Climate Policies under Dynamic Factor Adjustment *

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Abstract

This paper develops and estimates a dynamic general equilibrium model of the global economy, where imperfect and gradual adjustments of capital and labor across sectors influence the effectiveness of climate policies in reducing global emissions. The model's estimates indicate that (i) both capital assets and workers exhibit low responsiveness to carbon taxes, limiting the policy's ability to permanently shift capital and labor out of the fossil-fuel sector; (ii) small policy changes minimally alter the trajectory of sectoral capital allocation; and (iii) labor reallocation in response to carbon taxes takes at least a decade. Using a calibrated model, I show that if capital and labor are more responsive to China's removal of fossil fuel subsidies, the policy could further reduce cumulative global emissions by at least 4.29 percent of 2022 levels by 2030 and 64.96 percent by 2100. Additionally, I demonstrate that under imperfect factor mobility, China's subsidy on the electricity sector may inadvertently increase global emissions by boosting aggregate consumption and investment without significantly reallocating capital and labor away from the fossil-fuel sector. These findings underscore that a carbon tax and an electricity subsidy should be implemented together to reduce global emissions while enhancing aggregate consumption.

JEL Codes: F18, Q56, Q58.

Keywords: Environmental Policy, Labor Mobility, Capital Mobility, International Trade

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1 Introduction

To meet the Paris Agreement's goal of limiting global warming to 1.5°C above pre-industrial levels by the end of the century, countries must quickly reallocate capital and workers from fossil-fuel sectors to renewable energy sectors.[1] However, economic lock-ins and institutional path dependencies slow this swift transition by preserving existing infrastructure and systems, delaying the shift to renewable energy (Lee et al. (2023)). Despite the emphasis on these challenges in policy discussions, many integrated assessment frameworks analyzing climate policies assume capital and workers can move freely across sectors, contradicting empirical evidence showing that capital and labor imperfectly and gradually respond to policy changes (Artuç et al. (2010), Dix-Carneiro (2014), Dix-Carneiro and Kovak (2017)).

To address this gap, I develop a quantitative framework to examine how imperfect and dynamic adjustments of capital and labor influence the effectiveness of climate policies in reducing global emissions. The framework emphasizes two key margins of adjustment: (i) the extent of adjustment, which determines the long-term allocation of capital and labor across sectors, and (ii) the speed of adjustment, which governs the transition process.

First, the extent of capital adjustment to changes in sectoral rental rates may be limited, as heterogeneous capital assets (e.g., drilling rigs, semiconductor plants) tend to be more productive in specific sectors. Similarly, the extent of labor adjustment to wage changes may be constrained by non-pecuniary factors (e.g., family reasons, personal preferences) that affect workers' mobility across sectors.

Second, the persistence of initial capital allocations may slow the adjustment process, as sector-specific productivity depends on past capital allocation (Aghion et al. (2016)). Additionally, workers face costs (e.g., training costs) when moving between sectors, which may further decelerate the speed of adjustment. I estimate the structural parameters governing these two margins of adjustment and show that both capital and labor exhibit low responsiveness to changes in factor prices and would adjust slowly to climate policy changes.

I begin by presenting motivational evidence on the imperfect and dynamic responses of capital

[1] If global emissions remain at 2023 levels, global temperatures will rise to 1.5°C above pre-industrial levels within 10 years and exceed 2°C in 27 years (Lee et al. (2023)). According to Lee et al. (2023), the best estimates of the remaining carbon budgets are 500 $GtCO_2$ for a 50 percent likelihood of limiting global warming to 1.5°C, and 1150 $GtCO_2$ for a 67 percent likelihood of limiting warming to 2°C. Global carbon emissions in 2020, 2021, 2022, and 2023 were 35.01 $GtCO_2$, 36.82 $GtCO_2$, 37.15 $GtCO_2$, and 37.40 $GtCO_2$, respectively.

and employment in the fossil-fuel sector to environmental policy changes. Using data from the 2016 release of the World Input-Output Database, I apply the local projection method (Jordà, 2005) to estimate the impulse response of capital and employment in the fossil-fuel sector to an increase in the Environmental Policy Stringency (EPS) index, constructed by the OECD. I find that the long-run adjustments to stricter environmental policies vary between low-income and high-income countries. Furthermore, I demonstrate that the short-run responses of capital and employment differ significantly from the long-run outcomes.

I then develop a dynamic general equilibrium model of the global economy, where imperfect and dynamic adjustments of capital and labor across sectors influence both global emissions and global output through sectoral and international production linkages. The model features multiple sectors, divided into three groups: (i) a brown sector that directly uses raw fossil fuels for energy production, (ii) an electricity sector that produces an alternative to fossil fuel-based energy using intermediate inputs such as solar panels and wind turbines, and (iii) non-energy sectors reliant on intermediate inputs from the brown and electricity sectors. The use of raw fossil fuels by the brown sector generates carbon emissions, which reduce economy-specific environmental quality and, in turn, aggregate productivity (Barrage and Nordhaus (2024)). Workers make dynamic decisions about which sector to work in for the next period, while capitalists decide how to accumulate aggregate capital and allocate heterogeneous capital assets across sectors.

The government can reduce emissions using two types of policy instruments: a carbon tax and an electricity subsidy. A carbon tax, applied as a consumption tax on brown sector goods, reduces emissions by lowering demand for brown sector goods, which are directly responsible for generating emissions. On the other hand, an electricity subsidy, in the form of a consumption subsidy on electricity, reduces emissions by encouraging consumers and producers to substitute brown sector goods with electricity.

The overall impact of climate policies on global emissions depends on both substitution and scale effects. In the case of a carbon tax, which increases the price of brown sector goods, emissions decrease not only due to the substitution effect, as demand shifts from the brown sector to the electricity sector, but also due to the scale effect, driven by the overall decline in production caused by increased intermediate input costs faced by non-energy sector producers. Conversely, with an electricity subsidy, which decreases the price of electricity, the substitution effect reduces emissions by depressing the brown sector. However, emissions can rise due to the scale effect, as the overall production and demand for energy increase from lower intermediate input costs faced by non-energy sector producers.

The model generates key relationships between capital and labor supply and their underlying determinants. I show that optimal sectoral capital allocation depends on rental rates of capital, sector-specific capital-use efficiency, and aggregate capital. The static response to environmental policy depends on the dispersion of capital-use efficiencies, which governs the specificity of capital assets. The dynamic response depends on how sector-specific capital-use efficiency evolves, driven by the dynamic agglomeration externality from past capital allocation (e.g., within-sector knowledge spillovers, learning-by-doing), which is not internalized by capitalists.

Additionally, I show that labor flows across sectors depend on future wage differentials, sector-specific non-pecuniary preferences of workers, and bilateral mobility costs. The extent of labor adjustment to environmental policy is determined by the dispersion of workers' idiosyncratic preferences across sectors, while the speed of adjustment—or the magnitude of labor flows—depends on the size of bilateral mobility costs. These capital and labor supply conditions can be mapped to data and used to estimate the key structural parameters.

To estimate the capital supply parameters, I use the World Input-Output Database 2016 Release, which provides data on capital stocks and capital compensation for 43 countries and 56 industries from 2000 to 2014. The dataset includes industry-level capital stock measures, constructed as a weighted average of eight asset types, consistent with the theoretical interpretation of capital being heterogeneous in its uses across sectors.

To estimate labor supply parameters, I merge individual-level longitudinal labor surveys from eight countries — United States, China, India, France, United Kingdom, Australia, Republic of Korea, and Argentina. This merged dataset contains information on labor income, industry affiliation, and employment status over time, enabling the estimation of country-level inter-sectoral mobility costs and the global average dispersion of non-pecuniary motives for mobility.

The estimates indicate (i) a significant variation in efficiencies across capital assets, with many being highly productive in specific sectors but less useful in others, and (ii) strong non-pecuniary motives for worker mobility. These non-price mechanisms limit the responsiveness of factor flows to changes in sectoral factor prices: a 1 percent increase in sectoral rental rate of capital is associated with just a 0.02 percent increase in capital allocation to the sector within the same period, while a 1 percent increase in sectoral wages raises annual labor inflows by only 0.55 percent.

Furthermore, the estimates indicate that for workers in the brown sector across all economies, the cost of transitioning out of the sector is equivalent to sacrificing at least 97.2 percent of their current real wage, suggesting extreme difficulty in moving across sectors. Additionally, a 1 percent increase in capital allocation in the previous period is associated with a 0.97 percent increase in sector-specific capital-use efficiency in the subsequent period, implying that without government intervention, initial capital allocations across sectors would persist over time.

I calibrate the initial conditions of the model using data for the year 2022, or the most recent available year, on production, international trade, fossil fuel production, emissions, and global temperature. I then compare the impact of unilateral climate policies on global emissions across models with varying degrees of capital and labor mobility but identical initial conditions. The two main policy scenarios I consider for the quantitative analysis are: (i) China's unexpected, permanent implementation of a carbon tax equivalent to removing its currently implemented fossil fuel subsidies, and (ii) China's unexpected, permanent implementation of an electricity subsidy equivalent to the level of its currently implemented fossil fuel subsidies.

The key quantitative result is that imperfect factor mobility significantly dampens the substitution effect of climate policies on global emissions. I show that if capital assets and workers were to rapidly adjust to China's removal of fossil fuel subsidies, the policy would further reduce cumulative global emissions by at least 4.29 percent of 2022 global emission levels by 2030 and 63.96 percent by 2100. This reduction stems from both a drop in global emissions due to reduced aggregate production from higher energy costs (the *scale effect*) and a shift in demand from the brown sector to the electricity sector (the *substitution effect*).

Conversely, if capital assets and workers were to rapidly adjust to China's electricity subsidy, the policy would reduce the increase in cumulative global emissions by at least 5.34 percent of 2022 global emission levels by 2030 and 164.42 percent by 2100. In this scenario, the electricity subsidy still leads to an overall rise in global emissions, as the increase in emissions from higher aggregate production, driven by lower energy costs (the *scale effect*), outweighs the emissions reduction from the shift in demand from the brown sector to the electricity sector (the *substitution effect*). However, when factors become more mobile, the substitution effect strengthens, mitigating the increase in global emissions.

I find that China's unilateral carbon tax results in a significant long-term decline in domestic consumption, as the tax depresses aggregate investment. Although the carbon tax initially boosts consumption through tax revenues, it leads to a long-term reduction due to increased intermediate input costs and lower investment. In contrast, global consumption rises as improved environmental quality follows China's substantial emission reductions. This outcome suggests the possibility of a Kaldor-Hicks improvement, where economies benefiting from these environmental gains could compensate China for its economic losses through international transfers.

On the other hand, I find that the impact of China's unilateral electricity subsidy on aggregate consumption depends on the strength of the dynamic agglomeration externality, which influences the evolution of capital-use efficiency. When the scale effect dominates the substitution effect, the electricity subsidy increases global emissions and worsens environmental quality across all economies. However, if the dynamic agglomeration externality is strong, the consumption gains from internalizing this externality can offset the losses from deteriorating environmental quality. In this case, the government could reduce emissions and boost consumption by implementing both a carbon tax and an electricity subsidy. Conversely, if the agglomeration externality is weak, a unilateral electricity subsidy results in a decline in global consumption in both the short and long run.

I further calculate the magnitude of a permanent global carbon tax required to meet the Paris Agreement's 1.5 °C target by 2100. I find that the global temperature will remain below 1.5 °C if all economies immediately and permanently increase the consumption tax on the brown sector by 293 percentage points. Additionally, if capital assets and workers rapidly respond to the policy, the global carbon tax could delay reaching 1.5 °C by an additional 22 years.

Finally, to evaluate the long-run effects of temporary climate policies, I simulate multiple scenarios where China introduces electricity subsidies for five years. The results show that small subsidies have no lasting impact on capital allocation, as capital in the electricity sector gradually returns to levels projected in the no-policy scenario once the subsidies end. However, larger subsidies of 15 percentage points or more lead to a permanent reallocation of capital toward the electricity sector. This lasting impact on capital allocation is driven by a strong dynamic agglomeration externality that affects capital-use efficiencies; hence, it does not result in long-term changes in employment.

Related Literature This paper contributes to the literature on the economics of climate change (Nordhaus (1992), Golosov et al. (2014), Nordhaus (2015), Hémous (2016), Shapiro (2021), Kortum and Weisbach (2022), Cruz (2023), Känzig (2023), Cruz and Rossi-Hansberg (2023), Arkolakis and Walsh (2023), Xiang (2023), Bilal and Känzig (2024), Barrage and Nordhaus (2024), Farrokhi and Lashkaripour (2024)). It closely relates to the work of Cruz and Rossi-Hansberg (2023), who examine how labor mobility frictions and agents' idiosyncratic non-pecuniary preferences for different regions influence the welfare impact of climate change. In contrast to the existing literature, which primarily focuses on labor mobility across regions, this paper incorporates and estimates the imperfect and gradual adjustments of both capital and labor across sectors, particularly the energy sectors, which are crucial for understanding the effects of climate policies on emissions.

This paper also relates to a large body of literature on imperfect capital mobility, which generally focuses on capital allocation across firms within industries (Dixit (1989); Dixit and Pindyck (1994); Ramey and Shapiro (2001); Lanteri (2018); Artuc et al. (2022); Lanteri et al. (2023)). The key contribution of the paper is the tractable incorporation of imperfect and dynamic capital adjustment into the analysis of climate policies. This is achieved by extending the capital allocation framework of Kleinman et al. (2023) to a multi-sector setting and incorporating dynamic capital adjustments driven by the dynamic agglomeration externality, in the spirit of the directed technical change literature (Acemoglu et al. (2012), Acemoglu et al. (2016), Aghion et al. (2016), Acemoglu et al. (2023)). This approach enables the estimation of key parameters that govern the extent and speed of capital adjustment in response to climate policies. The extent and speed of adjustment are especially important in the context of understanding the impact of climate policies: the low responsiveness of capital and labor to policy changes affects long-run allocations of capital and employment across sectors, and the speed of adjustment significantly affects long-term environmental quality.

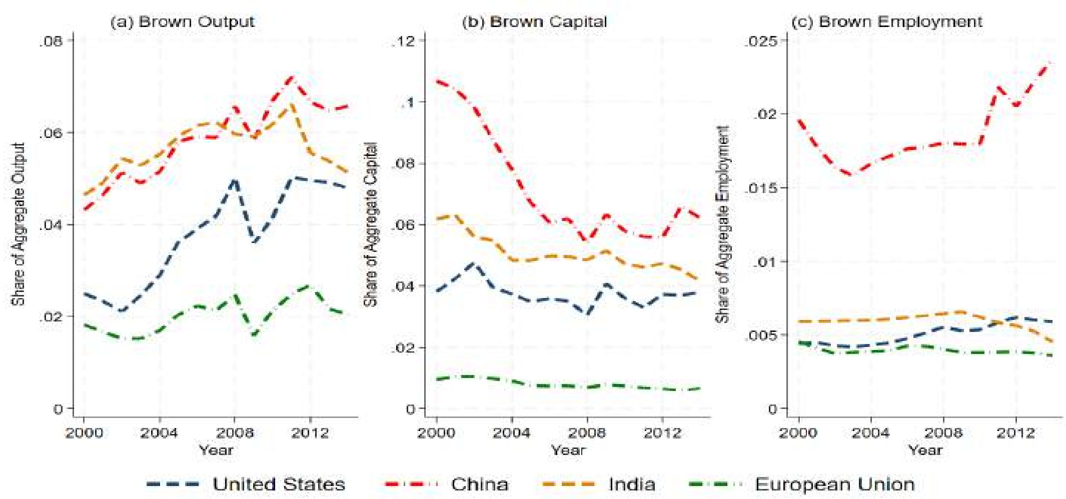
Moreover, this paper relates to the quantitative international trade literature that analyzes the global impact of policies (Armington (1969); Anderson (1979); Eaton and Kortum (2002); Arkolakis et al. (2012); Head and Mayer (2014); Caliendo and Parro (2015)). Specifically, it contributes to the trade literature studying the impact of policies on labor market adjustment processes (Artuç et al. (2010); Dix-Carneiro (2014); Artuç and McLaren (2015); Caliendo et al. (2019); Traiberman (2019); Caliendo et al. (2021); Dix-Carneiro et al. (2023)). Understanding the extent and speed of labor adjustment in major economies responsible for global emissions is central to this paper. Hence, compared to previous work that estimated labor supply parameters for a single country, I merge longitudinal labor surveys from eight major economies to estimate country-level bilateral labor mobility costs and non-pecuniary motives for worker mobility.

The remainder of the paper is organized as follows. Section 2 documents the motivating facts. Section 3 introduces a dynamic general equilibrium model of a global economy that features imperfect capital and labor mobility, production linkages, and environmental damage due to carbon emissions. In Section 4, I describe the data, the estimation procedure, and the estimation results. Section 5 presents the quantitative results. Section 6 concludes the paper.

2 Motivating Facts

In this section, I present descriptive facts about fossil fuel-producing industries in major economies and provide motivational evidence for two key margins of imperfect capital and labor adjustment in response to changes in environmental policies: (i) the extent of adjustment and (ii) the speed of adjustment.

Figure 1: Output, Capital, and Employment in the Brown Sector



Notes: Subplot (a) shows the gross output in the brown sector as a share of aggregate gross output in the United States, China, India, and the European Union from 2000 to 2014. Subplot (b) shows the capital stock in the brown sector as a share of aggregate capital stock in the same regions from 2000 to 2014. Subplot (c) shows employment in the brown sector as a share of aggregate employment in these regions over the same period. Source: World Input-Output Database 2016 Release.

Brown Sector The primary data source for this analysis is the World Input-Output Database (WIOD) 2016 Release (Timmer et al. (2015)), covering 43 countries and 56 industries from 2000 to 2014. The dataset provides industry-level data on gross output, capital stock, employment, and compensation for capital and labor.

In this paper, I define the brown sector as industries that extract and process raw fossil fuels, such as crude oil, natural gas, and coal. As a producer of fossil fuel energy, the brown sector is directly responsible for global emissions. Following O'Mahony and Timmer (2009), it is classified as a subset of the energy sector, including the mining and quarrying industry and the industry that manufactures coke and refined petroleum products.

As shown in subplot (a) of Figure 1, the gross output of the brown sector accounted for 2 percent to 6 percent of aggregate output in four major economies that accounted for approximately 60 percent of global emissions in 2022: the United States (US), China, India, and the European Union (EU). Subplots (a) and (b) demonstrate that the share of aggregate capital stock allocated to the brown sector is at least twice the share of aggregate employment allocated to the brown sector for all economies. This suggests that the brown sector is capital-intensive, and that reducing the size of the brown sector requires greater reallocation of capital than labor from the brown sector.

Table 1: Summary Statistics for Annual Changes in EPS ($EPS_{n,t} - EPS_{n,t-1}$)

Statistic	**EPS**	**Market-Based EPS**	**Non-Market-Based EPS**
25th Percentile	0.000	0.000	0.000
Median (50th)	0.000	0.000	0.000
75th Percentile	0.133	0.083	0.125
Minimum	-1.400	-2.000	-2.000
Maximum	1.112	2.083	2.000
Mean	0.079	0.060	0.098
Variance	0.066	0.113	0.136
Observations	771	771	799

Notes: Summary statistics for annual changes in the environmental policy stringency index (EPS), market-based EPS, and non-market-based EPS for 40 countries from 1990 to 2020. Source: OECD (Kruse et al. (2022))

Environmental Policy To understand the impact of changes in environmental policies on capital and employment in the brown sector, I employ the Environmental Policy Stringency index (EPS) constructed by the OECD (Kruse et al. (2022)), which measures the degree to which environmental policies impose an explicit or implicit price on polluting activities. It covers 40 countries from 1990 to 2020 and is constructed by assessing 13 types of environmental policies divided into three categories: market-based, non-market-based, and technology support policies. Each policy is assigned a score ranging from 0 to 6, where 0 indicates the weakest implementation and 6 indicates the most stringent implementation (refer to the Appendix A.3 for details on thresholds for each policy). The index $EPS_{n,t}$ for country n in year t is computed as the average of the scores assigned to each environmental policy.

Table 1 presents the summary statistics for the annual changes in $EPS_{n,t}$, as well as those for the market-based EPS and non-market-based EPS. The table shows that changes in $EPS_{n,t}$ are rare, with the median being zero for EPS, market-based EPS, and non-market-based EPS. Hence, I define the environmental policy shock $x_{n,t}$ as follows:

$$x_{n,t} = \mathbb{1}[EPS_{n,t} - EPS_{n,t-1} > 0, EPS_{n,t-1} - EPS_{n,t-2} = 0, EPS_{n,t-2} - EPS_{n,t-3} = 0].$$

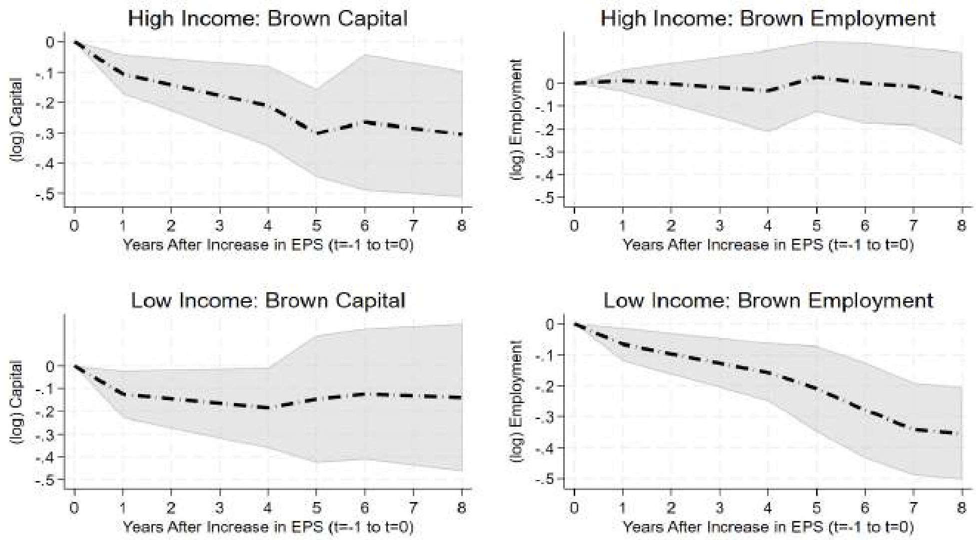
In words, a country n experiences an environmental policy shock in year t if $EPS_{n,t}$ increased compared to the previous year, and there was no change in $EPS_{n,t}$ for two years prior to such a change.

Empirical Specification I employ the local projection method (Jordà (2005)) to estimate the impulse response of capital and employment in industry j to the environmental policy shock. I specify the regression specification as follows:

$$\begin{aligned}
\underbrace{y^j_{n,t+h}}_{\text{Outcome}} = \alpha + \underbrace{\beta_h}_{\text{Average Effect}} \cdot \underbrace{x_{n,t}}_{\text{Policy Change}} + \underbrace{\gamma_h}_{\text{Differential Effect}} \cdot x_{n,t} \times \mathbb{1}[j = \text{Brown Sector}] \\
+ \underbrace{\delta_{n,t}}_{\text{Country-Year FE}} + \sum_{k=1}^{2} \lambda_k \underbrace{y_{n,t-k}}_{\text{Lagged Outcome}} + \underbrace{\Psi^j_{n,t}}_{\text{Vector of Controls}} + \varepsilon^j_{n,t}
\end{aligned}$$

where $y^j_{n,t+h}$ is the outcome variable of interest (e.g., log capital, log employment) h years after the shock; β_h captures the average effect of the shock on the outcome variable in year $t+h$ across all industries; γ_h captures the differential impact of the shock on the outcome variable in the brown sector in year $t+h$; $\delta_{n,t}$ are country-year fixed effects; $\{y_{n-k}\}$ are lagged outcomes, and $\Psi^j_{n,t}$ is a vector of controls such as k-lagged annual changes in $EPS_{n,t}$, k-lagged log industry outputs, k-lagged log wages, capital input share, k-lagged capital input shares where k ranges from 1 to 2; $\varepsilon^j_{n,t}$ is a residual term.

Figure 2: The Impact of the Environmental Policy Shock on the Brown Sector



Notes: The top plots show the impulse responses of capital and employment in the brown sector following an increase in EPS for countries with above-average gross output per capita. The bottom plots display the impulse responses of capital and employment in the brown sector for countries with below-average gross output per capita. Standard errors are clustered at the country-industry level. The shaded area represents the 95% confidence interval. Source: WIOD 2016 Release, OECD (Kruse et al. (2022)).

Results Figure 2 displays the coefficients γ_h for capital and employment in the brown sector, where h ranges from 0 to 8. The top plots show the impulse responses of capital and employment in the brown sector following an increase in EPS for countries with above-average gross output per capita (high-income countries). The bottom plots show the corresponding responses for countries with below-average gross output per capita (low-income countries).

The figure highlights notable differences in the extent of adjustment between the two groups. In low-income countries, capital in the brown sector declines by 14 log points 8 years after the shock, while in high-income countries, the decline reaches 30 log points. Looking at the employment responses in the brown sector, high-income countries experience a drop of 7 log points 8 years after the shock, whereas employment in low-income countries falls by 35 log points.

Additionally, the figure demonstrates that the full impact of environmental policy on capital and employment takes time to materialize. For example, in high-income countries, the decline in capital after 8 years is more than 2.8 times larger than the decline after 1 year. Similarly, in low-income countries, the decline in employment after 8 years is over 5.3 times the initial drop.

These results suggest that both capital and labor adjust imperfectly and gradually to changes in environmental policies. Motivated by these findings, I develop a quantitative framework in the next section to analyze the impact of environmental policies on global emissions when capital and labor adjust imperfectly and dynamically to such policies.

3 Model

In this section, I develop a dynamic general equilibrium model of the global economy to examine the effects of climate policies on emissions, accounting for the imperfect and gradual adjustment of capital and labor to environmental policy changes. Subsection 3.1 outlines the setting. Subsection 3.2 describes consumers' demand for sector-specific differentiated products (Armington (1969)) and the aggregation of these products into sectoral, energy, and final goods. Subsection 3.3 then extends the production framework of Caliendo and Parro (2015) to explicitly include energy production. Subsection 3.4 links production to carbon emissions and environmental quality, following the work of Cruz and Rossi-Hansberg (2023) and Barrage and Nordhaus (2024). Subsection 3.5 extends the capital allocation framework developed in Kleinman et al. (2023) to a multi-sector setting, where capitalists accumulate aggregate capital and allocate capital assets across sectors. Subsection 3.6 describes the workers' dynamic problem which follows Artuç et al. (2010). Finally, Subsection 3.7 describes the role of governments in regulating emissions and implementing environmental policies. The theoretical derivations for all the equations that appear in the model are provided in Appendix A.4.

3.1 Setting

Time is discrete and is indexed by $t \in \{0, 1, ...\} \equiv \mathcal{T}$. The world economy consists of N economies, indexed by $n, m \in \mathcal{N}$, where $\mathcal{N}$ represents the set of economies. Each economy contains J sectors, indexed by $j, k \in \mathcal{J}$, where $\mathcal{J}$ denotes the set of sectors. I categorize these sectors into three categories: non-employment (NE) sector, which does not produce output; energy sectors, denoted by $\mathcal{J}_E$, which produce energy goods; and non-energy sectors, denoted by $\mathcal{J}_N$, which use energy inputs for production. The set $\mathcal{J}_E$ includes both the brown (B) sector, which extracts and processes raw fossil fuels such as crude oil, natural gas, and coal, and the non-brown energy (NB) sector, which produces secondary energy (e.g., electricity) without directly extracting and processing raw fossil fuels.

Within each sector, a continuum of perfectly competitive producers (i.e., firms) produce economy-sector-specific varieties.[2] The numeraire of the model is one unit of final good in economy n_0, i.e., $P_{n_0,t} = 1$ for all $t \in \mathcal{T}$. Within each economy, a domestic market for riskless assets exists where only domestic agents can participate. Finally, there is no aggregate uncertainty; thus, all agents possess perfect foresight over aggregate variables.

3.2 Demand

Sectoral Goods Sectoral goods are bundles that represent consumers' demand for sector-specific products (or varieties) differentiated by the economy of origin (Armington (1969), Dixit and Stiglitz (1977)). These varities are either (i) consumed by final goods consumers or (ii) used as intermediate inputs for the production of varieties. Formally, the sectoral good $Y^j_{n,t}$ in sector j of economy n at time t is a Constant Elasticity of Substitution (CES) aggregate of sector-specific varieties $\{y^j_{nm,t}\}$ produced in economy m and consumed in economy n:

$$Y^j_{n,t} = \left(\sum_{m \in \mathcal{N}} \left(y^j_{nm,t} \right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \tag{1}$$

where $\sigma > 1$ dictates the elasticity of substitution across varieties.

International Trade The share of sector j expenditures of economy n ($\pi^j_{nm,t}$) spent on varieties produced in sector j of economy m is given by:

$$\pi^j_{nm,t} \equiv \frac{p^j_{nm,t} y^j_{nm,t}}{P^j_{n,t} Y^j_{n,t}} = \left(\frac{d^j_{nm,t} p^j_{nn,t}}{P^j_{n,t}} \right)^{1-\sigma} \tag{2}$$

where $p^j_{nm,t}$ denotes the price of variety produced in sector j of economy m and consumed in economy n at time t; $P^j_{n,t}$ is the sector-level price index in economy n at time t given by $P^j_{n,t} \equiv$

[2] I assume perfect competition to avoid distortions due to market power. In a model with market power, government intervention through production subsidies would be necessary. Since this paper focuses on environmental externalities, I abstract from market power considerations.

$\left(\sum_{m\in\mathcal{N}}(p^j_{nm,t})^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$.

The term $d^j_{nm,t} \geq 1$ captures the sector-level bilateral iceberg trade cost of shipping goods from economy m to economy n, which can be further decomposed into the bilateral import tax $e^j_{nm,t}$ and an exogenous bilateral trade cost $\varrho^j_{nm,t}$, i.e., $d^j_{nm,t} \equiv e^j_{nm,t}\varrho^j_{nm,t}$ (Krugman (1980)).[3] The elasticity of trade flows with respect to changes in trade costs (i.e., trade elasticity) is $1-\sigma$. Equation (2) is commonly referred to as the gravity equation in the international trade literature (Armington (1969), Anderson (1979)).

Energy Composite Goods Energy composite goods are bundles that represent consumers' demand for energy goods produced by brown (B) and non-brown energy (NB) sectors. The energy composite goods are either (i) consumed by final goods consumers, or (ii) used as a factor of production in sectors other than the brown sector. Formally, the energy composite good $Y^E_{n,t}$ in economy n at time t is a Constant Elasticity of Substitution (CES) aggregate of the sectoral goods $Y^{NB}_{n,t}$ and $Y^B_{n,t}$:

$$Y^E_{n,t} = \left((\alpha^{NB}_n)^{\frac{1}{\eta_1}}(Y^{NB}_{n,t})^{\frac{\eta_1-1}{\eta_1}} + (\alpha^B_n)^{\frac{1}{\eta_1}}(Y^B_{n,t})^{\frac{\eta_1-1}{\eta_1}}\right)^{\frac{\eta_1}{\eta_1-1}} \tag{3}$$

where the weights α^{NB}_n and α^B_n represent the reliance of economy n on non-brown energy and brown sectors, respectively, with $\alpha^{NB}_n + \alpha^B_n = 1$; the parameter η_1 governs the elasticity of substitution across these energy goods. Equation (3) highlights that an economy can reduce emissions through substitution if brown and non-brown energy goods are substitutes, i.e., $\eta_1 > 1$.

The energy price index $P^E_{n,t}$ is given by:

$$P^E_{n,t} = \left(\alpha^{NB}_n\left((1+\tau^{NB}_{n,t})P^{NB}_{n,t}\right)^{1-\eta_1} + \alpha^B_n\left((1+\tau^B_{n,t})P^B_{n,t}\right)^{1-\eta_1}\right)^{\frac{1}{1-\eta_1}} \tag{4}$$

where $\tau^{NB}_{n,t}$ and $\tau^B_{n,t}$ are consumption taxes imposed on the non-brown energy sector and the brown sector, respectively. Equation (4) implies that the government can shift demand away from the brown sector (the *substitution effect*) either by increasing the consumption tax, $\tau^B_{n,t}$, on brown goods (i.e., a carbon tax) or by reducing the consumption tax, $\tau^{NB}_{n,t}$, on non-brown energy goods (i.e., a non-brown energy subsidy).

[3] Also, the triangle inequality holds, i.e., $d^j_{nm,t} \leq d^j_{nh,t} \cdot d^j_{hm,t}$ for all $m, n, h \in \mathcal{N}$.

