility of a worker in sector j at time t

$$V^j_{n,t} = \ln(c^j_{n,t}) + \log(\mu^j_n) + \max_{k \in \mathcal{J}} \left\{ \beta V^k_{n,t+1} + \ln(1-\kappa^{j,k}_n) + \rho_L \varepsilon^k_{n,t} \right\}$$

- Denote $A^{j,k}_{n,t} \equiv \beta V^k_{n,t+1} + \ln(1-\kappa^{j,k}_n)$
- Decision variable

$$D(\vec{\varepsilon}_{n,t}, j, k) = \mathbb{1}\left[k = \arg\max_{h \in \mathcal{J}} \left\{ A^{j,h}_{n,t} + \rho_L \varepsilon^h_{n,t} \right\}\right]$$

- Gumbel cumulative distribution function:

$$F(x) = \exp(-\exp(-(x+\gamma)))$$

where γ is a Euler–Mascheroni constant

- Gumble probability density function: $f(x) = \exp(-x-\gamma)\exp(-e^{-x-\gamma})$

- Probability of choosing sector k

$$\begin{aligned}
\Pr\Big(D(\vec{\varepsilon}_{n,t}, j, k) = 1\Big) &= \Pr\Big(A^{j,k}_{n,t} + \rho_L \varepsilon^k_{n,t} \geq \max\{A^{j,h}_{n,t} + \rho_L \varepsilon^h_{n,t}\}\Big) \\
&= \int_{-\infty}^{\infty} f(\varepsilon^k_{n,t}) \prod_{h \neq k} F\Big(\frac{A^{j,k}_{n,t} - A^{j,h}_{n,t}}{\rho_L} + \varepsilon^k_{n,t}\Big) d\varepsilon^k_{n,t} \\
&= \int_{-\infty}^{\infty} e^{-\varepsilon^k_{n,t} - \gamma} e^{-\exp(-\varepsilon^k_{n,t} - \gamma)} \times \prod_{h \neq k} e^{-\exp(-(\frac{A^{j,k}_{n,t} - A^{j,h}_{n,t}}{\rho_L}) - \varepsilon^k_{n,t} - \gamma)} \\
&= \int_{-\infty}^{\infty} \exp(-\varepsilon^k_{n,t} - \gamma) \\
&\times \exp\Big(- \exp(-\varepsilon^k_{n,t} - \gamma) \times \sum_{h \in \mathcal{J}} \exp(\frac{A^{j,h}_{n,t} - A^{j,k}_{n,t}}{\rho_L})\Big) d\varepsilon^k_{n,t}
\end{aligned}$$

- Change of variables: $x = \varepsilon^k_{n,t} + \gamma$, $y = \log\Big(\sum_{h \in \mathcal{J}} \exp(\frac{A^{j,h}_{n,t} - A^{j,k}_{n,t}}{\rho_L})\Big)$
- Given that $dx = d\varepsilon^k_{n,t}$,

$$\Pr\Big(D(\vec{\varepsilon}_{n,t}, j, k) = 1\Big) = \int_{-\infty}^{\infty} e^{-x - e^{y-x}} dx$$

- Change of variable: $z = x - y$ and $dz = dx$,

$$\begin{aligned}
\Pr\Big(D(\vec{\varepsilon}_{n,t}, j, k) = 1\Big) &= \int_{-\infty}^{\infty} e^{-z - y - e^{-z}} dz = e^{-y} \int_{-\infty}^{\infty} e^{-z - e^{-z}} dz \\
&= e^{-y} = \Big(\sum_{h \in \mathcal{J}} \exp(A^{j,h}_{n,t} - A^{j,k}_{n,t})^{\frac{1}{\rho_L}}\Big)^{-1} \\
&= \frac{\exp(A^{j,k}_{n,t})^{\frac{1}{\rho_L}}}{\sum_{h \in \mathcal{J}} \exp(A^{j,h}_{n,t})^{\frac{1}{\rho_L}}} \\
&= \frac{\exp(\beta V^k_{n,t+1} + \ln(1 - \kappa^{j,k}_n))^{\frac{1}{\rho_L}}}{\sum_{h \in \mathcal{J}} \exp(\beta V^h_{n,t+1} + \ln(1 - \kappa^{j,h}_n))^{\frac{1}{\rho_L}}}
\end{aligned}$$

- Therefore, since $m_{n,t}^{j,k} = \Pr\Big(D(\vec{\varepsilon}_{n,t}, j, k) = 1\Big)$,

$$m_{n,t}^{j,k} = \frac{\exp(\beta V_{n,t+1}^{k} + \ln(1-\kappa_n^{j,k}))^{\frac{1}{\rho_L}}}{\sum_{h\in\mathcal{J}} \exp(\beta V_{n,t+1}^{h} + \ln(1-\kappa_n^{j,h}))^{\frac{1}{\rho_L}}}$$

A.12.2 Expected Life-time Utility of a Worker

- Expected future value of choosing sector k for a worker in sector j

$$\begin{aligned}
\tilde{V}_{n,t+1}^{j,k} &= \int_{-\infty}^{\infty} \Big(A_{n,t}^{j,k} + \rho_L \varepsilon_{n,t}^{k}\Big) f(\varepsilon_{n,t}^{k}) \prod_{h\neq k} F\Big(\frac{A_{n,t}^{j,k} - A_{n,t}^{j,h}}{\rho_L} + \varepsilon_{n,t}^{k}\Big) d\varepsilon_{n,t}^{k} \\
&= \int_{-\infty}^{\infty} \Big(A_{n,t}^{j,k} + \rho_L \varepsilon_{n,t}^{k}\Big) \times \exp(-\varepsilon_{n,t}^{k} - \gamma) \\
&\times \exp\Big(-\exp(-\varepsilon_{n,t}^{k} - \gamma) \times \sum_{h\in\mathcal{J}} \exp(\frac{A_{n,t}^{j,h} - A_{n,t}^{j,k}}{\rho_L})\Big) d\varepsilon_{n,t}^{k}
\end{aligned}$$

- Change of variables: $x = \varepsilon_{n,t}^{k} + \gamma$, $y = \log\Big(\sum_{h\in\mathcal{J}} \exp(\frac{A_{n,t}^{j,h} - A_{n,t}^{j,k}}{\rho_L})\Big)$

$$\tilde{V}_{n,t+1}^{j,k} = \int_{-\infty}^{\infty} \Big(A_{n,t}^{j,k} + \rho_L(x-\gamma)\Big) \exp(-x - e^{y-x}) dx$$

- Change of variable: $z = x - y$ and $dz = dx$,

$$\begin{aligned}
\tilde{V}_{n,t+1}^{j,k} &= \int_{-\infty}^{\infty} \Big(A_{n,t}^{j,k} + \rho_L(z+y-\gamma)\Big) \exp(-(z+y) - e^{-z}) dz \\
&= A_{n,t}^{j,k} e^{-y} \int_{-\infty}^{\infty} \exp(-z - e^{-z}) dz \\
&+ \rho_L e^{-y} \int_{-\infty}^{\infty} z \exp(-z - e^{-z}) dz \\
&+ \rho_L e^{-y}(y-\gamma) \int_{-\infty}^{\infty} \exp(-z - e^{-z}) dz \\
&= A_{n,t}^{j,k} e^{-y} + \rho_L e^{-y}\gamma + \rho_L e^{-y}(y-\gamma) \\
&= e^{-y}(A_{n,t}^{j,k} + \rho_L y)
\end{aligned}$$

