Understanding Trade Imbalances *

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Abstract

Are persistent trade imbalances signs of distortions? This paper investigates the drivers of persistent trade imbalances in eight advanced and emerging economies from 2000 to 2014. Using a dynamic general equilibrium framework, I rationalize the observed imbalances as outcomes of six key forces commonly discussed in the literature: (i) productivity growth, (ii) trade costs, (iii) financial frictions, (iv) life expectancy, (v) population growth, and (vi) economy-specific intertemporal distortions. The findings highlight that economy-specific intertemporal distortions, particularly in the United States, are the most significant driver of global imbalances. Additionally, changes in trade costs between China and its trading partners—the China shock—had a substantial impact on the trade imbalances of both China and the U.S. The results suggest that, without the China shock, the U.S. would have sustained persistent trade surpluses, while China would have experienced persistent deficits, emphasizing the importance of bilateral trade frictions in understanding global imbalances.

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1 Introduction

Over the past decades, many advanced and emerging economies have deviated from balanced trade. In advanced economies, the U.S. has run persistent deficits since 1976, while Germany has maintained surpluses since 2000. In emerging economies, China has consistently run surpluses since 1994, while India has run deficits over the same period.

Given the prevalence of persistent trade imbalances, understanding their drivers is crucial. Without knowing whether imbalances are driven by distortions or economic fundamentals, policy decisions can exacerbate distortions rather than enhance efficiency and welfare. For instance, policies aimed at reducing deficits driven by distortions may help prevent unsustainable borrowing that increases foreign liabilities and financial instability (Obstfeld (2012)). On the other hand, when deficits reflect economic fundamentals, such policies could distort investment and innovation.

Identifying the drivers behind persistent trade imbalances is challenging. Traditional explanations, such as those focused on productivity growth, often fall short in fully accounting for trade balance dynamics (Gourinchas and Jeanne (2013)). This gap has prompted the literature to explore other factors in isolation, including financial frictions (Caballero et al. (2008), Mendoza et al. (2009), Coeurdacier et al. (2015)), demographic changes (Sposi (2021), Bárány et al. (2023)), and trade costs (Reyes-Heroles (2016)). While these factors illuminate some aspects of imbalances, questions remain about their sufficiency and relative importance in explaining trade dynamics across economies.

To address this gap, this paper develops a unified framework to assess the contributions of (i) productivity growth, (ii) trade costs, (iii) domestic financial frictions, (iv) life expectancy, (v) population growth, and (vi) economy-specific intertemporal distortions to persistent trade imbalances. Using this model, I evaluate whether the trade imbalances in advanced and emerging economies are driven by established factors or unexplored intertemporal distortions.

The model integrates workhorse frameworks from the international trade and global imbalances literature, capturing trade imbalances driven by (i) resource flows influenced by endogenous trade activities and (ii) financial flows shaped by endogenous saving decisions. Following Caliendo and Parro (2015), bilateral trade flows between industries and economies are governed by Ricardian comparative advantage, trade costs, input-output linkages, and Heckscher–Ohlin motives arising from differences in labor and capital supplies across economies. To incorporate demographic factors, the model includes overlapping generations within each economy, reflecting various stages of the life cycle with differing life expectancies and supplies of effective labor and capital, following Bárány et al. (2023). As populations and age structures evolve, these demographic transitions influence the enter and exit of individuals in the economy, further impacting aggregate saving. Finally, the model accounts for financial frictions as borrowing costs, where the interest rate faced by borrowers differs from that for savers, reflecting economy-specific financial intermediation costs.

In the model, each age cohort experiences distinct paths of income, prices, interest rates, and survival probabilities, which introduce several channels influencing households' saving decisions. Productivity growth varies across economies and over time, affecting the income paths of age cohorts differently. Similarly, changes in export and import costs over time impact the income and price levels for households in different economies, leading to differences in their saving decisions. Additionally, the presence of varying borrowing costs (i.e., financial frictions) across economies prompts diverse saving decisions among households, even when they face similar income and price paths. Finally, life expectancy differences across economies cause these cohorts to value future consumption differently, thereby altering their saving incentives.

Aggregate saving behavior in an economy results from the combined choices of cohorts with varying sizes and at different life cycle stages. The entering cohort size changes over time within each economy. Young households, with upward-sloping income paths, tend to borrow to smooth consumption, while middle-aged households increase savings for retirement and voluntary bequests, counterbalancing the borrowing of younger generations. The composition of the economy, therefore, influences aggregate saving and expenditure.

I calibrate the model using various data sources to reflect a world comprising eight economies that have shown persistent trade imbalances from 2000 to 2014: the United States, China, Germany, the United Kingdom, the Republic of Korea, Mexico, India, and the Rest of the World (ROW).[1] Using model-implied equations and data, I compute changes in productivity, trade costs, interest rate spreads, population inflows, and survival probabilities, without incorporating information on trade imbalances. These factors are treated as exogenous throughout the analysis. Additionally, I calibrate economy-specific intertemporal distortions to precisely replicate the observed evolution of trade imbalances, capturing saving behaviors not addressed by other model components. This calibration ensures the model accurately matches the trade imbalances of all economies from 2000 to 2014.

[1] The data sources include but are not limited to International Financial Statistics, UN Comtrade database, UN World Population Prospects, and Luxembourg Income Study.

Through the lens of the model, I examine how the time variation of a factor originating from a specific economy influences the trade balances of all economies. Specifically, I analyze the counterfactual trade imbalances that emerge when the effect of a particular economy-specific factor is excluded from the model. This approach identifies factors that have contributed to persistent surpluses or deficits. Furthermore, it facilitates an evaluation of their relative significance in shaping trade imbalances.

Among 48 factors considered in the analysis, the intertemporal distortions associated with the United States emerge as a significant global driver of trade imbalances from 2000 to 2014. These distortions alone raised the trade balance as a percentage of gross output by 4.78 points for China, 3.72 points for Germany, and 3.44 points for the Republic of Korea, contributing to their persistent surpluses. Conversely, they reduced the U.S. trade balance by 9.30 points. This result suggests that global trade imbalances may be shaped by distortions, particularly those originating in the United States, rather than being solely the product of optimal saving decisions. The broad impact of U.S.-specific distortions on both surplus and deficit economies underscores their potential global significance and highlights the role these distortions might play in driving persistent trade imbalances.

The results further show considerable variation in the factors affecting trade imbalances across countries. For example, changes in trade costs between the United States and its trading partners substantially influenced the trade imbalances of China and the United States. Meanwhile, the primary factor for the persistent trade surpluses of Korea and Germany was changes in domestic productivity relative to the rest of the world.

Finally, I assess the impact of international trade on trade imbalances by examining the China shock—China's accession to the World Trade Organization in 2001. Specifically, I evaluate a counterfactual scenario where trade costs between China and other economies remained at their 2000 levels. The model predicts that, without the China shock, the United States would have maintained persistent trade surpluses, while China would have faced persistent trade deficits, contrary to actual outcomes. Additionally, the China shock increased the trade balances of all economies except the United States, underscoring the importance of incorporating bilateral trade frictions to understand trade imbalances.

Related Literature This paper contributes to the literature on global imbalances by introducing the first unified framework that quantitatively assesses the distinct roles of cross-country differences in trade costs, financial frictions, and demographic factors as drivers of trade imbalances within a dynamic, multi-economy general equilibrium setting.

One strand of literature highlights the role of asymmetric changes in trade costs over time. For instance, Reyes-Heroles (2016) demonstrates that the global decline in trade costs accounted for 69 percent of the increase in global imbalances from 1970 to 2007 by facilitating inter-temporal trade and generating income effects. Similarly, Alessandria and Choi (2021) finds that changes in trade barriers between the United States and other countries explain two-thirds of the evolution of U.S. trade imbalances from 1991 to 2015. This paper extends this literature by evaluating how changes in trade costs compare to other factors in driving trade imbalances across multiple economies.

Another body of literature suggests that financial frictions can increase savings in fast-growing but financially underdeveloped economies by limiting domestic asset storage (Caballero et al. (2008)), restricting borrowing for households and firms (Song et al. (2011), Coeurdacier et al. (2015), Wang et al. (2017)), or hindering insurance against individual risks (Mendoza et al. (2009), Angeletos and Panousi (2011), Coeurdacier et al. (2015)). However, most existing models include only two countries, making it difficult to explain why some economies, like Germany and China, consistently run trade surpluses, while others, such as the United States and India, persistently run deficits. Moreover, previous work focusing on the transition from financial autarky to financial integration does not fully capture the role of changing financial frictions over time in shaping trade imbalances. To address this gap, this paper quantifies the impact of time-varying financial frictions, specifically exogenous interest rate spreads, in a multi-economy context.

A separate strand of literature examines how persistent global imbalances can arise from differences in demographic transitions across countries (Ferrero (2010), Backus et al. (2014), Sposi (2021), Bárány et al. (2023)). Increases in life expectancy, which often accompany economic growth, can lead to higher aggregate savings as households plan for longer lifespans. Similarly, declines in fertility rates can reduce the number of younger, borrowing-prone households, thereby increasing aggregate savings through a compositional effect. While demographic factors could explain trade imbalances, their significance relative to other influences remains unclear. This paper contributes to this discussion by quantitatively assessing the importance of demographic transitions in explaining trade imbalances.

To ensure the analysis remains manageable, this paper does not directly model other potentially important sources of global imbalances, such as cross-country differences in pension systems (Eugeni (2015), Niemeläinen (2021)), incentives to accumulate foreign reserves (e.g. Bacchetta et al. (2013), Alfaro et al. (2014), Benigno et al. (2022)), precautionary saving (Mendoza et al. (2009), Angeletos and Panousi (2011), Choi et al. (2017)), and investment (Anderson et al. (2019), Ravikumar et al. (2019)). Nonetheless, these channels are indirectly captured by the intertemporal distortions,[2] which represent the variation of trade imbalances unexplained by the model ingredients.

The paper is structured as follows: Section 2 presents the dynamic general equilibrium model that incorporates overlapping generations, financial frictions, and international trade. Section 3 outlines the data sources and the calibration procedure. In Section 4, I carry out quantitative exercises to quantify the impact of time-varying factors on trade imbalances. Finally, in Section 5, I conclude the paper and discuss directions for future work.

2 Model

The model builds on the existing quantitative models of international trade and global imbalances. The international trade block closely follows the structure of Caliendo and Parro (2015). In addition, similar to Bárány et al. (2023), I model financial frictions, changes in population, and survival probabilities. Compared to their model, I incorporate multiple sectors, asymmetric changes in trade costs across economies, and model financial frictions as time-varying borrowing costs instead of credit constraints.[3]

The model features a world economy comprising N economies indexed by $n \in \mathcal{N}$. Within each economy, there are I sectors indexed by $i \in \mathcal{I}$. Time is discrete and is indexed by $t \in \{0, 1, ...\} \equiv \mathcal{T}$. Within each economy, households are classified by age, indexed by $j \in \{0, ..., J\}$. The number of households in economy n at time t and of age j is denoted by $L_{n,j,t}$, while the total population of economy n at time t is given by $L_{n,t} = \sum_{j=0}^{J} L_{n,j,t}$. Each economy n possesses a fixed endowment of aggregate capital stock, denoted by K_n, and the ownership of capital is exogenously distributed to different age groups.[4] Labor and capital are fully mobile within an

[2] Exogenous country-specific intertemporal distortions are widely used in the literature to perfectly match the evolution of trade imbalances. For examples, see Stockman and Tesar (1995), Eaton et al. (2016), Reyes-Heroles (2016), Kehoe et al. (2018), Dix-Carneiro et al. (2022).

[3] Credit constraints are stricter than borrowing costs as they strictly limit the amount of borrowing. I use borrowing costs to model financial frictions because they have clearer empirical counterparts (i.e., interest rate spreads) and for computational convenience. Financial frictions that are not captured by interest rate spreads will be absorbed into economy-level intertemporal distortions, the residuals of the model.

[4] The model abstracts from capital accumulation decisions for tractability. Capital is nonetheless included to capture

economy but not across economies. There is no aggregate uncertainty, and as a result, all agents possess perfect foresight over aggregate variables.

2.1 Firms

Final Goods In each economy, households and capital owners purchase economy-level bundles of goods referred to as final goods. The final good $Y_{n,t}$ in economy n at time t is a Cobb-Douglas aggregate of sector-level bundles $\{Y^i_{n,t}\}$, hereafter denoted as sectoral goods. Formally, final good $Y_{n,t}$ is given by

$$Y_{n,t} = \prod_{i \in \mathcal{I}} \left(Y^i_{n,t} \right)^{\alpha^i_n} \tag{1}$$

where α^i_n determines the share of expenditure spent on sectoral i goods in economy n, with $\sum_{i \in \mathcal{I}} \alpha^i_n = 1$.

Given the prices of sectoral goods $\{P^i_{n,t}\}$, the price index of a final good $P_{n,t}$ in economy n at time t is given by

$$P_{n,t} = \prod_{i \in \mathcal{I}} \left(\frac{P^i_{n,t}}{\alpha^i_n} \right)^{\alpha^i_n}. \tag{2}$$

Equation (2) shows that the aggregate price index depends on the price indices of sectors as well as the expenditure shares which vary across countries and sectors.

Sectoral Goods Sectoral goods are bundles that aggregate sector-specific tradable varieties. These goods are either used to produce final goods or used by variety producers as intermediate inputs for production. Formally, the sectoral good $Y^i_{n,t}$ in sector i of economy n at time t is a Constant Elasticity of Substitution (CES) aggregate of tradable sector-specific varieties $y^i_{n,t}(\omega)$:

$$Y^i_{n,t} = \int_0^1 \left(\left(y^i_{n,t}(\omega) \right)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}} \tag{3}$$

where $\sigma > 1$ dictates the elasticity of substitution across varieties.

The demand for the variety ω, denoted as $y^i_{n,t}(\omega)$, in sector i needed to produce $Y^i_{n,t}$ units of

the observed distribution of non-financial asset income across age groups.

sectoral-i goods in economy n at time t is given by

$$y^i_{n,t}(\omega) = \Big(\frac{p^i_{n,t}(\omega)}{P^i_{n,t}}\Big)^{-\sigma} Y^i_{n,t} \tag{4}$$

where $p^i_{n,t}(\omega)$ represents the price of variety ω in economy n at time t.

Moreover, the price of sector-i good, $P^i_{n,t}$, in economy n at time t is is given by

$$P^i_{n,t} = \Big(\int_0^1 \big(p^i_{n,t}(\omega)\big)^{1-\sigma} d\omega\Big)^{\frac{1}{1-\sigma}}. \tag{5}$$

Equation (5) shows that the sectoral price index depends on the distribution of prices of varieties. Since varieties are tradable, the sectoral price index will depend on the prices of both domestic and foreign varieties.

Variety Producers Perfectly competitive variety producers in industry i produce tradable varieties using a Cobb-Douglas production function that depends on the economy-variety-specific productivity $z^i_{n,t}(\omega)$, effective labor input $l^i_{n,t}(\omega)$, capital $k^i_{n,t}(\omega)$, and intermediate goods $m^i_{n,t}(\omega)$. A firm's profit maximization problem is given by:

$$\max_{l^i_{n,t}(\omega),k^i_{n,t}(\omega),m^i_{n,t}(\omega)} \quad p^i_{n,t}(\omega)q^i_{n,t}(\omega) - w_{n,t}l^i_{n,t}(\omega) - r^k_{n,t}k^i_{n,t}(\omega) - P^{i,M}_{n,t}m^i_{n,t}(\omega) \tag{6}$$

$$\text{s.t.} \qquad q^i_{n,t}(\omega) = z^i_{n,t}(\omega)\Big(\big(l^i_{n,t}(\omega)\big)^{\eta^i_n}\big(k^i_{n,t}(\omega)\big)^{1-\eta^i_n}\Big)^{1-\gamma^i_n}\Big(m^i_{n,t}(\omega)\Big)^{\gamma^i_n} \tag{7}$$

where $p^i_{n,t}(\omega)$ denotes the price of variety ω, $w_{n,t}$ denotes the wage per efficiency unit of labor in economy n at time t, $r^k_{n,t}$ denotes the rental rate of capital, and $P^{i,M}_{n,t}$ denotes the price of an intermediate good used in sector i of economy n. Parameters $\{\eta^i_n\}$ and $\{\gamma^i_n\}$ respectively govern the shares of total production costs spent on labor inputs and intermediate inputs which vary across sectors and countries.

The intermediate good $m^i_{n,t}(\omega)$ for variety producers is a Cobb-Douglas bundle of sectoral goods $\{m^{i,k}_{n,t}(\omega)\}$:

$$m^i_{n,t}(\omega) = \prod_{k\in\mathcal{I}}\Big(m^{i,k}_{n,t}(\omega)\Big)^{\gamma^{i,k}_n} \tag{8}$$

where $\gamma_n^{i,k} \geq 0$ governs the share of intermediate expenditure of producers in industry i of economy n spent on sector k, with $\sum_{k \in \mathcal{I}} \gamma_n^{i,k} = 1$. The sectoral linkages in production allow for the effects of demand or supply shocks in one sector to propagate to other sectors. The shares $\{\gamma_n^{i,k}\}$ governing input-output linkages vary across sectors and economies.