In addition, a carbon tax would further decrease demand for the brown sector by reducing the demand for the energy composite good (the *scale effect*). Conversely, the non-brown energy subsidy would have the additional effect of increasing demand for the brown sector by raising the demand for the energy composite good (the *scale effect*).

Final Goods Final goods are bundles of sectoral and energy composite goods consumed by workers and capitalists, and used by capitalists for investment. Formally, the final good $Y_{n,t}$ in economy n at time t is a Cobb-Douglas aggregate of sectoral goods $\{C^j_{n,t}\}$ produced in non-energy sectors and the energy composite good $C^E_{n,t}$:

$$Y_{n,t} = \mathcal{D}_n(\mathcal{C}_t)\Big(\prod_{j\in\mathcal{J}_N}(C^j_{n,t})^{\alpha^j_n}\Big)^{\alpha_n}(C^E_{n,t})^{1-\alpha_n}. \tag{5}$$

where α^j_n governs the share of final goods expenditure spent on downstream sector j, with $\sum_{j\in\mathcal{J}_N}\alpha^j_n = 1$; $\alpha_n \in [0,1]$ represents the share of total final goods expenditure allocated to non-energy goods.

Following Barrage and Nordhaus (2024), environmental quality $\mathcal{D}_n(\mathcal{C}_t)$, which depends on the global stock of carbon dioxide $\mathcal{C}_t$, influences the total factor productivity (TFP) of an economy, thereby affecting consumption by workers and capitalists, as well as capitalists' investment decisions. This term will be explained in detail in Section 3.4. Since consumers and producers do not account for the impact of their activities on the global carbon stock, a *global environmental externality* arises, necessitating government intervention.

Materials Materials are bundles of sectoral goods reflecting non-energy intermediate inputs used for production.[4] Formally, the material $M^j_{n,t}$ in sector j of economy n is a Cobb-Douglas aggregate of sectoral goods $\{M^{j,k}_{n,t}\}_{k\in\mathcal{J}}$: $M^j_{n,t} = \prod_{k\in\mathcal{J}}\left(M^{j,k}_{n,t}\right)^{\gamma^{j,k}_n}$ where intermediate input shares satisfy $\sum_{k\in\mathcal{J}}\gamma^{j,k}_n = 1$.

[4]Materials are included in the model for two main reasons. First, intermediate input expenditures account for 20 to 70 percent of gross revenue across all sectors in all economies, making it a quantitatively significant feature necessary to match the data. Second, materials capture the interconnections between sectors through production linkages. They illustrate how changes in the output price of one sector affect the economy by altering the input prices of other sectors that use these outputs for production. This is crucial for analyzing the impact of policies targeting specific sectors.

3.3 Production

Variety Production The output $y^B_{mn,t}$ of brown varieties produced in economy n at time t and consumed in economy m is produced using a Cobb-Douglas function that takes fossil fuel $(F_{mn,t})$ as an input for production in addition to labor $(L^B_{mn,t})$, capital $(K^B_{mn,t})$, and materials $(M^j_{mn,t})$:

$$y^B_{mn,t} = \left(\frac{z^B_{n,t}}{d^B_{mn,t}}\right)\left((F^B_{mn,t})^{\psi_n^{B,F}}(L^B_{nm,t})^{\psi_n^{B,L}}(K^B_{mn,t})^{\psi_n^{B,K}}\right)^{\gamma_n^{B,V}}\left(M^B_{mn,t}\right)^{1-\gamma_n^{B,V}} \tag{6}$$

where $z^B_{n,t}$ is the total factor productivity of the variety producer in sector j of economy n at time t, and input shares satisfy $(1-\gamma_n^{j,V}) + \sum_{h\in\{L,K,F\}} \gamma_n^{j,V}\psi_n^{j,h} = 1$.

In contrast to producers in the brown sector, the output $y^j_{mn,t}$ produced in sector $j \in \mathcal{J} \setminus \{B\}$ of economy n at time t and consumed in economy m requires energy inputs rather than raw fossil fuel inputs. In particular, the production function is expressed as a Cobb-Douglas function that takes energy composite $(Q^{j,E}_{mn,t})$, labor $(L^j_{mn,t})$, capital $(K^j_{mn,t})$, and materials $(M^j_{mn,t})$ as inputs:

$$y^j_{mn,t} = \left(\frac{z^j_{n,t}}{d^j_{mn,t}}\right)\left((Q^{j,E}_{mn,t})^{\psi_n^{j,E}}(L^j_{mn,t})^{\psi_n^{j,L}}(K^j_{mn,t})^{\psi_n^{j,K}}\right)^{\gamma_n^{j,V}}\left(M^j_{mn,t}\right)^{1-\gamma_n^{j,V}} \tag{7}$$

where $z^j_{n,t}$ is the total factor productivity of the variety producer in sector j of economy n at time t, and input shares satisfy $(1-\gamma_n^{j,V}) + \sum_{h\in\{L,K,E\}} \gamma_n^{j,V}\psi_n^{j,h} = 1$.

Fossil Fuel Extraction Variety producers in the brown sector of economy n use fossil fuel inputs $F_{n,t}$ at time t, which are a Constant Elasticity of Substitution (CES) composite of crude oil $(f^o_{n,t})$, natural gas $(f^g_{n,t})$, and coal $(f^c_{n,t})$:[5] $F_{n,t} = \left(\sum_{h\in\{o,g,c\}}(\omega_n^h)^{\frac{1}{\eta_2}}\left(f^h_{n,t}\right)^{\frac{\eta_2-1}{\eta_2}}\right)^{\frac{\eta_2}{\eta_2-1}}$ where the weights $\{\omega_n^h\}_{h\in\{o,g,c\}}$ represent the reliance of economy n on different types of fossil fuels for production with $\sum_{h\in\{o,g,c\}}\omega_n^h = 1$; η_2 governs the elasticity of substitution across these fossil fuels. When fossil fuels are substitutes (i.e., $\eta_2 > 1$), fuel-specific taxes can discourage the use of carbon-intensive fossil fuels (e.g., coal) and reduce carbon emissions.

To make the model tractable, I assume that either state-owned enterprises or perfectly compet-

[5] Including different types of fossil fuels is necessary to capture differing carbon emission trends across countries due to the changing composition of emissions. In 2022, more than 65% of the emissions in China and India were from coal, while less than 25% of the emissions in the United States and the European Union were from coal, driven by a shift to oil and natural gas.

itive and atomistic firms extract each type of fossil fuel.[6] These firms solve static extraction problems. Consequently, the before-tax price ($s^h_{n,t}$) of fossil fuel type $h \in \{o, g, c\}$ in economy n at time t equals the marginal cost of extraction φ^h_n, measured in units of the final good: $s^h_{n,t} = \varphi^h_n P_{n,t}$ where $P_{n,t}$ denotes the price of the final good in economy n at time t. Hence, the price of a fossil fuel input ($s_{n,t}$) in economy n at time t is given by:

$$s_{n,t} = \left(\sum_{h \in \{o,g,c\}} \omega^h_n \Big((1 + \tau^{f,h}_{n,t}) s^h_{n,t} \Big)^{1-\eta_2} \right)^{\frac{1}{1-\eta_2}} \tag{8}$$

where $\tau^{f,h}_{n,t}$ represents the consumption tax specific to fossil fuel type h. Equation (8) shows that the government can reduce carbon emissions through the use of fuel-specific consumption taxes that discourage the use of fuels emitting more carbon dioxide per unit (e.g., coal).

Finally, the law of motion for the reserve ($D^h_{n,t}$) of fossil fuel type $h \in \{o, g, c\}$ is given by: $D^h_{n,t+1} = D^h_{n,t} - f^h_{n,t}$ with the constraint $D^h_{n,t+1} \geq 0$. The existence of exhaustible fossil fuel resources implies that the steady state of the global economy can only be reached either when (i) fossil fuel reserves in all economies are depleted, or (ii) the non-brown energy sector becomes extremely productive, allowing the non-brown energy sector to completely, or asymptotically, dominate the brown sector in all economies. In both scenarios, global emissions will be zero or close to zero, and climate policies will no longer be necessary. This highlights the importance of understanding the impact of climate policies during the transition period, not just in the long-term steady state.

3.4 Carbon Emissions and Environmental Quality

Carbon dioxide (CO_2) emissions arise from the consumption of fossil fuels $\{f^h_{n,t}\}$ used in the brown sector. Specifically, consuming one unit of fossil fuel of type h emits ϕ^h units of CO_2. Hence, the total carbon emissions $E_{n,t}$ due to fossil fuel production in economy n at time t is given by: $E_{n,t} = \sum_{h \in \{o,g,c\}} \phi^h \cdot f^h_{n,t}$. The measure $E_{n,t}$ represents territorial carbon emissions based on

[6] A more realistic model would involve fossil fuel extracting firms with market power solving dynamic extraction problems, considering the future path of fossil fuel reserves. However, this is beyond the scope of this paper, as such an inclusion would introduce additional externalities and inefficiencies. For quantification, I account for any observed differences between retail prices and supply costs of fossil fuel using the fuel-specific consumption tax, which may also reflect firm markups.

fossil fuel production within the economy.[7] The global CO_2 emissions at time t, denoted by E_t, are given by: $E_t = \sum_{n \in \mathcal{N}} E_{n,t}$.

Following Cruz and Rossi-Hansberg (2023), I assume the global stock of carbon dioxide $\mathcal{C}_t$ in the atmosphere evolves as follows: $\mathcal{C}_t = \mathcal{S} + \sum_{\tau=T_{min}}^{t} (1 - \delta^C_{t-\tau}) \Big(E_\tau + E^L \Big)$ where $\mathcal{S}$ denotes the pre-industrial stock of carbon dioxide; T_{min} denotes the start period for measuring carbon emissions; $(1 - \delta^C_{t-\tau})$ represents the fraction of carbon emissions at time τ that remains in the atmosphere at time t, and E^L denotes exogenous carbon emissions due to land-use change.

Furthermore, following Golosov et al. (2014), the global average temperature $\mathbb{T}(\mathcal{C}_t)$ at time t relative to the global average temperature at time T_{min} is approximated as follows: $\mathbb{T}(\mathcal{C}_t) = \lambda_T \log_2 \left(\frac{\mathcal{C}_t}{\mathcal{S}} \right)$ where λ_T governs the increase in the global average temperature due to the increase in the global stock of carbon.

Following Barrage and Nordhaus (2024), I assume the economy-specific environmental quality function $\mathcal{D}_n(\cdot)$ decreases non-linearly with the global average temperature $\mathbb{T}(\mathcal{C}_t)$: $\mathcal{D}_n(\mathcal{C}_t) = 1 - \xi_n \Big(\mathbb{T}(\mathcal{C}_t) \Big)^2$ where $\xi_n > 0$ captures the environmental damage specific to economy n resulting from changes in global average temperature. The convexity of this function reflects the non-linear impact of rising global carbon stock on the climate system and the global economy, encompassing effects such as natural disasters and tipping points.

3.5 Capitalists

Now, I describe the capitalists' problem and the determinants of the extent and the speed of capital adjustment across sectors. Within each economy, there are infinitely-lived capitalists of measure one who are assumed to be homogeneous in their initial aggregate capital $W_{n,0}$. Given this homogeneity, I simplify the analysis by considering an infinitely-lived representative capitalist.

The representative capitalist in economy n aggregates all capital income ($R_{n,t} W_{n,t}$), profits ($\Pi_{n,t}$), and government transfers ($\Omega_{n,t}$) and makes collective decisions on aggregate consumption ($C^K_{n,t}$), aggregate capital accumulation ($W_{n,t+1}$), and capital allocation ($\{K^j_{n,t}\}$). For a given aggregate capital $W_{n,t}$ at time t, the capitalist solves a static discrete choice problem, similar to Kleinman et al. (2023), to allocate each unit of capital to the sector that generates the largest return.[8]. Then,

[7] In Appendix A.6.6, I present an alternative measure of carbon emissions based on the economy's total consumption of brown products, which includes both domestically produced and imported products. However, since countries typically report territorial carbon emissions, I use the production-based measure for the analysis.

[8] Kleinman et al. (2023) utilize this modeling approach in a one-sector model of international trade and capital holdings to match key facts about international capital holdings. I adapt this approach in a multi-sector model to match

the representative capitalist makes a dynamic decision to accumulate aggregate capital.

Capital Allocation Aggregate capital in economy n at time t, $W_{n,t}$, is an aggregate of heterogeneous capital assets, which have different uses across sectors, in economy n at time t. This assumption captures the concept of partial irreversibility, where capital in the brown sector cannot be easily repurposed for use in the non-brown energy sector due to the specialized nature of capital assets installed in the brown sector (Lanteri (2018)). For instance, oil rigs and petroleum refining plants, which are essential for the extraction and processing of fossil fuels, are not suitable for the production of solar panels and wind turbines. In contrast, computers, which serve general purposes, can be widely applied across many sectors.

To tractably capture this notion, I assume that the return on each unit of capital in sector j depends on (i) the sector-specific rental rate of capital ($r^j_{n,t}$) and (ii) the sector-specific capital-use efficiency ($\zeta^j_{n,t}$). The degree of heterogeneity in capital-use efficiencies ($\zeta^j_{n,t}$) across sectors governs the strength of irreversibility. Large heterogeneity in capital-use efficiencies implies that capital assets are inherently more efficient in some sectors than in others.

Formally, each unit of aggregate capital is characterized by its profile $\vec{\zeta}_{n,t} = \{\zeta^j_{n,t}\}_{j\in\mathcal{J}}$, which contains the capital-use efficiency for each sector, independently drawn from a Fréchet distribution $F^j_{n,t}(\zeta^j_{n,t})$ at each period: $F^j_{n,t}(\zeta^j_{n,t}) = e^{-(\zeta^j_{n,t}/a^j_{n,t})^{-\rho_K}}$ where the parameter $a^j_{n,t}$ captures the average usefulness of a given unit of capital in a given sector at time t. The dispersion parameter ρ_K reflects the variation in capital-use efficiency across capital assets; when ρ_K is low, indicating high dispersion, some capital assets, such as petroleum refining equipment, are highly efficient in specific sectors but less so in others. Conversely, when ρ_K is high, indicating low dispersion, most capital assets are like computers which retain their productivity across different sectors.

The probability that one unit of capital is allocated to sector j is given by:

$$\Phi^j_{n,t} = \frac{(r^j_{n,t}a^j_{n,t})^{\rho_K}}{\sum_{k\in\mathcal{J}}(r^k_{n,t}a^k_{n,t})^{\rho_K}}. \tag{9}$$

By the law of large numbers, the share of aggregate capital allocated to sector j coincides with $\Phi^j_{n,t}$ shown in Equation (9). This equation shows that the sector-level capital allocation depends not only on the sector-specific rental rate $r^j_{n,t}$ but also on the sector-specific capital-use efficiency $a^j_{n,t}$ and the dispersion of capital-use efficiencies captured by the parameter ρ_K.

the allocation of capital stock across sectors and the heterogeneous returns on capital observed in the data.

The capital allocation in sector j, $K^j_{n,t}$, is defined as the efficiency units of capital allocated to sector j and is given by: $K^j_{n,t} = \Gamma\Big(\frac{\rho_K-1}{\rho_K}\Big)\Big(a^j_{n,t}\Big)\Big(\Phi^j_{n,t}\Big)^{\frac{\rho_K-1}{\rho_K}} W_{n,t}$ where $\Gamma(\cdot)$ is a gamma function. Furthermore, the return on aggregate capital $\frac{v_{n,t}}{P_{n,t}} \equiv \frac{\sum_j r^j_{n,t} K^j_{n,t}}{W_{n,t}}$ can be expressed as follows: $\frac{v_{n,t}}{P_{n,t}} = \Gamma\Big(\frac{\rho_K-1}{\rho_K}\Big)\Big(\sum_{j\in\mathcal{J}}(a^j_{n,t} r^j_{n,t})^{\rho_K}\Big)^{\frac{1}{\rho_K}}$ which is a power mean of sector-specific rental rates with exponent ρ_K weighted by the sector-specific capital-use efficiencies $\{a^j_{n,t}\}$.

Lastly, to rationalize the persistence of sectoral capital allocation observed in the data, I introduce a dynamic capital allocation externality, where past capital allocation to a given sector impacts the current sector-specific capital-use efficiency. This externality captures external learning by doing and within-sector knowledge spillovers, which generate path dependence in sector-specific technology (Aghion et al. (2016), Bradt (2024)). It is considered an externality because atomistic capitalists do not account for the broader impact of their past capital allocations on the current capital-use efficiencies.

The law of motion for sector-specific capital-use efficiency, $a^j_{n,t}$, which captures this idea, is given by: $\ln(a^j_{n,t}) = \ln(a^j_n) + \rho_\Phi \ln\Big(\frac{K^j_{n,t-1}}{W_{n,t-1}}\Big)$. In this equation, a^j_n represents the time-invariant economy-sector-specific capital-use efficiency, while the parameter ρ_Φ captures the impact of the dynamic agglomeration externality. Specifically, ρ_Φ indicates the degree to which efficiency improves from allocating aggregate capital to sector j in the previous period. A high value ρ_Φ suggests that previous capital allocation decisions significantly influence current efficiency. On the other hand, the externality disappears when $\rho_\Phi = 0$. The inclusion of the past aggregate capital $W_{n,t-1}$ captures the diminishing gains in capital-use efficiencies across all sectors from aggregate investment.

The optimal sector-level capital allocation condition is given by:

$$\begin{aligned}\underbrace{\ln(K^j_{n,t})}_{\text{Capital Allocation}} &= \underbrace{(\rho_K-1)}_{\text{Responsiveness}} \cdot \underbrace{\ln\Big(\frac{P_{n,t} r^j_{n,t}}{v_{n,t}}\Big)}_{\text{Rental Rate of Capital}} + \underbrace{\ln(W_{n,t})}_{\text{Aggregate Capital}} - \underbrace{\rho_K \cdot \rho_\Phi \cdot \ln(W_{n,t-1})}_{\text{Lagged Wealth}} \\ &+ \underbrace{\rho_K \cdot \rho_\Phi \cdot \ln(K^j_{n,t-1})}_{\text{Dynamic Agglomeration Externality}} + \underbrace{\rho_K \cdot \ln(a^j_n)}_{\text{Sector-specific Efficiency}} + \rho_K \ln\Big(\Gamma\big(\frac{\rho_K-1}{\rho_K}\big)\Big).\end{aligned} \tag{10}$$

Equation (10) serves as the key moment condition used to estimate the capital supply parameters ρ_K and ρ_Φ. This condition indicates that sector-level capital allocation depends on (i) the rental rate of capital, (ii) aggregate capital, (iii) dynamic agglomeration externality, and (iv) time-invariant economy-sector-specific capital-use efficiency. It suggests that the extent of capital adjustment depends on the dispersion parameter ρ_K. If ρ_K is low, capital allocation responds inelastically to changes in the rental rate of capital induced by environmental policies because many capital assets are highly specialized and much more efficient in certain sectors than in others. Additionally, the speed of capital adjustment depends on the magnitude of ρ_Φ; a high ρ_Φ implies a greater persistence of past allocation and a minimal dynamic adjustment to small policy changes.

Aggregate Capital Accumulation The representative capitalist at time 0 with initial aggregate capital $W_{n,0}$ chooses a sequence of future aggregate capital $\{W_{n,t+1}\}$, in units of final good, to maximize her life-time utility $V^K_{n,0}$:

$$V^K_{n,0} = \max_{W_{n,t+1}} \sum_{t=0}^{\infty} \beta^t \mathbb{E}_0 \Big[\ln(C^K_{n,t}) \Big] \tag{11}$$

s.t.

$$C^K_{n,t} = \Big(\frac{v_{n,t}}{P_{n,t}} + (1 - \delta^k_{n,t}) \Big) W_{n,t} - W_{n,t+1} + \frac{\Omega_{n,t}}{P_{n,t}} + \frac{\Pi_{n,t}}{P_{n,t}}, \tag{12}$$

$$0 = \lim_{t\to\infty} \Big(\prod_{s=0}^{t} \Big(\frac{v_{n,s}}{P_{n,s}} + 1 - \delta^k_{n,t} \Big)^{-1} \Big) W_{n,t+1} \tag{13}$$

where $C^K_{n,t}$ denotes consumption of final good by the capitalist; β denotes the discount factor; $\delta^k_{n,t} \in [0, 1]$ captures the rate of capital depreciation in economy n; Equation (13) presents the transversality condition of the representative capitalist.

Although not explicitly shown in Equation (12), there exists a domestic financial asset market where capitalists can trade risk-free bonds, $A_{n,t+1}$, at a gross interest rate $R_{n,t+1}$. The no-arbitrage condition implies the gross interest rate on bonds equals the net return on wealth: $R_{n,t+1} = \frac{v_{n,t+1}}{P_{n,t+1}} + (1 - \delta^k_{n,t})$.

3.6 Workers

Here, I describe the workers' dynamic problem of moving across sectors, which highlights the extent and speed of labor adjustment across sectors. Within each economy, there are infinitely-lived hand-to-mouth workers of measure $L_{n,t}$ who are assumed to be homogeneous in their ability but heterogeneous in their sector of employment.[9] At each period, workers make dynamic discrete choice decisions regarding which sector to work in during the subsequent period. The decision to move from sector j (origin) to sector k (destination) depends on several factors, including (i) the destination-specific wage ($w^k_{n,t}$), (ii) the origin-destination-specific mobility costs ($\kappa^k_{n,t}$), (iii) the discounted expected life-time utility from working in sector k ($\beta \mathbb{E}_t[\mathcal{U}^k_{n,t+1}]$), (iv) the sector-specific non-pecuniary benefit (μ^j_n), and (v) the destination-specific idiosyncratic preference ($\varepsilon^k_{n,t}$).[10]

Formally, the lifetime utility $\mathcal{U}^j_{n,t}$ of a worker in sector j at time t is given by:

$$\mathcal{U}^j_{n,t} = \ln(c^j_{n,t}) + \ln(\mu^j_n) + \max_{k \in \mathcal{J}} \{\beta \mathbb{E}_t[\mathcal{U}^k_{n,t+1}] + \ln(1 - \kappa^{j,k}_n) + \rho_L \varepsilon^k_{n,t}\} \tag{14}$$

where $c^j_{n,t}$ denotes the consumption of final goods; The parameter $\kappa^{j,k}_n$ captures costs including the imperfect inter-sectoral transferability of experience and training costs (Dix-Carneiro (2014)). The cost of staying in the origin sector normalized to 0, i.e., $\kappa^{j,j}_n = 0$ for all $j \in \mathcal{J}$. As $\kappa^{j,k}_n$ approaches 1, the utility of moving from sector j to sector k approaches negative infinity, reflecting extreme difficulty in moving to sector k.[11]

At each period, a worker independently draws an idiosyncratic preference ($\varepsilon^k_{n,t}$) for each sector from an identical Gumbel distribution with mean zero, i.e., $\varepsilon^k_{n,t} \sim \text{Gumbel}(\gamma, 1)$.[12] The parameter ρ_L governs the dispersion of idiosyncratic non-pecuniary preferences and affects how responsive labor flows are to changes in economic incentives. A larger ρ_L indicates a greater influence of non-pecuniary reasons on labor mobility.

Since workers are assumed to be hand-to-mouth, the consumption of a worker in sector j in

[9] The assumption that workers are hand-to-mouth is a standard assumption made in the labor market dynamics literature for traceability (Artuç et al. (2010)). While allowing workers to make saving decisions is both desirable and realistic, it complicates the analysis because it requires keeping track of the asset distribution of workers, where each worker's asset would depend on the full path of her past income and employment history.

[10] The inclusion of the sector-specific non-pecuniary benefit primarily serves a quantitative purpose. Specifically, it captures factors like better work-life balance and job security that rationalize the inter-sectoral wage differentials that cannot be explained by estimated mobility frictions.

[11] Utility from consumption and the utility loss from moving can be combined as follows: $\ln\left((1 - \kappa^{j,k}_n)c^j_{n,t}\right) = \ln(1 - \kappa^{j,k}_n) + \ln(c^j_{n,t})$. Therefore, for a worker in sector j, moving from sector j to k is equivalent to sacrificing $\kappa^{j,k}_n$ fraction of her current consumption.

[12] The functional form is assumed for the tractability of the model. For examples of papers that use similar methodology, see Artuç et al. (2010), Dix-Carneiro (2014), Artuç and McLaren (2015), Caliendo et al. (2019), Dix-Carneiro et al. (2023). The idiosyncratic preference reflects various reasons associated with changing sectors, including but not limited to pursuing passion, relocation due to family reasons, and tedium.

economy n at time t only depends on the sector-specific wage $w^j_{n,t}$ and the price of final goods $P_{n,t}$, i.e., $c^j_{n,t} = \frac{w^j_{n,t}}{P_{n,t}}$. Moreover, I assume workers in the non-employment (NE) sector receive income $w^{NE}_{n,t}$ from the government. In particular, $w^{NE}_{n,t}$ is assumed to depend on the economy-specific component w^{NE}_n and the average wage of the economy: $w^{NE}_{n,t} = w^{NE}_n \times \Big(\frac{\sum_{j\in\mathcal{J}\setminus\{NE\}} w^j_{n,t} L^j_{n,t}}{L_{n,t}}\Big)$.

Denote $V^j_{n,t}$ as the expected lifetime utility of a worker in sector j of economy n at time t, i.e., $V^j_{n,t} \equiv \mathbb{E}_t[\mathcal{U}^j_{n,t}]$. The expected lifetime utility $V^j_{n,t}$ can be expressed as follows:

$$V^j_{n,t} = \ln\Big(\frac{w^j_{n,t}}{P_{n,t}}\Big) + \ln(\mu^j_n) + \rho_L \ln\left(\sum_{k\in\mathcal{J}} \exp\Big((\frac{\beta}{\rho_L})V^k_{n,t+1} + (\frac{1}{\rho_L})\ln(1-\kappa^{j,k}_n)\Big)\right). \tag{15}$$

Furthermore, the probability of a worker in sector j at time t moving to sector k at time $t+1$ is given by:

$$m^{j,k}_{n,t} = \frac{\exp\Big((\frac{\beta}{\rho_L})V^k_{n,t+1} + (\frac{1}{\rho_L})\ln(1-\kappa^{j,k}_n)\Big)}{\sum_{h\in\mathcal{J}} \exp\Big((\frac{\beta}{\rho_L})V^h_{n,t+1} + (\frac{1}{\rho_L})\ln(1-\kappa^{j,h}_n)\Big)}. \tag{16}$$

By the law of large numbers, $m^{j,k}_{n,t}$ is also the share of workers in sector j at time t moving to sector k at time $t+1$. Therefore, the law of motion for labor supply in sector j is given by $L^j_{n,t+1} = \sum_{k\in\mathcal{J}} L^{k,j}_{n,t}$ where $L^j_{n,t+1}$ denotes the employment in sector j at time $t+1$ and $L^{k,j}_{n,t}$ denotes the number of workers moving from sector k at time t to sector j at time $t+1$, i.e., $L^{k,j}_{n,t} \equiv m^{k,j}_{n,t} L^k_{n,t}$.

The inter-sectral labor flow condition can be derived combining equations (15) and (16):

$$\begin{aligned}\underbrace{\ln\Big(\frac{L^{j,k}_{n,t}}{L^{j,j}_{n,t}}\Big)}_{\text{Labor Flows}} = \underbrace{\Big(\frac{\beta}{\rho_L}\Big)}_{\text{Responsiveness}} \cdot \underbrace{\ln\Big(\frac{w^k_{n,t+1}}{w^j_{n,t+1}}\Big)}_{\text{Future Wage Differential}} + \underbrace{\Big(\frac{1-\beta}{\rho_L}\Big)\ln(1-\kappa^{j,k}_n)}_{\text{Bilateral Mobility Cost}} \\ + \underbrace{\beta \ln\Big(\frac{\mu^k_n}{\mu^j_n}\Big)}_{\text{Non-pecuniary Preferences}} + \underbrace{\beta\ln\Big(\frac{L^{j,k}_{n,t+1} L^k_{n,t+1}}{L^j_{n,t+1} L^{k,k}_{n,t+1}}\Big)}_{\text{Option Value}}.\end{aligned} \tag{17}$$

Equation (17) serves as the key moment condition describing the relationship between labor flows and key labor supply parameters. The extent of labor adjustment is governed by the dispersion of non-pecuniary preferences ρ_L. When ρ_L is high, workers will reallocate less to the destination sector because economic incentives matter less for labor mobility decisions. Furthermore, the speed of adjustment depends on the bilateral mobility costs. In particular, higher bilateral mobility costs ($\kappa_n^{j,k}$) decrease the magnitude of labor flows from the origin to the destination and, hence, make the labor adjustment in response to a sector-level policy to occur slowly over time.

3.7 Governments

Since this paper does not aim to solve for optimal climate policies, I assume that governments do not make endogenous decisions to maximize domestic social welfare. Instead, I model three types of pre-implemented sector-specific policy instruments that reflect climate policies used in practice: (i) sector-specific taxes/subsidies ($\{\tau_{n,t}^j\}$), (ii) fuel-specific taxes ($\{\tau_{n,t}^{f,h}\}$), and (iii) trade taxes ($\{e_{nm,t}^j\}$). I assume the government maintains a balanced budget each period through a lump-sum transfer $\Omega_{n,t}$ to the representative capitalist conditional on the set of policy instruments $\{\tau_{n,t}^{f,h}, \tau_{n,t}^j, e_{nm,t}^j, \}$:

$$
\begin{aligned}
\Omega_{n,t} = & \underbrace{\sum_{j \in \mathcal{J}_E} \tau_{n,t}^j P_{n,t}^j Y_{n,t}^j}_{\text{Consumption Tax Revenue}} + \underbrace{\sum_{h \in \{o,g,c\}} \tau_{n,t}^{f,h} s_{n,t}^h f_{n,t}^h}_{\text{Fuel Tax Revenue}} \\
& + \underbrace{\sum_{j \in \mathcal{J}} \sum_{m \in \mathcal{N}} \left(\frac{e_{nm,t}^j - 1}{e_{nm,t}^j} \right) p_{nm,t}^j y_{nm,t}^j}_{\text{Import Tariff Revenue}} - \underbrace{\sum_{j \in \mathcal{J}} \sum_{m \in \mathcal{N}} \left(\frac{e_{mn,t}^j - 1}{e_{mn,t}^j} \right) p_{mn,t}^j y_{mn,t}^j}_{\text{Foreign Tariff Expenditure}} \\
& - \underbrace{w_{n,t}^{NE} L_{n,t}^{\mathrm{NE}}}_{\text{Non-employment Expenditure}}, \quad \forall t.
\end{aligned} \tag{18}
$$

As shown in Equation (18), the lump-sum government transfer increases with tax revenues from consumption taxes and import tariffs and decreases with import tariffs imposed by foreign governments as well as expenditure towards workers in the non-employment sector.

3.8 General Equilibrium

In equilibrium, all markets in every economy must clear at every period. The final goods market clears in economy n at time t if the quantity of final goods equals the sum of aggregate consumption, aggregate investment, and extraction costs:

$$Y_{n,t} = \sum_{j \in \mathcal{J}} c^j_{n,t} L^j_{n,t} + C^K_{n,t} + W_{n,t+1} - (1 - \delta^k_{n,t}) W_{n,t} + \sum_{h \in \{o,g,c\}} \varphi^h_n f^h_{n,t}. \tag{19}$$

Denote by $X_{n,t}$ the aggregate expenditure of final goods consumers which equals the aggregate value, i.e., $X_{n,t} = P_{n,t} Y_{n,t}$.