- Re-arrange the expression for y

$$
\begin{aligned}
y &= \log\Big(\exp(\frac{-A^{j,k}_{n,t}}{\rho_L})\sum_{h\in\mathcal{J}}\exp(\frac{A^{j,h}_{n,t}}{\rho_L})\Big)\\
&= -\Big(\frac{A^{j,k}_{n,t}}{\rho_L}\Big) + \log\Big(\sum_{h\in\mathcal{J}}\exp(\frac{A^{j,h}_{n,t}}{\rho_L})\Big)
\end{aligned}
$$

- Evaluate e^{-y}

$$
e^{-y} = \exp\Big(\frac{A^{j,k}_{n,t}}{\rho_L}\Big)\Big(\sum_{h\in\mathcal{J}}\exp(\frac{A^{j,h}_{n,t}}{\rho_L})\Big)^{-1} = m^{j,k}_{n,t}
$$

- Plugging in the definition y,

$$
\begin{aligned}
\tilde{V}^{j,k}_{n,t+1} &= e^{-y}\Big(A^{j,k}_{n,t} - A^{j,k}_{n,t} + \rho_L\log\Big(\sum_{h\in\mathcal{J}}\exp(\frac{A^{j,h}_{n,t}}{\rho_L})\Big)\Big)\\
&= e^{-y}\Big(\rho_L\log\Big(\sum_{h\in\mathcal{J}}\exp(\frac{A^{j,h}_{n,t}}{\rho_L})\Big)\Big)\\
&= m^{j,k}_{n,t}\Big(\rho_L\log\Big(\sum_{h\in\mathcal{J}}\exp(\frac{A^{j,h}_{n,t}}{\rho_L})\Big)\Big)
\end{aligned}
$$

- Expected future value of worker in sector j

$$
\begin{aligned}
\tilde{V}^{j}_{n,t+1} &= \max_{k\in\mathcal{J}}\{\beta V^{k}_{n,t+1} + \ln(1-\kappa^{j,k}_{n}) + \rho_L\varepsilon^{k}_{n,t}\}\\
&= \sum_{k\in\mathcal{J}} V^{j,k}_{n,t+1} = \sum_{k\in\mathcal{J}} m^{j,k}_{n,t}\Big(\rho_L\log\Big(\sum_{h\in\mathcal{J}}\exp(\frac{A^{j,h}_{n,t}}{\rho_L})\Big)\Big)\\
&= \rho_L\log\Big(\sum_{h\in\mathcal{J}}\exp(\frac{A^{j,h}_{n,t}}{\rho_L})\Big)\\
&= \rho_L\ln\left(\sum_{k\in\mathcal{J}}\exp\Big((\frac{\beta}{\rho_L})V^{k}_{n,t+1} + (\frac{1}{\rho_L})\ln(1-\kappa^{j,k}_{n})\Big)\right)
\end{aligned}
$$

A.12.3 Fossil Fuel Extraction Problem

- Cost Minimization Problem

$$\min_{f^o_{n,t}, f^g_{n,t}, f^c_{n,t}} \quad (1+\tau^o_{n,t})s^o_{n,t}f^o_{n,t} + (1+\tau^g_{n,t})s^g_{n,t}f^g_{n,t} + (1+\tau^c_{n,t})s^c_{n,t}f^c_{n,t}$$

$$\text{s.t.}$$

$$F_{n,t} = \left(\sum_h (\omega^h_n)^{\frac{1}{\eta_2}} (f^h_{n,t})^{\frac{\eta_2-1}{\eta_2}}\right)^{\frac{\eta_2}{\eta_2-1}}$$

- First order condition

$$(1+\tau^h_{n,t})s^h_{n,t} = \lambda_{n,t}(\omega^h_n)^{\frac{1}{\eta_2}}(f^h_{n,t})^{\frac{-1}{\eta_2}}\left(\sum_h (\omega^h_n)^{\frac{1}{\eta_2}}(f^h_{n,t})^{\frac{\eta_2-1}{\eta_2}}\right)^{\frac{\eta_2}{\eta_2-1}-1}$$

$$\frac{(1+\tau^h_{n,t})s^h_{n,t}}{(1+\tau^k_{n,t})s^k_{n,t}} = \left(\frac{\omega^h_n}{\omega^k_n}\right)^{\frac{1}{\eta_2}}\left(\frac{f^h_n}{f^k_n}\right)^{-\frac{1}{\eta_2}}$$

- Re-arranging the equations, fossil fuel demand is obtained as follows:

$$f^h_{n,t} = f^k_{n,t}\left(\frac{\omega^h_n}{\omega^k_n}\right)\left(\frac{(1+\tau^k_{n,t})s^k_{n,t}}{(1+\tau^h_{n,t})s^h_{n,t}}\right)^{\eta_2}$$

$$F_{n,t} = f^k_{n,t}(\omega^k_n)^{-1}((1+\tau^k_{n,t})s^k_{n,t})^{\eta_2}\left(\sum_h (\omega^h_n)((1+\tau^h_{n,t})s^h_{n,t})^{1-\eta_2}\right)^{\frac{\eta_2}{\eta_2-1}}$$

$$f^k_{n,t} = \omega^k_n\left(\frac{(1+\tau^k_{n,t})s^k_{n,t}}{\left(\sum_h (\omega^h_n)((1+\tau^h_{n,t})s^h_{n,t})^{1-\eta_2}\right)^{\frac{1}{1-\eta_2}}}\right)^{-\eta_2}F_{n,t}$$

- Price of fossil fuel input is defined as follows:

$$s_{n,t} = \left(\sum_h \omega^h_n((1+\tau^h_{n,t})s^h_{n,t})^{1-\eta_2}\right)^{\frac{1}{1-\eta_2}}$$

- Then, the fossil fuel demand policy function is given by:

$$f^k_{n,t} = \omega^k_n \Big(\frac{(1+\tau^k_{n,t})s^k_{n,t}}{s_{n,t}}\Big)^{-\eta_2} F_{n,t}$$

with

$$\lambda_{n,t} = \sum_h (1+\tau^h_{n,t})s^h_{n,t} f^h_{n,t} = \frac{\sum_h \omega^h_n((1+\tau^h_{n,t})s^h_{n,t})^{1-\eta_2}}{(s_{n,t})^{-\eta_2}} F_{n,t}$$

A.12.4 Capitalist Optimization Problem

- Utility Maximization Problem

$$V^K_{n,0}(W_{n,0}) = \max_{W_{n,t+1}} \quad \sum_{t=0}^{\infty} \beta^t \mathbb{E}_0 \Big[u(C^K_{n,t}) \Big]$$

s.t.