Since the production function exhibits constant returns to scale and markets are perfectly competitive, the price set by a variety-ω producer in sector i equals the unit cost of production, $\frac{mc_{n,t}^i}{z_{n,t}^i(\omega)}$, where $mc_{n,t}^i$ denotes the cost of an input bundle given by

$$mc_{n,t}^i = \Upsilon_n^i \Big((w_{n,t})^{\eta_n^i} (r_{n,t}^k)^{1-\eta_n^i} \Big)^{1-\gamma_n^i} \left(\prod_{k \in \mathcal{I}} \Big(P_{n,t}^k(\omega) \Big)^{\gamma_n^{i,k}} \right)^{\gamma_n^i} \tag{9}$$

where Υ_n^i is a constant specific to industry i of economy n.[5]

Following Eaton and Kortum (2002), I assume that the productivity of a firm in economy n producing variety ω in sector i at time t is drawn from a Frechet distribution with the following cumulative distribution function:

$$F_{n,t}(z_{n,t}^i(\omega)) = e^{-T_{n,t}^i (z_{n,t}^i(\omega))^{-\theta}} \tag{10}$$

which depends on two parameters: $T_{n,t}^i$ and θ. $T_{n,t}^i$ represents the state of productivity in sector i of economy n at time t. As $T_{n,t}^i$ increases, firms in sector i of economy n become more likely to receive high productivity draws. θ governs the dispersion of productivity draws within sector j. As θ increases, firms in sector i of economy n have similar productivity levels. The parameter θ is crucial, as it is closely linked to Ricardian comparative advantage that promotes international trade between economies. If θ is small (i.e. productivity dispersion is large), economies more actively engage in international trade, as all economies have a comparative advantage in producing some varieties despite the differences in aggregate productivity levels.

2.2 International Trade

Shipping goods from industry i of economy n to economy m at time t entails an iceberg shipping cost $d_{mn,t}^i \geq 1$. This assumption implies that firms in economy n need to ship $d_{mn,t}^i$ units

[5] Specifically, $\Upsilon_n^i = (1-\gamma_n^i)^{-(1-\gamma_n^i)} (\eta_n^i)^{-\eta_n^i(1-\gamma_n^i)} (1-\eta_n^i)^{-(1-\eta_n^i)(1-\gamma_n^i)} \Big(\prod_{k \in \mathcal{I}} \Big(\gamma_n^{i,k} \gamma_n^i \Big)^{-\gamma_n^{i,k} \gamma_n^i} \Big)$.

of good for one unit of good to arrive in economy m. There is no shipping cost for transactions within economies: $d^i_{nn,t} = 1, \forall n \in \mathcal{N}, i \in \mathcal{I}, t \in \mathcal{T}$. Also, the triangle inequality always holds: $d^i_{nm,t} \leq d^i_{nh,t} d^i_{hm,t}, \forall n, m, h \in \mathcal{N}, i \in \mathcal{I}, t \in \mathcal{T}$.

Consumers derive the same amount of utility from same varieties produced in different countries. Hence, consumers shop around the world for producers with the best deals. Then, the price of variety ω in economy n at time t is given by

$$p^i_{n,t}(\omega) = \min_{m \in \mathcal{N}} \left\{ \frac{mc^i_{m,t}}{z^i_{m,t}(\omega)} d^i_{nm,t} \right\}. \tag{11}$$

This expression shows that the producer of variety ω in sector i of economy m who sells to economy n is either a supplier with a low input cost $mc^i_{m,t}$, a supplier with a high productivity draw $z^i_{m,t}(\omega)$, or a supplier who faces a low bilateral iceberg shipping cost $d^i_{nm,t}$.

Under these assumptions, the amount economy m imports from industry i of economy n at time t, $X^i_{mn,t}$, can be expressed as

$$X^i_{mn,t} = \pi^i_{mn,t} X^i_{m,t} \tag{12}$$

where

$$\pi^i_{mn,t} = \frac{T^i_{n,t} \left(mc^i_{n,t} d^i_{mn,t} \right)^{-\theta}}{\sum_{k \in \mathcal{N}} T^i_{k,t} \left(mc^i_{k,t} d^i_{mk,t} \right)^{-\theta}} \tag{13}$$

denotes the fraction of expenditure of economy m spent on varieties produced by industry i of economy n at time t, and $X^i_{m,t}$ is the aggregate expenditure of economy i at time t. The gravity equation expressed above shows that lower θ (i.e. greater comparative advantage) and higher $T^i_{n,t}$ (i.e. better productivity) promote bilateral trade between economies. On the other hand, higher $d^i_{mn,t}$ (i.e. more severe trade barriers) inhibits bilateral trade between economies, as in Eaton and Kortum (2002).

Denote $\Phi^i_{n,t} \equiv \sum_{k \in \mathcal{N}} T^i_{k,t} \left(w^i_{k,t} d^i_{nk,t} \right)^{-\theta}$. $\Phi^i_{n,t}$ is a parameter that captures the distribution of absolute levels of productivity, input costs, and trade barriers of all economies in the world. Assuming $\sigma < 1 + \theta$, the price index of a sectoral good of sector i of economy n at time t is given by

$$P^i_{n,t} = \Psi^i \Big(\Phi^i_{n,t}\Big)^{-\frac{1}{\theta}}, \quad \Psi^i = \left[\Gamma\Big(\frac{\theta + 1 - \sigma}{\theta}\Big)\right]^{\frac{1}{1-\sigma}} \tag{14}$$

where $\Gamma(\cdot)$ is a Gamma function.[6] The above expression shows that the price of a sectoral good $P^i_{n,t}$ depends on the states of productivity $\{T^i_{n,t}\}$, comparative advantage θ, trade costs $\{d^i_{nm,t}\}$, and input costs $\{mc^i_{n,t}\}$.

2.3 International Asset Market and Global Banks

There exists a global market for riskless financial assets, where each unit of such asset has a face value equal to the final good of a base economy $n^\star$, denoted by $P_{n^\star,t}$. That is, the numeraire is one unit of final good in economy $n^\star$.[7] However, financial transactions in this market can only be facilitated through global banks, which operate in a perfectly competitive environment. These banks search for households worldwide that are willing to lend at the lowest possible interest rate, which then becomes the world interest rate, represented by r_t. The net global supply of assets is zero.

Financial intermediation involves various costs, such as assessing default risks of households, complying with economy-specific financial regulations, and paying fees to governments and managers. When a household in economy n demands an asset position of $b < 0$, the financial intermediation cost is given by $\phi_{n,t} b$ which depends on the economy-specific cost $\phi_{n,t}$ and the amount borrowed by the household, b. Given perfect competition among global banks, global banks offer economy-specific interest rate, $\kappa_{n,t}(b)$, to households demanding asset position b such that banks make zero profit: $\kappa_{n,t}(b) = r_t + \phi_{n,t}\mathbb{1}[b < 0]$. That is, global banks set the interest rate to cover their financing costs.

The key feature of my model is that the interest rates faced by households with negative asset positions (borrowers) and those with positive asset positions (savers) differ due to heterogeneity in the financial environment across economies. This modeling choice is similar to the debt-elastic interest rate used in the international macroeconomics literature (Schmitt-Grohé and Uribe (2003),

[6] Since Ψ^i is a time-invariant constant that is common across economies, it can be ignored when analyzing differences in prices across economies and price changes over time.

[7] In the quantitative analysis section, I use the United States as the base economy.

Uribe and Yue (2006)).

Global banks pay financial intermediation costs in terms of final goods. The total expenditure $FX_{n,t}$ of global banks in economy n at time t is given by:

$$FX_{n,t} = -\sum_{j=0}^{J+1} \Big(\kappa_{n,t}(b_{n,j,t}) - r_t\Big) b_{n,j,t} L_{n,j-1,t-1} \tag{15}$$

where $b_{n,j,t}$ is the asset position of a household of age j in economy n at time t. These costs reflect inefficiencies arising from financial frictions.

2.4 Households

A household of age j, living in economy n at time t and holding an asset position b, makes decisions regarding consumption c and saving b' to maximize its lifetime utility. The decisions incorporate the probabilities of survival across different ages and the voluntary bequests left at the end of the life cycle. Specifically, the optimal utility function, $V(n, j, t, b)$, depends on several factors: the household's consumption of the final good c, the discount factor β, economy-specific intertemporal distortions $\{\varepsilon_{n,t}\}$, survival probabilities $\{\chi_{n,j+1,t+1}\}$, the price of the final good $\{P_{n,t}\}$, wages per effective labor $\{w_{n,t}\}$, the age-specific supply of effective labor $\{e_{n,j}\}$, rental rate of capital $\{r^k_{n,t}\}$, the age-specific capital stock per capita $\left(\frac{s_j K_n}{L_{n,j,t}}\right)$, and the financial income $\kappa_{n,t}(b)b$. The household also receives bequests $\Omega_{n,t}$, contingent on being younger than or equal to J_y, and considers leaving voluntary bequests at the end of the life cycle. Formally, a household of age j at time t living in economy n with asset position b solves the following problem:

$$V(n, j, t, b) = \max_{c,b'} \quad u(c, g) + \beta e^{\varepsilon_{n,t'}} \chi_{n,j',t'} V(n, j', t', b') \tag{16}$$

s.t.

$$P_{n,t}c + b' = \kappa_{n,t}(b)b + w_{n,t}e_{n,j} + r^k_{n,t}\Big(\frac{s_j K_n}{L_{n,j,t}}\Big) + \Omega_{n,t}\mathbb{1}[j \le J_y], \quad e_{n,j} = 0 \quad \forall j > \bar{J}_n$$

$$V(n, J+1, \tilde{t}, \tilde{b}) = \frac{\tau^\nu}{1-\nu}\Big(\frac{\tilde{b}}{p_{n,\tilde{t}}}\Big)^{1-\nu}, \quad \tilde{b} \ge 0, \quad \tilde{t} \equiv t + (J+1-j)$$

where $u(c) = \frac{c^{1-\nu}}{1-\nu}$ is the household's instantaneous utility function which depends on the consumption of final good c; The parameter ν governs the elasticity of inter-temporal substitution.

$\bar{J}_n$ represents the retirement age in economy n. s^j denotes the share of aggregate capital income allocated to households of age j. The function $V(n, J+1, \tilde{t}, \tilde{b}) = \frac{\tau^\nu}{1-\nu}\left(\frac{\tilde{b}}{p_{n,\tilde{t}}}\right)^{1-\nu}$ represents the subjective value of voluntary bequest $\tilde{b}$, where the bequest parameter τ governs the strength of emotional rewards of voluntary bequest.[8] Lastly, I assume that households are not allowed to choose a negative asset position at the end of their life cycle, i.e., $b_{n,J+1,\tilde{t}} \geq 0$.

The term $\varepsilon_{n,t}$ represents economy-specific intertemporal distortions, which are included to account for aggregate saving behavior not explained by other factors in the model. For instance, trade surpluses in certain economies that cannot be attributed to changes in productivity, trade costs, financial frictions, or demographic factors are rationalized by positive intertemporal distortions. Such distortions, in the form of time preferences, incentivize households to reduce current consumption and save for future consumption. It is important to note that these intertemporal distortions will be close to zero if the model's components sufficiently explain the observed trade imbalances.

Households supply effective labor and capital inelastically. Their labor income $w_{n,t}e_{n,j}$ and capital income $r^k_{n,t}\left(\frac{s_j K_n}{L_{n,j,t}}\right)$ vary over their life cycle, reflecting changes in the aggregate state of the economy, captured by $w_{n,t}$ and $r^k_{n,t}$, as well as changes in the supply of effective labor $e_{n,j}$ and capital $\left(\frac{s_j K_n}{L_{n,j,t}}\right)$ over the life cycle.[9] Furthermore, households cease to supply labor after the retirement age $\bar{J}_n$, i.e. $e_{n,j} = 0 \quad \forall j > \bar{J}_n$. These features lead households to choose different asset positions at different stages of their life cycle. Young households tend to borrow in anticipation of future increases in income, while middle-aged households save in preparation for retirement and voluntary bequest.

The extent to which households respond to changes in future prices depends on their elasticity of intertemporal substitution (EIS), denoted by $\frac{1}{\nu}$. Furthermore, the value that households place on future consumption is influenced by several factors, including the probability of surviving to future periods $\chi_{n,j+1,t+1}$ at different stages of the life cycle, the strength of the voluntary bequest motive τ, and intertemporal distortions $\{\varepsilon_{n,t+1}\}_t$. An increase in $\chi_{n,j+1,t+1}$, or life expectancy, encourages middle-aged households to save more.

[8] I have incorporated a voluntary bequest motive in the model to account for the sluggish decline in financial asset position after retirement and the significant amount of assets held by older households, as observed in data sources such as the Survey of Consumer Finances. While it is theoretically possible to also include utility from accidental bequest, this extension presents challenges due to limited data availability.

[9] The age-specific effective labor supply can be interpreted as a result of human capital accumulation, experience, and/or seniority. It can differ across economies due to differences in labor market institutions. The age-specific supply of capital may arise from capital accumulation or the purchase of real estate.

2.5 Bequests

There are two sources of bequests: accidental and voluntary. Accidental bequests arise from the presence of survival probabilities $\{\chi_{n,j,t}\}$, while voluntary bequests are chosen by households themselves. The total value of bequest $BV_{n,t}$ in economy n at time t is given by:

$$BV_{n,t} = \underbrace{\sum_{j=1}^{J} \kappa_{n,t}(b_{n,j,t}) b_{n,j,t} \Big(L_{n,j-1,t-1} - L_{n,j,t} \Big)}_{\text{Accidental Bequest}} + \underbrace{\kappa_{n,t}(b_{n,J+1,t}) b_{n,J+1,t} L_{n,J,t-1}}_{\text{Voluntary Bequest}} \tag{17}$$

where the right-hand side of Equation (17) is the sum of the asset positions, $b_{n,j,t}$, and interest payments, $(\kappa_{n,t}(b_{n,j,t}) - 1) b_{n,j,t}$, of households of age $j-1$ at time $t-1$ who are deceased at time t. For tractability, the bequests are assumed to be evenly distributed to households of age less than or equal to $J_y < \bar{J}_n$. Hence, the bequest received by a household with age less than or equal to $J_j < \bar{J}_n$ is given by $\Omega_{n,t} = \frac{BV_{n,t}}{\sum_{j=0}^{J_y} L_{n,j,t}}$.

2.6 Aggregate Variables

The aggregate final expenditures $\{X_{n,t}\}$, industry-level aggregate expenditures $\{X^i_{n,t}\}$, industry-level aggregate revenues/gross outputs $\{GO^i_{n,t}\}$, and the trade balance (or net export) of economy n at time t $\{TB_{n,t}\}$ are defined as follows:

$$X_{n,t} \equiv \underbrace{\Big(\sum_{j=0}^{J} P_{n,t} c_{n,j,t} L_{n,j,t} \Big)}_{\text{Aggregate Consumption}} + \underbrace{FX_{n,t}}_{\text{Financial Costs}} , \tag{18}$$

$$X^i_{n,t} = \underbrace{\alpha^i_n X_{n,t}}_{\text{Final Good Expenditure}} + \underbrace{\sum_{k \in \mathcal{I}} \gamma^k_n \gamma^{k,i}_n \sum_{m \in \mathcal{N}} \pi^k_{mn,t} X^k_{m,t}}_{\text{Intermediate Good Expenditure}}, \tag{19}$$

$$GO^i_{n,t} \equiv \underbrace{\sum_{n \in \mathcal{N} \setminus \{n\}} \pi^i_{mn,t} X^i_{m,t}}_{\text{Exports}} + \underbrace{\pi^i_{nn,t} X^i_{n,t}}_{\text{Domestic Absoprtion}} , \tag{20}$$

$$TB_{n,t} \equiv \underbrace{\sum_{i \in \mathcal{I}} GO^i_{n,t}}_{\text{Aggregate Revenue}} - \underbrace{\sum_{i \in \mathcal{I}} X^i_{n,t}}_{\text{Aggregate Expenditure}} , \tag{21}$$

where $\sum_{n\in\mathcal{N}} \pi^i_{mn,t} X^i_{m,t}$ represents the total revenue generated by industry i of economy n from selling to all economies including itself; $\sum_{i\in\mathcal{N}} \pi^i_{nm,t} X^i_{n,t}$, is the total expenditure of industry j of economy n on all economies including itself. The net foreign asset position $NFA_{n,t+1}$ and total liabilities $TL_{n,t+1}$ of economy n at time $t+1$ is defined as follows:

$$NFA_{n,t+1} = \sum_{j=0}^{J} b_{n,j+1,t+1} L_{n,j,t}, \tag{22}$$

$$TL_{n,t+1} = -\Big(\sum_{j=0}^{J} b_{n,j+1,t+1} \mathbb{1}[b_{n,j+1,t+1} < 0] L_{n,j,t}\Big) \tag{23}$$

where $b_{n,j+1,t+1}$ is the asset position of a household of age $j+1$ at time $t+1$, and $L_{n,j,t}$ is the population of households of age j at time t.

The law of motion of the age structure of economy n at time t is governed by the inflow of population $L_{n,0,t}$ and conditional survival probabilities $\{\chi_{n,j,t}\}_{j=0}^{J}$. Notably, for all $j \in \{1, ..., J\}$, $L_{n,j,t} = \chi_{n,j,t} L_{n,j-1,t-1}$. As a result, the population of economy n at time t ($L_{n,t}$) can be expressed as follows: $L_{n,t} = \sum_{j=0}^{J} L_{n,j,t} = L_{n,0,t} + \sum_{j=1}^{J} \chi_{n,j,t} L_{n,j-1,t-1}$.