The sectoral goods market clears in sector $j \in \mathcal{J}$ of economy n at time t if the aggregate quantity of sectoral goods equals the sum of the demand by final goods consumers and variety producers for sector j goods:

$$Y^j_{n,t} = \begin{cases} C^j_{n,t} + \sum_{k \in \mathcal{J}} M^{k,j}_{n,t}, & j \in \mathcal{J}_N, \\ \alpha^j_n \Big(\frac{(1+\tau^j_{n,t}) P^j_{n,t}}{P^E_{n,t}} \Big)^{-\eta_1} C^E_{n,t} + \sum_{k \in \mathcal{J}} M^{k,j}_{n,t}, & j \in \mathcal{J}_E. \end{cases} \tag{20}$$

The market for varieties produced in sector j of economy n at time clears if the gross revenue equals the sum of demand from all economies:

$$\sum_{m \in \mathcal{N}} p^j_{mn,t} y^j_{mn,t} = \sum_{m \in \mathcal{N}} \left(\frac{p^j_{mn,t}}{P^j_{m,t}} \right)^{1-\sigma} P^j_{m,t} Y^j_{m,t}. \tag{21}$$

The markets for labor, capital, and fossil fuel clear in sector j of economy n at time t if the aggregate factor income equals the total expenditure of variety producers on given inputs:

$$w^j_{n,t} L^j_{n,t} = \left(\psi^{j,V}_n \psi^{j,L}_n \right) \sum_{m \in \mathcal{N}} p^j_{mn,t} y^j_{mn,t}, \tag{22}$$

$$r^j_{n,t} P_{n,t} K^j_{n,t} = \left(\psi^{j,V}_n \psi^{j,K}_n \right) \sum_{m \in \mathcal{N}} p^j_{mn,t} y^j_{mn,t}, \tag{23}$$

$$s_{n,t} F_{n,t} = \left(\psi^{J,V}_n \psi^{B,F}_n \right) \sum_{m \in \mathcal{N}} p^B_{mn,t} y^B_{mn,t}. \tag{24}$$

Finally, given the net supply of risk-free financial assets is zero, the domestic financial asset market at time t clears if the financial asset position decision of the representative capitalist equals zero:

$$A_{n,t+1} = 0, \quad \forall n \in \mathcal{N}, t \in \mathcal{T}. \tag{25}$$

Now, I define the sequential equilibrium of the model. For readability, detailed descriptions of the sets of parameters Θ, prices $\mathbb{P}$, and endogenous variables $\mathbb{Y}$ are relegated to Appendix A.8.

Definition 1. ***Sequential Equilibrium.***
Given an initial distribution of aggregate capital and labor allocations $\{W_{n,0}, L^j_{n,0}\}_{n\in\mathcal{N}, j\in\mathcal{J}}$*, fossil fuel reserves* $\{D^h_{n,0}\}_{h\in\{o,g,c\}, n\in\mathcal{N}}$*, and the set of parameters* Θ*, the sequential equilibrium of the model is characterized by a sequence of factor prices, interest rates, goods prices, and taxes* $\mathbb{P}$*, and the sequence of consumption, investment, saving, output, supply and demand for labor, capital, and fossil fuel, trade shares, and transfers* $\mathbb{Y}$*, that (i) maximize the profits of producers* (6)*,* (7)*, (ii) maximize the life-time utilities of capitalists* (11) *and workers* (14) *(iii) satisfy governments' balanced budget conditions* (18)*, and (iv) satisfy the conditions for bilateral trade shares* (2)*, output market clearing* (19)*,* (20)*,* (21)*, factor market clearing* (22)*,* (23)*,* (25) *as well as the evolution of capital and labor allocation across sectors, and the law of motion for the stock of global carbon dioxide.*

4 Quantification

The model is calibrated to a global economy consisting of three countries and two country aggregates: the United States (US), China, India, the European Union (EU), and the Rest of the World (ROW).[13] I employ the industrial classification from the World Input-Output Database (WIOD) 2016 Release (Timmer et al. (2015)) to group 56 industries into five sectors, excluding the non-employment sector: Agriculture (AG), Manufacturing (MN), Services (SR), Non-Brown Energy (NB), and Brown (B). Following O'Mahony and Timmer (2009), the non-brown energy sector includes the industry supplying electricity, and the brown sector includes the mining and quarrying industry as well as the industry manufacturing coke and refined petroleum products. See Appendix A for a detailed description of how economies and sectors are categorized.

I categorize the model parameters into three categories: (i) parameters that are imposed based on values previously used in the literature (summarized in Table 2), (ii) parameters estimated outside of the model (summarized in Table 3), and (iii) parameters estimated using the method of simulated moments (MSM) (summarized in Table 9). The parameters are calibrated using various datasets from the year 2022 or the most recent year available.

[13] The selection of these economies is motivated by the fact that the US, China, India, and the EU accounted for approximately 60% of global carbon emissions in 2022.

4.1 Imposed Parameters

Table 2: Summary of Imposed Parameters

Parameter	Value	Description	Source
β	0.98	Discount factor	Rennert et al.
σ	6.03	Trade elasticity	HM
δ_τ^C	See Text	Carbon Stock Depreciation	CR
$\mathcal{S}$	2200	Pre-industrial carbon stock (Gt)	GHKT

Notes: HM: Head and Mayer (2014), Rennert et al.: Rennert et al. (2022), CR: Cruz and Rossi-Hansberg (2023), GHKT: Golosov et al. (2014).

The imposed parameters are derived from established values in the literature. These parameters, detailed in Table 2, include the discount factor (β) set at 0.98, following Rennert et al. (2022), and the trade elasticity (σ) set at 6.03, following Head and Mayer (2014).[14]

Additionally, the carbon stock depreciation rate (δ_τ^C) is calibrated following the approach of Cruz and Rossi-Hansberg (2023). In particular, I parameterize δ_τ^C as follows: $\delta_\tau^C = 1 - a_0 - \sum_{s=1}^{3} a_s e^{-\tau/b_s}$ with $a_0 = 0.2173$, $a_1 = 0.2240$, $a_2 = 0.2824$, $a_3 = 0.2763$, $b_1 = 394.4$, $b_2 = 36.54$, and $b_3 = 4.304$. This approximation reflects the finding of Stocker et al. (2013) that more than 20% of carbon emissions will remain in the atmosphere for more than 1000 years. Following Golosov et al. (2014), the pre-industrial carbon stock ($\mathcal{S}$) is set to 2200 Gt of CO_2, with the baseline year for the measurement being 1850 ($T_{min} = 1850$).

4.2 Parameters Estimated Outside of the Model

In this section, I describe the estimation procedure and results for the key parameters of the model: labor mobility costs $\kappa_n^{j,k}$, the dispersion of non-pecuniary preferences ρ_L, the dispersion of capital-use efficiencies ρ_K, the dynamic agglomeration externality ρ_Φ, energy demand elasticity η_1, and fossil fuel elasticity η_2.The estimation procedure and results for the other parameters listed in Table 3 are described in Appendix A.1.

Labor Mobility Costs To estimate the inter-sectoral labor mobility costs $\kappa_n^{j,k}$, I merge longitudinal labor force surveys from the following eight countries: the United States (Current Population

[14] Head and Mayer (2014) study 744 estimates of trade elasticity from a sample of 32 papers. The median of estimates is -5.03, i.e., $1 - \sigma = -5.03$.

Table 3: Summary of Parameters Estimated Outside of the Model

Parameter	Description	Source
$\kappa_n^{j,k}$	Labor mobility costs	See Text
ρ_L	Dispersion of non-pecuniary preferences	See Text
ρ_K	Dispersion of capital-use efficiencies	WIOD
ρ_Φ	Dynamic agglomeration externality	WIOD
η_1	Energy demand elasticity	See Text
η_2	Fossil fuel elasticity	See Text
w_n^{NE}	Non-employment income	See Appendix
$\varrho_{nm,0}^j$	Exogenous trade costs	See Appendix
δ_n^K	Capital depreciation	PWT
φ_n^h	Supply cost of fossil fuel by type	IMF
ϕ^h	Carbon emission by fossil fuel	RR2020
E_L	Land-change emissions	RR2020
λ_T	Carbon-temperature conversion	MET
α_n	Non-energy expenditure share	WIOD
α_n^j	Non-energy expenditure share by sector	WIOD
$\gamma_n^{j,V}$	Value added share	WIOD
$\gamma_n^{j,k}$	Material input share	WIOD
$\psi_n^{j,L}$	Labor input share	WIOD
$\psi_n^{j,K}$	Capital input share	WIOD
$\psi_n^{j,E}$	Energy demand share	WIOD
$\psi_n^{B,F}$	Fossil fuel input share	WIOD

Notes: RR2020: Ritchie and Roser (2020), MET: Met Office Hadley Centre (2024), PWT: Penn World Table, WIOD: World Input-Output Database 2016 Release, WIOD 2013: World Input-Output Database 2013 Release, IMF: Black et al. (2023), MacMap: Guimbard et al. (2012).

Survey, 2000-2023), China (China Family Panel Studies, 2010-2020), India (Periodic Labour Force Survey, 2018-2020), France (Continuous Labour Force Survey, 2014-2020), Argentina (Permanent Household Survey, 2003-2019), Australia (The Household, Income and Labour Dynamics in Australia, 2002-2022), South Korea (Korean Labor & Income Panel Study, 1999-2022), and the United Kingdom (Labour Force Survey, 2016-2022). These datasets track the movement of individuals ($L_{n,t}^{j,k}$) across sectors and employment status over time and report their income as well as individual characteristics such as gender, age, and education. Details on the data construction procedure are provided in Appendix A.9.

Following Artuç and McLaren (2015), I estimate the following regression specification (see

Appendix A.7 for the derivation):

$$L_{n,t}^{j,k} = \exp\left(C_n^{j,k} + \lambda_{n,t}^k + \lambda_{n,t}^j\right) + \xi_{n,t}^{j,k}. \tag{26}$$

where $C_n^{j,k}$, $\lambda_{n,t}^j$, $\lambda_{n,t}^k$ are, respectively, economy-origin-destination fixed effects, economy-origin-year fixed effects, and economy-destination-year fixed effects defined as $C_n^{j,k} \equiv \frac{\ln(1-\kappa_n^{j,k})}{\rho_L}$, $\lambda_{n,t}^k \equiv \frac{\beta}{\rho_L} V_{n,t+1}^k$, and $\lambda_{n,t}^j \equiv \ln(L_{n,t}^j) - \ln\left(\sum_{h\in\mathcal{J}} \exp\left(\frac{\beta}{\rho_L} V_{n,t+1}^h + \frac{1}{\rho_L}\ln(1-\kappa_n^{j,h})\right)\right)$, and $\xi_{n,t}^{j,k}$ is the residual term.

Given data on the labor flows across sectors over time ($L_{n,t}^{j,k}$), I use the Poisson Pseudo Maximum Likelihood (PPML) method of Silva and Tenreyro (2010) to estimate $C_n^{j,k}$ using the regression specification (26). For estimates of $C_n^{j,k}$ which are not statistically significant at the 5% level, I re-estimate these mobility costs using the following specification (see Appendix A.7 for the derivation):

$$\frac{L_{n,t}^{j,k} L_{n,t}^{k,j}}{L_{n,t}^{j,j} L_{n,t}^{k,k}} = \exp(\tilde{C}_n^{j,k}) + \tilde{\xi}_{n,t}^{j,k} \tag{27}$$

where $\tilde{C}_n^{j,k}$ are economy-origin-destination fixed effects defined as $\tilde{C}_n^{j,k} \equiv \frac{1}{\rho_L}\ln(1-\kappa_{n,t}^{j,k}) + \frac{1}{\rho_L}\ln(1-\kappa_{n,t}^{k,j})$, and $\tilde{\xi}_{n,t}^{j,k}$ is a residual term. The regression specification (27) cannot separately identify $\frac{1}{\rho_L}\ln(1-\kappa_{n,t}^{j,k})$ and $\frac{1}{\rho_L}\ln(1-\kappa_{n,t}^{k,j})$. Hence, for the estimates re-estimated using the second method, I assume that bilateral mobility costs are symmetric, i.e., $\kappa_n^{j,k} = \kappa_n^{k,j}$. Refer to the Appendix A for the estimates of $\frac{\ln(1-\kappa_n^{j,k})}{\rho_L}$. I discuss the estimates of $\kappa_n^{j,k}$ in the following section, after detailing the estimation procedure for ρ_L.

Dispersion of Non-pecuniary Preferences To estimate the dispersion of non-pecuniary preferences ρ_L, I use the following specification (see Appendix A.7 for the derivation):

$$y_{n,t}^{j,k} = \omega_n^k + \omega_n^j + \left(\frac{\beta}{\rho_L}\right)\ln\left(\frac{w_{n,t+1}^k}{w_{n,t+1}^j}\right) + \eta_{n,t+1}^{j,k} \tag{28}$$

where $y_{n,t}^{j,k} \equiv \ln\left(\frac{L_{n,t}^{j,k}}{L_{n,t}^{j,j}}\right) - (1-\beta)C_n^{j,k} - \beta\ln\left(\frac{L_{n,t+1}^k L_{n,t+1}^{j,k}}{L_{n,t+1}^j L_{n,t+1}^{k,k}}\right)$; ω_n^j and ω_n^k are, respectively, economy-origin and economy-destination fixed effects defined as $\omega_n^j \equiv -\beta\ln(\mu_n^j)$ and $\omega_n^k \equiv$

$\beta \ln(\mu_n^k)$; $\ln(\frac{w_{n,t+1}^k}{w_{n,t+1}^j})$ captures the (log) residualized wage of the destination sector k relative to the residualized wage of the origin sector j at time $t+1$; and, $\eta_{n,t+1}^{j,k}$ is a forecast error. For the estimation, I assume the discount factor β equals 0.98.

To address the potential endogeneity concern that forecast errors may be correlated with the regressors, I estimate the regression specification (28) using an Instrumental Variable Generalized Method of Moments (IV-GMM) approach. Two instruments are employed: (i) two-period lagged residualized wages $\ln(\frac{w_{n,t-1}^k}{w_{n,t-1}^j})$, as suggested by Artuç et al. (2010), and (ii) sector-level exposure to economy-level environmental policies, $\text{EPS}_{n,t+1}^j$.

The sector-level exposure to environmental policies, $\text{EPS}_{n,t+1}^j$, is calculated as the product of the country-level Environmental Policy Stringency (EPS) index, constructed by the OECD (Kruse et al. (2022)), and the country-sector-level carbon emissions per output in the year 2000, based on data from the World Input-Output Database Environmental Accounts (Corsatea et al. (2019)). As discussed in Section 2, the EPS index, available for 40 countries from 1990 to 2023, evaluates the stringency of 13 policy instruments—including market-based policies, non-market-based policies, and technology support policies—on a scale from 0 to 6, with higher scores indicating greater stringency. These scores are averaged to form the EPS index. To address the endogeneity concern that sector-level carbon emissions per output might respond to the EPS index over time, I use sector-level carbon emissions per output from the year 2000. The parameter ρ_L is then identified under the assumption that the environmental policy $\text{EPS}_{n,t+1}^j$ influences future labor flows solely by affecting producers' demand for labor.

The results are presented in Table 4. Column 3 provides the preferred IV estimate for $\frac{\beta}{\rho_L}$, yielding $\rho_L = 1.791$. This estimate suggests that a 1 percent increase in the next-period wage of the destination sector relative to the next-period wage of the origin sector increases labor flows to the destination sector by 0.547 percent. Hence, labor flows are not highly responsive to changes in future wage differentials and the dispersion of non-pecuniary motives for worker mobility is significant.

The labor mobility costs $\kappa_n^{j,k}$ can be determined using the estimates of ρ_L and $\frac{\ln(1-\kappa_n^{j,k})}{\rho_L}$. Table 5 shows the average costs faced by workers to move out of the brown sector. For workers in the brown sector in all economies, the cost of moving out of the brown sector is equivalent to sacrificing at least 97.2% of current real wage, i.e., $\kappa_n^{B,k} \geq 0.972$ for all $n \in \mathcal{N}$ and $k \in \mathcal{J} \setminus \{B\}$. This indicates that labor reallocation in response to the increase in carbon taxes, which decreases

Table 4: Estimation Result for Dispersion of Non-pecuniary Preferences $\left(\frac{\beta}{\rho_L}\right)$ with $\beta = 0.98$

Dependent: $y_{n,t}^{j,k}$	(1) OLS	(2) IV: $\ln\left(w_{n,t-1}^{k}/w_{n,t-1}^{j}\right)$	(3) IV: $\text{EPS}_{n,t+1}^{j}$
$\ln\left(\frac{w_{n,t+1}^{k}}{w_{n,t+1}^{j}}\right)$	0.270***	0.424***	0.547*
	(0.048)	(0.068)	(0.323)
Country-Origin FE	YES	YES	YES
Country-Destination FE	YES	YES	YES
Observations	3,466	2,906	2,356
First Stage F-Statistic		111.93	59.42
R-squared	0.067		

Notes: $y_{n,t}^{j,k}$ denotes the (log) difference of labor flows adjusted for mobility costs and option values. $\ln\left(w_{n,t+1}^{k}/w_{n,t+1}^{j}\right)$ denotes the (log) residualized wage ratio between sectors k and j at time $t+1$. Sources: Panel labor force surveys of the US, China, India, France, Argentina, Australia, South Korea, and the UK. Column 2 uses two-period lagged wages as an instrument. Column 3 uses the sector-level exposure to environmental policies as an instrument. Clustered standard errors (country-origin-destination) in parentheses. *** p < 0.01, ** p < 0.05, * p < 0.1

the demand for the brown sector, will be minimal and slow.

Table 5: Average Cost of Moving Out of the Brown Sector

Economy	Average Mobility Cost (share of real wage)
United States	0.995
China	0.997
India	0.999
European Union	0.999
Rest of the World	0.993

Notes: Average mobility costs faced by workers to move out of the brown sector, displayed as a share of the real wage in the brown sector. Sources: Panel labor force surveys of the US, China, India, France, Argentina, Australia, South Korea, and the UK.

Capital Supply Elasticities I use the following specification to estimate the capital supply elasticities ρ_K and ρ_Φ (see Appendix A.7 for the derivation):

$$\ln\left(\frac{K^j_{n,t}}{K^o_{n,t}}\right) = \tilde{\omega}^j_n + \left(\rho_K - 1\right)\ln\left(\frac{r^j_{n,t}}{r^o_{n,t}}\right) + \left(\rho_K \cdot \rho_\Phi\right)\ln\left(\frac{K^j_{n,t-1}}{K^o_{n,t-1}}\right) + \tilde{\eta}^j_{n,t} \tag{29}$$

where $\tilde{\omega}^j_n$ are economy-sector fixed effects defined as $\tilde{\omega}^j_n \equiv \rho_K \ln(a^j_n/a^o_n)$ for some base sector o, and $\tilde{\eta}^j_{n,t}$ is the residual term. To run the regression, I obtain data on sector-level capital stock ($K^j_{n,t}$) and capital compensation ($r^j_{n,t}P_{n,t}K^j_{n,t}$) from the Socio-Economic Accounts (SEA) of the WIOD 2016 Release for 56 sectors in 43 countries from years 2000 to 2014. Rental rates of capital ($r^j_{n,t}$) are computed by dividing sector-level capital compensation by sector-level capital stocks. I consider the petroleum and coke manufacturing (P) sector as the base sector for the estimation.

The specification in Equation (29) may suffer from simultaneity bias since capital demand and capital supply jointly determine the equilibrium capital allocation and the rental rate. To address this issue, I estimate the regression using two instruments: (i) lagged wages $\ln(\frac{w^j_{n,t-1}}{w^o_{n,t-1}})$ and (ii) sector-level exposure to environmental policies $EPS^j_{n,t}$. The coefficient $(\rho_K - 1)$ is identified under the assumption that lagged wages correlate with the persistent sectoral productivity process, which influences producers' demand for capital but does not directly affect the current supply of capital. Additionally, sectoral exposure to environmental policies $EPS^j_{n,t}$—discussed in the estimation of the parameter governing the dispersion of non-pecuniary preferences—is assumed to impact the capitalists' supply decisions only through its effect on producers' demand for capital.

The results are presented in Table 6. Column 3 provides the preferred IV estimates for $\rho_K - 1$ and $\rho_K \cdot \rho_\Phi$, yielding $\rho_K = 1.021$ and $\rho_\Phi = 0.981$. This result indicates that a 1 percent increase in the rate of return for a given sector relative to the petroleum and coke manufacturing sector leads to a 0.021 percent increase in the allocation of capital to that sector, implying that capital assets are highly specialized and more useful in some sectors than others. Moreover, the estimate for ρ_Φ suggests that a 1 percent increase in the past capital allocation to a given sector relative to the petroleum and coke manufacturing sector is associated with a 1.001 percent increase in the current capital allocation. This implies that the evolution of sector-specific capital-use efficiency is highly path dependent.

Table 6: Estimation Result for Capital Supply Elasticities ($\rho_K - 1$, $\rho_K \cdot \rho_\Phi$)

Dependent: $\ln\left(\frac{K^j_{n,t}}{K^P_{n,t}}\right)$	(1) OLS	(2) IV: $\ln(w^j_{n,t-1}/w^P_{n,t-1})$	(3) IV: $\text{EPS}^j_{n,t}$
$\ln\left(\frac{r^j_{n,t}}{r^P_{n,t}}\right)$	-0.002**	0.123***	0.0209*
	(0.0011)	(0.0435)	(0.0113)
$\ln\left(\frac{K^j_{n,t-1}}{K^P_{n,t-1}}\right)$	0.992***	1.032***	1.001***
	(0.0008)	(0.0134)	(0.0041)
Country-Sector FE	YES	YES	YES
Observations	24,183	22,740	20,616
First Stage F-Statistic		50.10	56.84
R-squared	0.987		

Notes: $\ln(K^j_{n,t}/K^P_{n,t})$ denotes the (log) ratio of sector-level capital stocks relative to the petroleum and coke manufacturing sector (P). $\ln(r^j_{n,t}/r^P_{n,t})$ denotes the (log) ratio of rental rates of capital relative to the petroleum and coke manufacturing sector (P). Source: WIOD 2016 Release. Clustered standard errors (country-sector) in parentheses. *** p < 0.01, ** p < 0.05, * p < 0.1

Energy Demand Elasticity I estimate the energy demand elasticity using the following specification (see Appendix A.7.3 for the derivation):

$$\ln\left(\frac{Y^{NB}_{n,t}}{Y^{NB}_{n,t-1}}\right) - \ln\left(\frac{Y^{B}_{n,t}}{Y^{B}_{n,t-1}}\right) = -\eta_1\left(\ln\left(\frac{P^{NB}_{n,t}}{P^{NB}_{n,t-1}}\right) - \ln\left(\frac{P^{B}_{n,t}}{P^{B}_{n,t-1}}\right)\right) + \delta_t + \iota_{n,t} \tag{30}$$

where δ_t is the year fixed effects, and $\iota_{n,t}$ is the residual term.[15] In words, I regress the difference in output growth between the brown and non-brown energy sectors on the difference in price growth between these two sectors.

To estimate the specification (30), I use the Socio-Economic Accounts (SEA) of the WIOD 2016 release, which provides sector-level output and output price indices normalized to 2010 levels. To address the potential endogeneity issue where prices may be correlated with the demand shifters in the residuals, I use the one-period lagged (log) difference between the growth rates of rental rates

[15] In particular, $\delta_t \equiv \frac{\eta_1}{|\mathcal{N}|}\sum_{m\in\mathcal{N}} \ln\left(\frac{(1+\tau^{NB}_{m,t})(1+\tau^{B}_{m,t-1})}{(1+\tau^{B}_{m,s})(1+\tau^{NB}_{m,t-1})}\right)$ and $\iota_{n,t} \equiv -\eta_1 \ln\left(\frac{(1+\tau^{NB}_{m,t})(1+\tau^{B}_{m,t-1})}{(1+\tau^{B}_{m,s})(1+\tau^{NB}_{m,t-1})}\right) - \frac{\eta_1}{|\mathcal{N}|}\sum_{m\in\mathcal{N}} \ln\left(\frac{(1+\tau^{NB}_{m,t})(1+\tau^{B}_{m,t-1})}{(1+\tau^{B}_{m,s})(1+\tau^{NB}_{m,t-1})}\right)$.

Table 7: Estimation Result for Energy Demand Elasticity ($-\eta_1$)

Dependent: $\ln\left(\frac{Y^{NB}_{n,t}}{Y^{NB}_{n,t-1}}\right) - \ln\left(\frac{Y^{B}_{n,t}}{Y^{B}_{n,t-1}}\right)$	(1) OLS	(2) IV: $\ln\left(\frac{r^{NB}_{n,t-1}}{r^{NB}_{n,t-2}}\right) - \ln\left(\frac{r^{B}_{n,t-1}}{r^{B}_{n,t-2}}\right)$
$\ln\left(\frac{P^{NB}_{n,t}}{P^{NB}_{n,t-1}}\right) - \ln\left(\frac{P^{B}_{n,t}}{P^{B}_{n,t-1}}\right)$	-0.37**	-2.28***
	(0.19)	(0.77)
Year FE	YES	YES
Observations	630	574
First Stage F-Statistic		9.89
R-squared	0.262	

Notes: The dependent variable is the one-period lagged (log) difference in the growth rates of non-brown energy sector output and brown sector output. Column 2 uses the difference in the rental rates of capital as the instrument. Robust standard errors are in parentheses. *** p < 0.01, ** p < 0.05, * p < 0.1

of capital in the non-brown energy sector and the brown sector, $\ln\left(\frac{r^{NB}_{n,t-1}}{r^{NB}_{n,t-2}}\right) - \ln\left(\frac{r^{B}_{n,t-1}}{r^{B}_{n,t-2}}\right)$, as an instrument. The parameter η_1 is identified under the assumption that variations in growth rates of rental rates of capital influence the relative output of the energy sectors only by affecting the production costs across these sectors and not the demand for energy. The results are presented in Table 7. Column 2 displays the preferred estimate for $-\eta_1$, yielding $\eta_1 = 2.28$, which suggests that brown and non-brown energy goods are substitutes.

Fossil Fuel Elasticity I estimate the elasticity of substitution across fossil fuels using the following specification (see Appendix A.7.4 for the derivation):

$$\ln\left(\frac{f^k_{n,t}}{f^h_{n,t}}\right) = -\eta_2 \ln\left(\frac{s^k_{n,t}}{s^h_{n,t}}\right) - \eta_2 \ln\left(\frac{1+\tau^k_{n,t}}{1+\tau^h_{n,t}}\right) + \delta^k_n + \tilde{\iota}^k_{n,t} \tag{31}$$

where δ^k_n are country-fuel fixed effects defined as $\delta^k_n \equiv \ln\left(\frac{\omega^k_n}{\omega^h_n}\right)$ for some base fuel type h; and, $\tilde{\iota}^k_{n,t}$ is an error term. In words, I regress the relative fossil fuel production on the relative supply costs and fuel-specific taxes. For the estimation, I consider crude oil (o) as the base fuel type.

To estimate the specification (31), I use the IMF Fossil Fuel Subsidies Data (Black et al. (2023)) which provides information on supply costs ($s^h_{n,t}$) and retail prices ($(1+\tau^h_{n,t})s^h_{n,t}$) for 168 countries

Table 8: Estimation Result for Fossil Fuel Elasticity ($-\eta_2$)

Dependent: $\ln\left(\frac{f^k_{n,t}}{f^o_{n,t}}\right)$	(1)	(2)
$\ln\left(\frac{s^k_{n,t}}{s^o_{n,t}}\right)$	-2.56***	-2.67***
	(0.41)	(0.31)
Country-Fuel FE	YES	YES
Year FE	NO	YES
Observations	1,219	1,219
R-squared	0.985	0.985

Notes: The dependent variable is the (log) difference in the production of fuel type h relative to the production of crude oil (o). Clustered standard errors (country-fuel) are in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

from 2015 to 2019. The extraction costs ($\{s^h_{n,t}\}$) for oil, natural gas, and coal are set equal to the supply costs of gasoline (bil. USD/bil. m^3), natural gas (bil. USD/bil. m^3), and coal (bil. USD/Gt). The fuel-specific taxes ($\{1 + \tau^h_{n,t}\}$) are computed by dividing the retail prices by the supply costs. Production levels of crude oil, natural gas, and coal ($\{f^h_{n,t}\}$) are sourced from Ritchie and Rosado (2017) for 169 countries from 2015 to 2019, with units of production for crude oil, natural gas, and coal provided in billion m^3, billion m^3, and billion tons, respectively.

The results are presented in Table 8. Column 1 shows the baseline estimate for $-\eta_2$, yielding $\eta_2 = 2.56$. Column 2 presents the preferred estimate for $-\eta_2$ when I additionally control for year fixed effects. The estimate is 2.67, showing minimal change from the estimate in Column 1. These estimates suggest that fossil fuel inputs are substitutes.

4.3 Parameters Estimated by Method of Simulated Moments

The parameters listed in Table 9 are estimated using the Method of Simulated Moments (MSM), which is particularly suitable for estimating model parameters where traditional estimation techniques may be infeasible. This method involves simulating the full dynamic general equilibrium model and matching model-generated moments with moments observed in the data. The MSM procedure involves the following steps: First, I specify the moments from the data that the model should replicate, such as sectoral output, sectoral factor prices, trade shares, fossil fuel production, and energy consumption. Next, I simulate the model using initial parameter guesses and compute

Table 9: Summary of Parameters Estimated by Method of Simulated Moments

Parameter	Description	Targeted Moments
a_n^j	Capital-use efficiency	Real interest, capital allocation, capital compensation
μ_n^j	Non-pecuniary benefits	Sector-level wages
$z_{n,0}^j$	Sector-specific TFP	Bilateral trade shares, sector-level gross output
ω_n^h	Weights on Fossil Fuels	Fossil fuel production by type
α_n^{NB}	Weight on Non-Brown Energy	Expenditure on the Non-Brown Energy
ξ_n	Environmental Damage	GDP response to 1°C increase in global temperature

Notes: Parameters are estimated by matching model moments to observed data moments using the Method of Simulated Moments.

the corresponding model moments. The parameters are then adjusted iteratively to minimize the distance between the simulated model moments and the observed data moments. The parameters are jointly estimated conditional on the parameters imposed or estimated outside of the model. Now, I discuss the targeted moments for the parameters estimated using the MSM. For a detailed description of the MSM procedure, refer to the Appendix A.11.

Capital-use Efficiencies and Non-pecuniary Benefits Capital-use efficiencies (a_n^j) target real interest rates, capital allocation shares, and economy-level capital compensation in the year 2014. Real interest rates are sourced from International Financial Statistics (IFS), and the capital allocation shares and economy-level capital compensation are calculated using the WIOD 2016 Release. Non-pecuniary benefits (μ_n^j) are matched to sector-level wages in the year 2014 computed using the WIOD 2016 Release.

Productivity Sector-specific total factor productivity ($z_{n,0}^j$) is estimated by targeting bilateral trade shares and sector-level output in 2014, which is the most recent year available in the WIOD 2016 Release. Bilateral trade shares pin down relative sector-level productivities across economies for a given sector. To pin down the relative productivities across sectors within each economy, I exploit the positive and monotonic relationship between sector-level output and sector-level productivity. In the estimation, I assume that relative productivity levels across sectors and economies remain constant over time, although I do not assume the global economy is initially in steady state.

Weights on Fossil Fuels and Non-Brown Energy Weights on fossil fuels (ω_n^h) target the production of fossil fuels at the economy level by fuel type sourced from Ritchie and Rosado (2017). The relative fossil fuel demand is given by: $\frac{\omega_n^h}{\omega_n^k} = \left(\frac{\varphi_n^h}{\varphi_n^k}\right)^{\eta_2}\left(\frac{f_{n,t}^h}{f_{n,t}^k}\right)$ which shows that weights are a function of fossil fuel production ($f_{n,t}^k$,$f_{n,t}^h$), elasticity η_2, and the efficiencies (φ_n^k, φ_n^h). Similarly, the energy demand can be re-arranged as follows: $\frac{\alpha_n^{NB}}{\alpha_n^B} = \left(\frac{(1+\tau_{n,t}^{NB})P_{n,t}^{NB}}{(1+\tau_{n,t}^{B})P_{n,t}^{B}}\right)^{\eta_1-1}\left(\frac{P_{n,t}^{NB}Y_{n,t}^{NB}}{P_{n,t}^{NB}Y_{n,t}^{B}}\right)$. Th equation implies that the weight on the non-brown energy sector (α_n^{NB}) can be identified by the share of final energy expenditure spent on the non-brown energy relative to the brown sector calculated using the WIOD 2016 Release.

Environmental Damage The environmental damage parameter (ξ_n) is estimated to match the impulse response of real GDP per capital of an economy to a 1 °C increase in global temperature, as estimated by Bilal and Känzig (2024). Specifically, I simulate an unexpected increase in exogenous carbon emissions E_0^X in year 2023, which results in an increase in global temperature by 1 °C in 2023. The shock can be computed as follows: $E_0^X = \mathcal{S} \cdot 2^{\frac{\mathbb{T}(\mathcal{C}_0)}{\lambda_T}}\left(2^{\frac{1}{\lambda_T}} - 1\right)$. I then match the drop in real GDP per capita of each economy, relative to the baseline simulation without the shock, for each economy after 5 years with the estimates of Bilal and Känzig (2024).