$$C^K_{n,t} + W_{n,t+1} = R_{n,t} W_{n,t} + \frac{\Omega_{n,t}}{P_{n,t}},$$

- Return on aggregate capital: $R_{n,t} = \Big(\frac{v_{n,t}}{P_{n,t}} + (1-\delta^K_n)\Big)$
- First order conditions

$$\begin{aligned} \frac{\beta^t}{C^K_{n,t}} &= \lambda_{n,t} P_{n,t} \\ \lambda_{n,t+1}\Big(v_{n,t+1} + (1-\delta^K_n)P_{n,t+1}\Big) &= \lambda_{n,t} P_{n,t} \\ \frac{\lambda_{n,t+1}P_{n,t+1}}{\lambda_{n,t}P_{n,t}}\Big(\frac{v_{n,t+1}}{P_{n,t+1}} + (1-\delta^K_n)\Big) &= 1 \\ \Big(\frac{v_{n,t+1}}{P_{n,t+1}} + (1-\delta^K_n)\Big) &= \frac{C^K_{n,t+1}}{\beta C^K_{n,t}} \end{aligned}$$

- Optimality condition

$$C^K_{n,t+1} = \beta R_{n,t+1} C^K_{n,t}$$

- Present discounted value of future non-capital income

$$
\begin{aligned}
h_{n,t} &= \sum_{s=1}^{\infty}\Big(\prod_{u=1}^{s} R_{n,t+u}\Big)^{-1}\Big(\frac{\Pi_{n,t+s}+\Omega_{n,t+s}}{P_{n,t+s}}\Big)\\
&= \Big(R_{n,t+1}\Big)^{-1}\Big(\frac{\Pi_{n,t+1}+\Omega_{n,t+1}}{P_{n,t+1}}\Big) + \Big(R_{n,t+1}\Big)^{-1}\sum_{s=1}^{\infty}\Big(\prod_{u=1}^{s} R_{n,t+1+u}\Big)^{-1}\Big(\frac{\Pi_{n,t+1+s}+\Omega_{n,t+s}}{P_{n,t+1+s}}\Big)\\
&= \Big(R_{n,t+1}\Big)^{-1}\Big(\frac{\Pi_{n,t+1}+\Omega_{n,t+1}}{P_{n,t+1}}\Big) + \Big(R_{n,t+1}\Big)^{-1} h_{n,t+1}
\end{aligned}
$$

- Conjecture (guess and verify)

$$
\begin{aligned}
C_{n,t}^{K} &= (1-\beta)\Big(R_{n,t}W_{n,t} + \Big(\frac{+\Omega_{n,t}}{P_{n,t}}\Big) + h_{n,t}\Big)\\
W_{n,t+1} &= \beta\Big(R_{n,t}W_{n,t} + \Big(\frac{\Omega_{n,t}}{P_{n,t}}\Big) + h_{n,t}\Big) - h_{n,t}\\
W_{n,t+1} &= R_{n,t}W_{n,t} + \Big(\frac{\Omega_{n,t}}{P_{n,t}}\Big) - C_{n,t}^{K}
\end{aligned}
$$

- Check optimality condition

$$
\begin{aligned}
C_{n,t}^{K} &= \Big(\beta R_{n,t+1}\Big)^{-1} C_{n,t+1}^{K}\\
\Big(R_{n,t}W_{n,t} + \Big(\frac{\Omega_{n,t}}{P_{n,t}}\Big) + h_{n,t}\Big) &= \Big(\beta R_{n,t+1}\Big)^{-1}\Big(R_{n,t+1}W_{n,t+1} + \Big(\frac{\Pi_{n,t+1}+\Omega_{n,t+1}}{P_{n,t+1}}\Big) + h_{n,t+1}\Big)\\
\Big(R_{n,t}W_{n,t} + \Big(\frac{\Omega_{n,t}}{P_{n,t}}\Big) + h_{n,t}\Big) &= \beta^{-1}W_{n,t+1} + \Big(\beta R_{n,t+1}\Big)^{-1}\Big(\Big(\frac{\Pi_{n,t+1}+\Omega_{n,t+1}}{P_{n,t+1}}\Big) + h_{n,t+1}\Big)\\
(\beta)^{-1}h_{n,t} &= \Big(\beta R_{n,t+1}\Big)^{-1}\Big(\Big(\frac{\Pi_{n,t+1}+\Omega_{n,t+1}}{P_{n,t+1}}\Big) + h_{n,t+1}\Big)\\
R_{n,t+1}h_{n,t} &= \Big(\frac{\Pi_{n,t+1}+\Omega_{n,t+1}}{P_{n,t+1}}\Big) + h_{n,t+1}\\
\frac{\Pi_{n,t+1}+\Omega_{n,t+1}}{P_{n,t+1}} &= R_{n,t+1}h_{n,t} - h_{n,t+1}
\end{aligned}
$$

A.12.5 Capital Allocation Problem

- Decision variable for capital allocated to sector k

$$D(k, \vec{\zeta}_{n,t}) = \mathbf{1}\Big[k = \arg\max_{h \in \mathcal{J}} \Big\{\zeta^h_{n,t} P_{n,t} r^h_{n,t}\Big\}\Big]$$

- Fréchet distribution for idiosyncratic capital use efficiency $\zeta^j_{n,t}$

$$\begin{aligned}
F^j(\zeta^j_{n,t}) &= e^{-((\zeta^j_{n,t})/a^j_n)^{-\rho_K}} \\
f^j(\zeta^j_{n,t}) &= \Big(\rho_K (a^j_{n,t})^{\rho_K} (\zeta^j_{n,t})^{-\rho_K - 1}\Big) e^{-((\zeta^j_{n,t})/a^j_n)^{-\rho_K}}
\end{aligned}$$

- Probability of investing in sector j

$$\begin{aligned}
\text{Pr(invest in sector } j) &= \text{Pr}\Big(j = \arg\max\{P_{n,t}(\zeta^j_{n,t}) a^j_{n,t}\}\Big) \\
&= \int_0^\infty \Big(\prod_{k \neq j} F^k\Big(\frac{r^j_{n,t}(\zeta^j_{n,t})}{r^k_{n,t}}\Big)\Big) f^j(\zeta^j_{n,t}) d(\zeta^j_{n,t}) \\
&= \int_0^\infty \Big(\prod_{k \neq j} e^{-\left(\frac{r^j_{n,t}(\zeta^j_{n,t})}{r^k_{n,t} a^k_{n,t}}\right)^{-\rho_K}}\Big)\Big(\rho_K (a^j_{n,t})^{\rho_K} (\zeta^j_{n,t})^{-\rho_K - 1}\Big) e^{-((\zeta^j_{n,t})/a^j_n)^{-\rho_K}} d(\zeta^j_{n,t}) \\
&= \int_0^\infty \rho_K (a^j_{n,t})^{\rho_K} (\zeta^j_{n,t})^{-\rho_K - 1} e^{-\left(\sum_{k \neq j}\left(\frac{r^j_{n,t}}{r^k_{n,t} a^k_{n,t}}\right)^{-\rho_K} + (a^j_{n,t})^{\rho_K}\right)(\zeta^j_{n,t})^{-\rho_K}} d(\zeta^j_{n,t})
\end{aligned}$$