2.7 Equilibrium

An equilibrium in this model is a set of consumption decisions by households $\{c_{n,j,t}\}$, asset position decisions by households $\{b_{n,j+1,t+1}\}$, firms' input sourcing decisions $\{l^i_{n,t}(\omega), k^i_{n,t}(\omega), m^i_{n,t}(\omega)\}$, interest rates $\{r_t, \kappa_{n,t}(\cdot)\}$, wages, rental rates, and price indices $\{w_{n,t}, r^k_{n,t}, P^i_{n,t}\}$, trade shares $\{\pi^i_{mn,t}\}$, and bequests $\{\Omega_{n,t}\}$ such that given a set of initial asset distributions over age $\{b_{n,j,0}\}$ and capital stocks $\{K_n\}$:

1. Households' consumption and saving decisions $\{c_{n,j,t}, b_{n,j+1,t+1}\}$ maximize their life-time utilities given prices $\Big\{w_{n,t}, r^k_{n,t}, P_{n,t}, r_t, \kappa_{n,t}(\cdot)\Big\}$ and initial asset positions $\{b_{n,j,0}\}$,

2. Firms' input sourcing decisions $\{l^i_{n,t}(\omega), k^i_{n,t}(\omega), m^i_{n,t}(\omega)\}$ maximize their profits every period given prices $\Big\{w_{n,t}, r^k_{n,t}, P^i_{n,t}\Big\}$,

3. Global banks make zero profit every period in all economies:

$$\kappa_{n,t}(b) = r_t + \phi_{n,t}\mathbb{1}[b < 0], \quad \forall b \in \mathbb{R} \quad \forall n \in \mathcal{N} \quad \forall t \in \mathcal{T},$$

4. Labor markets clear in all countries every period:

$$\sum_{j=0}^{J} e_{n,j} L_{n,j,t} = \int_0^1 l_{n,t}^i(\omega) ds, \quad \forall n \in \mathcal{N} \quad \forall t \in \mathcal{T},$$

5. Capital markets clear in all countries every period:

$$K_n = \int_0^1 k_{n,t}^i(\omega) ds, \quad \forall n \in \mathcal{N} \quad \forall i \in \mathcal{I} \quad \forall t \in \mathcal{T},$$

6. Product markets clear in all economies every period:

$$\int_0^1 p_{n,t}^i(\omega) y_{n,t}^i(\omega) d\omega = \alpha_n^i X_{n,t}^i + \sum_{k \in \mathcal{I}} \gamma_n^k \gamma_n^{k,i} \sum_{m \in \mathcal{N}} \pi_{mn,t}^k X_{m,t}^k, \quad \forall n \in \mathcal{N} \quad \forall i \in \mathcal{I} \quad \forall t \in \mathcal{T},$$

7. The international asset market clears every period:

$$\sum_{n \in \mathcal{N}} \sum_{j=0}^{J} b_{n,j+1,t+1} L_{n,j,t} = 0, \quad \forall t \in \mathcal{T}.$$

In equilibrium, the balance of payments (BOP) condition must hold for all economies: $\sum_{i \in \mathcal{I}} X_{n,t}^i + NFA_{n,t+1} = r_t NFA_{n,t} + \sum_{i \in \mathcal{I}} GO_{n,t}^i$ for all $n \in \mathcal{N}$ and $t \in \mathcal{T}$. The condition shows a close link between trade imbalances and net foreign asset positions:

$$TB_{n,t} = \Big(\sum_{i \in \mathcal{I}} GO_{n,t}^i \Big) - \Big(\sum_{i \in \mathcal{I}} X_{n,t}^i \Big) = NFA_{n,t+1} - r_t NFA_{n,t} \tag{24}$$

3 Data and Calibration

I calibrate the model to a world consisting of eight distinct economies: the United States, China, Germany, the United Kingdom, the Republic of Korea, Mexico, India, and the Rest of World (ROW). In addition, the industries within each economy are grouped into three categories: Agriculture/Mining (A), Manufacturing (M), and Services (S). The selection of economies is motivated by the observation that these economies exhibited persistent trade imbalances from 2000 to 2014. Specifically, the United States, the United Kingdom, and India experienced ongoing trade deficits throughout this period, whereas China, Germany, the Republic of Korea, and Mexico maintained persistent trade surpluses. The choice of these particular economies ensures the model remains computationally manageable while allowing for an in-depth analysis of the factors driving persistent trade imbalances across both emerging and advanced economies.

Table 1 provides a summary of the model's parameters. The parameters are classified into three categories: (1) parameter values borrowed from the literature, (2) parameters estimated without solving the model, and (3) parameters estimated using the simulated method of moments (SMM). The calibration of all parameters is based on datasets from 2000 to 2014.

Table 1: Summary of Parameters

Panel A. Imposed to Common Values			
Parameter	Value	Description	Source
θ	4.49	Frechet Scale Parameter	CP
J	60	Maximum Age (Age 80)	
$\bar{J}_n$	45	Retirement Age (Age 65)	
Panel B. Estimated Outside of the Model			
Parameter		Description	Source
α_n^i		Final Consumption Shares	WIOD
η_n^i		Labor Shares	WIOD
γ_n^i		Intermediate Input Shares	WIOD
$\gamma_n^{i,k}$		Input-Ouput Shares	WIOD
$\chi_{n,j,t}$		Conditional Survival Probabilities	WPP
$\phi_{n,t}$		Interest Rate Spread	IMF, IFS
$e_{n,j}$		Labor income by age	LIS
s_j		Capital income by age	SCF
$b_{n,j,0}$		Asset Position by Age in 2000	SCF, LM
Panel C. Estimated using the Simulated Method of Moments (SMM)			
Parameter	Value	Description	
β	0.955	Discount factor	
ν	2.600	Elasticity of Intertemporal Substitution	
τ	1.876	Voluntary Bequest Parameter	

Notes: WIOD = World Input-Output Database 2016 Release, WPP = UN World Population Prospects, IMF = International Monetary Fund, IFS = International Financial Statistics, LIS = Luxembourg Income Study, WB = World Bank National Accounts, SCF = Survey of Consumer Finances, CP = Caliendo and Parro (2015), LM = Lane and Milesi-Ferretti (2018)

Panel A of Table 1 reports parameters imposed to common values in the literature. The Frechet scale parameter θ, which governs the productivity dispersion within industries, is set to 4.49 following Caliendo and Parro (2015). Furthermore, I assume that households become economically active at age 20, retire at age 65 (i.e., $\bar{J} = 45$), and can live up to age 80; hence, there are 61 age cohorts in each economy, and age j in the model ranges from 0 to 60 (i.e. $J = 60$).[10]

Panel B of Table 1 reports parameters that are calibrated outside of the model. I utilize the World Input-Output Database (WIOD) November 2016 Release to estimate the production function parameters. The database provides information on bilateral trade flows between 56 industries in 28 EU countries and 15 other major countries in the world for the period from 2000 to 2014. Additionally, the Socio-Economic Accounts (SEA) of the WIOD offer industry-level information on gross output, value-added, employment, capital stock, labor compensation, capital compensation, intermediate input expenditure, and price indices for gross output, value-added, and intermediate inputs. I classify the 56 sectors in the dataset into three categories: Agriculture/Mining (A), Manufacturing (M), and Services (S).

The aggregate capital stocks were computed using the year 2000 values reported by the WIOD SEA. Furthermore, the final consumption shares α_n^i, labor shares η_n^i, intermediate input shares γ_n^i, and input-output shares $\gamma_n^{i,k}$ can be directly computed using the dataset. I obtain the estimates by calculating the values for each year and then averaging these annual values over the available time periods.

The conditional survival probabilities $\{\chi_{n,j,t}\}$ are computed based on the data on the population of age cohorts in order to realistically capture the changes in the age structure over time. That is, given the distribution of population $\{L_{n,j,t}\}$ in economy n, the conditional probabilities are computed as follows: $\chi_{n,j,t} = \frac{L_{n,j,t}}{L_{n,j-1,t-1}}$ for $j > 0$. The data on population by age $\{L_{n,j,t}\}$ from 2000 to 2014 come from UN World Population Prospects (WPP).[11] Figure A8 presents the life expectancy computed as the product of conditional survival probabilities ($\prod_{j=0}^{J} \chi_{n,j,t}$) of all ages for each time period relative to the levels in year 2000.

The interest rate spreads $\{\phi_{n,t}\}$ from 2000 to 2014 are computed as the percentage point differences between lending interest rates and the deposit interest rates, reported in IMF's International

[10]Due to limited data availability and the challenge of identifying the exact retirement age, I make the assumption that the retirement age is 65 and is common across all economies. This assumption is based on the fact that, in many countries, the retirement age specified by the law often differs from the actual retirement age defined in the model, which is the age at which households no longer earn labor income.

[11]In some cases, I observe the size of an age cohort increases over time potentially due to immigration. In such cases, the conditional survival probabilities are set to 1.

Financial Statistics.[12] The lending and deposit interest rates for the Rest of World (ROW) are calculated as a GDP-weighted average from ten countries: Japan, France, Russia, Canada, Italy, Brazil, Australia, Spain, Indonesia, and Argentina. Figure A9 in the appendix depicts the interest rate spreads from 2000 to 2014.

The profile of effective labor $\{e_{n,j}\}$ is computed using data from the Luxembourg Income Study (LIS), which provides household-level information on labor income by age for multiple countries. To calibrate the age-specific labor supply, I rely on the assumption of household homogeneity within age cohorts and compute the average labor income of different age groups relative to the average labor income of age 40 households, who are presumed to supply one unit of effective labor. This calibration leverages the premise that wages per effective labor are economy-specific, facilitating the identification of age-specific effective labor supply within an economy based on relative labor income across age cohorts.

Specifically, the effective labor profile $\{e_{n,j}\}$ for each economy is determined by averaging labor income profiles by age over all available time periods. For the Rest of the World (ROW), the profile is calculated using data from the following ten countries: Russia, Australia, Austria, Norway, Denmark, South Africa, Taiwan, Vietnam, Chile, and Colombia. Figure A1 illustrates the average effective labor by age for each economy. Consistent with the aforementioned assumption, I posit that the supply of effective labor ceases after age 65.

The capital ownership shares by age cohort, $\{s_j\}$, (shown in Figure A2) are calibrated to align with the distribution of non-financial assets across age groups found in the 1998 Survey of Consumer Finances (SCF). Given the constraints of data availability, I assume capital ownership shares do not vary across countries.

The initial asset positions by age cohort in 2000, $\{b_{n,j,0}\}$, (shown in Figure A3) are calibrated to match three key empirical moments: (i) the relative asset position by age, (ii) the total liabilities of the economy, and (iii) the net foreign asset position. To begin, I use the 1998 Survey of Consumer Finances (SCF) to compute the average asset position within each age group, calculated as the difference between the value of financial assets and the value of debt for U.S. households of the same age. To capture the relative standing of the asset position across age groups from 20 to 80, I normalize the average asset position by dividing it by the average labor income of age 40 households in the United States. This results in the calibration of the relative asset positions for the

[12] Lending interest rates refer to the bank rates that usually meet the short- and medium-term financing needs of the private sector. Deposit interest rate is the rate paid by commercial or similar banks for demand, time, or savings deposits.

United States, denoted as $\{\tilde{b}_{j,0}\}$.

Due to limited data availability for other economies, I make the assumption that the relative asset positions by age in all other economies are the same as those in the United States. To match total liabilities and net foreign asset positions, I introduce adjustment factors $\{\Delta_n\}$ that satisfy the net foreign asset position and total liabilities of each economy in 2000:

$$b_{n,j,0} \equiv \left(\tilde{b}_{j,0} - \Delta_n\right) w_{n,0}, \quad \forall n \in \mathcal{N} \tag{25}$$

$$NFA_{n,0}^{Data} = \sum_{j=0}^{J} b_{n,j,0} L_{n,j,0}, \quad \forall n \in \mathcal{N} \tag{26}$$

$$TL_{n,0}^{Data} = -\sum_{j=0}^{J} \mathbb{I}[b_{n,j,0} < 0] b_{n,j,0} L_{n,j,0}, \quad \forall n \in \mathcal{N} \tag{27}$$

where $\{NFA_{n,0}^{Data}\}_n$ and $\{TL_{n,0}^{Data}\}_n$, respectively, are the net foreign asset positions and total liabilities in 2000 reported by Lane and Milesi-Ferretti (2018).[13] Equations (26) and (27) are empirical counterparts of equations (22) and (23). Table A1 displays the estimates of net foreign asset position and total liabilities in 2000. Figure A3 presents the estimated per capita asset positions (in \$1000) by age in 2000.

Panel C of Table 1 displays the parameters estimated using the simulated method of moments (SMM). The discount factor β was targeted to align with the average of federal funds rates observed from 2000 to 2014, which was 2.01%. The elasticity of intertemporal substitution ν targets the variance of aggregate expenditure growth across all economies, with the aggregate expenditures computed using the WIOD. The voluntary bequest parameter τ targets the average asset holdings of U.S. households aged 80 in 2013 (as a share of the average labor income of households aged 40), which was 4.1047. This empirical moment was computed using the 2013 wave of the Survey of Consumer Finances (SCF).

3.1 Time-varying Factors

Time-varying factors used for the analysis are computed using both data and the model structure. I introduce the hat notation to denote the proportional deviations from levels in 2000 (i.e.

[13] The total liabilities is defined as the value of domestic assets owned by foreigners. The net foreign asset position is calculated as the total asset value minus the total liabilities.

$\hat{x}_{n,t} \equiv \frac{x_{n,t}}{x_{n,2000}}$). The hat notation is a useful tool for representing time-varying factors, such as productivity and trade costs, for which the initial levels are challenging to identify. There are six different types of time-varying factors in my model: productivity $\{\hat{T}^i_{n,t}\}$, trade costs $\{\hat{d}^i_{mn,t}\}$, interest rate spreads $\{\phi_{n,t}\}$, conditional survival probabilities $\{\chi_{n,j,t}\}$, age 20 population $\{L_{n,0,t}\}$, and intertemporal distortions $\{\varepsilon_{n,t}\}$.

Changes in productivity $\{\hat{T}^i_{n,t}\}$ and trade costs $\{\hat{d}^i_{mn,t}\}$ from 2000 to 2014 are extracted using the exact hat algebra technique developed in Dekle et al. (2007). As derived in the appendix, they can be computed using the Frechet scale parameter θ, proportional changes in observed bilateral trade shares $\{(\hat{\pi}^i_{mn,t})^{Data}\}$, and the proportional changes in observed price indices $\{(\hat{P}^i_{n,t})^{Data}\}$:

$$\hat{T}^i_{n,t} = (\hat{\pi}^i_{nn,t})^{Data} \Big(\frac{(\hat{mc}^i_{n,t})^{Data}}{(\hat{P}^i_{n,t})^{Data}} \Big)^{\theta} \tag{28}$$

$$\hat{d}^i_{mn,t} = \left(\frac{((\hat{\pi}^i_{nn,t})^{Data})^{\frac{1}{\theta}}}{(\hat{P}^i_{n,t})^{Data}} \right) \left(\frac{(\hat{P}^i_{n,t})^{Data}}{((\hat{\pi}^i_{mn,t})^{Data})^{\frac{1}{\theta}}} \right) \tag{29}$$

In words, Equation (28) shows that the change in productivity $\hat{T}^i_{n,t}$ is increasing in the expenditure share on domestically produced products $(\hat{\pi}^i_{nn,t})^{Data}$ and the change in the marginal cost of production $\frac{(\hat{mc}^i_{n,t})^{Data}}{(\hat{P}^i_{n,t})^{Data}}$. Equation (29) indicates that the change in import cost $\hat{d}^i_{mn,t}$ is increasing in the change in the importer's price index $(\hat{P}^i_{n,t})^{Data}$ and decreasing in the change in the share of the importer's expenditure on products from the exporting economy $(\hat{\pi}^i_{mn,t})^{Data}$, taking into account the change in the exporter's fundamentals captured by $\frac{((\hat{\pi}^i_{nn,t})^{Data})^{\frac{1}{\theta}}}{(\hat{P}^i_{n,t})^{Data}}$.[14]

The bilateral trade shares $\{\left(\pi^i_{mn,t}\right)^{Data}\}$, price indices $\{\left(P^i_{n,t}\right)^{Data}\}$, and factor prices used to compute the marginal cost of production $\left(mc^i_{n,t}\right)^{Data}$ are computed using the World Input-Output Table (WIOT) and the Socio-Economic Accounts (SEA) of the WIOD. To eliminate the common trend component across economies, I detrend the gross output using the GDP-weighted average real GDP growth rates of economies from 2000 to 2014, which is 2.6 percent. For the Rest of World (ROW), the values are calculated as the GDP-weighted average of 36 countries, excluding the United States, China, Germany, the United Kingdom, the Republic of Korea, Mexico, and India.

For periods after 2014, I assume the age 20 population, conditional survival probabilities, and

[14]Changes in trade costs obtained using Equation (29) may reflect any barriers to bilateral trade including but not limited to tariffs, non-tariff barriers, geographical distance, political distance, and exchange rates.

interest rate spreads remain constant. Furthermore, I make projections of productivity and trade costs after 2014 assuming the world economy reaches a steady state in 2150. Motivated by the persistence of changes in productivity and trade costs shown in Figures A4 and A6, I estimate AR(1) regression models of the changes in productivity $\{\hat{T}^i_{n,t}\}$ and trade costs $\{\hat{d}^i_{mn,t}\}_{i,n,t}$ from 2000 to 2014:

$$\log(\hat{T}^i_{n,t}/\hat{T}^i_{n,t-1}) = \mu^T_{n,i} + \rho^T_{n,i}\log(\hat{T}^i_{n,t-1}/\hat{T}^i_{n,t-2}) + \varepsilon^T_{n,i,t}, \quad \forall n \in \mathcal{N}, \tag{30}$$

$$\log(\hat{d}^i_{mn,t}/\hat{d}^i_{mn,t-1}) = \rho^d_{mn,i}\log(\hat{d}^i_{mn,t-1}/\hat{d}^i_{mn,t-2}) + \varepsilon^d_{mn,i,t}, \quad \forall i, n \in \mathcal{N}. \tag{31}$$

Motivated by observed sudden changes in trade costs, the AR(1) model for trade costs does not include a constant term. This choice implies that trade costs are expected to stabilize in the long term, with policy shifts and their subsequent adjustments being the main drivers of any persistent changes observed in the data.

Having estimated persistence parameters $\{\rho^T_{n,i}\}$, I make projections assuming $\frac{\hat{T}^i_{n,t+1}}{\hat{T}^i_{n,t}} = \exp\left(\mu^T_{n,i} + \rho^T_{n,i}\log(\frac{\hat{T}^i_{n,t}}{\hat{T}^i_{n,t-1}})\right)$ and $\frac{\hat{d}^i_{mn,t+1}}{\hat{d}^i_{mn,t}} = \exp\left(\rho^d_{mn,i}\log(\frac{\hat{d}^i_{mn,t}}{\hat{d}^i_{mn,t-1}})\right)$ for all $i, n \in \mathcal{N}$ and $t > 2014$. To ensure the world economy reaches a steady state in 2150, I set the trend components $\{\mu^T_{n,i}\}_n$ to zero after 2024. The estimates of $\{\mu^T_{n,i}\}$, $\{\rho^T_{n,i}\}$, and $\{\rho^d_{mn,i}\}$ are presented in Table A2, and Table A3 of the appendix.