4.4 Model Fit

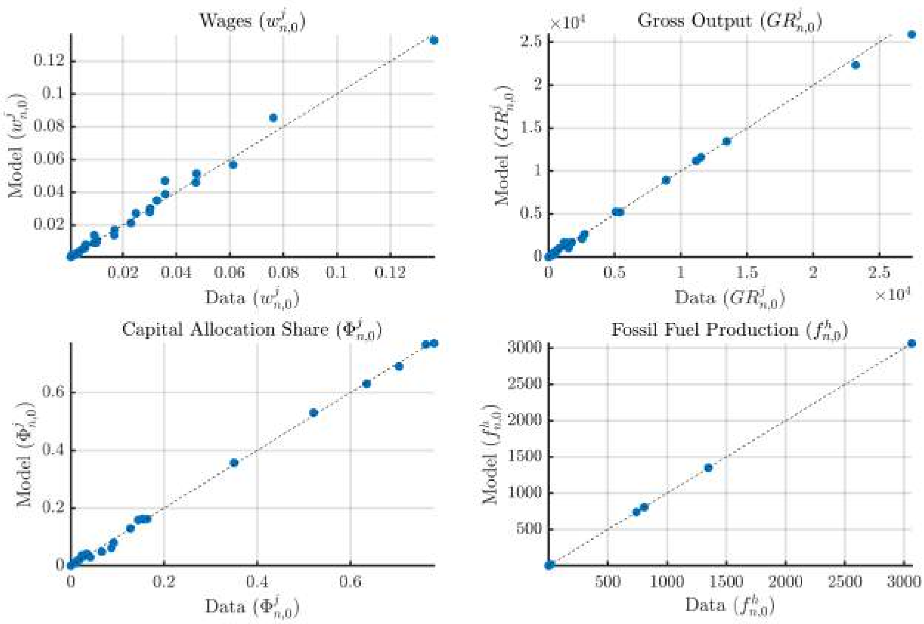
Figures 3 and 4 compare targeted data moments with the model-generated moments. Overall, the calibrated model provides a good fit for the data. The estimates of the parameters calibrated using the MSM procedure are presented in Appendix A.10.

4.5 Initial Conditions

The initial conditions of the model are computed for the year 2022 or the most recent year available. The initial distribution of wealth ($W_{n,0}$) and labor allocations ($L_{n,0}^j$) across sectors and economies are derived from the Socio-Economic Accounts (SEA) of the World Input-Output Database (WIOD) 2016 Release. I assume the labor force ($L_{n,t}$) in each economy remains constant over time.

The global carbon stock ($\mathcal{C}_0$) in the year 2022 is computed based on pre-industrial stock ($\mathcal{S}$), global carbon emissions from years 1850 to 2022 sourced from Ritchie and Roser (2020). The initial environmental quality ($\mathcal{D}_{n,0}$) for each economy is determined by the initial carbon stock and

Figure 3: Model Fit: Model vs. Data Moments



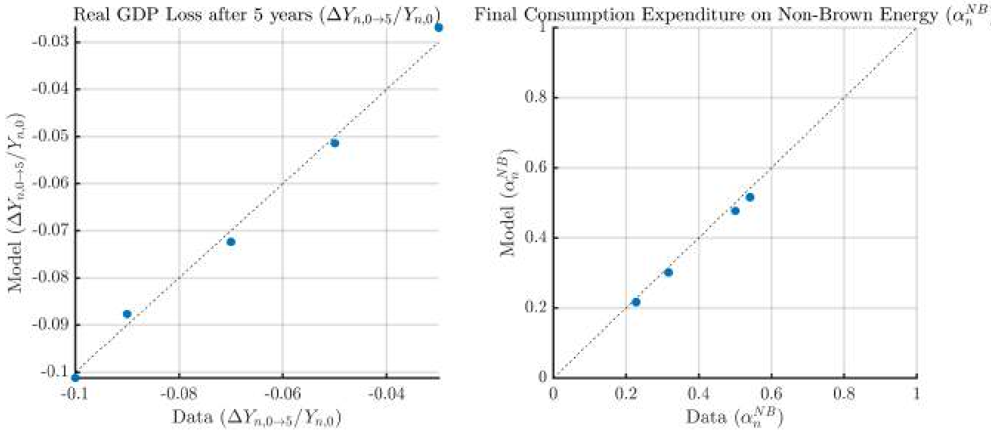
Notes: Comparison between the model-generated moments and the observed data moments for wages, gross output, capital allocation share, and fossil fuel production.

the estimated environmental damage parameter (ξ_n). Given the assumption that only domestic financial asset markets exist in each economy, the initial financial asset position of the representative capitalist in each economy is set to zero, i.e., $A_{n,0} = 0$ for all $n \in \mathcal{N}$.

Fossil fuel reserves ($D^h_{n,0}$) are estimated based on Ritchie and Rosado (2017), with units of reserves for crude oil, natural gas, and coal provided in billion m^3, billion m^3, and billion tons, respectively, for the year 2019. Using the same dataset, I compute the average annual discovery from 2010 to 2019 and calculate the reserves assuming new reserves will continue to be discovered at this average annual rate for the next 500 years. This assumption is primarily made to focus on the impacts of environmental policies rather than resource exhaustion; this assumption amplifies the environmental benefits of environmental policies, as larger reserves imply that emissions will continue unless the global economy achieves a clean transition. Estimates are presented in Appendix A.

In the baseline calibration, sector-specific consumption taxes ($\tau^j_{n,0}$) are set to zero for two main

Figure 4: Model Fit: Model vs. Data Moments



Notes: Comparison between the model-generated moments and the observed data moments for real GDP loss after 5 years in response to a 1 °C increase in the global temperature and final consumption expenditure on the non-brown energy sector.

reasons. First, countries primarily report value-added taxes (VAT) or sales taxes, which are applied uniformly across all or most products. Therefore, incorporating these taxes into the model would not meaningfully alter the distribution of economic activities across sectors. Second, taxes introduce distortions in the model. Removing uniform consumption taxes and redistributing through government transfers would enhance welfare by reducing these distortions, which is not the primary focus of this paper.

Fuel-specific tax rates are estimated based on the measures of supply costs of fossil fuels (φ_n^h) and retail prices of fossil fuels sourced from IMF Fossil Fuel Subsidies Data (Black et al. (2023)) for the year 2022. Fuel-specific tax rates are computed as the difference between the retail price after deducting value-added tax (VAT) of fossil fuels (gasoline, natural gas, coal) and the supply costs of fossil fuels.

Trade taxes ($e_{nm,0}^j$) are estimated based on the MAcMap-HS6 dataset (Guimbard et al. (2012)) for the year 2019. The MAcMap-HS6 (Market Access Map HS6) provides a detailed, comprehensive, and bilateral measurement of applied tariff duties for goods, considering regional agreements and trade preferences. Sector-specific trade taxes are computed as the average of the tariffs on products within each sector. See Appendix A for detailed estimates.

5 Quantitative Analysis

In this section, I present the key quantitative findings from the calibrated model. First, I compare how China's unilateral carbon tax affects emissions across models with varying levels of factor mobility. Second, I examine the impact of China's unilateral subsidy to the non-brown energy sector on emissions, again considering different degrees of factor mobility. Third, I evaluate the separate effects of the carbon tax, the non-brown energy subsidy, and their combination on emissions and consumption. Fourth, I calculate the global carbon tax required to limit warming to 1.5°C above pre-industrial levels by 2100 and assess how this changes when I relax the assumption of imperfect mobility. Finally, I show that large temporary subsidies to the non-brown energy sector result in a lasting shift of capital into the non-brown energy sector.

Counterfactual Models To understand the influence of factor mobility, I consider three counterfactual models in addition to the baseline: (i) a model with high labor responsiveness, (ii) a model with high capital responsiveness, and (iii) a model combining both modifications. For each counterfactual model, I re-calibrate the parameters estimated using the MSM to match the targeted moments and initial conditions presented in Section 4.3.

First, to assess the impact of labor mobility, I modify the baseline model by significantly reducing the dispersion of non-pecuniary preferences ρ_L by a factor of five, from 1.79 to 0.358. This reduction weakens the influence of non-pecuniary motives for worker mobility and increases the extent of labor adjustment by making labor flows more responsive to wage changes, as shown in the labor flow condition:

$$\underbrace{\ln\left(\frac{L_{n,t}^{j,k}}{L_{n,t}^{j,j}}\right)}_{\text{Labor Flows}} = \underbrace{\left(\frac{\beta}{\rho_L}\right)}_{\text{Responsiveness}} \cdot \underbrace{\ln\left(\frac{w_{n,t+1}^{k}}{w_{n,t+1}^{j}}\right)}_{\text{Future Wage Differential}} + \underbrace{\left(\frac{1-\beta}{\rho_L}\right)\ln(1-\kappa_n^{j,k})}_{\text{Bilateral Mobility Cost}}$$
$$+ \underbrace{\beta\ln\left(\frac{\mu_n^k}{\mu_n^j}\right)}_{\text{Non-pecuniary Preferences}} + \underbrace{\beta\ln\left(\frac{L_{n,t+1}^{j,k}L_{n,t+1}^{k}}{L_{n,t+1}^{j}L_{n,t+1}^{k,k}}\right)}_{\text{Option Value}}.$$

Second, to evaluate the impact of capital mobility, I consider a counterfactual model that significantly reduces differences in capital-use efficiencies across sectors by increasing ρ_K five-fold, from $\rho_K = 1.02$ to 5.10. The reduction of the dispersion of capital-use efficiencies increases the extent of capital adjustment as more capital assets preserve their efficiencies when re-allocated to other sectors, as shown in the capital allocation condition:

$$\underbrace{\ln\left(\frac{K^j_{n,t}}{W_{n,t}}\right)}_{\text{Capital Allocation}} = \underbrace{(\rho_K - 1)}_{\text{Responsiveness}} \underbrace{\ln\left(\frac{P_{n,t} r^j_{n,t}}{v_{n,t}}\right)}_{\text{Rental Rate}} + \underbrace{(\rho_K \cdot \rho_\Phi)}_{\text{Dynamic Agglomeration Externality}} \ln\left(\frac{K^j_{n,t-1}}{W_{n,t-1}}\right)$$
$$+ \rho_K \underbrace{\ln(a^j_n)}_{\text{Sector-specific Efficiency}} + \rho_K \ln\left(\Gamma\left(\frac{\rho_K - 1}{\rho_K}\right)\right).$$

Note that the increase in ρ_K not only increases the responsiveness but also affects the path dependence of capital allocation, governed by $\rho_K \cdot \rho_\Phi$. Hence, to match the persistence of capital allocation observed in the data, I also reduce the dynamic agglomeration externality parameter ρ_Φ by a factor of five so that $\rho_K \cdot \rho_\Phi$ remains unchanged.

Finally, I combine the modifications from the first two models to create a third counterfactual model, which simultaneously increases the responsiveness of capital and labor supply by reducing the dispersion of capital-use efficiencies and the dispersion of non-pecuniary preferences.

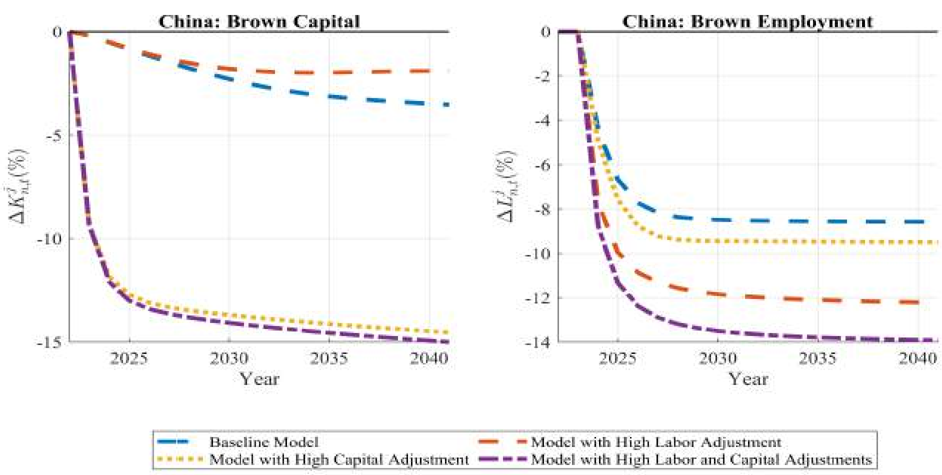
5.1 Impact of China's Carbon Tax

I now discuss the change in global emissions under a policy scenario where China— which accounts for 30.7 percent of global emissions as of year 2022— permanently increases the carbon tax by 13 percentage points, which is equivalent to removing fossil fuel subsidies implemented by China in 2022 (see Black et al. (2023) and Appendix A.6.8 for details). This scenario is compared to the business-as-usual (BAU) scenario where this policy is not implemented.

Capital and Employment in the Brown Sector The impact of China's policy on capital and employment in the brown sector across models is illustrated in Figure 6. The figure demonstrates that factor mobility significantly influences how much the carbon tax reduces capital and employment in the brown sector. In the baseline model, in response to the carbon tax, capital in the brown sector is 0.21 percent lower than the business-as-usual (BAU) level in 2023. In contrast, in the model with high capital adjustment, capital in the brown sector is 9.37 percent below the BAU level in 2023. This reduction becomes more pronounced over time: the difference in capital between the two models increases from 9.16 percentage points of the BAU level in 2023 to 10.99 percentage points by 2040.

Making workers more responsive to wage changes leads to a greater reduction in employment in China's brown sector. For all models, the carbon tax does not affect employment in 2023, as workers cannot move in the short run. In the baseline model, employment in the brown sector is 4.40 percent lower than the business-as-usual (BAU) level in 2024. In contrast, in the model with high labor adjustment, employment in the brown sector is 7.72 percent below the BAU level in 2024. This reduction becomes more pronounced over time: the difference in employment between the two models grows from 3.32 percentage points of the BAU level in 2024 to 3.63 percentage points by 2040.

Figure 5: Impact of China's Carbon Tax on Brown Capital and Employment in China

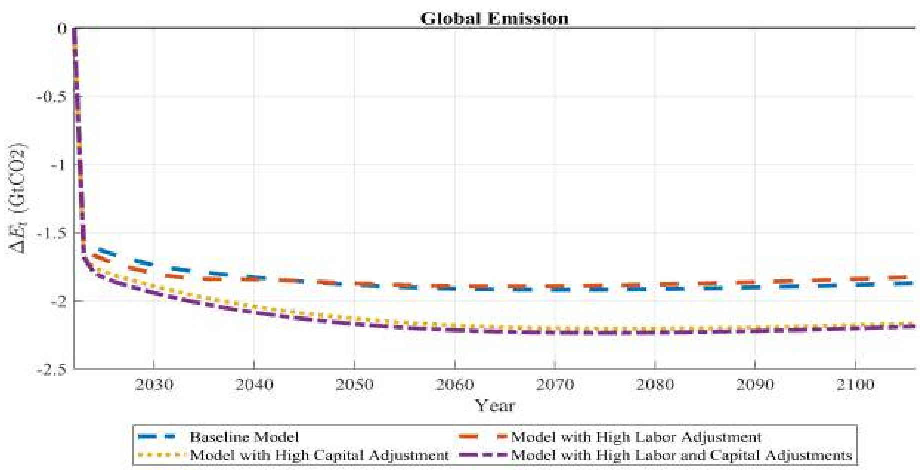


Notes: The impact of China's policy of unexpectedly and permanently increasing its consumption tax on the brown sector by 13 percentage points on capital and employment in China's brown sector. The blue line is the baseline model, and the red line represents the model with high labor mobility. The yellow line represents the model with high capital mobility, and the purple line indicates the model with high capital and labor mobility. The lines represent their values relative to the business-as-usual (BAU) case where the policy is not implemented.

Scale Effect The change in capital in the brown sector is influenced not only by the substitution effect but also by the scale effect. In the model with high labor adjustment, capital in the brown sector drops less than in the baseline model. This is due to the scale effect, where rapid labor adjustment allows workers in the brown sector to quickly adapt to the carbon tax, resulting in less consumption loss. A smaller drop in aggregate consumption leads to higher output and aggregate capital. Consequently, despite a larger decline in employment in the brown sector, the scale effect mitigates the decline in capital in that sector.

Capital-Labor Complementarity Figure 6 also highlights the complementarity between capital and labor. The marginal product of labor in the brown sector decreases more in the rapid capital adjustment model due to the larger decline in capital. Hence, China's carbon tax results in a greater reduction in employment in the model with rapid capital adjustment compared to the baseline model.

Figure 6: Impact of China's Carbon Tax on Global Emissions

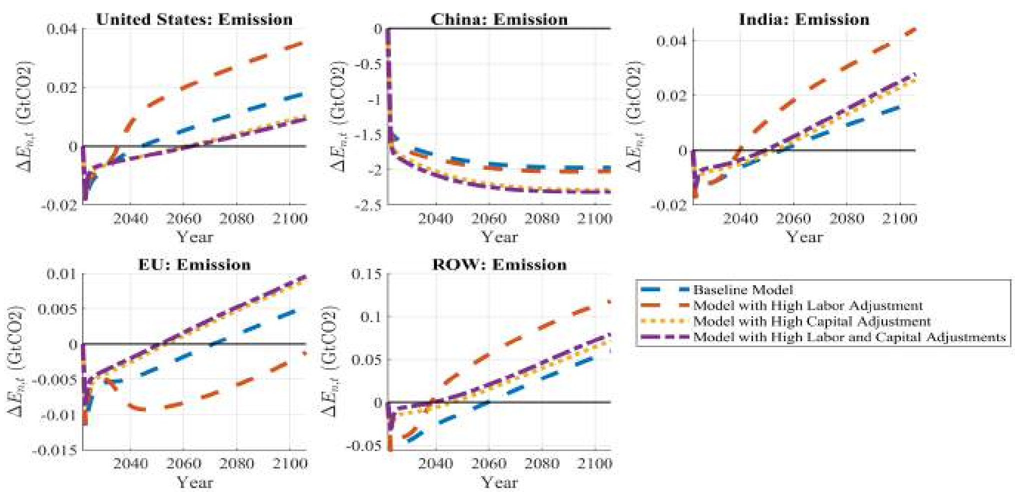


Notes: The impact of China's policy of unexpectedly and permanently increasing its consumption tax on the brown sector by 13 percentage points on global emissions. The blue line is the baseline model, and the red line represents the model with high labor mobility. The yellow line represents the model with high capital mobility, and the purple line indicates the model with high capital and labor mobility. The lines represent their values relative to the business-as-usual (BAU) case where the policy is not implemented.

Global Emissions The impact of China's unilateral carbon tax on global emissions is shown in Figure 6. In all models, the initial drop in global emissions is similar, with a reduction of 1.73

$GtCO_2$ (approximately 5 percent of global emissions in 2022) due to the increase in material prices and the subsequent decline in material usage and output. However, the figure demonstrates that the evolution of global emissions varies significantly across models. By 2030, the model with greater labor mobility results in a 3.55 percent larger reduction in global emissions compared to the baseline. However, it achieves 2.09 percent lower reduction by 2100 due to the scale effect.

Figure 7: Impact of China's Carbon Tax on Economy-level Emissions



Notes: The impact of China's policy of unexpectedly and permanently increasing its consumption tax on the brown sector by 13 percentage points on economy-level emissions. The blue line is the baseline model, and the red line represents the model with high labor mobility. The yellow line represents the model with high capital mobility, and the purple line indicates the model with high capital and labor mobility. The lines represent their values relative to the business-as-usual (BAU) case where the policy is not implemented.

I find that even greater emission reductions occur with increased capital mobility, driven by a further decline in employment and capital in the brown sector, as shown in Figure 6. In the model with high capital adjustment, the carbon tax achieves a 9.05 percent greater reduction in global emissions than the baseline by 2030, and 15.67 percent greater reductions by 2100. Compared to the model with both greater capital and labor mobility, the baseline achieves 10.59 percent less emission reduction by 2030 and 14.45 percent less by 2100.

Carbon Leakage Figure 6 shows the impact of China's implementation of the carbon tax on emissions across economies. Initially, emissions drop in all economies due to the increase in material prices, which propagates through sectoral and international linkages, leading to a decline in aggregate output. However, over time, emissions in economies outside of China rise as global demand for brown sector goods shifts from China to other economies, a phenomenon known as carbon leakage. The degree of factor mobility has an ambiguous effect on the magnitude of carbon leakage, as the size of the scale and substitution effects varies across economies.

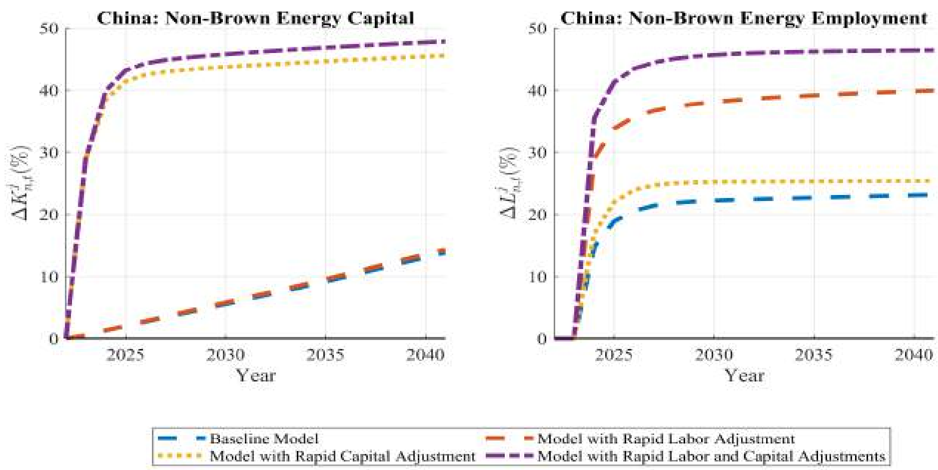
5.2 The Impact of China's Electricity Subsidy

To understand how subsidies on the non-brown energy (i.e., electricity) sector influence emissions, I consider a unilateral and permanent 13 percentage point reduction in the consumption tax on the non-brown energy sector implemented by China. This policy exercise aims to examine the impact of subsidizing the non-brown energy sector at the same magnitude as China's current subsidies for the fossil fuel sector. The policy scenairo is compared to the business-as-usual (BAU) scenario where this policy is not implemented.

Capital and Employment in the Non-Brown Energy Sector The impact of China's policy on capital and employment in the non-brown energy sector across models is illustrated in Figure 9. In the baseline model, in response to the subsidy, capital and employment in the non-brown energy sector are higher than the business-as-usual (BAU) level in both the short and long run. This is because the electricity subsidy incentivizes a shift in demand from the brown sector to the non-brown energy sector (substitution effect) and increases energy consumption by both producers and consumers, which leads to an overall rise in aggregate output and demand for both energy sectors. Models with greater factor mobility lead to higher capital and employment levels in the non-brown energy sector compared to the levels in the baseline model, indicating that greater factor mobility amplifies both the substitution and scale effects.

Capital and Employment in the Brown Sector The impact of China's electricity subsidy on capital and employment in the brown sector across models is illustrated in Figure 9. The figure shows that the effect of the subsidy on changes in capital and employment in the brown sector critically depends on the degree of factor mobility. In the baseline model, in response to the subsidy, capital and employment in the brown sector are higher than the business-as-usual (BAU) level in

Figure 8: Impact of China's Electricity Subsidy on its Non-Brown Energy Capital and Employment



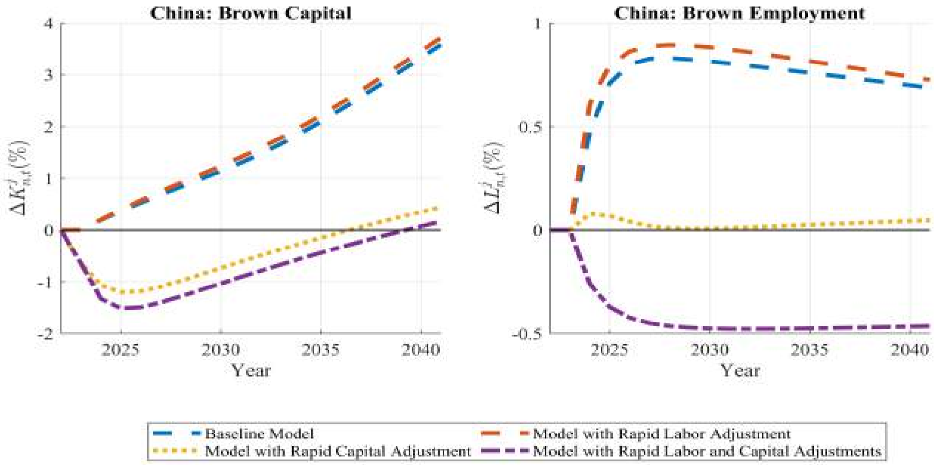
Notes: The impact of China's policy of unexpectedly and permanently reducing its consumption tax on the non-brown energy sector by 13 percentage points on capital and employment in China's non-brown energy sector. The blue line is the baseline model, and the red line represents the model with high labor mobility. The yellow line represents the model with high capital mobility, and the purple line indicates the model with high capital and labor mobility. The lines represent their values relative to the business-as-usual (BAU) case where the policy is not implemented.

both the short and long run. This happens because the subsidy encourages energy consumption by both producers and consumers, increasing aggregate output and raising demand for both brown and non-brown energy sectors.

In contrast, in the model with high capital and labor mobility, capital and employment in the non-brown energy sector are lower than the BAU level in both the short and long run. Greater factor mobility amplifies the shift in demand from the brown sector to the non-brown energy sector by significantly reallocating capital and labor away from the brown sector. In this model, the substitution effect outweighs the scale effect from increased energy consumption, resulting in lower capital and employment in the brown sector compared to the BAU level.

Global Emissions Figure 10 shows that global emissions rise across all models in response to China's electricity subsidy, both in the short and long run. Although models with greater capital mobility result in lower capital and employment in the brown sector than the BAU level, as shown

Figure 9: Impact of China's Electricity Subsidy on its Brown Capital and Employment

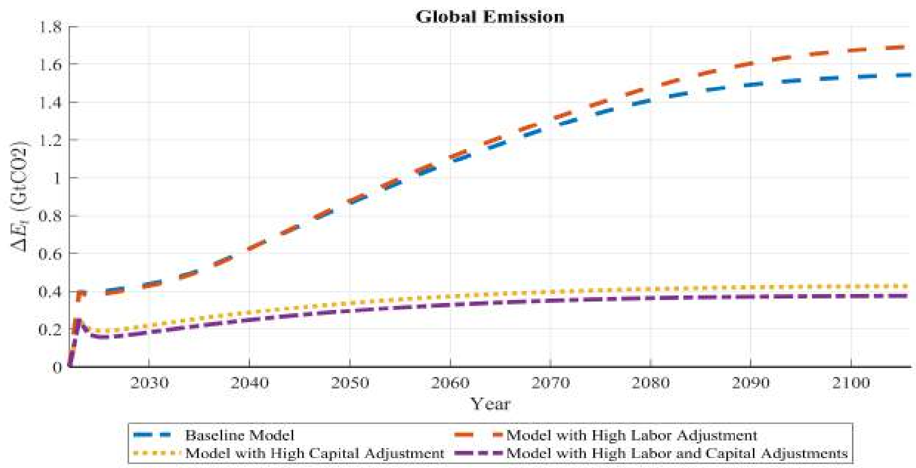


Notes: The impact of China's policy of unexpectedly and permanently decreasing its consumption tax on the brown sector by 13 percentage points on capital and employment in China's brown sector. The blue line is the baseline model, and the red line represents the model with high labor mobility. The yellow line represents the model with high capital mobility, and the purple line indicates the model with high capital and labor mobility. The lines represent their values relative to the business-as-usual (BAU) case where the policy is not implemented.

in Figure 9, the decline in material prices stimulates brown sector production. However, models with higher capital mobility still show a smaller overall increase in emissions, as greater mobility strengthens the substitution effect. Specifically, in the baseline model, global emissions in response to the subsidy are 0.39 percent higher in 2023 and 1.53 percent higher in 2100 compared to the BAU level. In contrast, in the model with high capital and labor adjustments, global emissions are 0.25 percent higher in 2023 and 0.37 percent higher in 2100 than the BAU level.

International Spillovers Figure 11 shows the impact of China's implementation of the non-brown energy subsidy on emissions across economies. In the baseline model, emissions increase in all economies except the EU in the long run due to the scale effect. However, in models with high capital mobility, emissions in economies outside of China decrease in the long run due to the decline in the price of the non-brown energy good imported from China which encourages a shift in demand from the brown sector to the non-brown energy sector. The figure highlights that such

Figure 10: The Impact of China's Electricity Subsidy on Global Emissions



Notes: The impact of China's implementation of a permanent 13 percentage point decrease in its consumption tax on the non-brown energy sector on global emissions. The lines represent economy-level contributions to the changes in global emissions compared to the global emissions in the business-as-usual (BAU) case where the policy is not implemented.

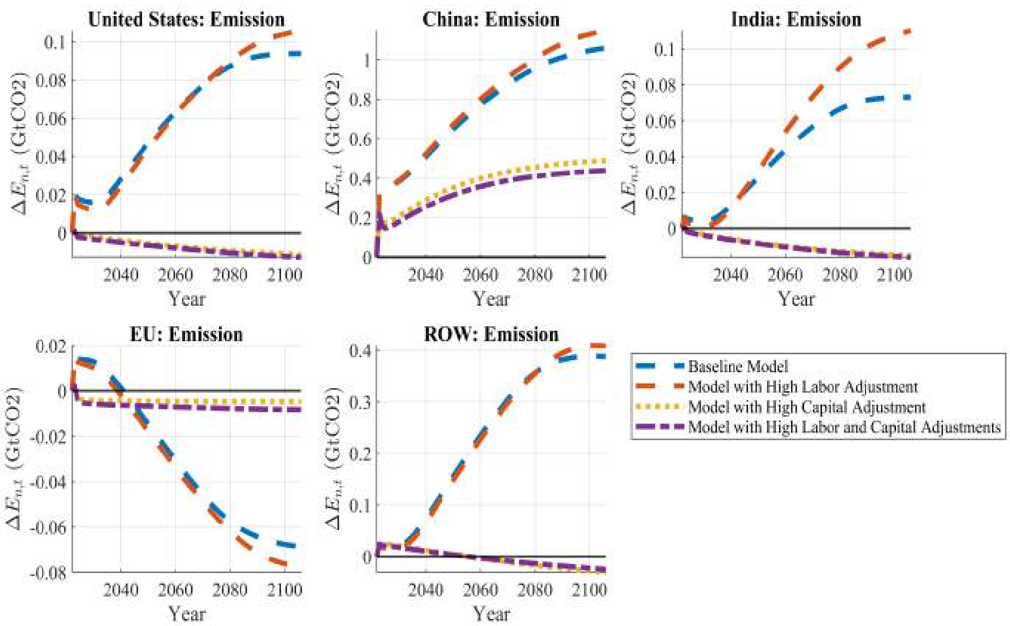
an effect exists only when capital is highly mobile across sectors.

5.3 Policy Comparison: Carbon Tax and Electricity Subsidy

In this section, I use the baseline model to compare the impact of China's carbon tax, electricity subsidy, and the combination of the two on global emissions and consumption. The combined policy of China's carbon tax and electricity subsidy can be seen as a hypothetical scenario where China subsidizes electricity— the secondary energy source— instead of fossil fuels— the primary energy source.

Global Emissions Figure 12 shows that the carbon tax reduces global emissions while the electricity subsidy increases them relative to the BAU level, as discussed earlier. When both policies are implemented, global emissions fall below the BAU level, but the reduction is smaller than with the carbon tax alone. The combination of the carbon tax and electricity subsidy amplifies the substitution effect, shifting demand and production from the brown sector to the non-brown energy sector. However, the positive scale effect from the subsidy and the negative scale effect from the

Figure 11: Impact of China's Electricity Subsidy on Economy-level Emissions

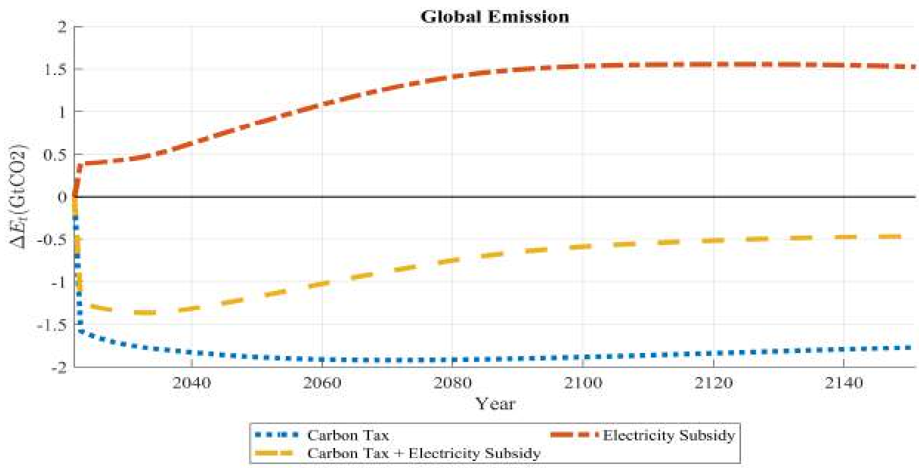


Notes: The impact of China's policy of unexpectedly and permanently increasing its consumption tax on the brown sector by 13 percentage points on economy-level emissions. The blue line is the baseline model, and the red line represents the model with high labor mobility. The yellow line represents the model with high capital mobility, and the purple line indicates the model with high capital and labor mobility. The lines represent their values relative to the business-as-usual (BAU) case where the policy is not implemented.

tax offset each other, resulting in an overall emissions reduction compared to the subsidy-only case. Nonetheless, due to the strong scale effect of the subsidy, the emissions reduction remains smaller than in the carbon tax-only scenario, both in the short and long run.