- Define variable

$$\begin{aligned}
\alpha &= \sum_{k \neq j} \Big(\frac{r^j_{n,t}}{r^k_{n,t} a^k_{n,t}}\Big)^{-\rho_K} + (a^j_{n,t})^{\rho_K} \\
&= \sum_{k \neq j} \Big(r^k_{n,t} a^k_{n,t}\Big)^{\rho_K} (r^j_{n,t})^{-\rho_K} + (a^j_{n,t})^{\rho_K} \\
&= (r^j_{n,t})^{-\rho_K} \Big(\sum_{k \neq j} (r^k_{n,t} a^k_{n,t})^{\rho_K} + (r^j_{n,t} a^j_{n,t})^{\rho_K}\Big) \\
&= (r^j_{n,t})^{-\rho_K} \Big(\sum_{k} (r^k_{n,t} a^k_{n,t})^{\rho_K}\Big)
\end{aligned}$$

- Simplification of the integral

$$
\begin{aligned}
\Pr(\text{invest in sector } j) &= \int_0^\infty \rho_K (a^j_{n,t})^{\rho_K} (\zeta^j_{n,t})^{-\rho_K - 1} e^{-\alpha(\zeta^j_{n,t})^{-\rho_K}} d(\zeta^j_{n,t}) \\
&= (a^j_{n,t})^{\rho_K} \int_0^\infty e^{-\alpha u} du \quad \text{(using substitution } u = (\zeta^j_{n,t})^{-\rho_K}) \\
&= (a^j_{n,t})^{\rho_K} \left[-\frac{1}{\alpha} e^{-\alpha u} \right]_0^\infty \\
&= \frac{(a^j_{n,t})^{\rho_K}}{\alpha}
\end{aligned}
$$

- Probability of investing in sector j

$$
\Phi^j_{n,t} = \Pr(\text{invest in sector } j) = \frac{(r^j_{n,t} a^j_{n,t})^{\rho_K}}{\sum_{k \in \mathcal{J}} (r^k_{n,t} a^k_{n,t})^{\rho_K}}
$$

- Denote $W^j_{n,t}$ aggregate capital allocated to sector j, i.e., $W^j_{n,t} \equiv \Phi^j_{n,t} W_{n,t}$
- Define return $v^j_{n,t} = \zeta^j_{n,t} P_{n,t} r^j_{n,t}$. Then, $\zeta^j_{n,t} = \frac{v^j_{n,t}}{P_{n,t} r^j_{n,t}}$.
- Distribution of return in sector j

$$
\begin{aligned}
\Psi^j_{n,t} &\equiv (a^j_{n,t} P_{n,t} r^j_{n,t})^{\rho_K} \\
F^j(v) &= e^{-\Psi^j_{n,t} v^{-\rho_K}} \\
f^j(\zeta^j_{n,t}) &= \Big(\rho_K \Psi^j_{n,t} v^{-\rho_K - 1} \Big) e^{-\Psi^j_{n,t} v^{-\rho_K}}
\end{aligned}
$$

- Distribution of return across all sectors

$$
\begin{aligned}
\Psi_{n,t} &\equiv \sum_{j \in \mathcal{J}} (a^j_{n,t} P_{n,t} r^j_{n,t})^{\rho_K} \\
F(v) &= \prod_{j \in \mathcal{J}} F^j(v) = e^{-\Psi_{n,t} v^{-\rho_K}} \\
f(v) &= \Big(\rho_K \Psi_{n,t} v^{-\rho_K - 1} \Big) e^{-\Psi_{n,t} v^{-\rho_K}}
\end{aligned}
$$

- Expected return

$$\begin{aligned}
v_{n,t} &= \int_0^\infty v f(v) dv \\
&= \int_0^\infty v \Big(\rho_K \Psi_{n,t} v^{-\rho_K - 1} \Big) e^{-\Psi_{n,t} v^{-\rho_K}} dv \\
&= \int_0^\infty \Big(\rho_K \Psi_{n,t} v^{-\rho_K} \Big) e^{-\Psi_{n,t} v^{-\rho_K}} dv
\end{aligned}$$

- Change of variable: $y = \Psi_{n,t} v^{-\rho_K}$, $v = \Psi_{n,t}^{\frac{1}{\rho_K}} y^{-\frac{1}{\rho_K}}$, $dy = (-\rho_K)\Psi_{n,t} v^{-\rho_K - 1} dv$

$$\begin{aligned}
v_{n,t} &= \int_0^\infty v e^{-y} dy \\
&= \int_0^\infty \Psi_{n,t}^{\frac{1}{\rho_K}} y^{-\frac{1}{\rho_K}} e^{-y} dy \\
&= \Psi_{n,t}^{\frac{1}{\rho_K}} \int_0^\infty y^{(1-\frac{1}{\rho_K})-1} e^{-y} dy \\
&= \Gamma\Big(1 - \frac{1}{\rho_K}\Big) \Psi_{n,t}^{\frac{1}{\rho_K}} \\
v_{n,t} &= \Gamma\Big(1 - \frac{1}{\rho_K}\Big) \Big(\sum_{j \in \mathcal{J}} (a_{n,t}^j P_{n,t} r_{n,t}^j)^{\rho_K} \Big)^{\frac{1}{\rho_K}}
\end{aligned}$$

- Therefore, the real return on aggregate capital is given by:

$$\frac{v_{n,t}}{P_{n,t}} = \Gamma\Big(1 - \frac{1}{\rho_K}\Big) \Big(\sum_{j \in \mathcal{J}} (a_{n,t}^j r_{n,t}^j)^{\rho_K} \Big)^{\frac{1}{\rho_K}}$$

- The allocation share can be expressed as follows:

$$\Phi_{n,t}^j = \frac{(r_{n,t}^j a_{n,t}^j)^{\rho_K}}{\sum_{k \in \mathcal{J}} (r_{n,t}^k a_{n,t}^k)^{\rho_K}} = \left(\Gamma\Big(\frac{\rho_K - 1}{\rho_K} \Big) \right)^{\rho_K} \Big(\frac{P_{n,t} r_{n,t}^j a_{n,t}^j}{v_{n,t}} \Big)^{\rho_K}$$

where $\sum_{j \in \mathcal{J}} (a_{n,t}^j r_{n,t}^j)^{\rho_K} = \left(\Big(\Gamma\Big(\frac{\rho_K - 1}{\rho_K} \Big) \Big)^{-1} \Big(\frac{v_{n,t}}{P_{n,t}} \Big) \right)^{\rho_K}$

- Distribution of return from sector j conditional on investing in sector j

$$
\begin{aligned}
\Pr(v|\text{invest in sector } j) &= \frac{1}{(\Phi^j_{n,t})}\Big(\int_0^v \Pi_{k\neq j}F^k(s)f^j(s)ds\Big)\\
&= \frac{1}{(\Phi^j_{n,t})}\Big(\int_0^v \big(\rho_K\Psi^j_{n,t}s^{-\rho_K-1}\big)F(s)ds\Big)\\
&= \frac{\Psi^j_{n,t}}{\Psi_{n,t}((\Phi^j_{n,t}))}\Big(\int_0^v \big(\rho_K\Psi_{n,t}s^{-\rho_K-1}\big)e^{-\Psi_{n,t}s^{-\rho_K-1}}ds\Big)\\
&= \int_0^v \big(\rho_K\Psi_{n,t}s^{-\rho_K-1}\big)e^{-\Psi_{n,t}s^{-\rho_K}}ds \quad \sim \quad \text{Frechet}(\Psi_{n,t}^{\frac{1}{\rho_K}},\rho_K)
\end{aligned}
$$