The intertemporal distortions $\{\varepsilon_{n,t}\}$ are introduced to perfectly match the evolution of trade imbalances from 2000 to 2014, which are computed using the WIOD. The calibration procedure for intertemporal distortions takes the paths of other time-varying factors as given. As trade imbalances must always sum up to zero, the model only requires intertemporal distortions of $||\mathcal{N}|| - 1$ economies to perfectly fit the observed imbalances. Therefore, I impose the intertemporal distortions of the United States to be zero for all years.

Intertemporal distortions capture the economy-specific time preferences of households required to match imbalances unexplained by other model ingredients. For instance, generating China's trade surpluses unexplained by other factors requires increasing China's intertemporal distortions which would encourage Chinese households to reduce consumption and save. Employing this idea, I obtain the intertemporal distortions of all economies that precisely fit the trade imbalances of all economies from 2000 to 2014.

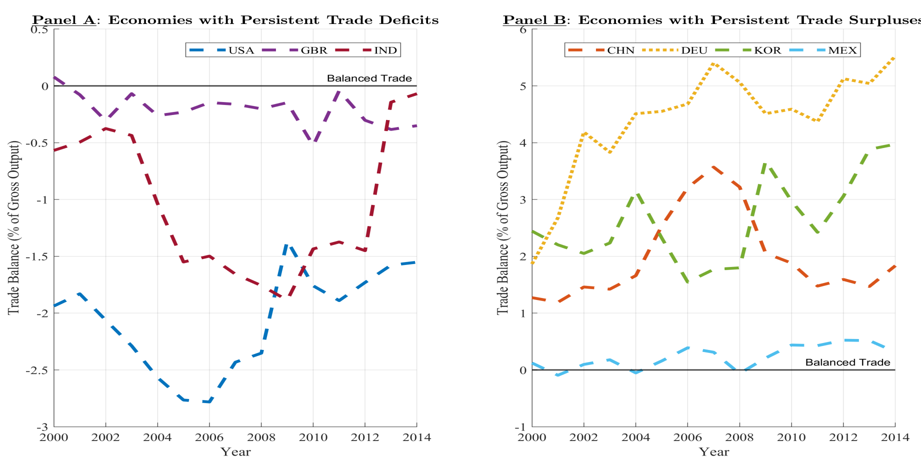
Given that households' saving decisions depend on the entire trajectory of intertemporal distortions throughout the life cycle, the calibration process requires the estimation of intertemporal distortions extending beyond 2014. To project these future intertemporal distortions, I employ an AR(1) regression model and utilize the model-generated estimates of intertemporal distortions from 2000 to 2014. The regression specification is given by

$$\varepsilon_{n,t} = \mu_n^\beta + \rho_n^\beta \varepsilon_{n,t-1} + \varepsilon_{n,t}^\beta, \quad \forall n \in \mathcal{N}. \tag{32}$$

I use the estimated persistence parameter ρ_n^β to make projections assuming $\varepsilon_{n,t+1} = \mu_n^\beta + \rho_n^\beta \varepsilon_{n,t}$ for all $n \in \mathcal{N}$ and $t > 2014$. To ensure the world economy reaches a steady state in 2150, I set the trend components to zero after 2024. Then, I update intertemporal distortions from 2000 to 2014 conditional on the projection. If the updated intertemporal distortions differ from the intertemporal distortions used for the projection, I iterate the procedure until convergence. The detailed calibration algorithm is described in section A.3 of the appendix. Table A5 displays the trend and persistence components $\{\mu^\beta, \rho_n^\beta\}$. Figure A10 plots the intertemporal distortions $\{\varepsilon_{n,t}\}$.

4 Quantitative Analysis

Figure 1: Trade balance (% of Gross Output) from 2000 to 2014



Notes: Trade balance (% of Gross Output) of the United States, China, Germany, United Kingdom, the Republic of Korea, Mexico, India, and the Rest of World from 2000 to 2014. Economies that ran persistent trade deficits (surpluses) are shown in Panel A (Panel B). Estimates are computed using the World Input-Output Database (WIOD) 2016 Release.

In this section, I explore the quantitative importance of six different types of time-varying factors in explaining the trade imbalances of eight economies in a dynamic general equilibrium setting. The six types of time-varying factors are productivity $\{\hat{T}^i_{n,t}\}$, trade costs $\{\hat{d}^i_{mn,t}\}_{i,n,t}$, financial frictions $\{\phi_{n,t}\}$, life expectancy $\{\chi_{n,j,t}\}_{n,j,t}$, age 20 population $\{L_{n,0,t}\}$, and intertemporal distortions $\{\varepsilon_{n,t}\}$.[15] Figure 1 depicts the trade balances (as a percentage of gross output) for eight economies considered in the analysis for the period from 2000 to 2014.

4.1 Measure of Impact

The paths of time-varying factors influence trade imbalances in a nonlinear way, as they alter the trajectories of prices. Therefore, directly comparing quantitative results is difficult. To ease this comparison, I introduce a measure that quantifies the impact of a specific factor on the levels of trade imbalance. The *level impact* $\mathcal{L}_i(\{\hat{x}_{n,\tau}\})$ of a factor $\{\hat{x}_{n,\tau}\}$ originating from economy n on economy i is defined as follows:

$$\mathcal{L}_i\big(\{\hat{x}_{n,\tau}\}\big) = \frac{1}{15}\sum_{t=2000}^{2014}\left(\frac{TB^{Data}_{i,t}}{GO^{Data}_{i,t}} - \frac{TB^{Model}_{i,t}\big(\mathcal{X}\backslash\{\hat{x}_{n,\tau}\}\big)}{GO^{Model}_{i,t}\big(\mathcal{X}\backslash\{\hat{x}_{n,\tau}\}\big)}\right) \times 100 \tag{33}$$

where $\mathcal{X}$ is a set of time-varying factors in a fully-saturated model; $TB^{Model}_{i,t}\big(\mathcal{X}\backslash\{\hat{x}_{n,\tau}\}\big)$ and $GO^{Model}_{i,t}\big(\mathcal{X}\backslash\{\hat{x}_{n,\tau}\}\big)$ are the model-generated trade imbalances and gross output of economy i at time t, excluding the influence of the factor $\{\hat{x}_{n,\tau}\}$, by setting it to the average value across economies. $TB^{Data}_{i,t}$ and $GO^{Data}_{i,t}$ are the actual trade imbalances and gross output of economy i at time t which can be observed in the data.

In words, the measure $\mathcal{L}_i(\{\hat{x}_{n,\tau}\})$ measures the average deviation of model-generated imbalances from data when the influence of a factor $\{\hat{x}_{n,\tau}\}$ is excluded. This measure is useful for understanding the extent to which each time-varying factor shifted the trade balance of each economy. A positive value of $\mathcal{L}_i\big(\{\hat{x}_{n,\tau}\}\big)$ implies the factor $\{\hat{x}_{n,\tau}\}$ is, on average, a "surplus-inducing" factor for economy i. It suggests that the evolution of $\{\hat{x}_{n,\tau}\}$ in economy n compared to other economies has shifted the trade balance for economy i upwards. On the other hand, a negative value of $\mathcal{L}_i\big(\{\hat{x}_{n,\tau}\}\big)$ implies the factor $\{\hat{x}_{n,\tau}\}$ is, on average, a "deficit-inducing" factor for economy i.

[15] In total, 48 ($= 6 \times 8$) time-varying factors are considered in the quantitative analysis.

4.2 Results

4.2.1 Surplus-inducing Factors for Surplus-running Economies

Table 2: Impact of Time-varying Factors on Surplus-running Economies

China (CHN)			Germany (DEU)		
Origin	Factor	Impact	Origin	Factor	Impact
USA	Distortion	4.78	DEU	Productivity	7.07
CHN	Distortion	4.49	DEU	Distortion	3.99
ROW	Distortion	4.10	USA	Distortion	3.72
ROW	Life Exp.	3.33	ROW	Distortion	3.17
USA	Trade Cost	3.12	CHN	Productivity	2.25
Republic of Korea (KOR)			**Mexico (MEX)**		
Origin	Factor	Impact	Origin	Factor	Impact
KOR	Productivity	9.61	USA	Distortion	9.70
USA	Distortion	3.44	ROW	Distortion	8.07
ROW	Distortion	2.98	ROW	Life Exp.	7.15
KOR	Life Exp.	2.84	DEU	Trade Cost	6.71
ROW	Life Exp.	2.17	IND	Trade Cost	6.70
Rest of World (ROW)					
Origin	Factor	Impact			
USA	Distortion	4.76			
ROW	Distortion	3.47			
USA	Financial Fric.	2.91			
ROW	Life Exp.	2.86			
GBR	Distortion	2.84			

Notes: Five time-varying factors with the highest level impact values on China, Germany, the Republic of Korea, Mexico, and the Rest of World. The level impact is measured by the time-averaged deviation between model-generated trade imbalances to gross output and trade imbalances to gross output observed in the data.

Table 2 reports the five time-varying factors with the highest level impact values for economies that recorded persistent trade surpluses between 2000 and 2014. As shown in Figure 1, the economies that maintained persistent trade surpluses between 2000 and 2014 are China, Germany, the Republic of Korea, Mexico, and the Rest of World.

For China, the result indicates the intertemporal distortions of China, the United States, and the Rest of World are key contributors to China's continuous trade surpluses. The finding reveals that removing the influence of the US-specific intertemporal distortions from the model leads to a prediction of China's trade balances as a percentage of gross output being, on average, 4.78 percentage points lower than the actual observed data. It is crucial to note that economy-specific intertemporal distortions include economy-specific factors not accounted for by the model, which influence households' decisions on consumption and saving. These factors would include institutions, norms, fiscal policies, monetary policies, capital control policies, and regulatory policies.

Besides intertemporal distortions, a slower increase in life expectancy in the Rest of World and the more rapid decline in trade costs faced by the United States have also played significant roles in augmenting China's trade surpluses by increasing savings among Chinese households. The changes in life expectancy in the Rest of World increased China's trade balance as a percentage of gross output by 3.33 percentage points. The changes in trade costs of the United States increased China's trade balance by 3.12 percentage points.

For Germany and the Republic of Korea, two primary factors have been pivotal in sustaining their trade surpluses from 2000 to 2014: (i) domestic productivity growth, and (ii) the intertemporal distortions of the United States. During this period, the productivity growth in these trade-dependent economies was significantly slower compared to China's remarkable expansion, prompting them to increase their savings relative to other economies. This behavior contributed to a notable rise in their trade balances as a percentage of gross output, with Germany and the Republic of Korea experiencing increases of 7.07 percentage points and 9.61 percentage points, respectively. Furthermore, the intertemporal distortions of the United States played a role in elevating the trade balances of both Germany and the Republic of Korea by 3.72 percentage points and 3.99 percentage points, respectively.

For Mexico and the Rest of World, the primary contributors to their trade surpluses were changes in the intertemporal distortions of the United States and the Rest of World. Specifically, for Mexico, changes in intertemporal distortions of the United States and the Rest of World led to increases in its trade balance as a percentage of gross output by 9.70 percentage points and 8.07 percentage points, respectively. Similarly, for the Rest of World, these changes boosted its trade balance by 4.76 percentage points and 3.47 percentage points, respectively. Notably, in both economies, changes in life expectancy of households within the Rest of World also played a role in increasing their trade surpluses.

4.2.2 Deficit-inducing Factors for Deficit-running Economies

Table 3: Impact of Time-varying Factors on Deficit-running Economies

United States (USA)			United Kingdom (GBR)		
Origin	Factor	Impact	Origin	Factor	Impact
USA	Distortion	-9.30	ROW	Productivity	-1.12
ROW	Distortion	-8.16	GBR	Distortion	0.48
USA	Trade Cost	-8.00	USA	Productivity	1.28
IND	Productivity	-7.95	MEX	Productivity	1.52
USA	Financial Fric.	-7.89	ROW	Financial Fric.	3.00
India (IND)					
Origin	Factor	Impact			
ROW	Productivity	-2.62			
CHN	Productivity	-1.51			
CHN	Distortion	-0.70			
MEX	Productivity	0.02			
USA	Productivity	0.15			

Notes: Five time-varying factors with the lowest level impact values on the United States, the United Kingdom, and India. The level impact is measured by the time-averaged deviation between model-generated trade imbalances to gross output and trade imbalances to gross output observed in the data.

Table 3 displays the five time-varying factors with the lowest level impact values for economies that have consistently run trade deficits: the United States, the United Kingdom, and India. Echoing the analysis from the previous section, the intertemporal distortions of the United States and the Rest of World are identified as the most significant factors contributing to the United States' sustained deficits. These factors decreased the United States' trade balance as a percentage of gross output by 9.30 percentage points, and 8.16 percentage points, respectively.

In addition to intertemporal distortions, the reduction in trade costs between the United States and its trade partners significantly influenced the trade balance of the United States. Specifically, the changes in the trade costs of the United States have decreased its trade balance as a percentage of gross output by 8.00 percentage points. This finding suggests households in the United States increased their consumption in response to the trade-driven gradual decline in the price level, resulting in trade deficits.

Among 48 factors evaluated, only the productivity changes in the Rest of the World have negatively affected the United Kingdom's trade balance as a percentage of gross output, resulting in a decrease of 1.12 percentage points. This impact is attributed to the slower productivity growth in the Rest of the World compared to the global average. The negative income effect encouraged households in the Rest of World to reduce consumption and increase savings. Given the United Kingdom's significant reliance on imports from the Rest of the World, this relative increase in savings within the Rest of the World has consequently diminished the United Kingdom's trade balance.

Similar to the United Kingdom, India's trade deficit was significantly influenced by productivity changes in the Rest of the World, leading to a 1.34 percentage point decrease in its trade balance as a percentage of gross output. Furthermore, the rapid productivity growth and intertemporal distortions in China also played a role in diminishing India's trade balance, with contributions to decreases of 1.51 percentage points and 0.70 percentage points, respectively.

4.2.3 China Shock and Trade Imbalances

Table 4: China Shock on Trade Imbalances

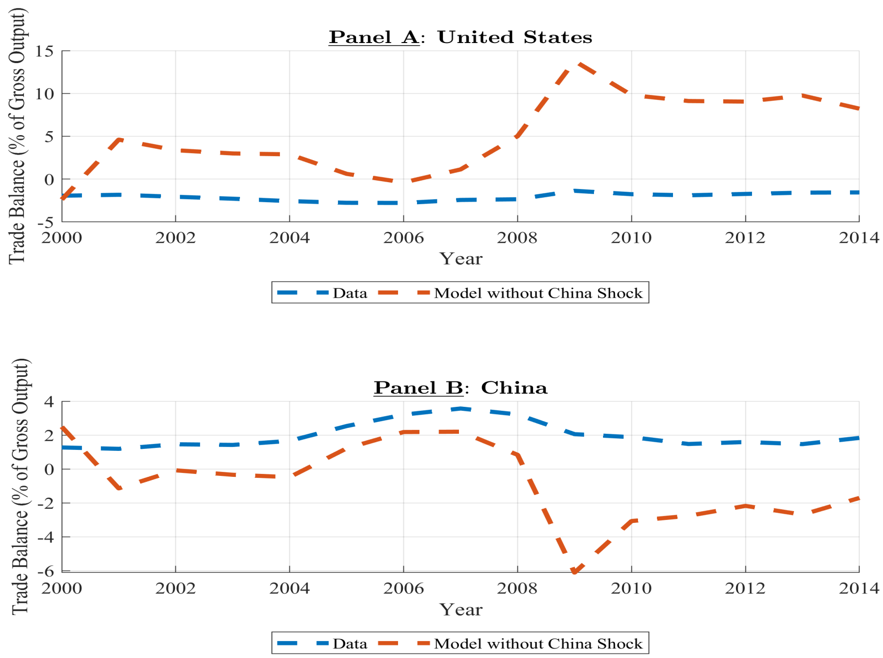
Economy	Impact
United States	-7.23
China	2.76
Germany	2.02
United Kingdom	3.71
Republic of Korea	1.55
Mexico	5.89
India	1.66
Rest of World	2.86

Notes: Level impact values of a model without the China shock. The level impact is measured by the time-averaged deviation between model-generated trade imbalances to gross output and trade imbalances to gross output observed in the data.

In this section, I evaluate the importance of international trade in understanding trade imbalances through the lens of the China shock — China's accession to the World Trade Organization (WTO) in 2001. Specifically, I explore a counterfactual scenario where the trade costs between China and the rest of the economies are assumed to have stayed at their levels from the year 2000.

Table 4 presents the level impact values for economies in response to the China shock. Figure

Figure 2: The Impact of the China Shock on the United States and China



Notes: Trade balance (% of Gross Output) of the United States and China from 2000 to 2014 with and without the China shock.

2 depicts the trade balances of the United States and China when the China shock is removed from the model. Table 4 and Figure 2 reveal that removing the China shock significantly shifted the trade balance of the United States upwards. On the other hand, the removal of the China shock shifted the trade balances of all other economies downward.

The findings reveal that changes in trade costs between China and other economies from 2000 to 2014 significantly influenced their trade balances. Specifically, the United States would likely have experienced persistent trade surpluses instead of deficits during this period. This is because economies with high initial trade costs with China (i.e., trade costs at 2000 levels) did not reap the benefits of China's exceptional productivity growth. This situation leads to reduced aggregate consumption and increased saving in these economies. As a result, a relative decline in consumption in the United States contributed to persistent trade surpluses, whereas elevated consumption in China steered it towards persistent trade deficits. In equilibrium, a rise in the world interest rate—stemming from reduced consumption and borrowing in the United States—prompted other economies to increase their borrowing, leading to lower trade balances for these economies.