Global Consumption Figure 13 shows the response of global consumption when climate policies are implemented relative to the BAU level. Global consumption is the sum of all consumption by workers and capitalists across sectors and economies. When subsidies are implemented, global consumption initially decreases for two reasons. First, as shown in Figure 12, global emissions rise, worsening environmental quality. Second, the subsidies are financed by lump-sum taxes on capitalists in China, causing a short-term drop in consumption.

Figure 12: China's Climate Policies on Global Emissions

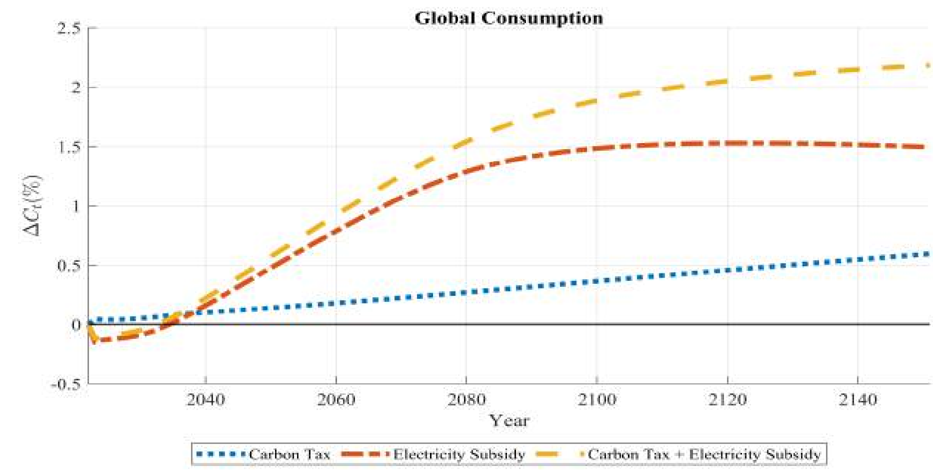


Notes: The impact of China's climates policies on global emissions. The blue line represents the response when China only implements a carbon tax; the red line represents the response when China implements a consumption subsidy on the non-brown energy sector. The yellow line represents the response when both the carbon tax and the subsidy on the non-brown energy sector are implemented. The lines represent their values relative to the business-as-usual (BAU) case where the policy is not implemented.

In the long run, all policies lead to higher global consumption compared to the BAU level, with the combination of the carbon tax and electricity subsidy generating the greatest increase. However, the channels through which global consumption rises differ across policies. The carbon tax boosts global consumption by reducing emissions and improving environmental quality, while the electricity subsidy allows capitalists to internalize the dynamic agglomeration externality, increasing production and consumption.

Dynamic Agglomeration Externality The long-term consumption gains from the subsidy depend largely on the strength of the dynamic agglomeration externality. Figure 14 illustrates the response of global consumption to climate policies in a model with high capital adjustment. In this model, the dynamic agglomeration externality ρ_Φ is reduced by a factor of five to match the observed persistence of capital allocation in the data. As shown in the figure, global consumption in response to the electricity subsidy falls below the BAU level in both the short and long run, as the

Figure 13: Baseline Model: China's Climate Policies on Global Consumption



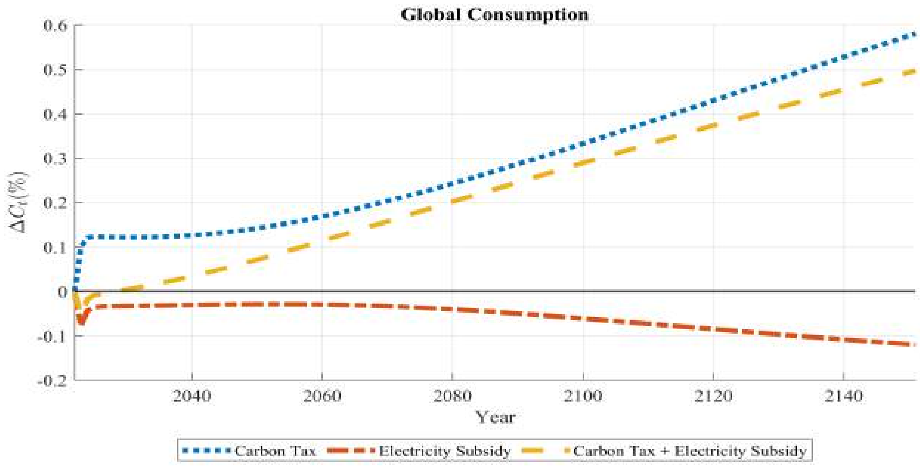
Notes: The impact of China's climates policies on global consumption. The blue line represents the response when China only implements a carbon tax; the red line represents the response when China implements a consumption subsidy on the non-brown energy sector. The yellow line represents the response when both the carbon tax and the subsidy on the non-brown energy sector are implemented. The lines represent their values relative to the business-as-usual (BAU) case where the policy is not implemented.

negative impact of environmental deterioration from increased emissions outweighs the benefits of internalizing the dynamic agglomeration externality.

Economy-level Consumption Figure 15 shows the consumption responses to China's climate policies in China and other economies. In China, the carbon tax initially raises consumption as capitalists benefit from the tax revenue. However, over time, the permanent carbon tax reduces aggregate output and investment by increasing energy costs faced by the non-energy sector, leading to a long-term decline in consumption.

In contrast, the electricity subsidy initially reduces consumption in China due to the lump-sum taxes imposed on capitalists to finance the subsidy. However, in the long run, China's aggregate consumption rises above the BAU level as the country's productivity improves from internalizing the dynamic agglomeration externality. When both policies are implemented together, the short-term drop in consumption is smaller, and long-term gains are similar to those from the subsidy

Figure 14: Model with High Capital Mobility: China's Climate Policies on Global Consumption



Notes: The impact of China's climates policies on global consumption in a model with high capital adjustment and low dynamic agglomeration externality. The blue line represents the response when China only implements a carbon tax; the red line represents the response when China implements a consumption subsidy on the non-brown energy sector. The yellow line represents the response when both the carbon tax and the subsidy on the non-brown energy sector are implemented. The lines represent their values relative to the business-as-usual (BAU) case where the policy is not implemented.

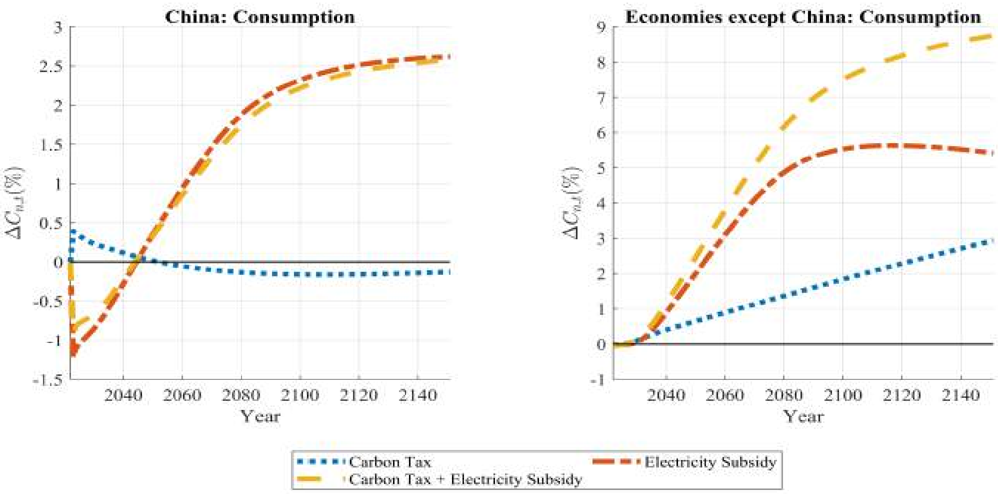
alone. These results, along with those in Figure 12, suggest that China could achieve both emissions reductions and consumption gains by implementing the carbon tax and electricity subsidy together.

In economies outside China, all policies lead to an increase in aggregate consumption. When China implements the carbon tax, the rise in consumption elsewhere results from improved environmental quality. In the case of the electricity subsidy, other economies benefit from lower energy prices and productivity spillovers from China through international trade.

5.4 Global Carbon Tax: Limiting Global Warming to 1.5 °C by 2100

I now compute the permanent global carbon tax required to meet the Paris Agreement's goal of limiting global warming to 1.5 °C above pre-industrial levels by 2100. Lee et al. (2023) estimates that, as of 2022, the remaining carbon budget is 428.17 $GtCO_2$ for a 50 percent chance of staying below the 1.5°C target. Using the baseline model, I calculate the minimum increase in a uniform,

Figure 15: China's Climate Policies on Economy-level Consumption



Notes: The impact of China's climates policies on consumption in China and economies other than China. The blue line represents the response when China only implements a carbon tax; the red line represents the response when China implements a consumption subsidy on the non-brown energy sector. The yellow line represents the response when both the carbon tax and the subsidy on the non-brown energy sector are implemented. The lines represent their values relative to the business-as-usual (BAU) case where the policy is not implemented.

permanent global carbon tax needed to keep cumulative emissions from 2022 to 2100 within this limit. I then apply the same tax in counterfactual models with varying degrees of factor mobility and compute how long it would take for cumulative emissions to reach 428.17 $GtCO_2$ in each model.

As shown in Table 10, the baseline model predicts that global temperatures will exceed 1.5 °C above pre-industrial levels by 2034. To limit global warming to 1.5 °C by 2100, all economies must increase their consumption taxes on the brown sector by at least 293 percentage points starting from 2023.

The table also shows that the same level of a global carbon tax in the model with high labor adjustment does not delay the time to reach 1.5 °C; in fact, the target is reached one year earlier. This is because rapid labor adjustment mitigates the scale effect that would otherwise push emissions downward, as workers in the brown sector adjust elastically to the shock.

In contrast, a model with high capital adjustment significantly delays the point at which global

Table 10: Global Carbon Tax Required to Limit Warming to 1.5°C Above Pre-Industrial Levels

Model	**Δ Global Carbon Tax (p.p.)**	**Year 1.5°C Reached**
Baseline	0	2034
Baseline	293	2100
High Labor Adjustment	293	2099
High Capital Adjustment	293	2120
High capital and labor Adjustments	293	2122

Notes: The table shows the permanent percentage points increase in the global carbon tax in 2023 and the projected year in which global warming is expected to exceed 1.5 °C above preindustrial levels, or when cumulative global emissions reach 428.17 $GtCO_2$, given the global carbon tax for each model.

temperatures exceed 1.5 °C. With a 293 percentage point increase in the global carbon tax, the 1.5 °C threshold is reached in 2120 in the high capital adjustment model and in 2122 in the model with both rapid capital and labor adjustments.

5.5 Big Push: Temporary Climate Policies

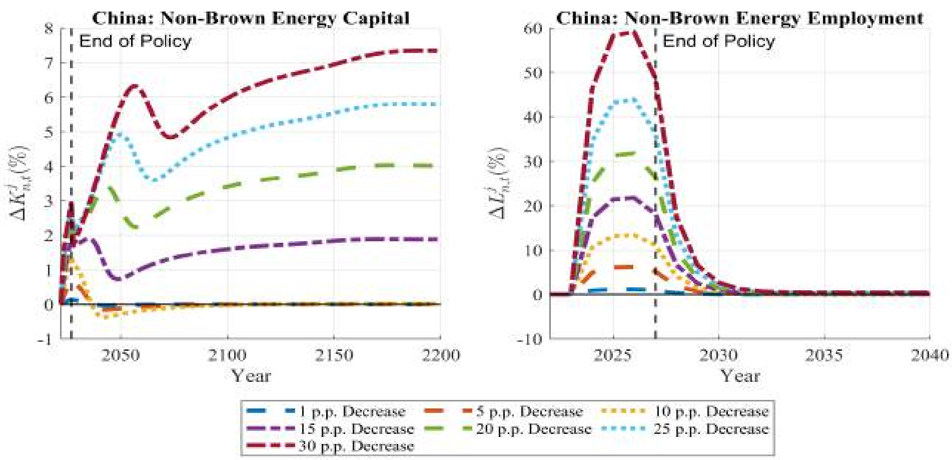
Temporary climate policies can have lasting effects due to the high costs of labor mobility and the strong path dependence of capital allocation. Even temporary policies, if substantial enough, can reshape the long-term distribution of capital and labor across sectors.

To explore this, I simulate multiple scenarios where China introduces electricity subsidies for five years, from 2023 to 2027, with consumption tax reductions on the electricity sector ranging from 1 to 30 percentage points. I then compare how capital and labor allocations evolve under these policies against a business-as-usual (BAU) scenario.

Figure 16 illustrates the impact of these temporary subsidies on capital and labor allocations in China. The results show that reductions in the consumption tax by 1 to 10 percentage points have no lasting impact on capital allocation, as non-brown energy capital gradually reverts to BAU levels after the policies end. However, subsidies of 15 percentage points or more lead to a more persistent reallocation of capital toward the electricity sector, even after the policy period. For instance, a 30 percentage point reduction results in over 7 percent more capital allocated to the non-brown energy sector by 2200 compared to the BAU scenario.

The long-term impact of these larger subsidies on capital allocation is driven by a strong dy-

Figure 16: 5-year Electricity Subsidies on Non-Brown Energy Capital and Employment in China



Notes: Employment and capital allocations in the non-brown energy sector relative to the business-as-usal (BAU) allocations under various temporary electricity subsidy scenarios in China from 2023 to 2027. The lines represent scenarios with consumption tax reductions on the non-brown energy sector by 1, 5, 10, 15, 20, 25, and 30 percentage points.

namic agglomeration externality, governed by parameter ρ_Φ, which reinforces the path dependence of capital. In terms of employment, labor in the non-brown energy sector remains above BAU levels after the subsidies expire, though this effect diminishes over time, with employment converging to BAU levels within five years.

6 Conclusion

This paper develops a dynamic general equilibrium model to evaluate the effectiveness of environmental policies in reducing global emissions under imperfect capital and labor mobility. The findings reveal that both capital and labor exhibit limited responsiveness to sector-specific wage and rental rate changes, constraining the large-scale resource reallocation needed to meet the Paris Agreement's emission reduction targets. The model shows that increasing capital and labor mobility significantly amplifies the effectiveness of carbon taxes in reducing global emissions, particularly when capital mobility is enhanced. However, the results also indicate that electricity subsidies can inadvertently increase emissions by lowering energy input prices and boosting aggregate production and investment.

The paper further explores the economic and environmental impacts of China's unilateral carbon tax. While the tax leads to a significant decline in China's domestic consumption due to its negative impact on the brown sector, global consumption rises as substantial emission reductions improve environmental quality. These findings suggest that international cooperation and compensation mechanisms may be necessary to enhance the effectiveness of carbon taxes and mitigate the economic costs for countries implementing such policies. Additionally, I show that if the productivity gains from reallocating capital are substantial, a country can reduce emissions and increase aggregate consumption by combining a carbon tax with a subsidy for the energy sector that produces substitutes for fossil fuel energy.

Although this paper provides a framework to understand the dynamic effects of environmental policies, it abstracts from optimal environmental policies. A fruitful avenue for future research is to develop methods for solving global optimal environmental policies in a dynamic setting, as this is a crucial yet underexplored area in the literature. Furthermore, strategic interactions between governments are critical for designing globally implementable environmental policies. The significant economic burden of unilateral carbon taxes, as highlighted in this paper, suggests that such policies may not be feasible without international cooperation and compensation from countries that benefit.

Another avenue for future research involves exploring the distributional consequences of environmental policies. This paper assumes homogeneous capitalists, thereby abstracting from the impact of ownership differences in heterogeneous capital assets across households. In practice, the ownership of sector-specific assets can lead to significant distributional effects, particularly between households with assets tied to the brown sector and those linked to the non-brown energy sector. Moreover, the assumption of infinitely lived agents overlooks potential generational conflicts. Addressing these distributional issues will be crucial for designing equitable and effective environmental policies.

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A Appendix

A.1 Detailed Estimation Procedure for Parameters

This section discusses the estimation procedure for parameters not discussed in the main text.

Non-Employment Income To estimate the non-employment income w_n^{NE} as a share of average labor income, I employ the panel labor force surveys described above which contain information on income of non-employed individuals. I begin by estimating the Mincerian regression of (log) wages on sector-year fixed effects, along with education, age, and gender fixed effects, weighted by individual weights to obtain residualized sector-year-level wages (Mincer (1974)):

$$\underbrace{\omega_{n,t}^{j,i}\left(\log(w_{n,t}^{j,i})\right)}_{\text{Weighted (log) wage in sector } j \text{ of individual } i} = \underbrace{\omega_{n,t}^{j,i}}_{\text{Weight}}\left(\underbrace{\delta_{jt}}_{\text{Sector-Year FE}} + \underbrace{\delta_e}_{\text{Education FE}} + \underbrace{\delta_a}_{\text{Age FE}} + \underbrace{\delta_g}_{\text{Gender FE}} + \underbrace{\varepsilon_{n,t}^{j,i}}_{\text{Residual}}\right)$$

Next, I compute the average residualized wage of employed workers, weighted by employment:

$$\underbrace{L_{n,t}^{j} \cdot \delta_{jt}}_{\text{Weighted (log) residualized wage}} = \underbrace{L_{n,t}^{j}}_{\text{Weight}}\left(\underbrace{\delta_t}_{\text{Year FE}} \mathbb{1}[j \neq \text{NE}] + \underbrace{\delta_t}_{\text{Year FE}} \mathbb{1}[j = \text{NE}] + \underbrace{\varepsilon_{n,t}^{j}}_{\text{Residual}}\right)$$

Finally, I estimate the non-employment income parameter w_n^{NE} by dividing the residualized non-employment income by the average residualized wage of employed workers and averaging over time.

Exogenous Trade Costs The gravity equation leads to the following regression specification:

$$\ln\left((\pi_{nm,t}^{j})^{\frac{1}{1-\sigma}}(e_{nm,t}^{j})^{-1}\right) = \ln(\varrho_{nm,t}^{j}) + \delta_{m,t}^{j} + \delta_{n,t}^{j} + \varepsilon_{nm,t}^{j}$$

where $\delta_{m,t}^{j}$ and $\delta_{n,t}^{j}$ are, respectively, exporter-sector-year fixed effects and importer-sector-year fixed effects defined as $\delta_{m,t}^{j} \equiv \ln(mc_{m,t}^{j}) - \ln(z_{m,t}^{j})$ and $\delta_{n,t}^{j} \equiv \frac{1}{\sigma-1}\ln\left(\sum_{i\in\mathcal{N}}(z_{i,t}^{j})^{\sigma-1}(d_{ni,t}^{j} mc_{i,t}^{j})^{1-\sigma}\right)$, and $\varepsilon_{nm,t+1}^{j}$ is the residual term.

Although the goal is to estimate trade costs for 2022 or the most recent year available, the WIOD only provides data up to 2014. Therefore, I assume the initial exogenous trade costs $\varrho_{nm,0}^{j}$

are equal to the time-averaged trade costs, calculated as $\rho^j_{nm,0} = \frac{1}{15}\sum_{s=0}^{14}\rho^j_{nm,-s}$ for the period 2000 - 2014. Under this assumption, the trade costs are estimated as the coefficients of the importer-exporter-sector fixed effects in the regression specification (**??**). The bilateral trade shares ($\pi^j_{nm,t}$) are sourced from the WIOD 2016 Release, while trade taxes ($e^j_{nm,t}$) are obtained from Guimbard et al. (2012). Refer to the Appendix A.10 for the detailed estimates.

Other Parameters The capital depreciation rates (δ^K_n) are taken from the Penn World Table 10.01 (Feenstra et al. (2015)) for the year 2019, which is the most recent year available. For country aggregates, I take the GDP-weighted average; the values range from 0.042 to 0.058. For detailed estimates, refer to the Appendix A.10.

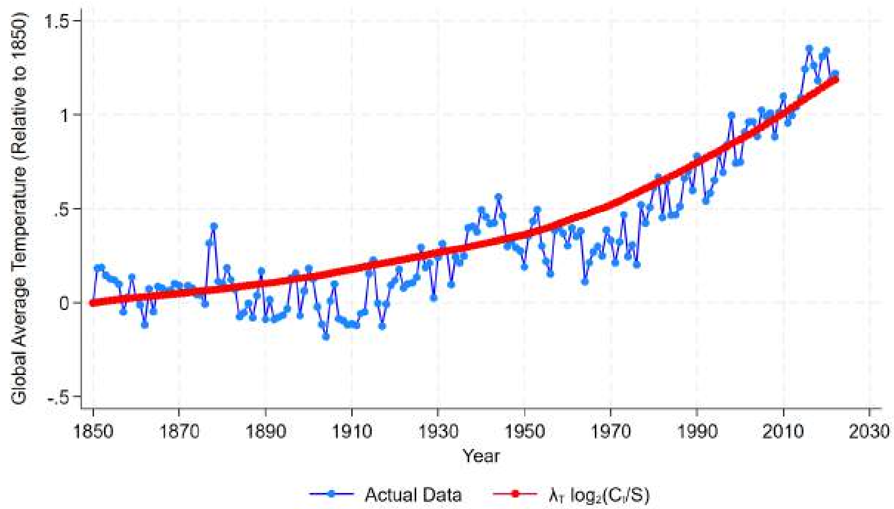
Supply costs of fossil fuels (φ^h_n) are estimated based on the IMF Fossil Fuel Subsidies Data (Black et al. (2023)) for the year 2022. The extraction costs for oil, natural gas, and coal are set equal to the supply costs of gasoline (bil. USD/bil. m^3), natural gas (bil. USD/bil. m^3), and coal (bil. USD/Gt). See Appendix A.10 for detailed estimates.

Carbon emissions data from years 1850 to 2022 are sourced from Ritchie and Roser (2020), with land-change emissions (E_L) computed as the average global land-change emissions over this period. The estimate for E_L is 4.9617 $GtCO_2$. The carbon-temperature conversion parameter (λ_T) is obtained by estimating Equation (**??**) using data on global temperature relative to the year 1850 obtained from Met Office Hadley Centre (2024) and global carbon stock computed based on Equation (**??**). The estimated value for λ_T is 1.671 with a robust standard error of 0.032 for the period 1980 - 2022. The model fit and the regression results are presented in Figure A1 and Table A14, respectively.

Consumption expenditure shares (α_n, α^j_n) and input shares (value added ($\gamma^{j,V}_n$), material ($\gamma^{j,k}_n$), labor ($\psi^{j,L}_n$), capital ($\psi^{j,K}_n$), energy ($\psi^{j,E}_n$), fossil fuel ($\psi^{B,F}_n$)) are directly computed using the World Input-Output Table (WIOT) and Socio-Economic Accounts (SEA) of the WIOD 2016 Release for the year 2014, which is the most recent year available. Refer to the Appendix A.10 for detailed estimates.

A.2 Figures

Figure A1: Global Average Temperature (°C) relative to Year 1850



Notes: The blue line represents the actual temperature (°C) data, while the red line represents the predicted temperature (°C) based on the carbon-temperature conversion parameter (λ_T). The prediction is based on the equation $\mathbb{T}(\mathcal{C}_t) = \lambda_T \log_2(\mathcal{C}_t/\mathcal{S})$, where $\mathbb{T}(\mathcal{C}_t)$ is the global temperature, $\mathcal{C}_t$ is the carbon stock and $\mathcal{S}$ is the pre-industrial carbon stock. The estimate for the parameter λ_T is 1.671 with a robust standard error of 0.032 for the period 1980 - 2022. Sources: Ritchie and Roser (2020) (carbon emissions), Met Office Hadley Centre (2024) (temperature).

A.3 Tables

Table A1: Threshold for Market-Based EPS

Score	Market based policies				
	CO_2 certificate	Renewable energy certificates (%)	CO_2 taxes (USD/tonne CO_2)	NO_x taxes (USD/tonne NO_x)	SO_x taxes (USD/tonne SO_x)
0	0	0	0	0	0
1	$0 < x \leq 10$	$0 < x \leq 0.05$	$0 < x \leq 10$	$0 < x \leq 90$	$0 < x \leq 116$
2	$10 < x \leq 20$	$0.05 < x \leq 0.08$	$10 < x \leq 20$	$90 < x \leq 137$	$116 < x \leq 180$
3	$20 < x \leq 30$	$0.08 < x \leq 0.11$	$20 < x \leq 30$	$137 < x \leq 184$	$180 < x \leq 244$
4	$30 < x \leq 40$	$0.11 < x \leq 0.14$	$30 < x \leq 40$	$184 < x \leq 231$	$244 < x \leq 308$
5	$40 < x \leq 50$	$0.14 < x \leq 0.17$	$40 < x \leq 50$	$231 < x \leq 278$	$308 < x \leq 372$
6	> 50	> 0.17	> 50	> 278	> 372

Source: OECD (Kruse et al. (2022))

Table A2: Threshold for Non-Market-Based EPS

Score	Non-market based policies			
	Emission Limit NO_x (mg/m^3)	Emission Limit SO_x (mg/m^3)	Emission Limit PM (mg/m^3)	Emission Limit Sulphur (mg/m^3)
0	No limit	No limit	No limit	No limit
1	>563	>643	>44	>1602
2	$458 \leq x \leq 563$	$518 \leq x \leq 643$	$39 \leq x \leq 44$	$1204 \leq x \leq 1602$
3	$353 \leq x \leq 458$	$393 \leq x \leq 518$	$32 \leq x \leq 39$	$806 \leq x \leq 1204$
4	$248 \leq x \leq 353$	$268 \leq x \leq 393$	$26 \leq x \leq 32$	$408 \leq x \leq 806$
5	$143 \leq x \leq 248$	$143 \leq x \leq 268$	$20 \leq x \leq 26$	$10 \leq x \leq 408$
6	$0 \leq x \leq 143$	$0 \leq x \leq 143$	$0 \leq x \leq 20$	$0 \leq x \leq 10$

Source: OECD (Kruse et al. (2022))

Table A3: Threshold for Technology Support EPS

Score	Technology support policies		
	R&D expenditure (1000 USD/GDP)	**FIT solar (USD/kWh)**	**FIT wind (USD/kWh)**
0	0	0	0
1	$0 < x \leq 0.14$	$0 < x \leq 0.41$	$0 < x \leq 0.95$
2	$0.14 < x \leq 0.27$	$0.41 < x \leq 0.81$	$0.95 < x \leq 1.27$
3	$0.27 < x \leq 0.4$	$0.81 < x \leq 1.21$	$1.27 < x \leq 1.59$
4	$0.4 < x \leq 0.53$	$1.21 < x \leq 1.61$	$1.59 < x \leq 1.91$
5	$0.53 < x \leq 0.66$	$1.61 < x \leq 2.01$	$1.91 < x \leq 2.23$
6	> 0.66	> 2.01	> 2.23

Source: OECD (Kruse et al. (2022))

Table A4: List of Economies and Countries in Country Aggregates

Economy	**Country**
United States (US)	United States
China (CHN)	China
India (IND)	India
European Union (EU)	Austria, Belgium, Bulgaria, Croatia, Czech Republic, Denmark, Germany, Estonia, Greece, Spain, France, Ireland, Italy, Cyprus, Latvia, Lithuania, Luxembourg, Hungary, Malta, Netherlands, Poland, Portugal, Romania, Slovenia, Slovakia, Finland, Sweden
Rest of the World (ROW)	Australia, Brazil, Canada, Switzerland, United Kingdom, Indonesia, Mexico, Malta, Norway, Russia, Turkey, Japan, South Korea, Taiwan

Notes: The European Union includes 27 member countries, and the Rest of the World (ROW) includes selected major economies and a composite of other countries. Source: World Input-Output Database (WIOD) 2016 Release.

Table A5: Industry Classification - Non-Brown Energy, Brown

Sector	**Industry**
Non-Brown Energy (NB)	Electricity, gas supply
Brown (B)	Mining and quarrying Manufacture of coke, petroleum

Notes: Industries are categorized into sectors based on the World Input-Output Database 2016 Release classification. The non-brown energy sector includes industries related to renewable energy and technology, while the Brown sector includes industries related to fossil fuels.

Table A6: Industry Classification - Agriculture, Manufacturing, Services

Sector	Industries
Agriculture (AG)	Crop and animal production
	Forestry and logging
	Fishing and aquaculture
Manufacturing (MN)	Manufacture of food, beverages, tobacco
	Manufacture of textiles, apparel, leather
	Manufacture of wood and paper products
	Printing and recorded media
	Manufacture of chemicals, pharmaceuticals
	Manufacture of rubber, plastic products
	Manufacture of non-metallic, basic, and fabricated minerals
	Manufacture of computer, electronics
	Manufacture of electrical equipment
	Manufacture of motor vehicles
Services (SR)	Motor vehicle trade and repair
	Wholesale trade, excl. motor vehicles
	Retail trade, excl. motor vehicles
	Land, Water, Air transport, pipelines
	Warehousing, support for transportation
	Postal and courier activities
	Accommodation, food services
	Publishing activities, Motion picture, video production
	Telecommunications
	Computer programming, consultancy
	Financial services, Insurance, pension funding
	Auxiliary financial services
	Real estate activities
	Legal, accounting, management consultancy
	Architectural, engineering activities
	Scientific research and development
	Advertising, market research
	Other professional, technical activities
	Administrative and support services
	Public administration, defence
	Education, Human health, social work
	Other service activities, Household activities as employers
	Activities of extraterritorial organizations

Notes: Industries are categorized into sectors based on the World Input-Output Database (WIOD) 2016 Release classification.