- Capital K^j_t invested in sector k

$$
K^j_{n,t} = W^j_{n,t}\int_0^\infty v\big(\rho_K\Psi_{n,t}v^{-\rho_K-1}\big)e^{-\Psi_{n,t}v^{-\rho_K}}dv
$$

- Efficiency: $v = P_{n,t}r^j_{n,t}(\zeta^j_{n,t})$

$$
\begin{aligned}
K^j_{n,t} &= W^j_{n,t}\int_0^\infty \big(\rho_K\Psi_{n,t}(P_{n,t}r^j_{n,t}(\zeta^j_{n,t}))^{-\rho_K}\big)e^{-\Psi_{n,t}(P_{n,t}r^j_{n,t}(\zeta^j_{n,t}))^{-\rho_K}}d(\zeta^j_{n,t})\\
&= W^j_{n,t}\int_0^\infty \big(\rho_K(a^j_{n,t})^{\rho_K}(\Phi^j_{n,t})^{-1}(\zeta^j_{n,t})^{-\rho_K}\big)e^{-(a^j_{n,t})^{\rho_K}(\Phi^j_{n,t})^{-1}(\zeta^j_{n,t})^{-\rho_K}}d(\zeta^j_{n,t})
\end{aligned}
$$

- Change of variable: $u = (a^j_{n,t})^{\rho_K}(\Phi^j_{n,t})^{-1}(\zeta^j_{n,t})^{-\rho_K}, du = -\rho_K(a^j_{n,t})^{\rho_K}(\Phi^j_{n,t})^{-1}(\zeta^j_{n,t})^{-\rho_K-1}d(\zeta^j_{n,t})$,

$$
K^j_{n,t} = W^j_{n,t}\int_0^\infty \big((\zeta^j_{n,t})\big)e^{-u}du
$$

- Change of Variable: $(\zeta^j_{n,t}) = a^j_{n,t}(\Phi^j_{n,t})^{-\frac{1}{\rho_K}}u^{-\frac{1}{\rho_K}}$

$$
\begin{aligned}
K^j_{n,t} &= W^j_{n,t}\Big(a^j_{n,t}(\Phi^j_{n,t})^{-\frac{1}{\rho_K}}\Big)\int_0^\infty \Big(u^{-\frac{1}{\rho_K}}\Big)e^{-u}du = W^j_{n,t}\Big(a^j_{n,t}(\Phi^j_{n,t})^{-\frac{1}{\rho_K}}\Big)\int_0^\infty \Big(u^{(1-\frac{1}{\rho_K})-1}\Big)e^{-u}du\\
&= \Gamma\Big(1-\frac{1}{\rho_K}\Big)\left(\frac{(a^j_{n,t})^{\rho_K}}{\Phi^j_{n,t}}\right)^{\frac{1}{\rho_K}}W^j_{n,t}
\end{aligned}
$$

- Sector-specific capital supply (using $W^j_{n,t} = \Phi^j_{n,t} W_{n,t}$ and $\Phi^j_{n,t} = \left(\Gamma\left(\frac{\rho_K-1}{\rho_K}\right)\right)^{\rho_K} \left(\frac{P_{n,t} r^j_{n,t} a^j_{n,t}}{v_{n,t}}\right)^{\rho_K}$)

$$\begin{aligned} K^j_{n,t} &= \Gamma\left(\frac{\rho_K - 1}{\rho_K}\right)\left(a^j_{n,t}\right)\left(\Phi^j_{n,t}\right)^{-\frac{1}{\rho_K}} W^j_{n,t} = \Gamma\left(\frac{\rho_K - 1}{\rho_K}\right)\left(a^j_{n,t}\right)\left(\Phi^j_{n,t}\right)^{\frac{\rho_K - 1}{\rho_K}} W_{n,t} \\ &= \left(\Gamma\left(\frac{\rho_K - 1}{\rho_K}\right)\right)^{\rho_K} \left(a^j_{n,t}\right)^{\rho_K} \left(\frac{P_{n,t} r^j_{n,t}}{v_{n,t}}\right)^{\rho_K - 1} W_{n,t} \end{aligned}$$

- This equation can be expressed as follows:

$$\ln(K^j_{n,t}) = (\rho_K - 1)\ln\left(\frac{P_{n,t} r^j_{n,t}}{v_{n,t}}\right) + \ln(W_{n,t}) + \rho_K \ln\left(a^j_{n,t}\right) + \rho_K \ln\left(\Gamma\left(\frac{\rho_K - 1}{\rho_K}\right)\right)$$

- Plugging in $\ln(a^j_{n,t}) = \ln(a^j_n) + \rho_\Phi \ln(\frac{K^j_{n,t-1}}{W_{n,t-1}})$, we get

$$\ln(K^j_{n,t}) = (\rho_K - 1)\ln\left(\frac{P_{n,t} r^j_{n,t}}{v_{n,t}}\right) + \ln(W_{n,t}) + \rho_K \rho_\Phi \ln(\frac{K^j_{n,t-1}}{W_{n,t-1}}) + \rho_K \ln(a^j_n) + \rho_K \ln\left(\Gamma\left(\frac{\rho_K - 1}{\rho_K}\right)\right)$$

A.12.6 Consumption-based Carbon Emissions

- Emission per fossil fuel input ($F_{n,t}$)

$$\phi^F_{n,t} = \frac{\sum_{h \in \{o,g,c\}} \overbrace{\phi^h \cdot f^h_{n,t}}^{\text{Emissions from fossil fuel type } h}}{F_{n,t}}$$

- Emission per brown output ($y^B_{n,t}$) where $y^B_{n,t} \equiv \sum_{m \in \mathcal{N}} y^B_{mn,t}$

$$\phi^B_{n,t} = \frac{\phi^F_{n,t} \cdot F_{n,t}}{y^B_{n,t}}$$

- Consumption-based Country-level Carbon Emissions ($E^C_{n,t}$)

$$E^C_{n,t} = \sum_{m \in \mathcal{N}} \phi^B_{m,t} \cdot y^B_{nm,t}$$

- Production-based Country-level Carbon Emissions ($E_{n,t}$)

$$E_{n,t} = \sum_{h \in \{o,g,c\}} \phi^h \cdot f_{n,t}^h$$

- Global Carbon Emissions (E_t)

$$E_t = \underbrace{\sum_{n \in \mathcal{N}} E_{n,t}}_{\text{Production-based}} = \underbrace{\sum_{n \in \mathcal{N}} E_{n,t}^C}_{\text{Consumption-based}}$$

A.12.7 Estimation Equation for Labor Mobility Costs

- The expected lifetime utility of a worker in sector j of economy n at time t is $V_{n,t}^j$ given by

$$V_{n,t}^j = \ln\left(\frac{w_{n,t}^j}{P_{n,t}}\right) + \ln(\mu_n^j) + \rho_L \ln\left(\sum_{k \in \mathcal{J}} \exp\left(\beta V_{n,t+1}^j + \ln(1-\kappa_n^{j,k})\right)^{\frac{1}{\rho_L}}\right).$$