5 Conclusion

This paper develops a unified framework to examine the drivers of trade imbalances: (i) productivity growth, (ii) trade costs, (iii) financial frictions, (iv) demographic factors, and (v) economy-specific intertemporal distortions. The quantitative analysis isolates the impact of these factors on the trade balances of advanced and emerging economies from 2000 to 2014. It reveals that the intertemporal distortion of the United States is a key global driver of persistent trade imbalances. Notably, this factor alone significantly influenced the trade balances of China, Germany, and the Republic of Korea, leading to their persistent surpluses. It also contributed to the trade deficits of the United States and the United Kingdom.

In addition, I find substantial variation in the factors affecting trade imbalances across countries. For example, changes in trade costs between the United States and its trading partners substantially influenced the trade imbalances of China and the United States. Meanwhile, the primary factor for the persistent trade surpluses of Korea and Germany was changes in domestic productivity relative to the rest of the world. Finally, I show that, absent the China shock, the United States would have maintained persistent trade surpluses, and China would have experienced persistent trade deficits, underscoring the importance of incorporating bilateral trade frictions to understand trade imbalances.

The findings suggest that although changes in trade costs, financial frictions, and demographic factors play important roles in explaining trade imbalances, the impact of intertemporal distortions on trade imbalances remains significant. Therefore, future research should explore additional factors contributing to global imbalances, including inter-generational transfers, foreign reserve accumulation, variations in social security systems, fiscal, exchange rate, and monetary policies, as well as international investment frictions. Importantly, the model presented in the paper assumes that economic agents have perfect foresight, excluding the role of precautionary saving, which may help explain high savings in China. Incorporating uncertainty, especially uncertainty correlated with economic growth, would be a promising avenue for future research seeking to reconcile the disparities between real-world data and the predictions of conventional neoclassical growth models.

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A Appendix

A.1 Tables and Figures

Table A1: Net Foreign Asset Position (NFA) and Domestic Credit in 2000

Economy	NFA/GDP	Domestic Credit/GDP
United States	-0.0301	1.2018
China	-0.0800	0.8588
West	-0.0073	1.0003
United Kingdom	0.1359	1.9559
Republic of Korea	-0.0788	0.6975
Mexico	-0.2377	0.6952
India	-0.2646	0.2365
Rest of World	0.0299	0.5597

Notes: Net foreign asset position and total liabilities in 2000 (relative to GDP). Estimates are from Lane and Milesi-Ferretti (2018), IMF, IFS.

Table A2: Productivity Parameters ($\rho^T_{n,i}$,$\mu^T_{n,i}$)

Economy	Parameter	Value	Parameter	Value
United States	ρ^T_{USA}	0.131	μ^T_{USA}	0.051
China	ρ^T_{CHN}	0.538	μ^T_{CHN}	0.180
Germany	ρ^T_{DEU}	0.248	μ^T_{DEU}	0.032
United Kingdom	ρ^T_{GBR}	0.302	μ^T_{GBR}	-0.011
Republic of Korea	ρ^T_{KOR}	-0.005	μ^T_{KOR}	-0.055
Mexico	ρ^T_{MEX}	0.247	μ^T_{MEX}	-0.150
India	ρ^T_{IND}	0.009	μ^T_{IND}	-0.009
Rest of World	ρ^T_{ROW}	0.456	μ^T_{ROW}	-0.113

Table A3: Persistence of Trade Costs ($\rho^d_{mn,i}$)

$\rho^d_{mn,i}$	USA	CHN	DEU	GBR	KOR	MEX	IND	ROW
USA	0.000	0.155	-0.201	-0.263	-0.197	0.732	0.123	0.311
CHN	0.506	0.000	0.625	0.446	0.312	0.518	0.298	0.415
DEU	-0.066	-0.164	0.000	0.035	0.035	0.637	0.460	0.276
GBR	-0.108	0.207	0.038	0.000	0.006	-0.078	0.024	0.223
KOR	0.173	0.179	0.008	0.032	0.000	0.580	0.171	0.200
MEX	0.610	0.430	0.696	0.431	0.003	0.000	0.143	0.273
IND	0.147	0.251	-0.027	0.106	0.012	0.250	0.000	0.237
ROW	0.202	0.049	0.462	0.420	0.250	-0.468	-0.057	0.000

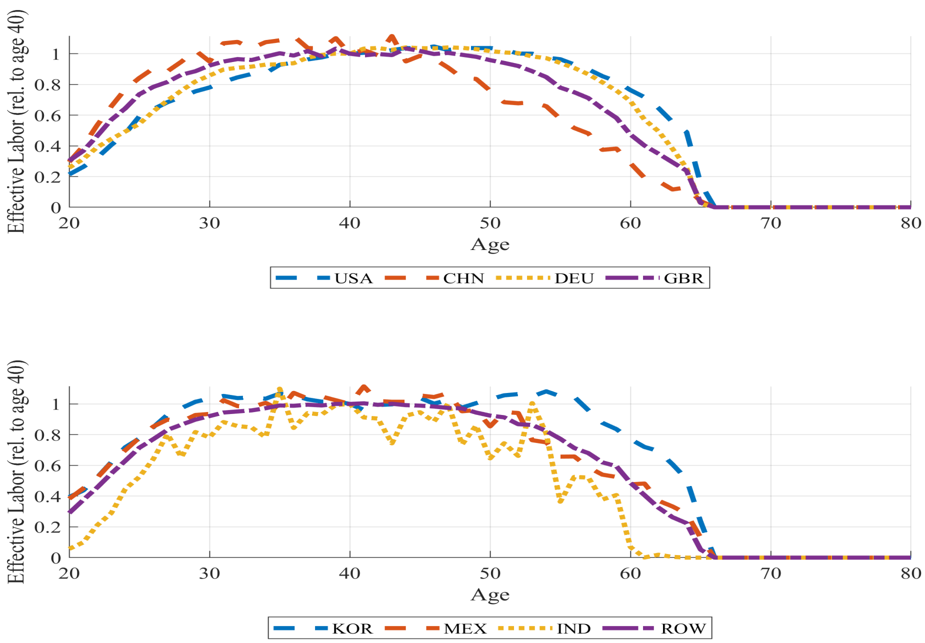
Table A4: Trend Components of Trade Costs (μ^d_{mn})

μ^d_{mn}	USA	CHN	DEU	GBR	KOR	MEX	IND	ROW
USA	0.000	-0.011	0.005	0.037	-0.027	-0.012	-0.049	-0.048
CHN	0.002	0.000	0.002	0.017	-0.013	-0.027	-0.036	-0.041
DEU	-0.007	-0.030	0.000	0.019	-0.031	-0.020	-0.032	-0.056
GBR	-0.025	-0.029	-0.023	0.000	-0.052	-0.078	-0.070	-0.077
KOR	0.026	0.013	0.033	0.053	0.000	-0.012	-0.021	-0.036
MEX	0.013	0.006	0.012	0.036	0.009	0.000	-0.021	-0.022
IND	0.032	0.014	0.046	0.058	0.009	-0.010	0.000	-0.025
ROW	0.052	-0.046	0.040	0.053	0.032	-0.028	0.005	0.000

Table A5: Intertemporal Distortion Parameters (ρ^β_n)

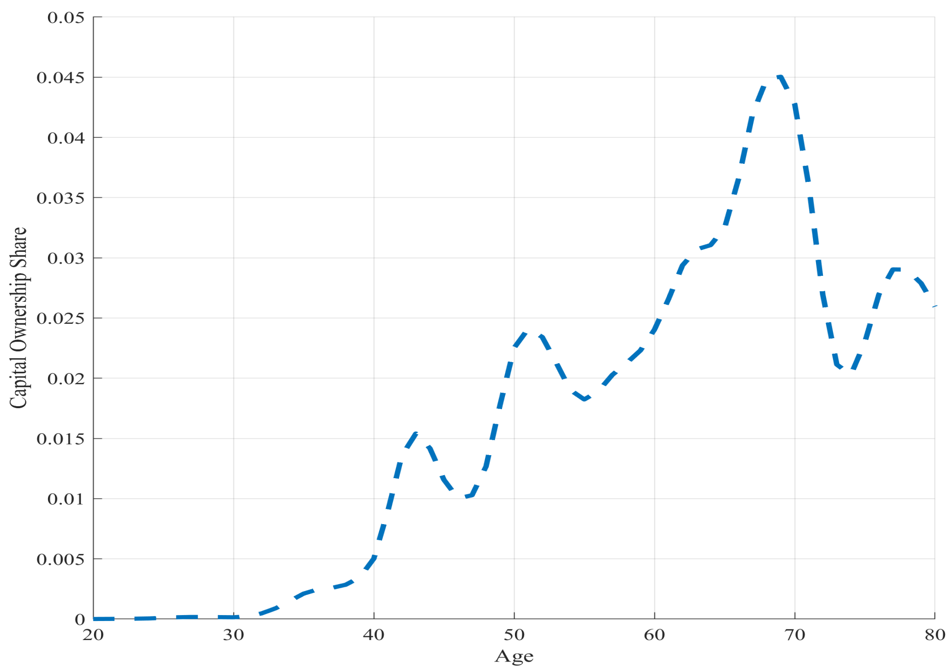
Economy	Parameter	Value	Parameter	Value
United States	ρ^β_{USA}	0.000	μ^β_{USA}	0.000
China	ρ^β_{CHN}	0.613	μ^β_{CHN}	0.048
Germany	ρ^β_{DEU}	0.309	μ^β_{DEU}	-0.005
United Kingdom	ρ^β_{GBR}	0.268	μ^β_{GBR}	-0.017
Republic of Korea	ρ^β_{KOR}	0.051	μ^β_{KOR}	-0.013
Mexico	ρ^β_{MEX}	0.256	μ^β_{MEX}	-0.071
India	ρ^β_{IND}	0.151	μ^β_{IND}	-0.012
Rest of World	ρ^β_{ROW}	0.553	μ^β_{ROW}	-0.023

Figure A1: Effective Labor Supply by Age



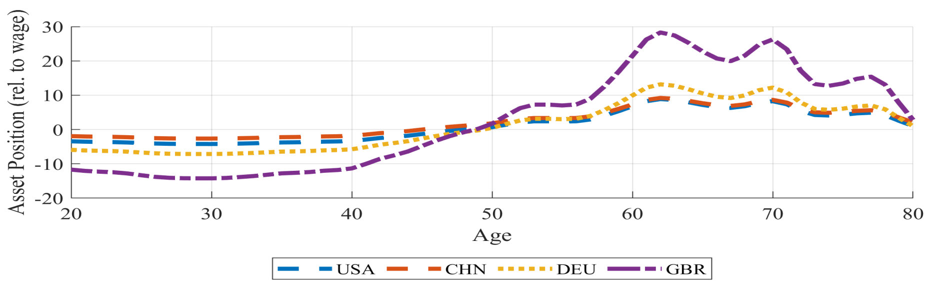
Notes: The supply of effective labor by age computed using income relative to age 40 income. It is assumed that a household of age 40 supplies one unit of effective labor. Estimates are based on the Luxembourg Income Study.

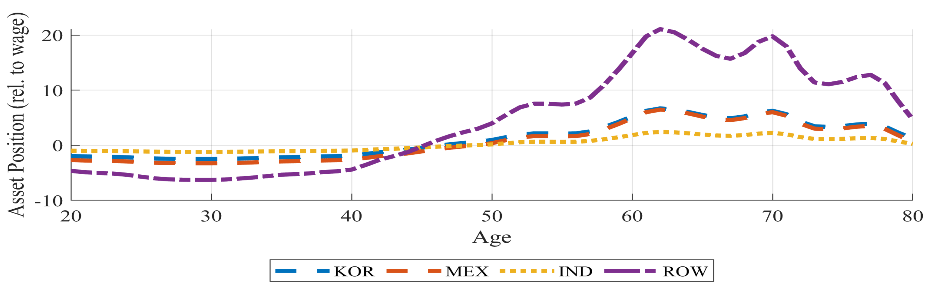
Figure A2: Capital Supply by Age



Notes: The supply of capital by age computed based on the relative non-financial asset position of households in the 1998 Survey of Consumer Finances (SCF).

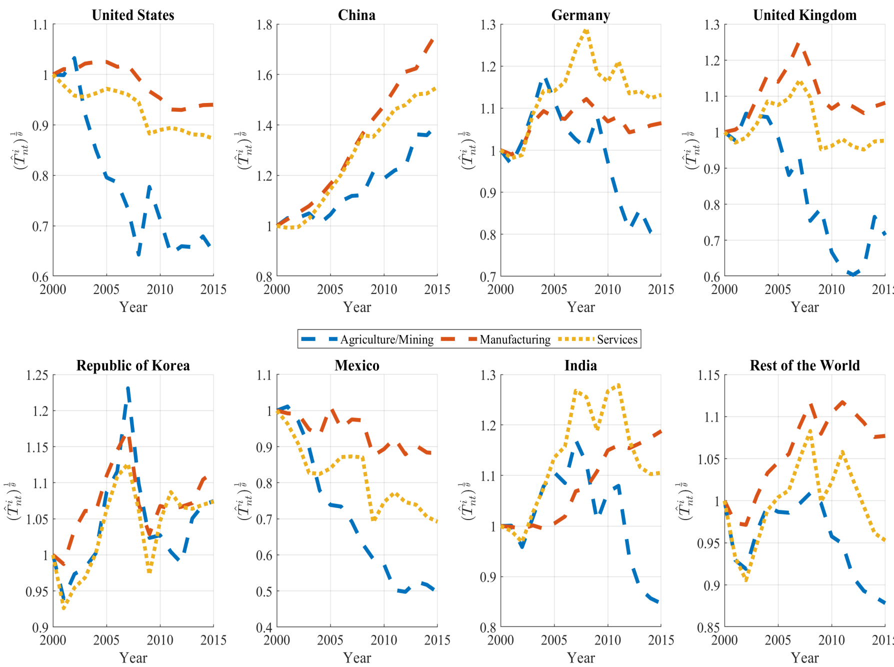
Figure A3: Asset Position by Age in 2000 (in current US$)





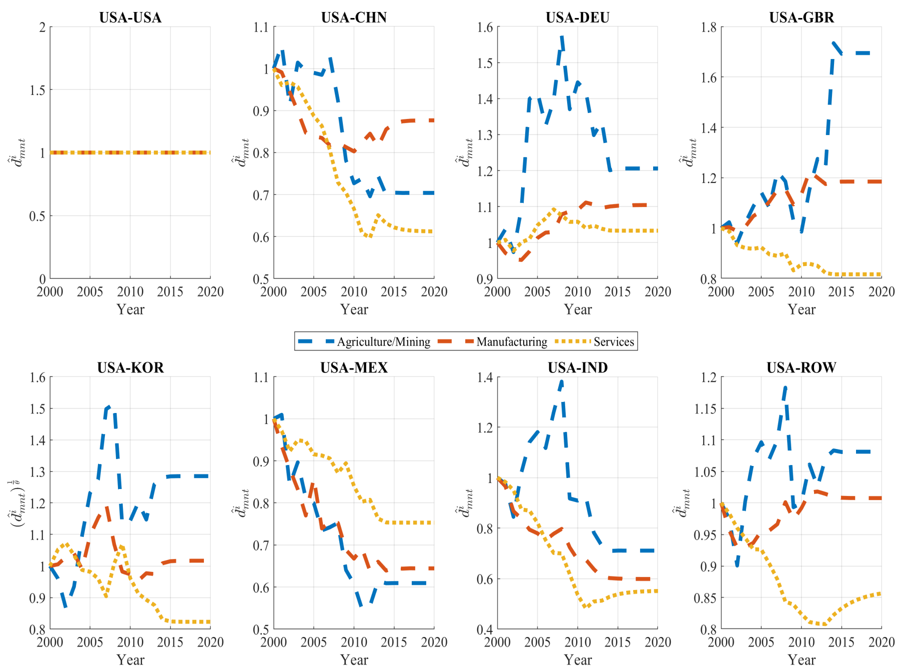
Notes: Asset position by age in 2000 (in current US$). The estimates are computed based on data from Lane and Milesi-Ferretti (2018), IMF, International Financial Statistics (IFS), 1998 Survey of Consumer Finances (SCF).

Figure A4: Changes in productivity ($\hat{T}_{n,t}^{i}{}^{\frac{1}{\theta}}$) from 2000 to 2014



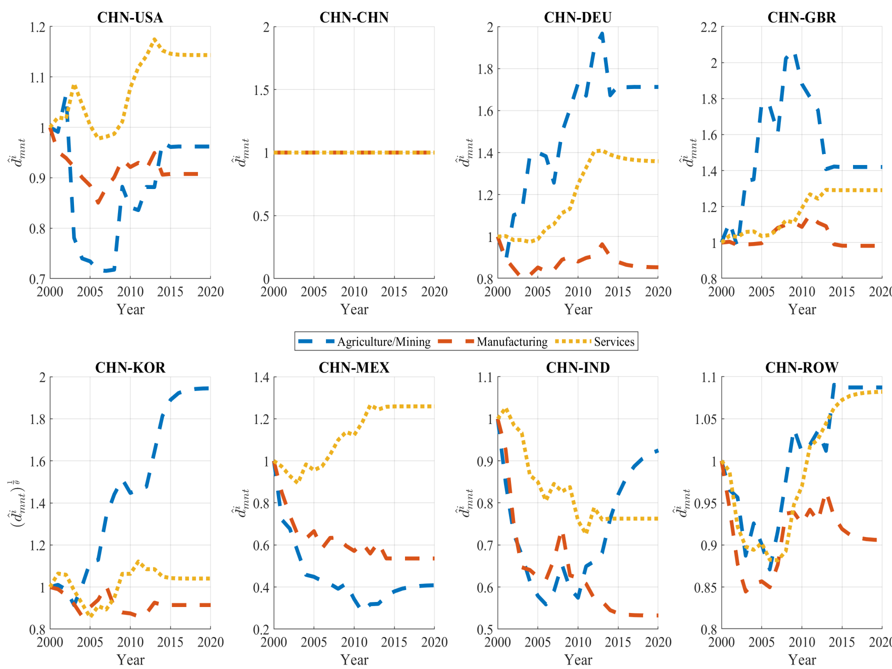
Notes: productivity relative to 2000 level from 2000 to 2014 (projected). Estimates are computed based on data from the World Input-Output Database 2016 Release. Productivity dispersion parameter θ equals 4.49.