A.4 Derivations

A.4.1 Sectoral Good and Bilateral Trade Share

- Given problem:

$$\min_{y^j_{nm,t}} \sum_{m\in\mathcal{N}} p^j_{nm,t} y^j_{nm,t}$$

- Subject to:

$$Y^j_{n,t} = \left(\sum_{m\in\mathcal{N}} \left(y^j_{nm,t}\right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

- Set up the Lagrangian:

$$\mathcal{L} = \sum_{m\in\mathcal{N}} p^j_{nm,t} y^j_{nm,t} + \lambda \left(Y^j_{n,t} - \left(\sum_{m\in\mathcal{N}} \left(y^j_{nm,t}\right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \right)$$

- First-order condition with respect to $y^j_{nm,t}$:

$$\frac{\partial \mathcal{L}}{\partial y^j_{nm,t}} = p^j_{nm,t} - \lambda \cdot \frac{\sigma}{\sigma-1} \left(\sum_{m\in\mathcal{N}} \left(y^j_{nm,t}\right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{1}{\sigma-1}} \cdot \frac{\sigma-1}{\sigma} \left(y^j_{nm,t}\right)^{-\frac{1}{\sigma}} = 0$$

- Simplify:

$$p^j_{nm,t} = \lambda \left(\sum_{m\in\mathcal{N}} \left(y^j_{nm,t}\right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{1}{\sigma-1}} \left(y^j_{nm,t}\right)^{-\frac{1}{\sigma}}$$

- Rearrange:

$$\left(y^j_{nm,t}\right)^{-\frac{1}{\sigma}} = \frac{p^j_{nm,t}}{\lambda} \left(\sum_{m\in\mathcal{N}} \left(y^j_{nm,t}\right)^{\frac{\sigma-1}{\sigma}} \right)^{-\frac{1}{\sigma-1}}$$

- Raise both sides to the power of $-\sigma$:

$$y^j_{nm,t} = \left(\frac{p^j_{nm,t}}{\lambda}\right)^{-\sigma} \left(\sum_{m\in\mathcal{N}} \left(y^j_{nm,t}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$$

- Substitute the constraint:

$$y^j_{nm,t} = \left(\frac{p^j_{nm,t}}{\lambda}\right)^{-\sigma} Y^j_{n,t}$$

- Sum over all m and use the constraint:

$$\begin{aligned} Y^j_{n,t} = \sum_{m\in\mathcal{N}} y^j_{nm,t} = Y^j_{n,t} \sum_{m\in\mathcal{N}} \left(\frac{p^j_{nm,t}}{\lambda}\right)^{-\sigma} \\ 1 = \sum_{m\in\mathcal{N}} \left(\frac{p^j_{nm,t}}{\lambda}\right)^{-\sigma} \\ \lambda = \left(\sum_{m\in\mathcal{N}} (p^j_{nm,t})^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \end{aligned}$$

- Substitute this back into the equation for $y^j_{nm,t}$:

$$y^j_{nm,t} = Y^j_{n,t} \frac{(p^j_{nm,t})^{-\sigma}}{\sum_{m\in\mathcal{N}} (p^j_{nm,t})^{1-\sigma}}$$

- Define the price index $P^j_{n,t}$:

$$P^j_{n,t} \equiv \left(\sum_{m\in\mathcal{N}} (p^j_{nm,t})^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$$

- Demand:

$$y^j_{nm,t} = \left(\frac{p^j_{nm,t}}{P^j_{n,t}}\right)^{-\sigma} Y^j_{n,t}$$

- Bilateral Trade Share:

$$\pi^j_{nm,t} \equiv \frac{p^j_{nm,t} y^j_{nm,t}}{P^j_{n,t} Y^j_{n,t}} = \Big(\frac{p^j_{nm,t}}{P^j_{n,t}}\Big)^{1-\sigma}$$

A.4.2 Energy Composite Good

- Problem: Minimize $(1+\tau^{NB}_{n,t})P^{NB}_{n,t} + (1+\tau^{B}_{n,t})P^{B}_{n,t}$ subject to the constraint:

$$Y^E_{n,t} = \Big((\alpha^{NB}_n)^{\frac{1}{\eta_1}}(Y^{NB}_{n,t})^{\frac{\eta_1-1}{\eta_1}} + (\alpha^{B}_n)^{\frac{1}{\eta_1}}(Y^{B}_{n,t})^{\frac{\eta_1-1}{\eta_1}}\Big)^{\frac{\eta_1}{\eta_1-1}}$$

- Form the Lagrangian:

$$\begin{aligned} L = &(1+\tau^{NB}_{n,t})P^{NB}_{n,t} + (1+\tau^{B}_{n,t})P^{B}_{n,t} \\ &+ \lambda\Big(Y^E_{n,t} - \Big((\alpha^{NB}_n)^{\frac{1}{\eta_1}}(Y^{NB}_{n,t})^{\frac{\eta_1-1}{\eta_1}} + (\alpha^{B}_n)^{\frac{1}{\eta_1}}(Y^{B}_{n,t})^{\frac{\eta_1-1}{\eta_1}}\Big)^{\frac{\eta_1}{\eta_1-1}}\Big) \end{aligned}$$

- Find the partial derivatives:

$$\begin{aligned} \frac{\partial L}{\partial Y^{NB}_{n,t}} &= (1+\tau^{NB}_{n,t})P^{NB}_{n,t} - \lambda\cdot(Y^E_{n,t})^{\frac{1}{\eta_1}}\cdot(\alpha^{NB}_n)^{\frac{1}{\eta_1}}\cdot(Y^{NB}_{n,t})^{-\frac{1}{\eta_1}} = 0 \\ \frac{\partial L}{\partial Y^{B}_{n,t}} &= (1+\tau^{B}_{n,t})P^{B}_{n,t} - \lambda\cdot(Y^E_{n,t})^{\frac{1}{\eta_1}}\cdot(\alpha^{B}_n)^{\frac{1}{\eta_1}}\cdot(Y^{B}_{n,t})^{-\frac{1}{\eta_1}} = 0 \\ \frac{\partial L}{\partial \lambda} &= Y^E_{n,t} - \Big((\alpha^{NB}_n)^{\frac{1}{\eta_1}}(Y^{NB}_{n,t})^{\frac{\eta_1-1}{\eta_1}} + (\alpha^{B}_n)^{\frac{1}{\eta_1}}(Y^{B}_{n,t})^{\frac{\eta_1-1}{\eta_1}}\Big)^{\frac{\eta_1}{\eta_1-1}} = 0 \end{aligned}$$

- Simplify the first two equations:

$$\begin{aligned} (1+\tau^{NB}_{n,t})P^{NB}_{n,t} &= \lambda\cdot(Y^E_{n,t})^{\frac{1}{\eta_1}}\cdot(\alpha^{NB}_n)^{\frac{1}{\eta_1}}\cdot(Y^{NB}_{n,t})^{-\frac{1}{\eta_1}} \\ (1+\tau^{B}_{n,t})P^{B}_{n,t} &= \lambda\cdot(Y^E_{n,t})^{\frac{1}{\eta_1}}\cdot(\alpha^{B}_n)^{\frac{1}{\eta_1}}\cdot(Y^{B}_{n,t})^{-\frac{1}{\eta_1}} \end{aligned}$$

- Divide these equations:

$$\frac{(1+\tau^{NB}_{n,t})P^{NB}_{n,t}}{(1+\tau^{B}_{n,t})P^{B}_{n,t}} = \frac{(\alpha^{NB}_n)^{\frac{1}{\eta_1}}\cdot(Y^{NB}_{n,t})^{-\frac{1}{\eta_1}}}{(\alpha^{B}_n)^{\frac{1}{\eta_1}}\cdot(Y^{B}_{n,t})^{-\frac{1}{\eta_1}}}$$

- Rearrange to get the ratio of $Y_{n,t}^{NB}$ to $Y_{n,t}^{B}$:

$$Y_{n,t}^{NB} = \Big(\frac{\alpha_n^{NB}}{\alpha_n^B}\Big) \cdot \Big(\frac{(1+\tau_{n,t}^{NB})P_{n,t}^{NB}}{(1+\tau_{n,t}^{B})P_{n,t}^{B}}\Big)^{-\eta_1} Y_{n,t}^{B}$$

- Substitute this ratio into the constraint equation:

$$\begin{aligned}
Y_{n,t}^{E} &= \Big((\alpha_n^{NB})^{\frac{1}{\eta_1}} (Y_{n,t}^{B}\Big(\frac{\alpha_n^{NB}}{\alpha_n^B}\Big) \cdot \Big(\frac{(1+\tau_{n,t}^{NB})P_{n,t}^{NB}}{(1+\tau_{n,t}^{B})P_{n,t}^{B}}\Big)^{-\eta_1})^{\frac{\eta_1-1}{\eta_1}} + (\alpha_n^B)^{\frac{1}{\eta_1}} (Y_{n,t}^{B})^{\frac{\eta_1-1}{\eta_1}}\Big)^{\frac{\eta_1}{\eta_1-1}} \\
&= Y_{n,t}^{B}\Big((\alpha_n^{NB})^{\frac{1}{\eta_1}} \Big(\frac{\alpha_n^{NB}}{\alpha_n^B}\Big)^{\frac{\eta_1-1}{\eta_1}} \cdot \Big(\frac{(1+\tau_{n,t}^{NB})P_{n,t}^{NB}}{(1+\tau_{n,t}^{B})P_{n,t}^{B}}\Big)^{1-\eta_1} + (\alpha_n^B)^{\frac{1}{\eta_1}}\Big)^{\frac{\eta_1}{\eta_1-1}} \\
&= (\alpha_n^B)^{-1}((1+\tau_{n,t}^{B})P_{n,t}^{B})^{\eta_1} Y_{n,t}^{B}\Big(\alpha_n^{NB}\Big((1+\tau_{n,t}^{NB})P_{n,t}^{NB}\Big)^{1-\eta_1} + \alpha_n^B\Big((1+\tau_{n,t}^{B})P_{n,t}^{B}\Big)^{1-\eta_1}\Big)^{\frac{\eta_1}{\eta_1-1}}
\end{aligned}$$

- Define the price index $P_{n,t}^{E}$:

$$P_{n,t}^{E} = \left(\alpha_n^{NB}\Big((1+\tau_{n,t}^{NB})P_{n,t}^{NB}\Big)^{1-\eta_1} + \alpha_n^B\Big((1+\tau_{n,t}^{B})P_{n,t}^{B}\Big)^{1-\eta_1}\right)^{\frac{1}{1-\eta_1}}$$

- Demand for brown sector

$$Y_{n,t}^{B} = \alpha_n^B\Big(\frac{(1+\tau_{n,t}^{B})P_{n,t}^{B}}{P_{n,t}^{E}}\Big)^{-\eta_1} Y_{n,t}^{E}$$

A.4.3 Material

- Problem: Minimize $\sum_{k\in\mathcal{J}}(1+\tau_{n,t}^{k})P_{n,t}^{k}M_{n,t}^{j,k}$ subject to the constraint:

$$M_{n,t}^{j} = \prod_{k\in\mathcal{J}} \Big(M_{n,t}^{j,k}\Big)^{\gamma_n^{j,k}}$$

- Form the Lagrangian:

$$L = \sum_{k\in\mathcal{J}}(1+\tau_{n,t}^{k})P_{n,t}^{k}M_{n,t}^{j,k} + \lambda\left(M_{n,t}^{j} - \prod_{k\in\mathcal{J}} \Big(M_{n,t}^{j,k}\Big)^{\gamma_n^{j,k}}\right)$$

- Find the partial derivatives:

$$\frac{\partial L}{\partial M_{n,t}^{j,k}} = (1+\tau_{n,t}^{k})P_{n,t}^{k} - \lambda \cdot \gamma_n^{j,k} \cdot \prod_{k' \in \mathcal{J}} \left(M_{n,t}^{j,k'}\right)^{\gamma_n^{j,k'}} \cdot \left(M_{n,t}^{j,k}\right)^{-1} = 0$$

$$\frac{\partial L}{\partial \lambda} = M_{n,t}^{j} - \prod_{k \in \mathcal{J}} \left(M_{n,t}^{j,k}\right)^{\gamma_n^{j,k}} = 0$$

- Simplify the first equation:

$$(1+\tau_{n,t}^{k})P_{n,t}^{k} = \lambda \cdot \gamma_n^{j,k} \cdot \prod_{k' \in \mathcal{J}} \left(M_{n,t}^{j,k'}\right)^{\gamma_n^{j,k'}} \cdot \left(M_{n,t}^{j,k}\right)^{-1}$$

$$(1+\tau_{n,t}^{k})P_{n,t}^{k} = \lambda \cdot \gamma_n^{j,k} \cdot M_{n,t}^{j} \cdot \left(M_{n,t}^{j,k}\right)^{-1}$$

- Rearrange the equation to solve for $M_{n,t}^{j,k}$:

$$M_{n,t}^{j,k} = M_{n,t}^{j} \cdot \frac{\gamma_n^{j,k}}{(1+\tau_{n,t}^{k})P_{n,t}^{k}} \cdot \lambda$$

- Sum the demand equations and solve for λ:

$$M_{n,t}^{j} = \prod_{k \in \mathcal{J}} \left(M_{n,t}^{j,k}\right)^{\gamma_n^{j,k}}$$

$$M_{n,t}^{j} = \prod_{k \in \mathcal{J}} \left(M_{n,t}^{j} \cdot \frac{\gamma_n^{j,k}}{(1+\tau_{n,t}^{k})P_{n,t}^{k}} \cdot \lambda\right)^{\gamma_n^{j,k}}$$

$$M_{n,t}^{j} = M_{n,t}^{j} \cdot \left(\prod_{k \in \mathcal{J}} \left(\frac{\gamma_n^{j,k}}{(1+\tau_{n,t}^{k})P_{n,t}^{k}}\right)^{\gamma_n^{j,k}} \cdot \lambda^{\sum_{k \in \mathcal{J}} \gamma_n^{j,k}}\right)$$

$$1 = \prod_{k \in \mathcal{J}} \left(\frac{\gamma_n^{j,k}}{(1+\tau_{n,t}^{k})P_{n,t}^{k}}\right)^{\gamma_n^{j,k}} \cdot \lambda$$

$$\lambda = \prod_{k \in \mathcal{J}} \left(\frac{(1+\tau_{n,t}^{k})P_{n,t}^{k}}{\gamma_n^{j,k}}\right)^{\gamma_n^{j,k}}$$

- Define the price index:

$$P_{n,t}^{M,j} = \prod_{k \in \mathcal{J}} \left(\frac{(1 + \tau_{n,t}^{k}) P_{n,t}^{k}}{\gamma_n^{j,k}} \right)^{\gamma_n^{j,k}}$$

- Substitute λ back into the demand equation:

$$M_{n,t}^{j,k} = M_{n,t}^{j} \cdot \frac{\gamma_n^{j,k} P_{n,t}^{M,j}}{(1 + \tau_{n,t}^{k}) P_{n,t}^{k}}$$

A.4.4 Final Good

- Problem: Minimize $\sum_{k \in \mathcal{J}_D} (1 + \tau_{n,t}^{k}) P_{n,t}^{j} C_{n,t}^{j} + P_{n,t}^{E} C_{n,t}^{E}$ subject to the constraint:

$$Y_{n,t} = \mathcal{D}_n(\mathcal{C}_t) \Big(\prod_{j \in \mathcal{J}_D} (C_{n,t}^{j})^{\alpha_n^j} \Big)^{\alpha_n} (C_{n,t}^{E})^{1-\alpha_n}$$

- Form the Lagrangian:

$$L = \sum_{j \in \mathcal{J}_D} (1 + \tau_{n,t}^{j}) P_{n,t}^{j} C_{n,t}^{j} + P_{n,t}^{E} C_{n,t}^{E} + \lambda \left(Y_{n,t} - \mathcal{D}_n(\mathcal{C}_t) \Big(\prod_{j \in \mathcal{J}_D} (C_{n,t}^{j})^{\alpha_n^j} \Big)^{\alpha_n} (C_{n,t}^{E})^{1-\alpha_n} \right)$$

- Find the partial derivatives:

$$\begin{aligned}
\frac{\partial L}{\partial C_{n,t}^{j}} &= (1 + \tau_{n,t}^{j}) P_{n,t}^{j} - \lambda \cdot \mathcal{D}_n(\mathcal{C}_t) \cdot \alpha_n \cdot \alpha_n^j \cdot \Big(\prod_{j' \in \mathcal{J}_D} (C_{n,t}^{j'})^{\alpha_n^{j'}} \Big)^{\alpha_n} \cdot (C_{n,t}^{j})^{-1} (C_{n,t}^{E})^{1-\alpha_n} = 0 \\
\frac{\partial L}{\partial C_{n,t}^{E}} &= P_{n,t}^{E} - \lambda \cdot \mathcal{D}_n(\mathcal{C}_t) \cdot (1 - \alpha_n) \cdot \Big(\prod_{j \in \mathcal{J}_D} (C_{n,t}^{j})^{\alpha_n^j} \Big)^{\alpha_n} \cdot (C_{n,t}^{E})^{-\alpha_n} = 0 \\
\frac{\partial L}{\partial \lambda} &= Y_{n,t} - \mathcal{D}_n(\mathcal{C}_t) \Big(\prod_{j \in \mathcal{J}_D} (C_{n,t}^{j})^{\alpha_n^j} \Big)^{\alpha_n} (C_{n,t}^{E})^{1-\alpha_n} = 0
\end{aligned}$$

- Simplify the first equation:

$$(1+\tau_{n,t}^{j})P_{n,t}^{j} = \lambda \cdot \mathcal{D}_n(\mathcal{C}_t) \cdot \alpha_n \cdot \alpha_n^j \cdot \Big(\prod_{j' \in \mathcal{J}_D} (C_{n,t}^{j'})^{\alpha_n^{j'}} \Big)^{\alpha_n} \cdot (C_{n,t}^{j})^{-1} (C_{n,t}^{E})^{1-\alpha_n}$$

$$(1+\tau_{n,t}^{j})P_{n,t}^{j} = \lambda \cdot \mathcal{D}_n(\mathcal{C}_t) \cdot \alpha_n \cdot \alpha_n^j \cdot Y_{n,t} \cdot (C_{n,t}^{j})^{-1}$$

- Rearrange the equations to solve for $C_{n,t}^{j}$, $C_{n,t}^{E}$:

$$C_{n,t}^{j} = Y_{n,t} \cdot \frac{\alpha_n \cdot \alpha_n^j}{(1+\tau_{n,t}^{j})P_{n,t}^{j}} \cdot \lambda$$

$$C_{n,t}^{E} = Y_{n,t} \cdot \frac{(1-\alpha_n)}{P_{n,t}^{E}} \cdot \lambda$$

- Sum the demand equations and solve for λ:

$$Y_{n,t} = \mathcal{D}_n(\mathcal{C}_t)\Big(\prod_{j \in \mathcal{J}_D} (C_{n,t}^{j})^{\alpha_n^j} \Big)^{\alpha_n} (C_{n,t}^{E})^{1-\alpha_n}$$

$$Y_{n,t} = \lambda Y_{n,t} \mathcal{D}_n(\mathcal{C}_t)\Big(\prod_{j \in \mathcal{J}_D} \Big(\frac{\alpha_n \cdot \alpha_n^j}{(1+\tau_{n,t}^{j})P_{n,t}^{j}} \cdot \Big)^{\alpha_n^j} \Big)^{\alpha_n} \Big(\frac{(1-\alpha_n)}{P_{n,t}^{E}} \Big)^{1-\alpha_n}$$

$$1 = \lambda \mathcal{D}_n(\mathcal{C}_t)\Big(\prod_{j \in \mathcal{J}_D} \Big(\frac{\alpha_n \cdot \alpha_n^j}{(1+\tau_{n,t}^{j})P_{n,t}^{j}} \cdot \Big)^{\alpha_n^j} \Big)^{\alpha_n} \Big(\frac{(1-\alpha_n)}{P_{n,t}^{E}} \Big)^{1-\alpha_n}$$

$$\lambda = \Big(\mathcal{D}_n(\mathcal{C}_t) \Big)^{-1} \Big(\prod_{j \in \mathcal{J}_D} \Big(\frac{(1+\tau_{n,t}^{j})P_{n,t}^{j}}{\alpha_n \cdot \alpha_n^j} \cdot \Big)^{\alpha_n^j} \Big)^{\alpha_n} \Big(\frac{P_{n,t}^{E}}{(1-\alpha_n)} \Big)^{1-\alpha_n}$$

- Define the price index:

$$P_{n,t} = (\mathcal{D}_n(\mathcal{C}_t))^{-1} \left(\prod_{j \in \mathcal{J}_D} \Big((1+\tau_{n,t}^{j})P_{n,t}^{j} \Big)^{\alpha_n^j} \right)^{\alpha_n} \big(P_{n,t}^{E} \big)^{1-\alpha_n}$$

- Substitute λ back into the demand equation:

$$C_{n,t}^{j} = Y_{n,t} \cdot \frac{\alpha_n \cdot \alpha_n^j P_{n,t}}{(1+\tau_{n,t}^{j})P_{n,t}^{j}}$$
$$C_{n,t}^{E} = Y_{n,t} \cdot \frac{(1-\alpha_n)P_{n,t}}{P_{n,t}^{E}}$$

A.5 Brown Variety

- Problem: Minimize

$$w_{n,t}^{B} L_{mn,t}^{B} + r_{n,t}^{B} K_{mn,t}^{B} + s_{n,t} F_{mn,t}^{B} + P_{n,t}^{M,B} M_{mn,t}^{B}$$

subject to the constraint:

$$y_{mn,t}^{B} = (\frac{z_{n,t}^{B}}{d_{mn,t}^{B}})((F_{mn,t}^{B})^{\psi_n^{B,F}}(L_{mn,t}^{B})^{\psi_n^{B,L}}(K_{mn,t}^{B})^{\psi_n^{B,K}})^{\gamma_n^{B,V}}(M_{mn,t}^{B})^{1-\gamma_n^{B,V}}$$

- First-order conditions:

$$L_{mn,t}^{B} = \lambda_{n,t} \cdot y_{mn,t}^{B}(\frac{\gamma_n^{B,V} \cdot \psi_n^{B,L}}{w_{n,t}^{B}})$$
$$K_{mn,t}^{B} = \lambda_{n,t} \cdot y_{mn,t}^{B}(\frac{\gamma_n^{B,V} \cdot \psi_n^{B,K}}{r_{n,t}^{B}})$$
$$F_{mn,t}^{B} = \lambda_{n,t} \cdot y_{mn,t}^{B}(\frac{\gamma_n^{B,V} \cdot \psi_n^{B,F}}{s_{n,t}})$$
$$M_{mn,t}^{B} = \lambda_{n,t} \cdot y_{mn,t}^{B}(\frac{1-\gamma_n^{B,V}}{P_{n,t}^{M,B}})$$

- Lagrangian multiplier equals marginal cost:

$$\lambda_{n,t} \cdot y_{mn,t}^{B} = w_{n,t}^{B} L_{mn,t}^{B} + r_{n,t}^{B} K_{mn,t}^{B} + s_{n,t} F_{mn,t}^{B} + P_{n,t}^{M,B} M_{mn,t}^{B}$$
$$\lambda_{n,t} = \frac{1}{y_{mn,t}^{B}}(w_{n,t}^{B} L_{mn,t}^{B} + r_{n,t}^{B} K_{mn,t}^{B} + s_{n,t} F_{mn,t}^{B} + P_{n,t}^{M,B} M_{mn,t}^{B})$$

- Plugging into the constraint:

$$1 = \lambda_{n,t} \cdot (\frac{z^B_{n,t}}{d^B_{mn,t}})((\frac{\gamma^{B,V}_n \cdot \psi^{B,F}_n}{s_{n,t}})^{\psi^{B,F}_n}(\frac{\gamma^{B,V}_n \cdot \psi^{B,L}_n}{w^B_{n,t}})^{\psi^{B,L}_n}(\frac{\gamma^{B,V}_n \cdot \psi^{B,K}_n}{r^B_{n,t}})^{\psi^{B,K}_n})^{\gamma^{B,V}_n}(\frac{1-\gamma^{B,V}_n}{P^{M,B}_{n,t}})^{1-\gamma^{B,V}_n}$$

$$\lambda_{n,t} = (\frac{d^B_{mn,t}}{z^B_{n,t}})((\frac{s_{n,t}}{\gamma^{B,V}_n \cdot \psi^{B,F}_n})^{\psi^{B,F}_n}(\frac{w^B_{n,t}}{\gamma^{B,V}_n \cdot \psi^{B,L}_n})^{\psi^{B,L}_n}(\frac{r^B_{n,t}}{\gamma^{B,V}_n \cdot \psi^{B,K}_n})^{\psi^{B,K}_n})^{\gamma^{B,V}_n}(\frac{P^{M,B}_{n,t}}{1-\gamma^{B,V}_n})^{1-\gamma^{B,V}_n}$$

$$mc^B_{mn,t} = (\frac{d^B_{mn,t}}{z^B_{n,t}})((\frac{s_{n,t}}{\gamma^{B,V}_n \cdot \psi^{B,F}_n})^{\psi^{B,F}_n}(\frac{w^B_{n,t}}{\gamma^{B,V}_n \cdot \psi^{B,L}_n})^{\psi^{B,L}_n}(\frac{r^B_{n,t}}{\gamma^{B,V}_n \cdot \psi^{B,K}_n})^{\psi^{B,K}_n})^{\gamma^{B,V}_n}(\frac{P^{M,B}_{n,t}}{1-\gamma^{B,V}_n})^{1-\gamma^{B,V}_n}$$

A.6 Non-Brown Energy and Non-energy Varieties

- Problem: Minimize

$$w^j_{n,t}L^j_{mn,t} + r^j_{n,t}K^j_{mn,t} + P^E_{n,t}Q^j_{mn,t} + P^{M,j}_{n,t}M^j_{mn,t}$$

subject to the constraint:

$$y^j_{mn,t} = (\frac{z^j_{n,t}}{d^j_{mn,t}})[(Q^j_{mn,t})^{\psi^{j,E}_n}(L^j_{mn,t})^{\psi^{j,L}_n}(K^j_{mn,t})^{\psi^{j,K}_n}]^{\gamma^{j,V}_n}(M^j_{mn,t})^{1-\gamma^{j,V}_n}$$

- First-order conditions:

$$L^j_{mn,t} = \lambda_{n,t} \cdot y^j_{mn,t}(\frac{\gamma^{j,V}_n \cdot \psi^{j,L}_n}{w^j_{n,t}})$$

$$K^j_{mn,t} = \lambda_{n,t} \cdot y^j_{mn,t}(\frac{\gamma^{j,V}_n \cdot \psi^{j,K}_n}{r^j_{n,t}})$$

$$Q^j_{mn,t} = \lambda_{n,t} \cdot y^j_{mn,t}(\frac{\gamma^{j,V}_n \cdot \psi^{j,E}_n}{P^E_{n,t}})$$

$$M^j_{mn,t} = \lambda_{n,t} \cdot y^j_{mn,t}(\frac{1-\gamma^{j,V}_n}{P^{M,j}_{n,t}})$$

- Lagrangian multiplier equals marginal cost:

$$\lambda_{n,t} \cdot y^j_{mn,t} = w^j_{n,t} L^j_{mn,t} + r^j_{n,t} K^j_{mn,t} + P^E_{n,t} Q^j_{mn,t} + P^{M,j}_{n,t} M^j_{mn,t}$$
$$\lambda_{n,t} = \frac{1}{y^j_{mn,t}} (w^j_{n,t} L^j_{mn,t} + r^j_{n,t} K^j_{mn,t} + P^E_{n,t} Q^j_{mn,t} + P^{M,j}_{n,t} M^j_{mn,t})$$

- Plugging into the constraint:

$$1 = \lambda_{n,t} \cdot (\frac{z^j_{n,t}}{d^j_{mn,t}}) \cdot [(\frac{\gamma^{j,V}_n \cdot \psi^{j,E}_n}{P^E_{n,t}})^{\psi^{j,E}_n} (\frac{\gamma^{j,V}_n \cdot \psi^{j,L}_n}{w^j_{n,t}})^{\psi^{j,L}_n} (\frac{\gamma^{j,V}_n \cdot \psi^{j,K}_n}{r^j_{n,t}})^{\psi^{j,K}_n}]^{\gamma^{j,V}_n} (\frac{1-\gamma^{j,V}_n}{P^{M,j}_{n,t}})^{1-\gamma^{j,V}_n}$$
$$\lambda_{n,t} = (\frac{d^j_{mn,t}}{z^j_{n,t}}) \cdot [(\frac{P^E_{n,t}}{\gamma^{j,V}_n \cdot \psi^{j,E}_n})^{\psi^{j,E}_n} (\frac{w^j_{n,t}}{\gamma^{j,V}_n \cdot \psi^{j,L}_n})^{\psi^{j,L}_n} (\frac{r^j_{n,t}}{\gamma^{j,V}_n \cdot \psi^{j,K}_n})^{\psi^{j,K}_n}]^{\gamma^{j,V}_n} (\frac{P^{M,j}_{n,t}}{1-\gamma^{j,V}_n})^{1-\gamma^{j,V}_n}$$
$$mc^j_{mn,t} = (\frac{d^j_{mn,t}}{z^j_{n,t}}) \cdot [(\frac{P^E_{n,t}}{\gamma^{j,V}_n \cdot \psi^{j,E}_n})^{\psi^{j,E}_n} (\frac{w^j_{n,t}}{\gamma^{j,V}_n \cdot \psi^{j,L}_n})^{\psi^{j,L}_n} (\frac{r^j_{n,t}}{\gamma^{j,V}_n \cdot \psi^{j,K}_n})^{\psi^{j,K}_n}]^{\gamma^{j,V}_n} (\frac{P^{M,j}_{n,t}}{1-\gamma^{j,V}_n})^{1-\gamma^{j,V}_n}$$

A.6.1 Labor Mobility Share

- Expected lifetime utility of a worker in sector j at time t

$$V^j_{n,t} = \ln(c^j_{n,t}) + \log(\mu^j_n) + \max_{k \in \mathcal{J}} \left\{ \beta V^k_{n,t+1} + \ln(1-\kappa^{j,k}_n) + \rho_L \varepsilon^k_{n,t} \right\}$$

- Denote $A^{j,k}_{n,t} \equiv \beta V^k_{n,t+1} + \ln(1-\kappa^{j,k}_n)$

- Decision variable

$$D(\vec{\varepsilon}_{n,t}, j, k) = \mathbb{1}\left[k = \arg\max_{h \in \mathcal{J}} \left\{ A^{j,h}_{n,t} + \rho_L \varepsilon^h_{n,t} \right\} \right]$$

- Gumbel cumulative distribution function:

$$F(x) = \exp(-\exp(-(x+\gamma)))$$

  where γ is a Euler–Mascheroni constant

- Gumble probability density function: $f(x) = \exp(-x-\gamma)\exp(-e^{-x-\gamma})$

- Probability of choosing sector k

$$
\begin{aligned}
\Pr\Big(D(\vec{\varepsilon}_{n,t}, j, k) = 1\Big) &= \Pr\Big(A^{j,k}_{n,t} + \rho_L \varepsilon^k_{n,t} \geq \max\{A^{j,h}_{n,t} + \rho_L \varepsilon^h_{n,t}\Big\}\Big) \\
&= \int_{-\infty}^{\infty} f(\varepsilon^k_{n,t}) \prod_{h \neq k} F\Big(\frac{A^{j,k}_{n,t} - A^{j,h}_{n,t}}{\rho_L} + \varepsilon^k_{n,t}\Big) d\varepsilon^k_{n,t} \\
&= \int_{-\infty}^{\infty} e^{-\varepsilon^k_{n,t} - \gamma} e^{-\exp(-\varepsilon^k_{n,t} - \gamma)} \times \prod_{h \neq k} e^{-\exp(-(\frac{A^{j,k}_{n,t} - A^{j,h}_{n,t}}{\rho_L}) - \varepsilon^k_{n,t} - \gamma)} \\
&= \int_{-\infty}^{\infty} \exp(-\varepsilon^k_{n,t} - \gamma) \\
&\times \exp\Big(- \exp(-\varepsilon^k_{n,t} - \gamma) \times \sum_{h \in \mathcal{J}} \exp(\frac{A^{j,h}_{n,t} - A^{j,k}_{n,t}}{\rho_L})\Big) d\varepsilon^k_{n,t}
\end{aligned}
$$

- Change of variables: $x = \varepsilon^k_{n,t} + \gamma$, $y = \log\Big(\sum_{h \in \mathcal{J}} \exp(\frac{A^{j,h}_{n,t} - A^{j,k}_{n,t}}{\rho_L})\Big)$
- Given that $dx = d\varepsilon^k_{n,t}$,

$$
\Pr\Big(D(\vec{\varepsilon}_{n,t}, j, k) = 1\Big) = \int_{-\infty}^{\infty} e^{-x - e^{y-x}} dx
$$

- Change of variable: $z = x - y$ and $dz = dx$,

$$
\begin{aligned}
\Pr\Big(D(\vec{\varepsilon}_{n,t}, j, k) = 1\Big) &= \int_{-\infty}^{\infty} e^{-z - y - e^{-z}} dz = e^{-y} \int_{-\infty}^{\infty} e^{-z - e^{-z}} dz \\
&= e^{-y} = \Big(\sum_{h \in \mathcal{J}} \exp(A^{j,h}_{n,t} - A^{j,k}_{n,t})^{\frac{1}{\rho_L}}\Big)^{-1} \\
&= \frac{\exp(A^{j,k}_{n,t})^{\frac{1}{\rho_L}}}{\sum_{h \in \mathcal{J}} \exp(A^{j,h}_{n,t})^{\frac{1}{\rho_L}}} \\
&= \frac{\exp(\beta V^k_{n,t+1} + \ln(1 - \kappa^{j,k}_n))^{\frac{1}{\rho_L}}}{\sum_{h \in \mathcal{J}} \exp(\beta V^h_{n,t+1} + \ln(1 - \kappa^{j,h}_n))^{\frac{1}{\rho_L}}}
\end{aligned}
$$

- Therefore, since $m_{n,t}^{j,k} = \Pr\Big(D(\vec{\varepsilon}_{n,t}, j, k) = 1\Big)$,

$$m_{n,t}^{j,k} = \frac{\exp(\beta V_{n,t+1}^{k} + \ln(1-\kappa_{n}^{j,k}))^{\frac{1}{\rho_L}}}{\sum_{h\in\mathcal{J}} \exp(\beta V_{n,t+1}^{h} + \ln(1-\kappa_{n}^{j,h}))^{\frac{1}{\rho_L}}}$$