- Labor mobility share $m_{n,t}^{j,k}$ is given by

$$m_{n,t}^{j,k} = \frac{\exp\left(\beta V_{n,t+1}^k + \ln(1-\kappa_n^{j,k})\right)^{\frac{1}{\rho_L}}}{\sum_{h \in \mathcal{J}} \exp\left(\beta V_{n,t+1}^h + \ln(1-\kappa_n^{j,h})\right)^{\frac{1}{\rho_L}}}.$$

- The equation can be arranged as follows:

$$\rho_L \ln(m_{n,t}^{j,k}) = \beta V_{n,t+1}^k + \ln(1-\kappa_n^{j,k}) - \rho_L \ln\left(\sum_{k \in \mathcal{J}} \exp\left(\beta V_{n,t+1}^j + \ln(1-\kappa_n^{j,k})\right)^{\frac{1}{\rho_L}}\right)$$

$$\ln\left(\frac{L_{n,t}^{j,k}}{L_{n,t}^j}\right) = \frac{\beta}{\rho_L} V_{n,t+1}^k + \frac{1}{\rho_L}\ln(1-\kappa_n^{j,k}) - \ln\left(\sum_{k \in \mathcal{J}} \exp\left(\beta V_{n,t+1}^j + \ln(1-\kappa_n^{j,k})\right)^{\frac{1}{\rho_L}}\right)$$

$$\ln\left(L_{n,t}^{j,k}\right) = \frac{\beta}{\rho_L} V_{n,t+1}^k + \frac{1}{\rho_L}\ln(1-\kappa_n^{j,k}) + \ln\left(L_{n,t}^j\right) - \ln\left(\sum_{k \in \mathcal{J}} \exp\left(\beta V_{n,t+1}^j + \ln(1-\kappa_n^{j,k})\right)^{\frac{1}{\rho_L}}\right)$$

- The equation implies the following regression specification:

$$L_{n,t}^{j,k} = \exp\left(C_n^{j,k} + \lambda_{n,t}^k + \lambda_{n,t}^j\right) + \xi_{n,t}^{j,k}.$$

  where

  - $C_n^{j,k} \equiv \frac{\ln(1-\kappa_n^{j,k})}{\rho_L}$,
  - $\lambda_{n,t}^k \equiv \frac{\beta}{\rho_L} V_{n,t+1}^k$,
  - $\lambda_{n,t}^j \equiv \ln(L_{n,t}^j) - \ln\left(\sum_{h\in\mathcal{J}} \exp\left(\frac{\beta}{\rho_L} V_{n,t+1}^h + \frac{1}{\rho_L}\ln(1-\kappa_n^{j,h})\right)\right)$
  - $\xi_{n,t}^{j,k}$: the residual term.

- Also, the expected life-time utility $V_{n,t}^j$ can be expressed as follows:

$$V_{n,t}^j = \ln\left(\frac{w_{n,t}^j}{P_{n,t}}\right) + \ln(\mu_n^j) + \beta V_{n,t+1}^k + \ln(1-\kappa_n^{j,k}) - \rho_L \ln(m_{n,t}^{j,k})$$
$$V_{n,t}^j = \ln\left(\frac{w_{n,t}^j}{P_{n,t}}\right) + \ln(\mu_n^j) + \beta V_{n,t+1}^j - \rho_L \ln(m_{n,t}^{j,j})$$
$$V_{n,t}^k = \ln\left(\frac{w_{n,t}^k}{P_{n,t}}\right) + \ln(\mu_n^k) + \beta V_{n,t+1}^j + \ln(1-\kappa_n^{k,j}) - \rho_L \ln(m_{n,t}^{k,j})$$
$$V_{n,t}^k = \ln\left(\frac{w_{n,t}^k}{P_{n,t}}\right) + \ln(\mu_n^k) + \beta V_{n,t+1}^k - \rho_L \ln(m_{n,t}^{k,k})$$

- Taking differences of these equations leads to the following equations:

$$0 = \beta V_{n,t+1}^k - \beta V_{n,t+1}^j + \ln(1-\kappa_n^{j,k}) - \rho_L \ln\left(\frac{m_{n,t}^{j,k}}{m_{n,t}^{j,j}}\right)$$
$$0 = \beta V_{n,t+1}^j - \beta V_{n,t+1}^k + \ln(1-\kappa_n^{k,j}) - \rho_L \ln\left(\frac{m_{n,t}^{k,j}}{m_{n,t}^{k,k}}\right)$$
$$0 = \beta V_{n,t+1}^k - \beta V_{n,t+1}^j + \ln(1-\kappa_n^{j,k}) - \rho_L \ln\left(\frac{L_{n,t}^{j,k}}{L_{n,t}^{j,j}}\right)$$
$$0 = \beta V_{n,t+1}^j - \beta V_{n,t+1}^k + \ln(1-\kappa_n^{k,j}) - \rho_L \ln\left(\frac{L_{n,t}^{k,j}}{L_{n,t}^{k,k}}\right)$$

- Adding last two equations leads to the following equations:

$$0 = \ln(1-\kappa_n^{j,k}) + \ln(1-\kappa_n^{k,j}) - \rho_L \ln\Big(\frac{L_{n,t}^{k,j} L_{n,t}^{j,k}}{L_{n,t}^{j,j} L_{n,t}^{k,k}}\Big)$$

$$\rho_L \ln\Big(\frac{L_{n,t}^{k,j} L_{n,t}^{j,k}}{L_{n,t}^{j,j} L_{n,t}^{k,k}}\Big) = \ln(1-\kappa_n^{j,k}) + \ln(1-\kappa_n^{k,j})$$

- The equation implies the following regression specification:

$$\frac{L_{n,t}^{j,k} L_{n,t}^{k,j}}{L_{n,t}^{j,j} L_{n,t}^{k,k}} = \exp(\tilde{C}_n^{j,k}) + \tilde{\xi}_{n,t}^{j,k}$$

  where

  - $\tilde{C}_n^{j,k} \equiv \frac{1}{\rho_L} \ln(1-\kappa_{n,t}^{j,k}) + \frac{1}{\rho_L} \ln(1-\kappa_{n,t}^{k,j})$,
  - $\tilde{\xi}_{n,t}^{j,k}$: residual term.