Figure A5: Changes in Trade Costs for the United States from 2000 to 2020



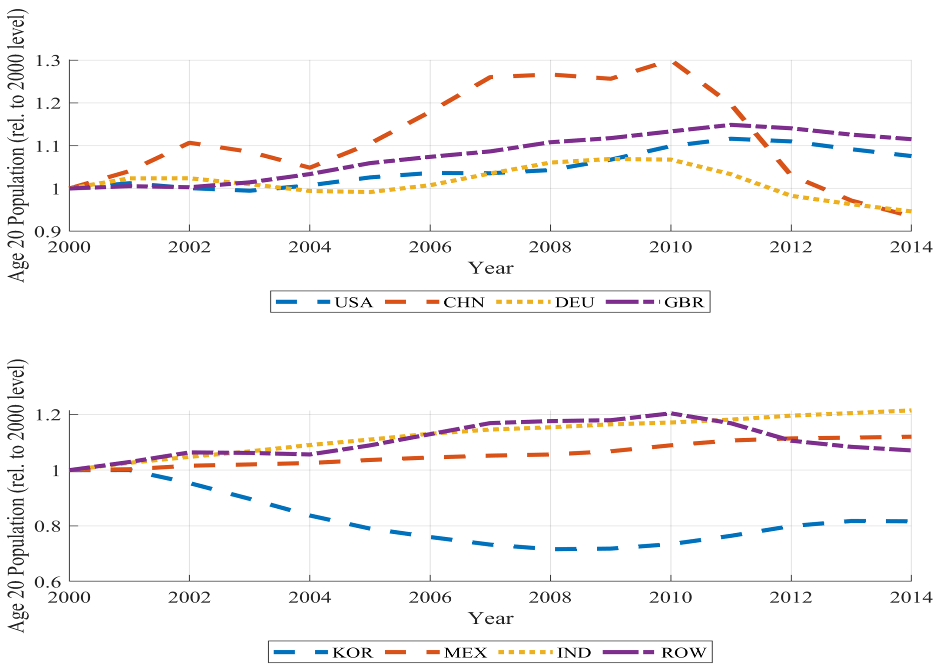
Notes: The United States' costs of importing from trade partners relative to 2000 levels from 2000 to 2020 (projected). Estimates are computed based on data from the World Input-Output Database 2016 Release.

Figure A6: Changes in Trade Costs for China from 2000 to 2020 (projected)



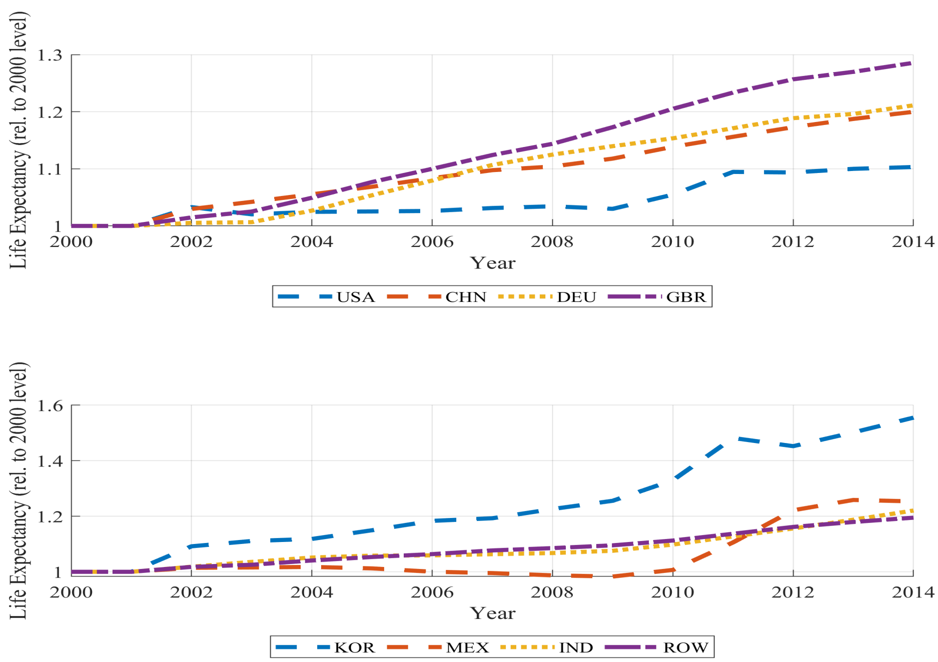
Notes: China's costs of importing from trade partners relative to 2000 levels from 2000 to 2020 (projected). Estimates are computed based on data from the World Input-Output Database 2016 Release.

Figure A7: Age 20 Population (in 2000 level) from 2000 to 2014



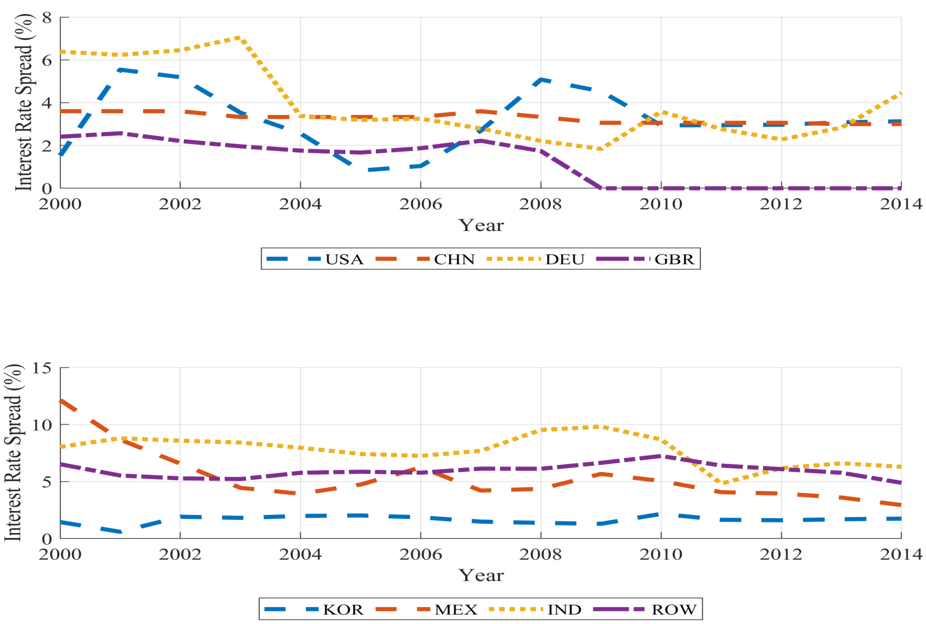
Notes: Age 20 population relative to 2000 level from 2000 to 2014. Estimates are computed based on data from UN World Population Prospects (WPP).

Figure A8: Conditional Survival Probability (in 2000 level) from 2000 to 2014



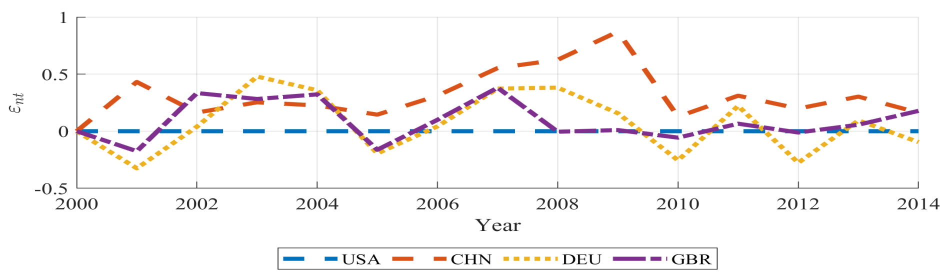
Notes: Life expectancy relative to 2000 levels from 2000 to 2014. Life expectancy is computed as the product of conditional survival probabilities of all ages for each time period. Estimates are computed based on data from UN World Population Prospects (WPP).

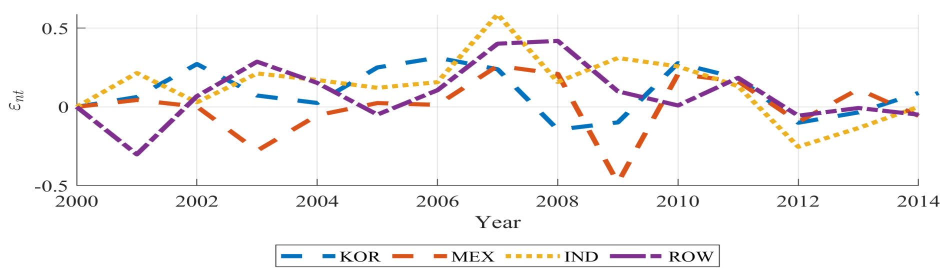
Figure A9: Interest Rate Spread (in 2000 level) from 2000 to 2014



Notes: Interest rate spread relative to 2000 level from 2000 to 2014. Estimates are computed based on lending interest rates and deposit interest rates reported in IMF's International Financial Statistics and FRED.

Figure A10: Intertemporal Distortions (ε_{nt}) from 2000 to 2014





Notes: intertemporal distortions from 2000 to 2014 that rationalize the trade imbalances that cannot be rationalized by changes in productivity, trade costs, demographic factors and financial frictions.

A.2 Derivations

A.2.1 Exact Hat Algebra

- Bilateral import share (i.e. gravity equation)

$$\pi^j_{in,t} = \left(\frac{T^j_{n,t} \big(mc^j_{n,t} d^j_{in,t} \big)^{-\theta}}{\sum_{k=1}^N T^j_{k,t} \big(mc^j_{k,t} d^j_{ik,t} \big)^{-\theta}} \right)$$

- Price index

$$\begin{aligned} p^j_{n,t} &= \gamma \Big(\sum_{k=1}^N T^j_{k,t} \big(mc^j_{k,t} d^j_{nk,t} \big)^{-\theta} \Big)^{-\frac{1}{\theta}} \equiv \gamma (\Phi^j_{n,t})^{-\frac{1}{\theta}} \\ \iff \quad \gamma^\theta (p^j_{n,t})^{-\theta} &= \sum_{k=1}^N T^j_{k,t} \big(mc^j_{k,t} d^j_{nk,t} \big)^{-\theta} \end{aligned}$$

- Derivation of changes in trade costs

$$\begin{aligned} \pi^j_{in,t} &= \left(\frac{T^j_{n,t} \big(mc^j_{n,t} d^j_{in,t} \big)^{-\theta}}{\sum_{k=1}^N T^j_{k,t} \big(mc^j_{k,t} d^j_{ik,t} \big)^{-\theta}} \right) \\ \iff \quad \pi^j_{in,t} \sum_{k=1}^N T^j_{k,t} \big(mc^j_{k,t} d^j_{ik,t} \big)^{-\theta} &= T^j_{n,t} \big(mc^j_{n,t} d^j_{in,t} \big)^{-\theta} \\ \iff \quad \pi^j_{in,t} \gamma^\theta (p^j_{i,t})^{-\theta} &= T^j_{n,t} \big(mc^j_{n,t} d^j_{in,t} \big)^{-\theta} \\ \implies \quad \pi^j_{nn,t} \gamma^\theta (p^j_{n,t})^{-\theta} &= T^j_{n,t} \big(mc^j_{n,t} d^j_{nn,t} \big)^{-\theta} \\ \implies \quad \frac{\pi^j_{in,t}}{\pi^j_{nn,t}} \Big(\frac{p^j_{i,t}}{p^j_{n,t}} \Big)^{-\theta} &= \Big(\frac{d^j_{in,t}}{d^j_{nn,t}} \Big)^{-\theta} \\ d^j_{in,t} = \Big(\frac{d^j_{in,t}}{d^j_{nn,t}} \Big) &= \Big(\frac{\pi^j_{in,t}}{\pi^j_{nn,t}} \Big)^{-\frac{1}{\theta}} \Big(\frac{p^j_{i,t}}{p^j_{n,t}} \Big) \\ \implies \quad \hat{d}^j_{in,t} &= \Big(\frac{\hat{p}^j_{i,t}}{\hat{p}^j_{n,t}} \Big) \Big(\frac{\hat{\pi}^j_{nn,t}}{\hat{\pi}^j_{in,t}} \Big)^{\frac{1}{\theta}} \end{aligned}$$

- Derivation of changes in productivity

$$
\begin{aligned}
\pi^j_{nn,t}\gamma^\theta(p^j_{n,t})^{-\theta} &= T^j_{n,t}\big(mc^j_{n,t}d^j_{nn,t}\big)^{-\theta} \\
\Longrightarrow \quad \hat{\pi}^j_{nn,t}(\hat{p}^j_{n,t})^{-\theta} &= \hat{T}^{j}{}^{i}_{n,t}{}^{j}\big(\hat{mc}^j_{n,t}\big)^{-\theta} \\
\Longleftrightarrow \quad \hat{T}^j_{n,t} &= \hat{\pi}^j_{nn,t}\Big(\frac{\hat{mc}^j_{n,t}}{\hat{p}^j_{n,t}}\Big)^{\theta}
\end{aligned}
$$

A.3 Algorithms

In this section, I present the algorithms used for calibration and counterfactual exercises. First, I describe the algorithm I used to calibrate economy-specific intertemporal distortions $\{\varepsilon_{n,t}\}$ and ω. Then, I describe the algorithm I used to quantify the contribution of time-varying factors on trade imbalances.

A.3.1 Calibration Algorithm

The calibration procedure takes the following information as given:

- $\{b_{n,j,0}\}$: Asset distributions in 2000
- $\{L_{n,j,0}\}_{n,j}$: Population by age in 2000
- $\{e_{n,j}\}$: Effective labor supply by age
- $\{s_j\}$: Capital supply by age
- $\{(\hat{T}^i_{n,t})^{Data}\}$: Changes in productivity from 2000 to 2014
- $\{(\hat{d}^i_{in,t})^{Data}\}$: Changes in trade costs from 2000 to 2014
- $\{\hat{\chi}^{Data}_{n,j,t}\}$: Changes in conditional survival probabilities from 2000 to 2014
- $\{\hat{L}^{Data}_{n,0,t}\}$: Changes in age 20 population from 2000 to 2014
- $\{\phi^{Data}_{n,t}\}$: Interest rate spreads from 2000 to 2014

I make the following assumptions for the factors after 2014 ($= T_{Data}$):

- Changes in conditional survival probability

$$\hat{\chi}_{n,j,t} = \hat{\chi}_{n,j,t-1} \quad \forall t > T_{Data}$$

- Changes in age 20 population

$$\{\hat{L}^{Data}_{n,0,t}\} = \{\hat{L}^{Data}_{n,0,t}\}_{n,t-1} \quad \forall t > T_{Data}$$

- Changes in interest rate spreads

$$\hat{\phi}_{n,t} = \hat{\phi}_{n,t-1} \quad \forall t > T_{Data}$$

Step 0. The numeraire of the model is the final good of the U.S. (i.e. $P_{US,t} = 1$ for all t)

Step 1. Set the end of the time frame $\bar{T}$ (e.g. $\bar{T} = 300$).

Step 2. Guess the saving wedges $\{\varepsilon_{n,t}\}_{n,t}$ from 2000 to $\bar{T}$ (Note: $\varepsilon_{n,t} = 0$ for all n in 2000)

Step 3. $\boxed{\textbf{Loop 1} \rightarrow}$ Guess economy-specific persistent intertemporal distortion parameters $\{\mu^{\beta}_{n}, \rho^{\beta}_{n}\}_{n}$

Step 4. $\boxed{\textbf{Loop 2} \rightarrow}$ Guess the bequest parameter τ and financial frictions $\{\phi_{n,t}\}$

Step 5. $\boxed{\textbf{Loop 3} \rightarrow}$ Guess $\{w_{n,t}, r^{k}_{n,t}, r_t, P^{i}_{n,t}, K_{n,t}\}$ from 2000 to $\bar{T}$

- From now on, $t = 0$ when year is 2000

Step 6.1. Compute intermediate good price index $P^{i,M}_{n,t}$ for $t \leq T_{Data}$

$$P^{i,M}_{n,t} = \left(P^{i,M}_{n,t}\right)^{Datat}$$
$$\hat{P}^{i,M}_{n,t} = \left(\hat{P}^{i,M}_{n,t}\right)^{Data}$$

Step 6.2. Compute intermediate good price index $P_{n,t}^{i,M}$ for $t > T_{Data}$

$$P_{n,t}^{i,M} = \Big(\prod_{k=1}^{I} \frac{P_{n,t}^{k}}{\gamma_n^{i,k}}\Big)^{\gamma_n^{i,k}}$$
$$\hat{P}_{n,t}^{i,M} = \Big(\prod_{k=1}^{I} \hat{P}_{n,t}^{k}\Big)^{\gamma_n^{i,k}}$$

Step 7.1. Compute marginal cost of production $mc_{n,t}^i$ for $t \leq T_{Data}$

$$mc_{n,t}^i = \Upsilon_n^i\Big(((w_{n,t})^{Data})^{\eta_n^i}((r_{n,t}^k)^{Data})^{1-\eta_n^i}\Big)^{1-\gamma_n^i}\Big((P_{n,t}^{i,M})^{Data}\Big)^{\gamma_n^i},$$
$$\Upsilon_n^i = (1-\gamma_n^i)^{-(1-\gamma_n^i)}(\eta_n^i)^{-\eta_n^i(1-\gamma_n^i)}(1-\eta_n^i)^{-(1-\eta_n^i)(1-\gamma_n^i)}(\gamma_n^i)^{-\gamma_n^i},$$
$$\hat{mc}_{n,t}^i = \Big(((\hat{w}_{n,t})^{Data})^{\eta_n^i}((\hat{r}_{n,t}^k)^{Data})^{1-\eta_n^i}\Big)^{1-\gamma_n^i}\Big((\hat{P}_{n,t}^{i,M})^{Data}\Big)^{\gamma_n^i}$$