A.6.2 Expected Life-time Utility of a Worker

- Expected future value of choosing sector k for a worker in sector j

$$\begin{aligned}
\tilde{V}_{n,t+1}^{j,k} &= \int_{-\infty}^{\infty} \Big(A_{n,t}^{j,k} + \rho_L \varepsilon_{n,t}^{k}\Big) f(\varepsilon_{n,t}^{k}) \prod_{h\neq k} F\Big(\frac{A_{n,t}^{j,k} - A_{n,t}^{j,h}}{\rho_L} + \varepsilon_{n,t}^{k}\Big) d\varepsilon_{n,t}^{k} \\
&= \int_{-\infty}^{\infty} \Big(A_{n,t}^{j,k} + \rho_L \varepsilon_{n,t}^{k}\Big) \times \exp(-\varepsilon_{n,t}^{k} - \gamma) \\
&\times \exp\Big(- \exp(-\varepsilon_{n,t}^{k} - \gamma) \times \sum_{h\in\mathcal{J}} \exp(\frac{A_{n,t}^{j,h} - A_{n,t}^{j,k}}{\rho_L})\Big) d\varepsilon_{n,t}^{k}
\end{aligned}$$

- Change of variables: $x = \varepsilon_{n,t}^{k} + \gamma$, $y = \log\Big(\sum_{h\in\mathcal{J}} \exp(\frac{A_{n,t}^{j,h} - A_{n,t}^{j,k}}{\rho_L})\Big)$

$$\tilde{V}_{n,t+1}^{j,k} = \int_{-\infty}^{\infty} \Big(A_{n,t}^{j,k} + \rho_L (x-\gamma)\Big) \exp(-x - e^{y-x}) dx$$

- Change of variable: $z = x - y$ and $dz = dx$,

$$\begin{aligned}
\tilde{V}_{n,t+1}^{j,k} &= \int_{-\infty}^{\infty} \Big(A_{n,t}^{j,k} + \rho_L (z + y - \gamma)\Big) \exp(-(z+y) - e^{-z}) dz \\
&= A_{n,t}^{j,k} e^{-y} \int_{-\infty}^{\infty} \exp(-z - e^{-z}) dz \\
&+ \rho_L e^{-y} \int_{-\infty}^{\infty} z \exp(-z - e^{-z}) dz \\
&+ \rho_L e^{-y} (y-\gamma) \int_{-\infty}^{\infty} \exp(-z - e^{-z}) dz \\
&= A_{n,t}^{j,k} e^{-y} + \rho_L e^{-y} \gamma + \rho_L e^{-y} (y - \gamma) \\
&= e^{-y} (A_{n,t}^{j,k} + \rho_L y)
\end{aligned}$$

- Re-arrange the expression for y

$$
\begin{aligned}
y &= \log\Big(\exp(\frac{-A^{j,k}_{n,t}}{\rho_L})\sum_{h\in\mathcal{J}}\exp(\frac{A^{j,h}_{n,t}}{\rho_L})\Big) \\
&= -\Big(\frac{A^{j,k}_{n,t}}{\rho_L}\Big) + \log\Big(\sum_{h\in\mathcal{J}}\exp(\frac{A^{j,h}_{n,t}}{\rho_L})\Big)
\end{aligned}
$$

- Evaluate e^{-y}

$$
e^{-y} = \exp\Big(\frac{A^{j,k}_{n,t}}{\rho_L}\Big)\Big(\sum_{h\in\mathcal{J}}\exp(\frac{A^{j,h}_{n,t}}{\rho_L})\Big)^{-1} = m^{j,k}_{n,t}
$$

- Plugging in the definition y,

$$
\begin{aligned}
\tilde{V}^{j,k}_{n,t+1} &= e^{-y}\Big(A^{j,k}_{n,t} - A^{j,k}_{n,t} + \rho_L \log\Big(\sum_{h\in\mathcal{J}}\exp(\frac{A^{j,h}_{n,t}}{\rho_L})\Big)\Big) \\
&= e^{-y}\Big(\rho_L \log\Big(\sum_{h\in\mathcal{J}}\exp(\frac{A^{j,h}_{n,t}}{\rho_L})\Big)\Big) \\
&= m^{j,k}_{n,t}\Big(\rho_L \log\Big(\sum_{h\in\mathcal{J}}\exp(\frac{A^{j,h}_{n,t}}{\rho_L})\Big)\Big)
\end{aligned}
$$

- Expected future value of worker in sector j

$$
\begin{aligned}
\tilde{V}^{j}_{n,t+1} &= \max_{k\in\mathcal{J}}\{\beta V^{k}_{n,t+1} + \ln(1-\kappa^{j,k}_{n}) + \rho_L \varepsilon^{k}_{n,t}\} \\
&= \sum_{k\in\mathcal{J}} V^{j,k}_{n,t+1} = \sum_{k\in\mathcal{J}} m^{j,k}_{n,t}\Big(\rho_L \log\Big(\sum_{h\in\mathcal{J}}\exp(\frac{A^{j,h}_{n,t}}{\rho_L})\Big)\Big) \\
&= \rho_L \log\Big(\sum_{h\in\mathcal{J}}\exp(\frac{A^{j,h}_{n,t}}{\rho_L})\Big) \\
&= \rho_L \ln\left(\sum_{k\in\mathcal{J}}\exp\Big((\frac{\beta}{\rho_L})V^{k}_{n,t+1} + (\frac{1}{\rho_L})\ln(1-\kappa^{j,k}_{n})\Big)\right)
\end{aligned}
$$

A.6.3 Fossil Fuel Extraction Problem

- Cost Minimization Problem

$$\min_{f^o_{n,t}, f^g_{n,t}, f^c_{n,t}} \quad (1+\tau^o_{n,t})s^o_{n,t}f^o_{n,t} + (1+\tau^g_{n,t})s^g_{n,t}f^g_{n,t} + (1+\tau^c_{n,t})s^c_{n,t}f^c_{n,t}$$

$$\text{s.t.}$$

$$F_{n,t} = \left(\sum_h (\omega^h_n)^{\frac{1}{\eta_2}} (f^h_{n,t})^{\frac{\eta_2-1}{\eta_2}} \right)^{\frac{\eta_2}{\eta_2-1}}$$

- First order condition

$$(1+\tau^h_{n,t})s^h_{n,t} = \lambda_{n,t}(\omega^h_n)^{\frac{1}{\eta_2}} (f^h_{n,t})^{\frac{-1}{\eta_2}} \left(\sum_h (\omega^h_n)^{\frac{1}{\eta_2}} (f^h_{n,t})^{\frac{\eta_2-1}{\eta_2}} \right)^{\frac{\eta_2}{\eta_2-1}-1}$$

$$\frac{(1+\tau^h_{n,t})s^h_{n,t}}{(1+\tau^k_{n,t})s^k_{n,t}} = \left(\frac{\omega^h_n}{\omega^k_n}\right)^{\frac{1}{\eta_2}} \left(\frac{f^h_n}{f^k_n}\right)^{-\frac{1}{\eta_2}}$$

- Re-arranging the equations, fossil fuel demand is obtained as follows:

$$f^h_{n,t} = f^k_{n,t}\left(\frac{\omega^h_n}{\omega^k_n}\right)\left(\frac{(1+\tau^k_{n,t})s^k_{n,t}}{(1+\tau^h_{n,t})s^h_{n,t}}\right)^{\eta_2}$$

$$F_{n,t} = f^k_{n,t}(\omega^k_n)^{-1}((1+\tau^k_{n,t})s^k_{n,t})^{\eta_2} \left(\sum_h (\omega^h_n)((1+\tau^h_{n,t})s^h_{n,t})^{1-\eta_2} \right)^{\frac{\eta_2}{\eta_2-1}}$$

$$f^k_{n,t} = \omega^k_n \left(\frac{(1+\tau^k_{n,t})s^k_{n,t}}{\left(\sum_h (\omega^h_n)((1+\tau^h_{n,t})s^h_{n,t})^{1-\eta_2}\right)^{\frac{1}{1-\eta_2}}} \right)^{-\eta_2} F_{n,t}$$

- Price of fossil fuel input is defined as follows:

$$s_{n,t} = \left(\sum_h \omega^h_n ((1+\tau^h_{n,t})s^h_{n,t})^{1-\eta_2} \right)^{\frac{1}{1-\eta_2}}$$

- Then, the fossil fuel demand policy function is given by:

$$f^k_{n,t} = \omega^k_n \Big(\frac{(1+\tau^k_{n,t})s^k_{n,t}}{s_{n,t}}\Big)^{-\eta_2} F_{n,t}$$

with

$$\lambda_{n,t} = \sum_h (1+\tau^h_{n,t})s^h_{n,t} f^h_{n,t} = \frac{\sum_h \omega^h_n((1+\tau^h_{n,t})s^h_{n,t})^{1-\eta_2}}{(s_{n,t})^{-\eta_2}} F_{n,t}$$

A.6.4 Capitalist Optimization Problem

- Utility Maximization Problem

$$V^K_{n,0}(W_{n,0}) = \max_{W_{n,t+1}} \quad \sum_{t=0}^{\infty} \beta^t \mathbb{E}_0 \Big[u(C^K_{n,t})\Big]$$

s.t.

$$C^K_{n,t} + W_{n,t+1} = R_{n,t}W_{n,t} + \frac{\Pi_{n,t} + \Omega_{n,t}}{P_{n,t}},$$

- Return on Aggregate Capital: $R_{n,t} = \Big(\frac{v_{n,t}}{P_{n,t}} + (1-\delta^K_n)\Big)$

- First order conditions

$$\begin{aligned}
\frac{\beta^t}{C^K_{n,t}} &= \lambda_{n,t}P_{n,t} \\
\lambda_{n,t+1}\Big(v_{n,t+1} + (1-\delta^K_n)P_{n,t+1}\Big) &= \lambda_{n,t}P_{n,t} \\
\frac{\lambda_{n,t+1}P_{n,t+1}}{\lambda_{n,t}P_{n,t}}\Big(\frac{v_{n,t+1}}{P_{n,t+1}} + (1-\delta^K_n)\Big) &= 1 \\
\Big(\frac{v_{n,t+1}}{P_{n,t+1}} + (1-\delta^K_n)\Big) &= \frac{C^K_{n,t+1}}{\beta C^K_{n,t}}
\end{aligned}$$

- Optimality condition

$$C^K_{n,t+1} = \beta R_{n,t+1} C^K_{n,t}$$

- Present discounted value of future non-capital income

$$
\begin{aligned}
h_{n,t} &= \sum_{s=1}^{\infty} \Big(\prod_{u=1}^{s} R_{n,t+u} \Big)^{-1} \Big(\frac{\Pi_{n,t+s} + \Omega_{n,t+s}}{P_{n,t+s}} \Big) \\
&= \Big(R_{n,t+1} \Big)^{-1} \Big(\frac{\Pi_{n,t+1} + \Omega_{n,t+1}}{P_{n,t+1}} \Big) + \Big(R_{n,t+1} \Big)^{-1} \sum_{s=1}^{\infty} \Big(\prod_{u=1}^{s} R_{n,t+1+u} \Big)^{-1} \Big(\frac{\Pi_{n,t+1+s} + \Omega_{n,t+s}}{P_{n,t+1+s}} \Big) \\
&= \Big(R_{n,t+1} \Big)^{-1} \Big(\frac{\Pi_{n,t+1} + \Omega_{n,t+1}}{P_{n,t+1}} \Big) + \Big(R_{n,t+1} \Big)^{-1} h_{n,t+1}
\end{aligned}
$$

- Conjecture (guess and verify)

$$
\begin{aligned}
C^K_{n,t} &= (1-\beta)\Big(R_{n,t} W_{n,t} + \Big(\frac{\Pi_{n,t} + \Omega_{n,t}}{P_{n,t}} \Big) + h_{n,t} \Big) \\
W_{n,t+1} &= \beta\Big(R_{n,t} W_{n,t} + \Big(\frac{\Pi_{n,t} + \Omega_{n,t}}{P_{n,t}} \Big) + h_{n,t} \Big) - h_{n,t} \\
W_{n,t+1} &= R_{n,t} W_{n,t} + \Big(\frac{\Pi_{n,t} + \Omega_{n,t}}{P_{n,t}} \Big) - C^K_{n,t}
\end{aligned}
$$

- Check optimality condition

$$
\begin{aligned}
C^K_{n,t} &= \Big(\beta R_{n,t+1} \Big)^{-1} C^K_{n,t+1} \\
\Big(R_{n,t} W_{n,t} + \Big(\frac{\Pi_{n,t} + \Omega_{n,t}}{P_{n,t}} \Big) + h_{n,t} \Big) &= \Big(\beta R_{n,t+1} \Big)^{-1} \Big(R_{n,t+1} W_{n,t+1} + \Big(\frac{\Pi_{n,t+1} + \Omega_{n,t+1}}{P_{n,t+1}} \Big) + h_{n,t+1} \Big) \\
\Big(R_{n,t} W_{n,t} + \Big(\frac{\Pi_{n,t} + \Omega_{n,t}}{P_{n,t}} \Big) + h_{n,t} \Big) &= \beta^{-1} W_{n,t+1} + \Big(\beta R_{n,t+1} \Big)^{-1} \Big(\Big(\frac{\Pi_{n,t+1} + \Omega_{n,t+1}}{P_{n,t+1}} \Big) + h_{n,t+1} \Big) \\
(\beta)^{-1} h_{n,t} &= \Big(\beta R_{n,t+1} \Big)^{-1} \Big(\Big(\frac{\Pi_{n,t+1} + \Omega_{n,t+1}}{P_{n,t+1}} \Big) + h_{n,t+1} \Big) \\
R_{n,t+1} h_{n,t} &= \Big(\frac{\Pi_{n,t+1} + \Omega_{n,t+1}}{P_{n,t+1}} \Big) + h_{n,t+1} \\
\frac{\Pi_{n,t+1} + \Omega_{n,t+1}}{P_{n,t+1}} &= R_{n,t+1} h_{n,t} - h_{n,t+1}
\end{aligned}
$$

A.6.5 Capital Allocation Problem

- Decision variable for Aggregate Capital allocation

$$D(k, \vec{\zeta}_{n,t}) = \mathbf{1}\Big[k = \arg\max_{h\in\mathcal{J}} \Big\{\zeta^h_{n,t} P_{n,t} r^h_{n,t}\Big\}\Big]$$

- Fréchet distribution for idiosyncratic capital use efficiency $\zeta^j_{n,t}$

$$\begin{aligned} F^j(\zeta^j_{n,t}) &= e^{-((\zeta^j_{n,t})/a^j_n)^{-\rho_K}} \\ f^j(\zeta^j_{n,t}) &= \Big(\rho_K (a^j_{n,t})^{\rho_K} (\zeta^j_{n,t})^{-\rho_K - 1}\Big) e^{-((\zeta^j_{n,t})/a^j_n)^{-\rho_K}} \end{aligned}$$

- Probability of investing in sector j

$$\begin{aligned} \text{Pr(invest in sector } j) &= \text{Pr}\Big(j = \arg\max\{P_{n,t}(\zeta^j_{n,t}) a^j_{n,t}\}\Big) \\ &= \int_0^\infty \Big(\prod_{k\neq j} F^k\Big(\frac{r^j_{n,t}(\zeta^j_{n,t})}{r^k_{n,t}}\Big)\Big) f^j(\zeta^j_{n,t}) d(\zeta^j_{n,t}) \\ &= \int_0^\infty \Big(\prod_{k\neq j} e^{-\left(\frac{r^j_{n,t}(\zeta^j_{n,t})}{r^k_{n,t}a^k_{n,t}}\right)^{-\rho_K}}\Big)\Big(\rho_K (a^j_{n,t})^{\rho_K}(\zeta^j_{n,t})^{-\rho_K-1}\Big) e^{-((\zeta^j_{n,t})/a^j_n)^{-\rho_K}} d(\zeta^j_{n,t}) \\ &= \int_0^\infty \rho_K (a^j_{n,t})^{\rho_K} (\zeta^j_{n,t})^{-\rho_K - 1} e^{-\left(\sum_{k\neq j}\left(\frac{r^j_{n,t}}{r^k_{n,t}a^k_{n,t}}\right)^{-\rho_K} + (a^j_{n,t})^{\rho_K}\right)(\zeta^j_{n,t})^{-\rho_K}} d(\zeta^j_{n,t}) \end{aligned}$$

- Define variable

$$\begin{aligned} \alpha &= \sum_{k\neq j}\Big(\frac{r^j_{n,t}}{r^k_{n,t}a^k_{n,t}}\Big)^{-\rho_K} + (a^j_{n,t})^{\rho_K} \\ \alpha &= \sum_{k\neq j}\Big(r^k_{n,t}a^k_{n,t}\Big)^{\rho_K}(r^j_{n,t})^{-\rho_K} + (a^j_{n,t})^{\rho_K} \\ \alpha &= (r^j_{n,t})^{-\rho_K}\Big(\sum_{k\neq j}(r^k_{n,t}a^k_{n,t})^{\rho_K} + (r^j_{n,t}a^j_{n,t})^{\rho_K}\Big) \\ \alpha &= (r^j_{n,t})^{-\rho_K}\Big(\sum_{k}(r^k_{n,t}a^k_{n,t})^{\rho_K}\Big) \end{aligned}$$

- Simplification of the integral

$$
\begin{aligned}
\Pr(\text{invest in sector } j) &= \int_0^\infty \rho_K (a_{n,t}^j)^{\rho_K} (\zeta_{n,t}^j)^{-\rho_K - 1} e^{-\alpha (\zeta_{n,t}^j)^{-\rho_K}} d(\zeta_{n,t}^j) \\
&= (a_{n,t}^j)^{\rho_K} \int_0^\infty e^{-\alpha u} du \quad \text{(using substitution } u = (\zeta_{n,t}^j)^{-\rho_K}\text{)} \\
&= (a_{n,t}^j)^{\rho_K} \left[-\frac{1}{\alpha} e^{-\alpha u} \right]_0^\infty \\
&= \frac{(a_{n,t}^j)^{\rho_K}}{\alpha}
\end{aligned}
$$

- Probability of investing in sector j

$$
\Phi_{n,t}^j = \Pr(\text{invest in sector } j) = \frac{(r_{n,t}^j a_{n,t}^j)^{\rho_K}}{\sum_{k \in \mathcal{J}} (r_{n,t}^k a_{n,t}^k)^{\rho_K}}
$$

- Denote $W_{n,t}^j$ aggregate capital allocated to sector j, i.e., $W_{n,t}^j \equiv \Phi_{n,t}^j W_{n,t}$
- Define return $v_{n,t}^j = \zeta_{n,t}^j P_{n,t} r_{n,t}^j$. Then, $\zeta_{n,t}^j = \frac{v_{n,t}^j}{P_{n,t} r_{n,t}^j}$.
- Distribution of return in sector j

$$
\begin{aligned}
\Psi_{n,t}^j &\equiv (a_{n,t}^j P_{n,t} r_{n,t}^j)^{\rho_K} \\
F^j(v) &= e^{-\Psi_{n,t}^j v^{-\rho_K}} \\
f^j(\zeta_{n,t}^j) &= \left(\rho_K \Psi_{n,t}^j v^{-\rho_K - 1} \right) e^{-\Psi_{n,t}^j v^{-\rho_K}}
\end{aligned}
$$

- Distribution of return across all industries

$$
\begin{aligned}
\Psi_{n,t} &\equiv \sum_{j \in \mathcal{J}} (a_{n,t}^j P_{n,t} r_{n,t}^j)^{\rho_K} \\
F(v) &= \prod_{j \in \mathcal{J}} F^j(v) = e^{-\Psi_{n,t} v^{-\rho_K}} \\
f(v) &= \left(\rho_K \Psi_{n,t} v^{-\rho_K - 1} \right) e^{-\Psi_{n,t} v^{-\rho_K}}
\end{aligned}
$$

- Expected return

$$
\begin{aligned}
v_{n,t} &= \int_0^\infty v f(v) dv \\
&= \int_0^\infty v\Big(\rho_K \Psi_{n,t} v^{-\rho_K - 1}\Big) e^{-\Psi_{n,t} v^{-\rho_K}} dv \\
&= \int_0^\infty \Big(\rho_K \Psi_{n,t} v^{-\rho_K}\Big) e^{-\Psi_{n,t} v^{-\rho_K}} dv
\end{aligned}
$$

- Change of variable: $y = \Psi_{n,t} v^{-\rho_K}$, $v = \Psi_{n,t}^{\frac{1}{\rho_K}} y^{-\frac{1}{\rho_K}}$, $dy = (-\rho_K)\Psi_{n,t} v^{-\rho_K - 1} dv$

$$
\begin{aligned}
v_{n,t} &= \int_0^\infty v e^{-y} dy \\
&= \int_0^\infty \Psi_{n,t}^{\frac{1}{\rho_K}} y^{-\frac{1}{\rho_K}} e^{-y} dy \\
&= \Psi_{n,t}^{\frac{1}{\rho_K}} \int_0^\infty y^{(1-\frac{1}{\rho_K})-1} e^{-y} dy \\
&= \Gamma\Big(1 - \frac{1}{\rho_K}\Big) \Psi_{n,t}^{\frac{1}{\rho_K}} \\
v_{n,t} &= \Gamma\Big(1 - \frac{1}{\rho_K}\Big)\Big(\sum_{j \in \mathcal{J}} (a_{n,t}^j P_{n,t} r_{n,t}^j)^{\rho_K}\Big)^{\frac{1}{\rho_K}}
\end{aligned}
$$

- Therefore, the real return on Aggregate Capital is given by:

$$
\frac{v_{n,t}}{P_{n,t}} = \Gamma\Big(1 - \frac{1}{\rho_K}\Big)\Big(\sum_{j \in \mathcal{J}} (a_{n,t}^j r_{n,t}^j)^{\rho_K}\Big)^{\frac{1}{\rho_K}}
$$

- The allocation share can be expressed as follows:

$$
\Phi_{n,t}^j = \frac{(r_{n,t}^j a_{n,t}^j)^{\rho_K}}{\sum_{k \in \mathcal{J}} (r_{n,t}^k a_{n,t}^k)^{\rho_K}} = \left(\Gamma\Big(\frac{\rho_K - 1}{\rho_K}\Big)\right)^{\rho_K} \Big(\frac{P_{n,t} r_{n,t}^j a_{n,t}^j}{v_{n,t}}\Big)^{\rho_K}
$$

where $\sum_{j \in \mathcal{J}} (a_{n,t}^j r_{n,t}^j)^{\rho_K} = \left(\Big(\Gamma\Big(\frac{\rho_K - 1}{\rho_K}\Big)\Big)^{-1} \Big(\frac{v_{n,t}}{P_{n,t}}\Big)\right)^{\rho_K}$

- Distribution of return from sector j conditional on investing in sector j

$$
\begin{aligned}
\Pr(v|\text{invest in sector } j) &= \frac{1}{(\Phi^j_{n,t})}\Big(\int_0^v \Pi_{k\neq j} F^k(s) f^j(s) ds\Big) \\
&= \frac{1}{(\Phi^j_{n,t})}\Big(\int_0^v \big(\rho_K \Psi^j_{n,t} s^{-\rho_K - 1}\big) F(s) ds\Big) \\
&= \frac{\Psi^j_{n,t}}{\Psi_{n,t}((\Phi^j_{n,t}))}\Big(\int_0^v \big(\rho_K \Psi_{n,t} s^{-\rho_K - 1}\big) e^{-\Psi_{n,t} s^{-\rho_K - 1}} ds\Big) \\
&= \int_0^v \big(\rho_K \Psi_{n,t} s^{-\rho_K - 1}\big) e^{-\Psi_{n,t} s^{-\rho_K}} ds \quad \sim \quad \text{Frechet}(\Psi_{n,t}^{\frac{1}{\rho_K}}, \rho_K)
\end{aligned}
$$

- Capital K^j_t invested in sector k

$$
K^j_{n,t} = W^j_{n,t} \int_0^\infty v\big(\rho_K \Psi_{n,t} v^{-\rho_K - 1}\big) e^{-\Psi_{n,t} v^{-\rho_K}} dv
$$

- Efficiency: $v = P_{n,t} r^j_{n,t}(\zeta^j_{n,t})$

$$
\begin{aligned}
K^j_{n,t} &= W^j_{n,t} \int_0^\infty \big(\rho_K \Psi_{n,t}(P_{n,t} r^j_{n,t}(\zeta^j_{n,t}))^{-\rho_K}\big) e^{-\Psi_{n,t}(P_{n,t} r^j_{n,t}(\zeta^j_{n,t}))^{-\rho_K}} d(\zeta^j_{n,t}) \\
&= W^j_{n,t} \int_0^\infty \big(\rho_K (a^j_{n,t})^{\rho_K} (\Phi^j_{n,t})^{-1} (\zeta^j_{n,t})^{-\rho_K}\big) e^{-(a^j_{n,t})^{\rho_K} (\Phi^j_{n,t})^{-1} (\zeta^j_{n,t})^{-\rho_K}} d(\zeta^j_{n,t})
\end{aligned}
$$

- Change of variable: $u = (a^j_{n,t})^{\rho_K} (\Phi^j_{n,t})^{-1} (\zeta^j_{n,t})^{-\rho_K}, du = -\rho_K (a^j_{n,t})^{\rho_K} (\Phi^j_{n,t})^{-1} (\zeta^j_{n,t})^{-\rho_K - 1} d(\zeta^j_{n,t}),$

$$
K^j_{n,t} = W^j_{n,t} \int_0^\infty \big((\zeta^j_{n,t})\big) e^{-u} du
$$

- Change of Variable: $(\zeta^j_{n,t}) = a^j_{n,t}(\Phi^j_{n,t})^{-\frac{1}{\rho_K}} u^{-\frac{1}{\rho_K}}$

$$
\begin{aligned}
K^j_{n,t} &= W^j_{n,t}\Big(a^j_{n,t}(\Phi^j_{n,t})^{-\frac{1}{\rho_K}}\Big) \int_0^\infty \Big(u^{-\frac{1}{\rho_K}}\Big) e^{-u} du = W^j_{n,t}\Big(a^j_{n,t}(\Phi^j_{n,t})^{-\frac{1}{\rho_K}}\Big) \int_0^\infty \Big(u^{(1-\frac{1}{\rho_K})-1}\Big) e^{-u} du \\
&= \Gamma\Big(1 - \frac{1}{\rho_K}\Big)\left(\frac{(a^j_{n,t})^{\rho_K}}{\Phi^j_{n,t}}\right)^{\frac{1}{\rho_K}} W^j_{n,t}
\end{aligned}
$$

- Sector-specific capital supply (using $W^j_{n,t} = \Phi^j_{n,t} W_{n,t}$ and $\Phi^j_{n,t} = \left(\Gamma\left(\frac{\rho_K - 1}{\rho_K}\right)\right)^{\rho_K} \left(\frac{P_{n,t} r^j_{n,t} a^j_{n,t}}{v_{n,t}}\right)^{\rho_K}$)

$$
\begin{aligned}
K^j_{n,t} &= \Gamma\left(\frac{\rho_K - 1}{\rho_K}\right)\left(a^j_{n,t}\right)\left(\Phi^j_{n,t}\right)^{-\frac{1}{\rho_K}} W^j_{n,t} = \Gamma\left(\frac{\rho_K - 1}{\rho_K}\right)\left(a^j_{n,t}\right)\left(\Phi^j_{n,t}\right)^{\frac{\rho_K - 1}{\rho_K}} W_{n,t} \\
&= \left(\Gamma\left(\frac{\rho_K - 1}{\rho_K}\right)\right)^{\rho_K}\left(a^j_{n,t}\right)^{\rho_K}\left(\frac{P_{n,t} r^j_{n,t}}{v_{n,t}}\right)^{\rho_K - 1} W_{n,t}
\end{aligned}
$$

- Aggregate capital efficiency: $K_{n,t} = \sum_{j \in \mathcal{J}} K^j_{n,t} = \Gamma\left(\frac{\rho_K - 1}{\rho_K}\right) \sum_{j \in \mathcal{J}} a^j_{n,t} \left(\Phi^j_{n,t}\right)^{\frac{\rho_K - 1}{\rho_K}} W_{n,t}$

- Aggregate capital: $W_{n,t} = \left(\Gamma\left(\frac{\rho_K - 1}{\rho_K}\right) \sum_{j \in \mathcal{J}} a^j_{n,t} \left(\Phi^j_{n,t}\right)^{\frac{\rho_K - 1}{\rho_K}}\right)^{-1} K_{n,t}$

- Capital allocation share: $\Xi^j_{n,t} \equiv \frac{K^j_{n,t}}{K_{nt}}$

$$
\Xi^j_{n,t} = \frac{a^j_n \left(\Phi^j_{n,t}\right)^{\frac{\rho_K - 1}{\rho_K}}}{\sum_{j \in \mathcal{J}} a^h_n \left(\Phi^h_{n,t}\right)^{\frac{\rho_K - 1}{\rho_K}}}
$$

- Denote $\Psi_{n,t} \equiv \left(\Gamma\left(\frac{\rho_K - 1}{\rho_K}\right) \sum_{j \in \mathcal{J}} a^j_{n,t} \left(\Phi^j_{n,t}\right)^{\frac{\rho_K - 1}{\rho_K}}\right)^{-1}$, $\Psi^j_{n,t} \equiv \left(\Gamma\left(\frac{\rho_K - 1}{\rho_K}\right) a^j_{n,t} \left(\Phi^j_{n,t}\right)^{\frac{\rho_K - 1}{\rho_K}}\right)^{-1}$

- Law of motion for aggregate capital: $W_{n,t+1} = (1 - \delta^K_n) W_{n,t} + I_{n,t}$

- Law of motion for sector-specific capital: $\Psi^j_{n,t+1} K^j_{n,t+1} = (1 - \delta^K_n) \Psi^j_{n,t} K^j_{n,t} + I_{n,t}$

A.6.6 Consumption-based Carbon Emissions

- Emission per fossil fuel input ($F_{n,t}$)

$$
\phi^F_{n,t} = \frac{\sum_{h \in \{o,g,c\}} \overbrace{\phi^h \cdot f^h_{n,t}}^{\text{Emissions from fossil fuel type } h}}{F_{n,t}}
$$

- Emission per brown output ($y^B_{n,t}$) where $y^B_{n,t} \equiv \sum_{m\in\mathcal{N}} y^B_{mn,t}$

$$\phi^B_{n,t} = \frac{\phi^F_{n,t} \cdot F_{n,t}}{y^B_{n,t}}$$

- Consumption-based Country-level Carbon Emissions ($E^C_{n,t}$)

$$E^C_{n,t} = \sum_{m\in\mathcal{N}} \phi^B_{m,t} \cdot y^B_{nm,t}$$

- Production-based Country-level Carbon Emissions ($E_{n,t}$)

$$E_{n,t} = \sum_{h\in\{o,g,c\}} \phi^h \cdot f^h_{n,t}$$

- Global Carbon Emissions (E_t)

$$E_t = \underbrace{\sum_{n\in\mathcal{N}} E_{n,t}}_{\text{Production-based}} = \underbrace{\sum_{n\in\mathcal{N}} E^C_{n,t}}_{\text{Consumption-based}}$$

A.6.7 Estimation Equation for Labor Mobility Costs

- The expected lifetime utility of a worker in sector j of economy n at time t is $V^j_{n,t}$ given by

$$V^j_{n,t} = \ln\left(\frac{w^j_{n,t}}{P_{n,t}}\right) + \ln(\mu^j_n) + \rho_L \ln\left(\sum_{k\in\mathcal{J}} \exp\left(\beta V^j_{n,t+1} + \ln(1-\kappa^{j,k}_n)\right)^{\frac{1}{\rho_L}}\right).$$

- Labor mobility share $m^{j,k}_{n,t}$ is given by

$$m^{j,k}_{n,t} = \frac{\exp\left(\beta V^k_{n,t+1} + \ln(1-\kappa^{j,k}_n)\right)^{\frac{1}{\rho_L}}}{\sum_{h\in\mathcal{J}} \exp\left(\beta V^h_{n,t+1} + \ln(1-\kappa^{j,h}_n)\right)^{\frac{1}{\rho_L}}}.$$

- The equation can be arranged as follows:

$$\rho_L \ln(m_{n,t}^{j,k}) = \beta V_{n,t+1}^{k} + \ln(1-\kappa_n^{j,k}) - \rho_L \ln\left(\sum_{k\in\mathcal{J}} \exp\left(\beta V_{n,t+1}^{j} + \ln(1-\kappa_n^{j,k})\right)^{\frac{1}{\rho_L}}\right)$$

$$\ln\left(\frac{L_{n,t}^{j,k}}{L_{n,t}^{j}}\right) = \frac{\beta}{\rho_L} V_{n,t+1}^{k} + \frac{1}{\rho_L}\ln(1-\kappa_n^{j,k}) - \ln\left(\sum_{k\in\mathcal{J}} \exp\left(\beta V_{n,t+1}^{j} + \ln(1-\kappa_n^{j,k})\right)^{\frac{1}{\rho_L}}\right)$$

$$\ln\left(L_{n,t}^{j,k}\right) = \frac{\beta}{\rho_L} V_{n,t+1}^{k} + \frac{1}{\rho_L}\ln(1-\kappa_n^{j,k}) + \ln\left(L_{n,t}^{j}\right) - \ln\left(\sum_{k\in\mathcal{J}} \exp\left(\beta V_{n,t+1}^{j} + \ln(1-\kappa_n^{j,k})\right)^{\frac{1}{\rho_L}}\right)$$

- The equation implies the following regression specification:

$$L_{n,t}^{j,k} = \exp\left(C_n^{j,k} + \lambda_{n,t}^{k} + \lambda_{n,t}^{j}\right) + \xi_{n,t}^{j,k}.$$

  where

  - $C_n^{j,k} \equiv \frac{\ln(1-\kappa_n^{j,k})}{\rho_L}$,
  - $\lambda_{n,t}^{k} \equiv \frac{\beta}{\rho_L} V_{n,t+1}^{k}$,
  - $\lambda_{n,t}^{j} \equiv \ln(L_{n,t}^{j}) - \ln\left(\sum_{h\in\mathcal{J}} \exp\left(\frac{\beta}{\rho_L} V_{n,t+1}^{h} + \frac{1}{\rho_L}\ln(1-\kappa_n^{j,h})\right)\right)$
  - $\xi_{n,t}^{j,k}$: the residual term.