A.12.8 Carbon Tax Equivalent to Removing Fossil Fuel Subsidies

- Price of fossil fuel composite with fossil fuel subsides is given by:

$$s_{n,t} = \Big(\sum_h \omega_n^h ((1+\tau_{n,t}^h) s_{n,t}^h)^{1-\eta_2}\Big)^{\frac{1}{1-\eta_2}}$$

- Price of fossil fuel composite without fossil fuel subsides is given by:

$$s_{n,t}^0 = \Big(\sum_h \omega_n^h (s_{n,t}^h)^{1-\eta_2}\Big)^{\frac{1}{1-\eta_2}}$$

- Demand for fossil fuel composite by the brown variety producers is given by:

$$F_{mn,t}^B = \frac{(\gamma_n^{B,V} \cdot \psi_n^{B,F}) p_{mn,t}^B y_{mn,t}^B}{(1+\tau_{n,t}^B) s_{n,t}}$$

- Expressing the demand in terms of $s^0_{n,t}$ and $\tilde{\tau}^B_{n,t} \equiv \frac{s_{n,t}}{s^0_{n,t}} - 1$ (fossil fuel subsidy):

$$F^B_{mn,t} = \frac{(\gamma^{B,V}_n \cdot \psi^{B,F}_n) p^B_{mn,t} y^B_{mn,t}}{(1+\tau^B_{n,t})(1+\tilde{\tau}^B_{n,t}) s^0_{n,t}}$$

- As discussed in the main text, the initial carbon tax equals zero, i.e., $\tau^B_{n,t} = 0$.
- Carbon tax is equivalent to removing fossil fuel subsides if the following condition holds:

$$(1+\tau^B_{n,t})(1+\tilde{\tau}^B_{n,t}) = 1.$$

- Then, carbon tax equivalent to removing fossil fuel subsides is given by:

$$\tau^B_{n,t} = (1+\tilde{\tau}^B_{n,t})^{-1} - 1.$$

A.13 Derivations for Estimation Equations

A.13.1 Estimation Equation for Dispersion of Non-pecuniary Preferences

- As derived in Section A.12.7, the life-time expected utility $V^j_{n,t}$ can be expressed as follows:

$$V^j_{n,t} = \ln\left(\frac{w^j_{n,t}}{P_{n,t}}\right) + \ln(\mu^j_n) + \beta V^k_{n,t+1} + \ln(1-\kappa^{j,k}_n) - \rho_L \ln(m^{j,k}_{n,t})$$
$$V^j_{n,t} = \ln\left(\frac{w^j_{n,t}}{P_{n,t}}\right) + \ln(\mu^j_n) + \beta V^j_{n,t+1} - \rho_L \ln(m^{j,j}_{n,t})$$

- Then, taking the difference of two equations leads to the following equations

$$\rho_L \log(\frac{m^{j,k}_{n,t}}{m^{j,j}_{n,t}}) = \beta\Big(V^k_{n,t+1} - V^j_{n,t+1}\Big) + \log(1-\kappa^{j,k}_n)$$

- Future expected life-time utilities are given by

$$V^k_{n,t+1} = \log\left(\frac{w^k_{n,t+1}}{P_{n,t+1}}\right) + \log(\mu^k_n) - \rho_L \log(m^{k,k}_{n,t+1}) + \beta V^k_{n,t+2}$$
$$V^j_{n,t+1} = \log\left(\frac{w^j_{n,t+1}}{P_{n,t+1}}\right) + \log(\mu^j_n) - \rho_L \log(m^{j,k}_{n,t+1}) + \beta V^k_{n,t+2} + \log(1-\kappa^{j,k}_n)$$

- Taking the difference leads to the following equation

$$V^k_{n,t+1} - V^j_{n,t+1} = \log\left(\frac{w^k_{n,t+1}}{w^j_{n,t+1}}\right) + \log\left(\frac{\mu^k_n}{\mu^j_n}\right) + \rho_L \log(\frac{m^{j,k}_{n,t+1}}{m^{k,k}_{n,t+1}}) - \log(1-\kappa^{j,k}_n)$$

- Plugging this expression for the difference leads to the following equations:

$$\rho_L \log(\frac{m^{j,k}_{n,t}}{m^{j,j}_{n,t}}) = \beta \log\left(\frac{w^k_{n,t+1}}{w^j_{n,t+1}}\right) + \beta \log\left(\frac{\mu^k_n}{\mu^j_n}\right) + \beta\rho_L \log(\frac{m^{j,k}_{n,t+1}}{m^{k,k}_{n,t+1}}) + (1-\beta)\log(1-\kappa^{j,k}_n)$$
$$\log(\frac{m^{j,k}_{n,t}}{m^{j,j}_{n,t}}) = \left(\frac{1-\beta}{\rho_L}\right)\log(1-\kappa^{j,k}_n) + \left(\frac{\beta}{\rho_L}\right)\log\left(\frac{w^k_{n,t+1}}{w^j_{n,t+1}}\right) + \beta \log\left(\frac{\mu^k_n}{\mu^j_n}\right) + \beta \log(\frac{m^{j,k}_{n,t+1}}{m^{k,k}_{n,t+1}})$$
$$\log\left(\frac{L^{j,k}_{n,t}}{L^{j,j}_{n,t}}\right) = \left(\frac{1-\beta}{\rho_L}\right)\log(1-\kappa^{j,k}_n) + \left(\frac{\beta}{\rho_L}\right)\log\left(\frac{w^k_{n,t+1}}{w^j_{n,t+1}}\right) + \beta \log\left(\frac{\mu^k_n}{\mu^j_n}\right) + \beta \log\left(\frac{L^{j,k}_{n,t+1}L^k_{n,t+1}}{L^j_{n,t+1}L^{k,k}_{n,t+1}}\right)$$

- The equation leads to the following regression specification:

$$y_{n,t}^{j,k} = \omega_n^k + \omega_n^j + \left(\frac{\beta}{\rho_L}\right) \ln\left(\frac{w_{n,t+1}^k}{w_{n,t+1}^j}\right) + \eta_{n,t+1}^{j,k}$$

where

  - $y_{n,t}^{j,k} \equiv \ln\left(\frac{L_{n,t}^{j,k}}{L_{n,t}^{j,j}}\right) - (1-\beta)C_n^{j,k} - \beta \ln\left(\frac{L_{n,t+1}^{k} L_{n,t+1}^{j,k}}{L_{n,t+1}^{j} L_{n,t+1}^{k,k}}\right)$
  - $C_n^{j,k} \equiv \frac{\ln(1-\kappa_n^{j,k})}{\rho_L}$
  - $\omega_n^j \equiv -\beta \ln(\mu_n^j)$
  - $\omega_n^k \equiv \beta \ln(\mu_n^k)$
  - $\eta_{n,t+1}^{j,k}$: forecast error.

A.13.2 Estimation Equation for Capital Supply Elasticities

- As derived in Section A.12.5, the sector-specific capital allocation is given by

$$K_{n,t}^j = \Gamma\left(\frac{\rho_K - 1}{\rho_K}\right)\left(a_{n,t}^j\right)\left(\Phi_{n,t}^j\right)^{\frac{\rho_K-1}{\rho_K}} W_{n,t}$$

where $\Phi_{n,t}^j$ is the aggregate capital allocation share given by

$$\Phi_{n,t}^j = \frac{(r_{n,t}^j a_{n,t}^j)^{\rho_K}}{\sum_{k\in\mathcal{J}}(r_{n,t}^k a_{n,t}^k)^{\rho_K}}$$

- Taking log difference for sector j and sector k leads to the following equation:

$$\ln(K_{n,t}^j/K_{n,t}^k) = \ln(a_{n,t}^j/a_{n,t}^k) + \left(\frac{\rho_K - 1}{\rho_K}\right)\ln(\Phi_{n,t}^j/\Phi_{n,t}^k)$$

- The term $\ln(\Phi_{n,t}^j/\Phi_{n,t}^k)$ can be expressed as follows:

$$\ln(\Phi_{n,t}^j/\Phi_{n,t}^k) = \rho_K\Big(\ln(r_{n,t}^j a_{n,t}^j) - \ln(r_{n,t}^k a_{n,t}^k)\Big)$$
$$\ln(\Phi_{n,t}^j/\Phi_{n,t}^k) = \rho_K\Big(\ln(a_{n,t}^j/a_{n,t}^k) + \ln(r_{n,t}^j/r_{n,t}^k)\Big)$$