Step 7.2. Compute marginal cost of production $mc_{n,t}^i$ for $t > T_{Data}$

$$mc_{n,t}^i = \Upsilon_n^i\Big((w_{n,t})^{\eta_n^i}(r_{n,t}^k)^{1-\eta_n^i}\Big)^{1-\gamma_n^i}\Big(P_{n,t}^{i,M}\Big)^{\gamma_n^i},$$
$$\Upsilon_n^i = (1-\gamma_n^i)^{-(1-\gamma_n^i)}(\eta_n^i)^{-\eta_n^i(1-\gamma_n^i)}(1-\eta_n^i)^{-(1-\eta_n^i)(1-\gamma_n^i)}(\gamma_n^i)^{-\gamma_n^i},$$
$$\hat{mc}_{n,t}^i = \Big((\hat{w}_{n,t})^{\eta_n^i}(\hat{r}_{n,t}^k)^{1-\eta_n^i}\Big)^{1-\gamma_n^i}\Big(\hat{P}_{n,t}^{i,M}\Big)^{\gamma_n^i}$$

Step 8. Compute the proportional changes in bilateral import shares (relative to the 2000 level)

$$\hat{\pi}_{mn,t}^i = \frac{\hat{T}_{n,t}^i\big(\hat{d}_{mn,t}^i\hat{mc}_{n,t}^i\big)^{-\theta^i}}{\sum_k \pi_{mk,0}^i\hat{T}_{k,t}\big(\hat{d}_{mk,t}^i\hat{mc}_{k,t}^i\big)^{-\theta^i}}$$

Step 9. Compute bilateral import shares

$$\pi_{mn,t}^i = \hat{\pi}_{mn,t}^i \times (\pi_{mn,0}^i)^{Data}$$

Step 10. Compute industry price indices $P^i_{n,t}$

$$P^i_{n,t} = (P^i_{n,0}) \times (\hat{\pi}^i_{n,n,t})^{\frac{1}{\theta^i}} (\hat{T}^i_{n,t})^{-\frac{1}{\theta^i}} \hat{mc}^i_{n,t}$$

Step 11.1. Normalize with respect to numeraire (final good of US)

$$P^i_{n,t} \leftarrow \frac{\left(P^i_{n,t}\right)}{\left(\prod_{i\in\mathcal{I}}\left(\frac{P^i_{1,t}}{\alpha^i_1}\right)^{\alpha^i_1}\right)^{\frac{1}{\alpha^i_1}}}$$

Step 11.2. Compute final price index $P_{n,t}$

$$P_{n,t} = \prod_{i\in\mathcal{I}}\left(\frac{P^i_{n,t}}{\alpha^i_n}\right)^{\alpha^i_n},$$
$$\hat{P}_{n,t} = \prod_{i\in\mathcal{I}}\left(\hat{P}^i_{n,t}\right)^{\alpha^i_n}$$

Step 12. For a household of age j in economy n in year 2000 (i.e. $t = 0$), obtain consumption decisions $\{c_{n,j,0,q}\}_{q\geq 0}$ and asset positions $\{b_{n,j,0,q}\}_{q\geq 0}$ at time q given initial asset position $b_{n,j,0,0}$ and prices $\{p_{n,q}, w_{n,q}, r^k_{n,q}, r_q\}_{q\geq 0}$ by solving the following problem (economy subscript n dropped for exposition):

$$\max_{c_{j,0,q},b_{j,0,q+1}} \quad \frac{1}{1-\nu}\Big(c_{j,0}\Big)^{1-\nu}$$
$$+\sum_{q=1}^{J-j}\Big(\prod_{k=1}^{q}\beta e^{\varepsilon_k}\chi_{j+k,k}\Big)\frac{1}{1-\nu}\Big(c_{j,0,q}\Big)^{1-\nu} + \Big(\prod_{k=1}^{J-j+1}\beta e^{\varepsilon_k}\chi_{j+k,k}\Big)\frac{\tau^\nu}{1-\nu}\Big(\frac{b_{j,0,J-j+1}}{p_{J-j+1}}\Big)^{1-\nu}$$

s.t.

$$p_q c_{j,0,q} + b_{j,0,q+1} = \kappa_q(b_{j,0,q})b_{j,0,q} + w_q e_{j+q} + r^k_q k_{j,0,q} + \Omega_{n,t}\mathbb{1}[j+q \leq J_y],$$
$$k_{j,0,q} = s_{j+q}(\frac{K_q}{L_{j+q,q}}), \quad \kappa_q(b) = r_q + \phi_q\Big(0.5 - \frac{1}{\pi}\arctan(\zeta b)\Big), \quad \Psi_{j,0,q} \equiv \prod_{k=1}^{q}\beta e^{\varepsilon_k}\chi_{j+k,k}$$

Step 12.1. Guess $c_{j,0,0}$

Step 12.2. Compute $b_{j,0,1}$

$$\kappa_q(b_{j,0,0}) = r_q + \phi_q\Big(0.5 - \frac{1}{\pi}\arctan(\zeta b_{j,0,0})\Big),$$
$$b_{j,0,1} = \Big(\kappa_0(b_{j,0,0})b_{j,0,0} + w_0 e_j + r_0^k k_{j,0,0} + \Omega_{n,t}\mathbb{1}[j+q \leq J_y] - p_0 c_{j,0,0}\Big)$$

Step 12.3. Compute λ_1

$$\lambda_1 = r_1 - \frac{\phi_1}{\pi}\Big(\frac{\zeta}{1+(\zeta b_{j,0,1})^2}\Big)b_{j,0,1} + \phi_1\Big(0.5 - \frac{1}{\pi}\arctan(\zeta b_{j,0,1})\Big)$$

Step 12.4. Compute $c_{j,0,1}$

$$c_{j,0,1} = \lambda_1^{\frac{1}{\nu}}\Big(\frac{\Psi_{j,0,1}}{\Psi_{j,0,0}}\Big)^{\frac{1}{\nu}}\Big(\frac{P_1}{P_0}\Big)^{\frac{1}{\nu}}c_{j,0,0}$$

Step 12.5. Repeat steps 6.2 - 6.4. to obtain $\Big\{c_{j,0,q}, b_{j,0,q+1}\Big\}_{q=0}^{J-j}$

Step 12.6. Compute $a_{j+1,t+J-j+1}$ using budget constraint

$$b_{j,0,J-j+1} = \Big(\kappa_{J-j}(b_{j,0,J-j+1})b_{j,0,J-j+1} + w_{J-j}e_J + r_{J-j}^k k_{j,0,J-j} - p_{J-j}c_{j,0,J-j}\Big)$$

Step 12.7. Compute $\big(b_{j+1,t+J-j+1}\big)'$ using optimality condition

$$\big(b_{j,0,J-j+1}\big)' = \tau\Big(\frac{\Psi_{j,0,J-j+1}}{\Psi_{j,0,J-j}}\big(\frac{p_{J-j+1}}{p_{J-j}}\big)\Big)^{\frac{1}{\nu}}\Big(\frac{p_{J-j+1}}{r_{J-j+1}}\Big)c_{j,0,J-j}$$

Step 12.8. If $\Big((b_{j,0,J-j+1} - \big(b_{j,0,J-j+1}\big)'\Big)^2 > \varepsilon$ for small ε, update $x_{j,0,0}$ and go to Step 10.2 and repeat until convergence

Step 13. For a household of age 0 in economy n at time t, obtain consumption decisions $\{c_{n,0,t,q}\}_{q\geq t}$ and asset positions $\{b_{n,0,t,q}\}_{q\geq t}$ at time q given initial asset position $b_{n,0,t,t}(=0)$ and prices $\{p_{n,q}, w_{n,q}, r_q\}_{q\geq t}$ by solving the following problem (economy subscript n dropped for exposition):

$$\max_{x_{0,t,q},b_{0,t,q+1}} \quad \frac{1}{1-\nu}\Big(c_{0,t,t}\Big)^{1-\nu}$$
$$+\sum_{q=1}^{t+J}\Big(\prod_{k=1}^{q}\beta e^{\varepsilon_k}\chi_{k-t,k}\Big)\frac{1}{1-\nu}\Big(c_{0,t,q}\Big)^{1-\nu}+\Psi_{0,t,t+J+1}\frac{\tau^{\nu}}{1-\nu}\Big(\frac{b_{0,t,t+J+1}}{P_{t+J+1}}\Big)^{1-\nu}$$

s.t.

$$p_q c_{0,t,q}+b_{0,t,q+1}=\kappa_q(b_{0,t,q})b_{0,t,q}+w_q e_{q-t}+r_q^k k_{0,t,q}+a_{0,t,q}+\Omega_{n,q}\mathbb{1}[q-t<J_y],$$
$$k_{0,t,q}=s_{q-t}(\frac{K_q}{L_{q-t,q}}),\quad \kappa_q(b)=r_q+\phi_q\Big(0.5-\frac{1}{\pi}\arctan(\zeta b)\Big),\quad \Psi_{0,t,q}\equiv\prod_{k=1}^{q}\beta e^{\varepsilon_k}\chi_{k-t,k}$$

Step 13.1. Guess $c_{0,t,t}$

Step 13.2. Compute $b_{0,t,t+1}$

$$\kappa_q(b_{j,t,t})=r_t+\phi_t\Big(0.5-\frac{1}{\pi}\arctan(\zeta b_{j,t,t})\Big),$$
$$b_{0,t,t+1}=\Big(\kappa_q(b_{j,t,t})b_{j,t,t}+w_t e_0+r_t k_{0,t,t}+\Omega_{n,t}\mathbb{1}[0\le J_y]-p_t c_{0,t,t}\Big)$$

Step 13.3. Compute Ω_{t+1}

$$\Omega_{t+1}=r_{t+1}-\frac{\phi_{t+1}}{\pi}\Big(\frac{\zeta}{1+(\zeta a_{0,t,t+1})^2}\Big)a_{0,t,t+1}+\phi_{t+1}\Big(0.5-\frac{1}{\pi}\arctan(\zeta a_{0,t,t+1})\Big)$$

Step 13.4. Compute $x_{0,t,t+1}$

$$c_{0,t,t+1}=\Omega_{t+1}^{\frac{1}{\nu}}\Big(\frac{\Psi_{0,t,t+1}}{\Psi_{0,t,t}}\Big)^{\frac{1}{\nu}}\Big(\frac{p_t}{p_{t+1}}\Big)^{\frac{1}{\nu}}c_{0,t,t}$$

Step 13.5. Repeat steps 5.2 - 5.4. to obtain $\Big\{c_{0,t,q},b_{0,t,q+1}\Big\}_{q=t}^{t+J}$

Step 13.6. Compute $b_{0,t,J+1}$ using budget constraint,

$$b_{0,t,t+J+1}=\Big(\kappa_t(b_{0,t,J+1})b_{0,t,t+J+1}+w_{t+J}e_J+r_{t+J}k_{0,t,t+J}-p_{t+J}c_{0,t,t+J}\Big)$$

Step 13.7. Compute $\left(a_{0,t,t+J+1}\right)'$ using optimality condition

$$\left(b_{0,t,t+J+1}\right)' = \tau\Big(\frac{\Psi_{0,t,J+1}}{\Psi_{0,t,J}}\big(\frac{p_{t+J}}{p_{t+J+1}}\big)\Big)^{\frac{1}{\nu}}\big(\frac{p_{t+J+1}}{r_{t+J+1}}\big)c_{0,t,t+J}$$

Step 13.8. If $\Big(\big(b_{0,t,t+J+1} - \left(b_{0,t,t+J+1}\right)'\Big)^2 > \varepsilon$ for small ε, update $c_{0,t,t}$, go to Step 11.1, and repeat until convergence

Step 14. Compute total bequest value $BV_{n,t}$

$$BV_{n,t} = \sum_{j=1}^{J} a_{n,j,t}\Big(L_{n,j-1,t-1} - L_{n,j,t}\Big) + a_{n,J+1,t}L_{n,J,t-1}$$

Step 15. Compute bequest per household $\Omega_{n,t}$

$$\Omega_{n,j,t} = \frac{BV_{n,t}}{\sum_{j=1}^{J_y} L_{n,j,t}}\mathbb{1}[j \leq J_y]$$

Step 16. Compute financial intermediaries' expenditure $FX_{n,t}$

$$FX_{n,t} = -\left(\sum_{j=1}^{J+1}(\kappa_{n,t}(b_{n,j,t}) - r_t)b_{n,j,t}L_{n,j-1,t-1}\right)$$

Step 17. Compute aggregate capital $K_{n,t}$

$$K_{n,t} = \sum_{j=0}^{J} k_{n,j,t}L_{n,j,t}$$

Step 19. Compute aggregate expenditure $X_{n,t}$

$$X_{n,t} = P_{n,t}\Big(\sum_{j=0}^{J} c_{n,j,t}L_{n,j,t}\Big) + FX_{n,t}$$

Step 19.1 Compute industry-level aggregate expenditure $X_{n,t}^i$

$$X_{n,t}^i = \alpha_n^i X_{n,t} + \sum_{k\in\mathcal{I}} \gamma_n^k \gamma_n^{k,i} \sum_{m\in\mathcal{N}} \pi_{mn,t}^k X_{m,t}^k$$

Step 19.2 Compute industry-level aggregate expenditure matrix $X_{n,t}^i$

$$\begin{aligned}
\vec{X}_t(I(n-1)+i) &= [X_{n,t}^i]_{n,i} \\
\vec{A}_t(I(n-1)+i) &= [\alpha_n^i X_{n,t}]_{n,i} \\
\mathbb{L}_t(I(n-1)+i, I(m-1)+k) &= [\gamma_n^k \gamma_n^{k,i} \pi_{mn,t}^k]_{m,n,i,k} \\
\vec{X}_t &= \left[\mathbb{I}_{N\times I} - \mathbb{L}_t\right]^{-1} \vec{A}_t
\end{aligned}$$

Step 20. Compute gross domestic product $GDP_{n,t}^i$

$$GDP_{n,t}^i = \sum_m \pi_{mn,t}^i X_{m,t}^i$$

Step 21. Compute updated aggregate income $AI_{n,t}$

$$AI_{n,t} = w_{n,t} \sum_{j=0}^{J} e_{n,j} L_{n,j,t} + r_{n,t}^k K_{n,t}$$

Step 22. Compute net export $TB_{n,t}$

$$TB_{n,t} = \sum_{i=1}^{I} GDP_{n,t}^i - \sum_{i=1}^{I} X_{n,t}^i$$

Step 23. Compute aggregate bond position $B_{n,t+1}$

$$B_{n,t+1} \equiv \sum_{j=0}^{J} b_{n,j+1,t+1} L_{n,j,t}$$

Step 24. Compute domestic debt $DB_{n,t+1}$

$$DB_{n,t+1} = -\sum_{j=0}^{J} \mathbb{1}[b_{n,j+1,t+1} < 0] b_{n,j+1,t+1} L_{n,j,t}$$

Step 25. Compute domestic debt $DB_{n,t+1}$

$$DB_{n,t+1} = \sum_{j=0}^{J} \mathbb{1}[b_{n,j+1,t+1} > 0] b_{n,j+1,t+1} L_{n,j,t}$$

Step 26. Compute rental rate of capital $(r^k_{n,t})'$ for $t > T_{Data}$

$$(r^k_{n,t})' = \frac{\sum_{i\in\mathcal{I}}(1-\eta^i_n)(1-\gamma^i_n)GDP^i_{n,t}}{K_{n,t}}$$

Step 27. Compute wage $\{\big(w_{n,t}\big)'\}_{n,t}$ using labor market clearing for $t > T_{Data}$

$$\big(w_{n,t}\big)' = \frac{\sum_{i\in\mathcal{I}} \eta^i_n(1-\gamma^i_n)GDP^i_{n,t}}{\sum_{j=0}^{J} e_{n,j} L_{n,j,t}}$$

Step 28. Compute interest rate $\{\big(r_{t+1}\big)'\}_{n,t}$

$$\big(r_{n.t+1}\big)' = \Big(\frac{\log(\sum_n DB_{n,t+1})}{\log(\sum_n DS_{n,t+1})}\Big) \times r_{t+1}$$

Step 29. Update domestic financial frictions $\{\big(\phi_{n,t+1}\big)'\}_{n,t}$

$$\big(\phi_{n,t+1}\big)' = \Big(\frac{(NX_{n,t})^{Data}}{NX_{n,t}}\Big) \times \phi_{n,t+1}$$

Step 33. Compute dev_1

$$dev_1 \equiv \max(||\log(\big(w_{n,t}\big)') - \log(w_{n,t})||, ||\log(\big(r^k_{n,t}\big)') - \log(r^k_{n,t})||, ||\log(\big(r_{t+1}\big)') - \log(r_{t+1})||)$$

Step 34. Update $\{w_{n,t}, r_{t+1}\}_{n,t}$ (e.g. $\delta_1 = 0.99$)

$$
\begin{aligned}
w_{n,t} &\leftarrow \delta_1 w_{n,t} + (1-\delta_1)\big(w_{n,t}\big)' \\
r^k_{n,t} &\leftarrow \delta_1 r^k_{n,t} + (1-\delta_1)\big(r^k_{n,t}\big)' \\
r_{t+1} &\leftarrow \delta_1 r_{t+1} + (1-\delta_1)\big(r_{t+1}\big)'
\end{aligned}
$$

Step 34. If $dev_1 > \epsilon_1$ for small ϵ_1 (e.g. $\epsilon_1 = 10^{-6}$), go to step 5.2. $\boxed{\leftarrow \textbf{Loop 3}}$

A.3.2 Algorithm: Transitional evolution Without Factor $\hat{y}_{n,t}$

Step 0. The numeraire of the model is the final good of the U.S.
(i.e. $P_{US,t} = 1$ for all t)

Step 1. Set the end of the time frame $\bar{T}$ (e.g. $\bar{T} = 300$).

Step 2. Guess the saving wedges $\{\varepsilon_{n,t}\}_{n,t}$ from 2000 to $\bar{T}$ (Note: $\varepsilon_{n,t} = 0$ for all n in 2000)

Step 3. $\boxed{\textbf{Loop 1} \rightarrow}$ Guess economy-specific persistent intertemporal distortion parameters $\{\mu_n^{\beta}, \rho_n^{\beta}\}_n$

Step 4. $\boxed{\textbf{Loop 2} \rightarrow}$ Guess the bequest parameter τ and financial frictions $\{\phi_{n,t}\}$

Step 5. $\boxed{\textbf{Loop 3} \rightarrow}$ Guess $\{w_{n,t}, r_{n,t}^k, r_t, P_{n,t}^i, K_{n,t}\}$ from 2000 to $\bar{T}$

- From now on, $t = 0$ when year is 2000

Step 6. Compute intermediate good price index $P_{n,t}^{i,M}$

$$P_{n,t}^{i,M} = \Big(\prod_{k=1}^{I} \frac{P_{n,t}^k}{\gamma_n^{i,k}} \Big)^{\gamma_n^{i,k}}$$
$$\hat{P}_{n,t}^{i,M} = \Big(\prod_{k=1}^{I} \hat{P}_{n,t}^k \Big)^{\gamma_n^{i,k}}$$

Step 7. Compute marginal cost of production $mc_{n,t}^i$

$$mc_{n,t}^i = \Upsilon_n^i \Big((w_{n,t})^{\eta_n^i} (r_{n,t}^k)^{1-\eta_n^i} \Big)^{1-\gamma_n^i} \Big(P_{n,t}^{i,M} \Big)^{\gamma_n^i},$$
$$\Upsilon_n^i = (1-\gamma_n^i)^{-(1-\gamma_n^i)} (\eta_n^i)^{-\eta_n^i(1-\gamma_n^i)} (1-\eta_n^i)^{-(1-\eta_n^i)(1-\gamma_n^i)} (\gamma_n^i)^{-\gamma_n^i},$$
$$\hat{mc}_{n,t}^i = \Big((\hat{w}_{n,t})^{\eta_n^i} (\hat{r}_{n,t}^k)^{1-\eta_n^i} \Big)^{1-\gamma_n^i} \Big(\hat{P}_{n,t}^{i,M} \Big)^{\gamma_n^i}$$

Step 8. Compute the proportional changes in bilateral import shares (relative to the 2000 level)

$$\hat{\pi}_{mn,t}^i = \frac{\hat{T}_{n,t}^i \big(\hat{d}_{mn,t}^i \hat{mc}_{n,t}^i \big)^{-\theta^i}}{\sum_k \pi_{mk,0}^i \hat{T}_{k,t} \big(\hat{d}_{mk,t}^i \hat{mc}_{k,t}^i \big)^{-\theta^i}}$$

Step 9. Compute bilateral import shares

$$\pi^i_{mn,t} = \hat{\pi}^i_{mn,t} \times (\pi^i_{mn,0})^{Data}$$

Step 10. Compute industry price indices $P^i_{n,t}$

$$P^i_{n,t} = (P^i_{n,0}) \times (\hat{\pi}^i_{n,n,t})^{\frac{1}{\theta^i}} (\hat{T}^i_{n,t})^{-\frac{1}{\theta^i}} \hat{mc}^i_{n,t}$$

Step 11.1. Normalize with respect to numeraire (final good of US)

$$P^i_{n,t} \leftarrow \frac{\left(P^i_{n,t}\right)}{\left(\prod_{i \in \mathcal{I}} \left(\frac{P^i_{1,t}}{\alpha^i_1}\right)^{\alpha^i_1}\right)^{\frac{1}{\alpha^i_1}}}$$

Step 11.2. Compute final price index $P_{n,t}$

$$P_{n,t} = \prod_{i \in \mathcal{I}} \left(\frac{P^i_{n,t}}{\alpha^i_n}\right)^{\alpha^i_n},$$

$$\hat{P}_{n,t} = \prod_{i \in \mathcal{I}} \left(\hat{P}^i_{n,t}\right)^{\alpha^i_n}$$

Step 12. For a household of age j in economy n in year 2000 (i.e. $t = 0$), obtain consumption decisions $\{c_{n,j,0,q}\}_{q \geq 0}$ and asset positions $\{b_{n,j,0,q}\}_{q \geq 0}$ at time q given initial asset position $b_{n,j,0,0}$ and prices $\{p_{n,q}, w_{n,q}, r^k_{n,q}, r_q\}_{q \geq 0}$ by solving the following problem (economy subscript n dropped for exposition):

$$\max_{c_{j,0,q},b_{j,0,q+1}} \quad \frac{1}{1-\nu}\Big(c_{j,0}\Big)^{1-\nu}$$
$$+\sum_{q=1}^{J-j}\Big(\prod_{k=1}^{q}\beta e^{\varepsilon_k}\chi_{j+k,k}\Big)\frac{1}{1-\nu}\Big(c_{j,0,q}\Big)^{1-\nu}+\Big(\prod_{k=1}^{J-j+1}\beta e^{\varepsilon_k}\chi_{j+k,k}\Big)\frac{\tau^{\nu}}{1-\nu}\Big(\frac{b_{j,0,J-j+1}}{p_{J-j+1}}\Big)^{1-\nu}$$

s.t.

$$p_q c_{j,0,q}+b_{j,0,q+1}=\kappa_q(b_{j,0,q})b_{j,0,q}+w_q e_{j+q}+r_q^k k_{j,0,q}+\Omega_{n,t}\mathbb{1}[j+q\leq J_y],$$
$$k_{j,0,q}=s_{j+q}(\frac{K_q}{L_{j+q,q}}),\quad \kappa_q(b)=r_q+\phi_q\Big(0.5-\frac{1}{\pi}\arctan(\zeta b)\Big),\quad \Psi_{j,0,q}\equiv\prod_{k=1}^{q}\beta e^{\varepsilon_k}\chi_{j+k,k}$$

Step 12.1. Guess $c_{j,0,0}$

Step 12.2. Compute $b_{j,0,1}$

$$\kappa_q(b_{j,0,0})=r_q+\phi_q\Big(0.5-\frac{1}{\pi}\arctan(\zeta b_{j,0,0})\Big),$$
$$b_{j,0,1}=\Big(\kappa_0(b_{j,0,0})b_{j,0,0}+w_0 e_j+r_0^k k_{j,0,0}+\Omega_{n,t}\mathbb{1}[j+q\leq J_y]-p_0 c_{j,0,0}\Big)$$

Step 12.3. Compute λ_1

$$\lambda_1=r_1-\frac{\phi_1}{\pi}\Big(\frac{\zeta}{1+(\zeta b_{j,0,1})^2}\Big)b_{j,0,1}+\phi_1\Big(0.5-\frac{1}{\pi}\arctan(\zeta b_{j,0,1})\Big)$$

Step 12.4. Compute $c_{j,0,1}$

$$c_{j,0,1}=\lambda_1^{\frac{1}{\nu}}\Big(\frac{\Psi_{j,0,1}}{\Psi_{j,0,0}}\Big)^{\frac{1}{\nu}}\Big(\frac{P_1}{P_0}\Big)^{\frac{1}{\nu}}c_{j,0,0}$$

Step 12.5. Repeat steps 6.2 - 6.4. to obtain $\Big\{c_{j,0,q},b_{j,0,q+1}\Big\}_{q=0}^{J-j}$

Step 12.6. Compute $a_{j+1,t+J-j+1}$ using budget constraint

$$b_{j,0,J-j+1}=\Big(\kappa_{J-j}(b_{j,0,J-j+1})b_{j,0,J-j+1}+w_{J-j}e_J+r_{J-j}^k k_{j,0,J-j}-p_{J-j}c_{j,0,J-j}\Big)$$

Step 12.7. Compute $\big(b_{j+1,t+J-j+1}\big)'$ using optimality condition

$$\big(b_{j,0,J-j+1}\big)' = \tau\Big(\frac{\Psi_{j,0,J-j+1}}{\Psi_{j,0,J-j}}\big(\frac{p_{J-j+1}}{p_{J-j}}\big)\Big)^{\frac{1}{\nu}}\Big(\frac{p_{J-j+1}}{r_{J-j+1}}\Big)c_{j,0,J-j}$$

Step 12.8. If $\Big(\big(b_{j,0,J-j+1} - \big(b_{j,0,J-j+1}\big)'\Big)^2 > \varepsilon$ for small ε, update $x_{j,0,0}$ and go to Step 10.2 and repeat until convergence

Step 13. For a household of age 0 in economy n at time t, obtain consumption decisions $\{c_{n,0,t,q}\}_{q\geq t}$ and asset positions $\{b_{n,0,t,q}\}_{q\geq t}$ at time q given initial asset position $b_{n,0,t,t}(=0)$ and prices $\{p_{n,q}, w_{n,q}, r_q\}_{q\geq t}$ by solving the following problem (economy subscript n dropped for exposition):

$$\max_{x_{0,t,q},b_{0,t,q+1}} \quad \frac{1}{1-\nu}\Big(c_{0,t,t}\Big)^{1-\nu} + \sum_{q=1}^{t+J}\Big(\prod_{k=1}^{q}\beta e^{\varepsilon_k}\chi_{k-t,k}\Big)\frac{1}{1-\nu}\Big(c_{0,t,q}\Big)^{1-\nu} + \Psi_{0,t,t+J+1}\frac{\tau^\nu}{1-\nu}\Big(\frac{b_{0,t,t+J+1}}{P_{t+J+1}}\Big)^{1-\nu}$$

s.t.

$$p_q c_{0,t,q} + b_{0,t,q+1} = \kappa_q(b_{0,t,q})b_{0,t,q} + w_q e_{q-t} + r^k_q k_{0,t,q} + a_{0,t,q} + \Omega_{n,q}\mathbb{1}[q-t<J_y],$$

$$k_{0,t,q} = s_{q-t}(\frac{K_q}{L_{q-t,q}}), \quad \kappa_q(b) = r_q + \phi_q\Big(0.5 - \frac{1}{\pi}\arctan(\zeta b)\Big), \quad \Psi_{0,t,q} \equiv \prod_{k=1}^{q}\beta e^{\varepsilon_k}\chi_{k-t,k}$$

Step 13.1. Guess $c_{0,t,t}$

Step 13.2. Compute $b_{0,t,t+1}$

$$\kappa_q(b_{j,t,t}) = r_t + \phi_t\Big(0.5 - \frac{1}{\pi}\arctan(\zeta b_{j,t,t})\Big),$$
$$b_{0,t,t+1} = \Big(\kappa_q(b_{j,t,t})b_{j,t,t} + w_t e_0 + r_t k_{0,t,t} + \Omega_{n,t}\mathbb{1}[0 \leq J_y] - p_t c_{0,t,t}\Big)$$

Step 13.3. Compute Ω_{t+1}

$$\Omega_{t+1} = r_{t+1} - \frac{\phi_{t+1}}{\pi}\Big(\frac{\zeta}{1+(\zeta a_{0,t,t+1})^2}\Big)a_{0,t,t+1} + \phi_{t+1}\Big(0.5 - \frac{1}{\pi}\arctan(\zeta a_{0,t,t+1})\Big)$$

Step 13.4. Compute $x_{0,t,t+1}$

$$c_{0,t,t+1} = \Omega_{t+1}^{\frac{1}{\nu}} \Big(\frac{\Psi_{0,t,t+1}}{\Psi_{0,t,t}}\Big)^{\frac{1}{\nu}} \Big(\frac{p_t}{p_{t+1}}\Big)^{\frac{1}{\nu}} c_{0,t,t}$$

Step 13.5. Repeat steps 5.2 - 5.4. to obtain $\Big\{c_{0,t,q}, b_{0,t,q+1}\Big\}_{q=t}^{t+J}$

Step 13.6. Compute $b_{0,t,J+1}$ using budget constraint,

$$b_{0,t,t+J+1} = \Big(\kappa_t(b_{0,t,J+1})b_{0,t,t+J+1} + w_{t+J}e_J + r_{t+J}k_{0,t,t+J} - p_{t+J}c_{0,t,t+J}\Big)$$

Step 13.7. Compute $\big(a_{0,t,t+J+1}\big)'$ using optimality condition

$$\big(b_{0,t,t+J+1}\big)' = \tau\Big(\frac{\Psi_{0,t,J+1}}{\Psi_{0,t,J}}\big(\frac{p_{t+J}}{p_{t+J+1}}\big)\Big)^{\frac{1}{\nu}}\big(\frac{p_{t+J+1}}{r_{t+J+1}}\big)c_{0,t,t+J}$$

Step 13.8. If $\Big((b_{0,t,t+J+1} - \big(b_{0,t,t+J+1}\big)'\Big)^2 > \varepsilon$ for small ε, update $c_{0,t,t}$, go to Step 11.1, and repeat until convergence

Step 14. Compute total bequest value $BV_{n,t}$

$$BV_{n,t} = \sum_{j=1}^{J} a_{n,j,t}\Big(L_{n,j-1,t-1} - L_{n,j,t}\Big) + a_{n,J+1,t}L_{n,J,t-1}$$

Step 15. Compute bequest per household $\Omega_{n,t}$

$$\Omega_{n,j,t} = \frac{BV_{n,t}}{\sum_{j=1}^{J_y} L_{n,j,t}} \mathbb{1}[j \leq J_y]$$

Step 16. Compute financial intermediaries' expenditure $FX_{n,t}$

$$FX_{n,t} = -\Bigg(\sum_{j=1}^{J+1}(\kappa_{n,t}(b_{n,j,t}) - r_t)b_{n,j,t}L_{n,j-1,t-1}\Bigg)$$

Step 17. Compute aggregate capital $K_{n,t}$

$$K_{n,t} = \sum_{j=0}^{J} k_{n,j,t} L_{n,j,t}$$

Step 19. Compute aggregate expenditure $X_{n,t}$

$$X_{n,t} = P_{n,t}\Big(\sum_{j=0}^{J} c_{n,j,t} L_{n,j,t}\Big) + FX_{n,t}$$

Step 19.1 Compute industry-level aggregate expenditure $X_{n,t}^{i}$

$$X_{n,t}^{i} = \alpha_n^i X_{n,t} + \sum_{k\in\mathcal{I}} \gamma_n^k \gamma_n^{k,i} \sum_{m\in\mathcal{N}} \pi_{mn,t}^k X_{m,t}^k$$

Step 19.2 Compute industry-level aggregate expenditure matrix $X_{n,t}^{i}$

$$\begin{aligned}
\vec{X}_t(I(n-1)+i) &= [X_{n,t}^i]_{n,i} \\
\vec{A}_t(I(n-1)+i) &= [\alpha_n^i X_{n,t}]_{n,i} \\
\mathbb{L}_t(I(n-1)+i, I(m-1)+k) &= [\gamma_n^k \gamma_n^{k,i} \pi_{mn,t}^k]_{m,n,i,k} \\
\vec{X}_t &= \Big[\mathbb{I}_{N\times I} - \mathbb{L}_t\Big]^{-1} \vec{A}_t
\end{aligned}$$

Step 20. Compute gross domestic product $GDP_{n,t}^{i}$

$$GDP_{n,t}^{i} = \sum_m \pi_{mn,t}^i X_{m,t}^i$$

Step 21. Compute updated aggregate income $AI_{n,t}$

$$AI_{n,t} = w_{n,t} \sum_{j=0}^{J} e_{n,j} L_{n,j,t} + r_{n,t}^k K_{n,t}$$

Step 22. Compute net export $TB_{n,t}$

$$TB_{n,t} = \sum_{i=1}^{I} GDP_{n,t}^{i} - \sum_{i=1}^{I} X_{n,t}^{i}$$

Step 23. Compute aggregate bond position $B_{n,t+1}$

$$B_{n,t+1} \equiv \sum_{j=0}^{J} b_{n,j+1,t+1} L_{n,j,t}$$

Step 24. Compute domestic debt $DB_{n,t+1}$

$$DB_{n,t+1} = -\sum_{j=0}^{J} \mathbb{1}[b_{n,j+1,t+1} < 0] b_{n,j+1,t+1} L_{n,j,t}$$

Step 25. Compute domestic debt $DB_{n,t+1}$

$$DB_{n,t+1} = \sum_{j=0}^{J} \mathbb{1}[b_{n,j+1,t+1} > 0] b_{n,j+1,t+1} L_{n,j,t}$$

Step 26. Compute rental rate of capital $(r_{n,t}^{k})'$

$$(r_{n,t}^{k})' = \frac{\sum_{i\in\mathcal{I}} (1-\eta_n^i)(1-\gamma_n^i) GDP_{n,t}^{i}}{K_{n,t}}$$

Step 27. Compute wage $\{\big(w_{n,t}\big)'\}_{n,t}$ using labor market clearing

$$\big(w_{n,t}\big)' = \frac{\sum_{i\in\mathcal{I}} \eta_n^i (1-\gamma_n^i) GDP_{n,t}^{i}}{\sum_{j=0}^{J} e_{n,j} L_{n,j,t}}$$

Step 28. Compute interest rate $\{\big(r_{n,t+1}\big)'\}_{n,t}$

$$\big(r_{n.t+1}\big)' = \Big(\frac{\log(\sum_n DB_{n,t+1})}{\log(\sum_n DS_{n,t+1})}\Big) \times r_{t+1}$$

Step 33. Compute dev_1

$$dev_1 \equiv \max(|| \log(\big(w_{n,t}\big)') - \log(w_{n,t})||, || \log(\big(r^k_{n,t}\big)') - \log(r^k_{n,t})||, || \log(\big(r_{t+1}\big)') - \log(r_{t+1})||)$$

Step 34. Update $\{w_{n,t}, r_{t+1}\}_{n,t}$ (e.g. $\delta_1 = 0.99$)

$$\begin{aligned} w_{n,t} &\leftarrow \delta_1 w_{n,t} + (1-\delta_1)\big(w_{n,t}\big)' \\ r^k_{n,t} &\leftarrow \delta_1 r^k_{n,t} + (1-\delta_1)\big(r^k_{n,t}\big)' \\ r_{t+1} &\leftarrow \delta_1 r_{t+1} + (1-\delta_1)\big(r_{t+1}\big)' \end{aligned}$$

Step 34. If $dev_1 > \epsilon_1$ for small ϵ_1 (e.g. $\epsilon_1 = 10^{-6}$), go to step 5.2. $\boxed{\leftarrow \textbf{Loop 3}}$