- Plugging this expression in leads to the following equation:

$$\ln(K^j_{n,t}/K^k_{n,t}) = \rho_K \ln(a^j_{n,t}/a^k_{n,t}) + \Big(\rho_K - 1\Big)\ln(r^j_{n,t}/r^k_{n,t})\Big)$$

- Law of motion for sector-specific capital efficiency is given by:

$$\ln(a^j_{n,t}) = \ln(a^j_n) + \rho_\Phi \ln(K^j_{n,t-1}) - \rho_\Phi \ln(W_{n,t-1})$$

- Hence, the equation leads to the following regression specification:

$$\ln\Big(\frac{K^j_{n,t}}{K^o_{n,t}}\Big) = \tilde{\omega}^j_n + \big(\rho_K - 1\big)\ln\Big(\frac{r^j_{n,t}}{r^o_{n,t}}\Big) + \big(\rho_K \cdot \rho_\Phi\big)\ln\Big(\frac{K^j_{n,t-1}}{K^o_{n,t-1}}\Big) + \tilde{\eta}^j_{n,t}$$

where

  - $\omega^j_n = \tilde{\omega}^j_n \equiv \rho_K \ln(a^j_n/a^o_n)$
  - $\tilde{\eta}^j_{n,t}$: the residual term

A.13.3 Estimation Equation for Energy Demand Elasticity

- Relative energy demand is given by:

$$\frac{Y^{NB}_{n,t}}{Y^B_{n,t}} = \Big(\frac{\alpha^{NB}_n}{\alpha^B_n}\Big) \cdot \Big(\frac{(1+\tau^{NB}_{n,t})P^{NB}_{n,t}}{(1+\tau^B_{n,t})P^B_{n,t}}\Big)^{-\eta_1}$$

- Taking logs and re-arranging leads to the following equation:

$$\ln\Big(\frac{Y^{NB}_{n,t}}{Y^B_{n,t}}\Big) = \ln\Big(\frac{\alpha^{NB}_n}{\alpha^B_n}\Big) - \eta_1 \ln\Big(\frac{1+\tau^{NB}_{n,t}}{1+\tau^B_{n,t}}\Big) - \eta_1 \ln\Big(\frac{P^{NB}_{n,t}}{P^B_{n,t}}\Big)$$

- Taking the time difference leads to the following equation:

$$\Delta Y^{NB}_{n,t} - \Delta Y^B_{n,t} = -\eta_1 \Delta(1+\tau^{NB}_{n,t}) + \eta_1 \Delta(1+\tau^B_{n,t}) + \eta_1(\Delta P^B_{n,t} - \Delta P^{NB}_{n,t})$$

where $\Delta x_{n,t} \equiv \ln(x_{n,t}) - \ln(x_{n,0})$.

- This leads to the following regression specification:

$$\Delta Y_{n,t}^{NB} - \Delta Y_{n,t}^{B} = -\eta_1(\Delta P_{n,t}^{NB} - \Delta P_{n,t}^{B}) + \iota_{n,t}$$

where $\iota_{n,t} \equiv -\eta_1 \Delta(1+\tau_{n,t}^{NB}) + \eta_1 \Delta(1+\tau_{n,t}^{B})$.

A.13.4 Estimation Equation for Fossil Fuel Elasticity

- Relative fossil fuel demand is given by:

$$\frac{f_{n,t}^k}{f_{n,t}^h} = \frac{\omega_n^k}{\omega_n^h}\Big(\frac{(1+\tau_{n,t}^k)s_{n,t}^k}{(1+\tau_{n,t}^h)s_{n,t}^h}\Big)^{-\eta_2}$$

- Taking logs and re-arranging leads to the following equation:

$$\ln\Big(\frac{f_{n,t}^k}{f_{n,t}^h}\Big) = \ln\Big(\frac{\omega_n^k}{\omega_n^h}\Big) - \eta_2 \ln\Big(\frac{s_{n,t}^k}{s_{n,t}^h}\Big) - \eta_2 \ln\Big(\frac{1+\tau_{n,t}^k}{1+\tau_{n,t}^h}\Big)$$

- This leads to the following regression specification w.r.t. base sector k:

$$\ln\Big(\frac{f_{n,t}^k}{f_{n,t}^h}\Big) = \delta_n^k - \eta_2 \ln\Big(\frac{s_{n,t}^k}{s_{n,t}^h}\Big) - \eta_2 \ln\Big(\frac{1+\tau_{n,t}^k}{1+\tau_{n,t}^h}\Big) + \varepsilon_{n,t}^k$$

where δ_n^k are country-fuel fixed effects defined as $\delta_n^k \equiv \ln\Big(\frac{\omega_n^k}{\omega_n^h}\Big)$; $\varepsilon_{n,t}^k$ is an error term.

A.14 Solution Algorithm

This section describes the algorithm used to solve the model.

Solution Algorithm

Initialization: Load parameters and set loop variables: $dev_\star$, $iter_\star$, $itermax_\star$, $tol_\star$, $smth_\star$
Step 1: Initialize $w^j_{nt}, r^j_{nt}, m^{jk}_{nt}, (\varphi^\star)^h_{nt}$
Production Block:
while $dev_1 > tol_1$ **and** $iter_1 < itermax_1$ **do**
    while $dev_{NN} > tol_{NN}$ **and** $iter_{NN} < itermax_{NN}$ **do**
        Step 2: Compute fossil fuel price s^h_{nt}
        Step 3: Calculate marginal costs for non-energy, non-brown energy, and brown sectors
        Step 4: Compute trade costs d^j_{nmt} and variety prices p^j_{nmt}
        Step 5: Compute sector good prices P^j_{nt} and after-tax prices $\tilde{P}^j_{nt}$
        Step 6: Compute energy price P^E_{nt}, final good price P_{nt}, and material price $P^{M,j}_{nt}$
        Step 7: Compute bilateral trade share π^j_{nmt} and capital allocation share Φ^j_{nt}
        Step 8: Compute rental rate of aggregate capital v_{nt}
        Step 9: Normalize prices, update dev_{NN}, and increment $iter_{NN}$
    end while
    Step 10: Calculate present-discounted income and capitalist consumption C^K_{nt}
    Step 11: Compute capitalist investment W_{nt} and supply of capital K^j_{nt}
    Step 12: Compute worker consumption c^j_{nt} and demand for final goods
    Step 13: Update fossil fuel extraction F_{nt} and reserves D^h_{nt}
    Labor Mobility Block:
    while $dev_m > tol_m$ **and** $iter_m < itermax_m$ **do**
        Step 14: Compute worker utility V^j_{nt} and labor transition probability $(m')^{jk}_{nt}$
        Step 15: Update dev_m, smooth m^{jk}_{nt}
    end while
    Market Clearing:
    Step 16: Check labor and capital market clearing conditions
    Step 17: Update wages w^j_{nt} and rental rates r^j_{nt}
    Step 18: Update dev_1, smooth variables, and update increment $iter_1$
end while