- Also, the expected life-time utility $V_{n,t}^{j}$ can be expressed as follows:

$$V_{n,t}^{j} = \ln\left(\frac{w_{n,t}^{j}}{P_{n,t}}\right) + \ln(\mu_n^{j}) + \beta V_{n,t+1}^{k} + \ln(1-\kappa_n^{j,k}) - \rho_L \ln(m_{n,t}^{j,k})$$

$$V_{n,t}^{j} = \ln\left(\frac{w_{n,t}^{j}}{P_{n,t}}\right) + \ln(\mu_n^{j}) + \beta V_{n,t+1}^{j} - \rho_L \ln(m_{n,t}^{j,j})$$

$$V_{n,t}^{k} = \ln\left(\frac{w_{n,t}^{k}}{P_{n,t}}\right) + \ln(\mu_n^{k}) + \beta V_{n,t+1}^{j} + \ln(1-\kappa_n^{k,j}) - \rho_L \ln(m_{n,t}^{k,j})$$

$$V_{n,t}^{k} = \ln\left(\frac{w_{n,t}^{k}}{P_{n,t}}\right) + \ln(\mu_n^{k}) + \beta V_{n,t+1}^{k} - \rho_L \ln(m_{n,t}^{k,k})$$

- Taking differences of these equations leads to the following equations:

$$0 = \beta V^k_{n,t+1} - \beta V^j_{n,t+1} + \ln(1-\kappa^{j,k}_n) - \rho_L \ln\Big(\frac{m^{j,k}_{n,t}}{m^{j,j}_{n,t}}\Big)$$
$$0 = \beta V^j_{n,t+1} - \beta V^k_{n,t+1} + \ln(1-\kappa^{k,j}_n) - \rho_L \ln\Big(\frac{m^{k,j}_{n,t}}{m^{k,k}_{n,t}}\Big)$$
$$0 = \beta V^k_{n,t+1} - \beta V^j_{n,t+1} + \ln(1-\kappa^{j,k}_n) - \rho_L \ln\Big(\frac{L^{j,k}_{n,t}}{L^{j,j}_{n,t}}\Big)$$
$$0 = \beta V^j_{n,t+1} - \beta V^k_{n,t+1} + \ln(1-\kappa^{k,j}_n) - \rho_L \ln\Big(\frac{L^{k,j}_{n,t}}{L^{k,k}_{n,t}}\Big)$$

- Adding last two equations leads to the following equations:

$$0 = \ln(1-\kappa^{j,k}_n) + \ln(1-\kappa^{k,j}_n) - \rho_L \ln\Big(\frac{L^{k,j}_{n,t}L^{j,k}_{n,t}}{L^{j,j}_{n,t}L^{k,k}_{n,t}}\Big)$$
$$\rho_L \ln\Big(\frac{L^{k,j}_{n,t}L^{j,k}_{n,t}}{L^{j,j}_{n,t}L^{k,k}_{n,t}}\Big) = \ln(1-\kappa^{j,k}_n) + \ln(1-\kappa^{k,j}_n)$$

- The equation implies the following regression specification:

$$\frac{L^{j,k}_{n,t}L^{k,j}_{n,t}}{L^{j,j}_{n,t}L^{k,k}_{n,t}} = \exp(\tilde{C}^{j,k}_n) + \tilde{\xi}^{j,k}_{n,t}$$

  where

  - $\tilde{C}^{j,k}_n \equiv \frac{1}{\rho_L}\ln(1-\kappa^{j,k}_{n,t}) + \frac{1}{\rho_L}\ln(1-\kappa^{k,j}_{n,t})$,
  - $\tilde{\xi}^{j,k}_{n,t}$: residual term.

A.6.8 Carbon Tax Equivalent to Removing Fossil Fuel Subsidies

- Price of fossil fuel composite with fossil fuel subsides is given by:

$$s_{n,t} = \Big(\sum_h \omega^h_n((1+\tau^h_{n,t})s^h_{n,t})^{1-\eta_2}\Big)^{\frac{1}{1-\eta_2}}$$

- Price of fossil fuel composite without fossil fuel subsides is given by:

$$s_{n,t}^0 = \Big(\sum_h \omega_n^h (s_{n,t}^h)^{1-\eta_2}\Big)^{\frac{1}{1-\eta_2}}$$

- Demand for fossil fuel composite by the brown variety producers is given by:

$$F_{mn,t}^B = \frac{(\gamma_n^{B,V} \cdot \psi_n^{B,F}) p_{mn,t}^B y_{mn,t}^B}{(1+\tau_{n,t}^B) s_{n,t}}$$

- Expressing the demand in terms of $s_{n,t}^0$ and $\tilde{\tau}_{n,t}^B \equiv \frac{s_{n,t}}{s_{n,t}^0} - 1$ (fossil fuel subsidy):

$$F_{mn,t}^B = \frac{(\gamma_n^{B,V} \cdot \psi_n^{B,F}) p_{mn,t}^B y_{mn,t}^B}{(1+\tau_{n,t}^B)(1+\tilde{\tau}_{n,t}^B) s_{n,t}^0}$$

- As discussed in the main text, the initial carbon tax equals zero, i.e., $\tau_{n,t}^B = 0$.
- Carbon tax is equivalent to removing fossil fuel subsides if the following condition holds:

$$(1+\tau_{n,t}^B)(1+\tilde{\tau}_{n,t}^B) = 1.$$

- Then, carbon tax equivalent to removing fossil fuel subsides is given by:

$$\tau_{n,t}^B = (1+\tilde{\tau}_{n,t}^B)^{-1} - 1.$$

A.7 Derivations for Estimation Equations

A.7.1 Estimation Equation for Dispersion of Non-pecuniary Preferences

- As derived in Section A.6.7, the life-time expected utility $V_{n,t}^j$ can be expressed as follows:

$$V_{n,t}^j = \ln\Big(\frac{w_{n,t}^j}{P_{n,t}}\Big) + \ln(\mu_n^j) + \beta V_{n,t+1}^k + \ln(1-\kappa_n^{j,k}) - \rho_L \ln(m_{n,t}^{j,k})$$

$$V_{n,t}^j = \ln\Big(\frac{w_{n,t}^j}{P_{n,t}}\Big) + \ln(\mu_n^j) + \beta V_{n,t+1}^j - \rho_L \ln(m_{n,t}^{j,j})$$

- Then, taking the difference of two equations leads to the following equations

$$\rho_L \log(\frac{m^{j,k}_{n,t}}{m^{j,j}_{n,t}}) = \beta\Big(V^k_{n,t+1} - V^j_{n,t+1}\Big) + \log(1-\kappa^{j,k}_n)$$

- Future expected life-time utilities are given by

$$\begin{aligned} V^k_{n,t+1} &= \log\left(\frac{w^k_{n,t+1}}{P_{n,t+1}}\right) + \log(\mu^k_n) - \rho_L \log(m^{k,k}_{n,t+1}) + \beta V^k_{n,t+2} \\ V^j_{n,t+1} &= \log\left(\frac{w^j_{n,t+1}}{P_{n,t+1}}\right) + \log(\mu^j_n) - \rho_L \log(m^{j,k}_{n,t+1}) + \beta V^k_{n,t+2} + \log(1-\kappa^{j,k}_n) \end{aligned}$$

- Taking the difference leads to the following equation

$$V^k_{n,t+1} - V^j_{n,t+1} = \log\left(\frac{w^k_{n,t+1}}{w^j_{n,t+1}}\right) + \log\left(\frac{\mu^k_n}{\mu^j_n}\right) + \rho_L \log(\frac{m^{j,k}_{n,t+1}}{m^{k,k}_{n,t+1}}) - \log(1-\kappa^{j,k}_n)$$

- Plugging this expression for the difference leads to the following equations:

$$\begin{aligned} \rho_L \log(\frac{m^{j,k}_{n,t}}{m^{j,j}_{n,t}}) &= \beta \log\left(\frac{w^k_{n,t+1}}{w^j_{n,t+1}}\right) + \beta \log\left(\frac{\mu^k_n}{\mu^j_n}\right) + \beta\rho_L \log(\frac{m^{j,k}_{n,t+1}}{m^{k,k}_{n,t+1}}) + (1-\beta)\log(1-\kappa^{j,k}_n) \\ \log(\frac{m^{j,k}_{n,t}}{m^{j,j}_{n,t}}) &= \left(\frac{1-\beta}{\rho_L}\right)\log(1-\kappa^{j,k}_n) + \left(\frac{\beta}{\rho_L}\right)\log\left(\frac{w^k_{n,t+1}}{w^j_{n,t+1}}\right) + \beta \log\left(\frac{\mu^k_n}{\mu^j_n}\right) + \beta \log(\frac{m^{j,k}_{n,t+1}}{m^{k,k}_{n,t+1}}) \\ \log\left(\frac{L^{j,k}_{n,t}}{L^{j,j}_{n,t}}\right) &= \left(\frac{1-\beta}{\rho_L}\right)\log(1-\kappa^{j,k}_n) + \left(\frac{\beta}{\rho_L}\right)\log\left(\frac{w^k_{n,t+1}}{w^j_{n,t+1}}\right) + \beta \log\left(\frac{\mu^k_n}{\mu^j_n}\right) + \beta \log\left(\frac{L^{j,k}_{n,t+1}L^k_{n,t+1}}{L^j_{n,t+1}L^{k,k}_{n,t+1}}\right) \end{aligned}$$

- The equation leads to the following regression specification:

$$y^{j,k}_{n,t} = \omega^k_n + \omega^j_n + \left(\frac{\beta}{\rho_L}\right)\ln\left(\frac{w^k_{n,t+1}}{w^j_{n,t+1}}\right) + \eta^{j,k}_{n,t+1}$$

where

  - $y^{j,k}_{n,t} \equiv \ln\left(\frac{L^{j,k}_{n,t}}{L^{j,j}_{n,t}}\right) - (1-\beta)C^{j,k}_n - \beta\ln\left(\frac{L^k_{n,t+1}L^{j,k}_{n,t+1}}{L^j_{n,t+1}L^{k,k}_{n,t+1}}\right)$
  - $C^{j,k}_n \equiv \frac{\ln(1-\kappa^{j,k}_n)}{\rho_L}$

- $\omega_n^j \equiv -\beta \ln(\mu_n^j)$
- $\omega_n^k \equiv \beta \ln(\mu_n^k)$
- $\eta_{n,t+1}^{j,k}$: forecast error.

A.7.2 Estimation Equation for Capital Supply Elasticities

- As derived in Section A.6.5, the sector-specific capital allocation is given by

$$K_{n,t}^j = \Gamma\left(\frac{\rho_K - 1}{\rho_K}\right)\left(a_{n,t}^j\right)\left(\Phi_{n,t}^j\right)^{\frac{\rho_K-1}{\rho_K}} W_{n,t}$$

where $\Phi_{n,t}^j$ is the aggregate capital allocation share given by

$$\Phi_{n,t}^j = \frac{(r_{n,t}^j a_{n,t}^j)^{\rho_K}}{\sum_{k\in\mathcal{J}}(r_{n,t}^k a_{n,t}^k)^{\rho_K}}$$

- Taking log difference for sector j and sector k leads to the following equation:

$$\ln(K_{n,t}^j/K_{n,t}^k) = \ln(a_{n,t}^j/a_{n,t}^k) + \left(\frac{\rho_K - 1}{\rho_K}\right)\ln(\Phi_{n,t}^j/\Phi_{n,t}^k)$$

- The term $\ln(\Phi_{n,t}^j/\Phi_{n,t}^k)$ can be expressed as follows:

$$\ln(\Phi_{n,t}^j/\Phi_{n,t}^k) = \rho_K\Big(\ln(r_{n,t}^j a_{n,t}^j) - \ln(r_{n,t}^k a_{n,t}^k)\Big)$$
$$\ln(\Phi_{n,t}^j/\Phi_{n,t}^k) = \rho_K\Big(\ln(a_{n,t}^j/a_{n,t}^k) + \ln(r_{n,t}^j/r_{n,t}^k)\Big)$$

- Plugging this expression in leads to the following equation:

$$\ln(K_{n,t}^j/K_{n,t}^k) = \rho_K \ln(a_{n,t}^j/a_{n,t}^k) + \Big(\rho_K - 1\Big)\ln(r_{n,t}^j/r_{n,t}^k)\Big)$$

- Law of motion for sector-specific capital-use efficiency is given by:

$$\ln(a_{n,t}^j) = \ln(a_n^j) + \rho_\Phi \ln(K_{n,t-1}^j) - \rho_\Phi \ln(W_{n,t-1})$$

- Hence, the equation leads to the following regression specification:

$$\ln\Big(\frac{K^j_{n,t}}{K^o_{n,t}}\Big) = \tilde{\omega}^j_n + \big(\rho_K - 1\big)\ln\Big(\frac{r^j_{n,t}}{r^o_{n,t}}\Big) + \big(\rho_K \cdot \rho_\Phi\big)\ln\Big(\frac{K^j_{n,t-1}}{K^o_{n,t-1}}\Big) + \tilde{\eta}^j_{n,t}$$

where

  - $\omega^j_n = \tilde{\omega}^j_n \equiv \rho_K \ln(a^j_n/a^o_n)$
  - $\tilde{\eta}^j_{n,t}$: the residual term

A.7.3 Estimation Equation for Energy Demand Elasticity

- Relative energy demand is given by:

$$\frac{Y^{NB}_{n,t}}{Y^B_{n,t}} = \Big(\frac{\alpha^{NB}_n}{\alpha^B_n}\Big) \cdot \Big(\frac{(1+\tau^{NB}_{n,t})P^{NB}_{n,t}}{(1+\tau^B_{n,t})P^B_{n,t}}\Big)^{-\eta_1}$$

- Taking logs and re-arranging leads to the following equation:

$$\ln\Big(\frac{Y^{NB}_{n,t}}{Y^B_{n,t}}\Big) = \ln\Big(\frac{\alpha^{NB}_n}{\alpha^B_n}\Big) - \eta_1 \ln\Big(\frac{1+\tau^{NB}_{n,t}}{1+\tau^B_{n,t}}\Big) - \eta_1 \ln\Big(\frac{P^{NB}_{n,t}}{P^B_{n,t}}\Big)$$

- Taking the time difference leads to the following equation:

$$\Delta Y^{NB}_{n,t} - \Delta Y^B_{n,t} = -\eta_1 \Delta(1+\tau^{NB}_{n,t}) + \eta_1\Delta(1+\tau^B_{n,t}) + \eta_1(\Delta P^B_{n,t} - \Delta P^{NB}_{n,t})$$

where $\Delta x_{n,t} \equiv \ln(x_{n,t}) - \ln(x_{n,0})$.

- This leads to the following regression specification:

$$\Delta Y^{NB}_{n,t} - \Delta Y^B_{n,t} = -\eta_1(\Delta P^{NB}_{n,t} - \Delta P^B_{n,t}) + \iota_{n,t}$$

where $\iota_{n,t} \equiv -\eta_1\Delta(1+\tau^{NB}_{n,t}) + \eta_1\Delta(1+\tau^B_{n,t})$.

A.7.4 Estimation Equation for Fossil Fuel Elasticity

- Relative fossil fuel demand is given by:

$$\frac{f_{n,t}^k}{f_{n,t}^h} = \frac{\omega_n^k}{\omega_n^h}\Big(\frac{(1+\tau_{n,t}^k)s_{n,t}^k}{(1+\tau_{n,t}^h)s_{n,t}^h}\Big)^{-\eta_2}$$

- Taking logs and re-arranging leads to the following equation:

$$\ln\Big(\frac{f_{n,t}^k}{f_{n,t}^h}\Big) = \ln\Big(\frac{\omega_n^k}{\omega_n^h}\Big) - \eta_2 \ln\Big(\frac{s_{n,t}^k}{s_{n,t}^h}\Big) - \eta_2 \ln\Big(\frac{1+\tau_{n,t}^k}{1+\tau_{n,t}^h}\Big)$$

- This leads to the following regression specification w.r.t. base sector k:

$$\ln\Big(\frac{f_{n,t}^k}{f_{n,t}^h}\Big) = \delta_n^k - \eta_2 \ln\Big(\frac{s_{n,t}^k}{s_{n,t}^h}\Big) - \eta_2 \ln\Big(\frac{1+\tau_{n,t}^k}{1+\tau_{n,t}^h}\Big) + \varepsilon_{n,t}^k$$

  where δ_n^k are country-fuel fixed effects defined as $\delta_n^k \equiv \ln\Big(\frac{\omega_n^k}{\omega_n^h}\Big)$; $\varepsilon_{n,t}^k$ is an error term.

A.8 Description of Equilibrium Variables

In this section, I provide detailed descriptions of the sets of parameters Θ, prices $\mathbb{P}$, and endogenous variables $\mathbb{Y}$ shown in Section **??**. The variables in the set of parameters Θ are shown in Table A7. The variables in the set of prices $\mathbb{P}$ are the shown in Table A8. The variables in the set of endogenous variables $\mathbb{Y}$ are shown in Table A9.

Table A7: List of Model Parameters

Variable	**Description**
$\kappa_n^{j,k}$	Bilateral labor mobility costs
ρ_L	Dispersion of idiosyncratic non-pecuniary preferences
ρ_K	Dispersion of capital-use efficiencies
ρ_Φ	Dynamic capital allocation externality
η_1 (η_2)	Elasticity of substitution between energy goods (fossil fuels)
β (σ)	Discount factor (Elasticity of substitution across varieties)
δ_n^K (δ_t^C)	Economy-specific capital depreciation (Carbon stock depreciation rate)
ξ_n	Environmental damage in economy n
λ_T	Carbon-temperature conversion
α_n	Non-energy consumption share in economy n
α_n^j	Non-energy share by sector j in economy n
$\gamma_n^{j,V}$	Value added share for sector j in economy n
$\gamma_n^{j,k}$	Material input share for sector j in economy n
$\psi_n^{j,L}$	Labor input share for sector j in economy n
$\psi_n^{j,K}$	Capital input share for sector j in economy n
$\psi_n^{j,E}$	Energy input share for sector j in economy n
$\psi_n^{B,F}$	Fossil fuel input share in economy n
$\varrho_{nm,t}^j$	Exogenous trade costs between economies n and m for sector j at time t
z_n^j	Sector-specific TFP for sector j in economy n
μ_n^j	Non-pecuniary benefits for sector j in economy n
$a_{n,t}^j$	Capital-use efficiency for sector j in economy n
φ_n^h	Supply cost of fossil fuel by type h
ϕ^h	Carbon emission by fossil fuel type h
E_L	Land-change emissions
ω_n^h	Weight on fossil fuels for fuel type h in economy n
$\alpha_n^B(\alpha_n^{NB})$	Weights on brown (non-brown energy) in economy n
w_n^{NE}	Non-employment income in economy n

Table A8: List of Prices and Taxes

Variable	**Description**
$p^j_{mn,t}$	Variety price in economy m for variety produced in sector j of economy n at time t
$P^{M,j}_{n,t}$	Price of material in sector j of economy n at time t
$P^E_{n,t}$	Energy price index in economy n at time t
$s^h_{n,t}$	Before-tax price of fossil fuel type h in economy n at time t
$s_{n,t}$	Price of a fossil fuel input in economy n at time t
$P_{n,t}$	Price of the final good in economy n at time t
$P^j_{n,t}$	Sector-level price index for sector j in economy n at time t
$w^j_{n,t}$	Wage in sector j of economy n at time t
$r^j_{n,t}$	Rental rate of capital in sector j of economy n at time t
$R_{n,t+1}$	Gross interest rate in economy n at time $t+1$
$\tau^B_{n,t}$	Consumption tax on the brown sector in economy n at time t
$\tau^{NB}_{n,t}$	Consumption tax on the non-brown energy sector in economy n at time t
$\tau^{f,h}_{n,t}$	Consumption tax specific to fossil fuel type h in economy n at time t
$e^j_{nm,t}$	Bilateral import tax in sector j between economies n and m at time t

Table A9: List of Endogenous Variables

Variable	**Description**
$C^K_{n,t}$	Consumption of the representative capitalist in economy n at time t
$c^j_{n,t}$	Consumption of a worker in sector j of economy n at time t
$W_{n,t+1}$	Aggregate capital of the representative capitalist in economy n at time $t+1$
$A_{n,t+1}$	Risk-free assets held by capitalists in economy n at time $t+1$
$Y_{n,t}$	Output of final goods in economy n at time t
$L_{n,t}$	Aggregate supply of labor in economy n at time t
$L^j_{n,t}$	Supply of labor in sector j of economy n at time t
$K_{n,t}$	Aggregate stock of capital in economy n at time t
$K^j_{n,t}$	Stock of capital in sector j of economy n at time t
$F_{n,t}$	Aggregate supply of fossil fuels in economy n at time t
$F^h_{n,t}$	Supply of fossil fuel type h in economy n at time t
$y^j_{mn,t}$	Output produced in sector j of economy n and consumed in economy m at time t
$Y^j_{n,t}$	Sectoral output in sector j of economy n at time t
$C^j_{n,t}$	Consumption of sectoral goods from sector j in economy n at time t
$Q^{j,E}_{n,t}$	Energy input in sector j of economy n at time t
$M^j_{n,t}$	Material input in sector j of economy n at time t
$M^{j,k}_{n,t}$	Material input from sector k used in sector j of economy n at time t
$\pi^j_{nm,t}$	Share of sector j expenditure of economy n spent on sector j of economy m at time t
$E_{n,t}$	Total carbon emissions due to fossil fuel production in economy n at time t
E_t	Global CO_2 emissions at time t
$\mathcal{C}_t$	Global stock of carbon dioxide in the atmosphere at time t
$\mathbb{T}(\mathcal{C}_t)$	Global temperature at time t the global temperature at time T_{min}
$\mathcal{D}_n(\mathcal{C}_t)$	Environmental quality in economy n at time t
$\Phi^j_{n,t}$	Probability of allocating one unit of aggregate capital to sector j in economy n at time t
$V^j_{n,t}$	Expected lifetime utility of a worker in sector j of economy n at time t
$V^K_{n,t}$	Lifetime utility of the representative capitalist in economy n at time t
$m^{j,k}_{n,t}$	Share of workers moving from sector j to sector k in economy n at time t
$\Omega_{n,t}$	Lump-sum transfer in economy n at time t

A.9 Description of Labor Force Surveys and Variable Construction

In this section, I describe the labor force surveys used in the analysis which track the movement of individuals across sectors and employment status over time and report individuals' income as well as individual characteristics such as gender, age, and education.

United States The data for the United States is sourced from the IPUMS Current Population Survey (CPS) Annual Social and Economic Supplement (ASEC) for the years 2000 to 2023. The dataset is annual and includes variables such as individual panel ID, year, sex, age, marital status, individual weight, employment status, sector of employment, sector of employment last year, education, income, and unemployment benefits. The sample includes individuals aged 25 to 65, excluding retirees and individuals unable to work. Income values exceeding 99990 or less than zero are treated as missing. The dataset tracks employment status (unemployed, employed) and movements across sectors.

China The data for China comes from the China Family Panel Studies (CFPS), covering the years 2010 to 2020. The survey is biennial, and the data includes variables such as sector of origin (sector of employment last year) and sector of destination (sector of employment), with sectors categorized into Agriculture, Brown, Manufacturing, Non-Brown Energy, and Services. The labor mobility shares are computed by annualizing the labor flows using the method of Artuç et al. (2010). The dataset also includes individual-level data such as income and employment status (unemployed, employed).

India The data for India is sourced from the Periodic Labour Force Survey (PLFS) for the years 2018 to 2020. The survey is quarterly, and the data includes variables such as quarter, visit, stratum, subsample, individual panel ID, years of education, sector, income, individual weight, employment status, age, hours worked per week, wage, and sex. The sample includes individuals aged 25 to 65. Employment status is derived based on a combination of variables indicating self-employment, wage employment, unemployment, and inactivity.

France The data for France is sourced from the Continuous Labour Force Survey (EEEC) for the years 2014 to 2020. The survey is quarterly, and the data includes variables such as individual panel ID, year, quarter, age, individual weight, birth year, birth month, start year, sector, employment status, sex, education, income, and unemployment benefits. The sample includes individuals aged 25 to 65 who are either employed or actively seeking employment. Sector classification follows NAF Rev. 2, which aligns with ISIC Rev. 4. The labor mobility shares are computed by annualizing the labor flows using the method of Artuç et al. (2010).

Argentina The data for Argentina is derived from the Permanent Household Survey (EPH) for the years 2003 to 2019. The survey is quarterly, and the data includes variables such as year, quarter, age, individual weight, income, wage, sector, education, employment status inactivity status, sex, job search, and individual panel ID. The sample includes individuals aged 25 to 65 who are either employed or actively seeking employment. Sector classification follows CAES-Mercosur, which aligns with ISIC Rev. 4. The labor mobility shares are computed by annualizing the labor flows using the method of Artuç et al. (2010).

Australia The dataset for Australia is the the Household, Income and Labour Dynamics in Australia (HILDA) Survey, covering 2002 to 2022. The survey is annual and provides longitudinal information on individual movements across sectors and employment status, including income and personal characteristics. Variables include individual panel ID, age, sex, marital status, individual weight, sector, education, income, employment status, school years, and annual work hours. The sample includes individuals aged 25 to 65 who are either employed or actively seeking employment.

South Korea The data for South Korea is from the Korean Labor & Income Panel Study (KLIPS) for the years 1999 to 2022. The survey is annual, and the data includes variables such as individual panel ID, year, sector, employment status, income, education, and weekly work hours. The sample includes individuals aged 25 to 65 who are either employed or actively seeking employment. Sector classification follows the 3-digit Korean Standard Industry Classification (KSIC), which aligns with the 3-digit ISIC Rev. 4.

United Kingdom The data for the United Kingdom comes from the Labour Force Survey (LFS) for the years 2016 to 2022. The survey is both quarterly and annual, and the data includes variables such as individual panel ID, year, quarter, age, individual weight, employment status, sector, and income. The sample includes individuals aged 25 to 65 who are either employed or actively seeking employment. Sector classification follows the 2-digit UK Standard Industry Classification (SIC), which aligns with the 2-digit ISIC Rev. 4.

A.10 Parameter Estimates

In this section, I provide the estimates of parameters described in Section 4.

Table A10: Capital Depreciation Rates (δ_n^K) for 2019

Country/Aggregate	**Depreciation Rate** (δ_n^K)
United States (US)	0.0460
China	0.0523
India	0.0579
European Union (EU)	0.0424
Rest of the World (ROW)	0.0463

Source: Penn World Table 10.01 (Feenstra et al. (2015))

Table A11: Supply Costs of Fossil Fuels (2022)

Economy	**Crude Oil** (φ_n^o)	**Natural Gas** (φ_n^{NB})	**Coal** (φ_n^c)
United States (US)	1626.3	0.3	99.2
China	1741.5	0.9	156.1
India	1741.5	0.8	248.5
European Union (EU)	1771.1	1.3	265.8
Rest of the World (ROW)	1730.0	0.8	243.4

Notes: The supply costs for crude oil, natural gas, and coal are in units of billion USD per billion m^3 for gasoline, billion USD per billion m^3 for natural gas, and billion USD per gigaton for coal, respectively. Source: IMF Fossil Fuel Subsidies Data (Black et al. (2023))

Table A12: Fuel-Specific Taxes for Fossil Fuels (2022)

Economy	**Crude Oil** ($\tau^{f,o}_{n,0}$)	**Natural Gas** ($\tau^{f,g}_{n,0}$)	**Coal** ($\tau^{f,c}_{n,0}$)
United States (US)	0.3117	0.0146	-0.0411
China	0.2843	-0.5373	0.0121
India	0.3200	-0.0998	-0.0896
European Union (EU)	0.8667	-0.2865	0.3040
Rest of the World (ROW)	0.2889	-0.2547	-0.0139

Notes: Fuel-specific tax rates are computed as the difference between the retail price after deducting value-added tax (VAT) of fossil fuels (gasoline, natural gas, coal) and the supply costs of fossil fuels. Source: IMF Fossil Fuel Subsidies Data (Black et al. (2023))

Table A13: Fossil Fuel Reserves ($D^h_{n,0}$) for 2019

Economy	**Crude Oil** ($D^o_{n,0}$)	**Natural Gas** ($D^g_{n,0}$)	**Coal** ($D^c_{n,0}$)
United States (US)	471.19	750388.90	279.22
China	170.92	235184.33	3500.82
India	17.90	42669.44	1867.59
European Union (EU)	9.91	18646.31	716.86
Rest of the World (ROW)	4561.01	1836887.27	6446.29

Notes: The units of reserves for crude oil, natural gas, and coal are in billion m^3, billion m^3, and billion tons, respectively. Estimates are based on Ritchie and Rosado (2017) for the baseline year 2019. I compute the average annual discovery from 2010 to 2019; then, I calculate the reserves assuming new reserves will continue to be discovered at this average annual rate for the next 500 years.

Table A14: Regression Results: Carbon-Temperature Conversion Parameter (λ_T)

Dependent: Global Average Temperature $\left(\mathbb{T}(\mathcal{C}_t)\right)$	(1) 1850 - 2022	(2) 1980 - 2022
$\log_2(\mathcal{C}_t)$	1.556***	1.671***
	(0.037)	(0.032)
Observations	173	43
R-squared	0.919	0.984

Notes: Data on global temperature $\mathbb{T}(\mathcal{C}_t)$ in celsius (°C) relative to the year 1850 obtained from Met Office Hadley Centre (2024) and global carbon emissions from Ritchie and Roser (2020). Global carbon stock ($\mathcal{C}_t$) computed based on Equation (**??**). Column 1 presents the regression result for the period 1850 - 2022. Column 2 presents the regression result for the period 1980 - 2022. Robust standard errors in parentheses.
*** p <0.01, ** p <0.05, * p <0.1

Table A15: Non-energy Consumption Share (α_n)

	US	**China**	**India**	**EU**	**ROW**
α_n	0.9163	0.8941	0.9215	0.9214	0.9112

Notes: α_n represents the Non-energy consumption share in economy n. Source: WIOD 2016 Release. US: United States, EU: European Union, ROW: Rest of the World.

Table A16: Non-energy Share by Sector (α_n^j)

Sector	**US**	**China**	**India**	**EU**	**ROW**
AG	0.0051	0.0462	0.1157	0.0130	0.0337
MN	0.1534	0.2741	0.2796	0.1842	0.1978
SR	0.8416	0.6797	0.6047	0.8027	0.7685

Notes: α_n^j represents the Non-energy share of sector j in economy n. Source: WIOD 2016 Release. AG: Agriculture, MN: Manufacturing, SR: Services, NB: Non-Brown Energy, B: Brown. US: United States, EU: European Union, ROW: Rest of the World.

Table A17: Value Added Share ($\gamma_n^{j,V}$)

Sector	**US**	**China**	**India**	**EU**	**ROW**
AG	0.4978	0.6156	0.7844	0.4758	0.6458
MN	0.3907	0.2849	0.2827	0.3461	0.3814
SR	0.6405	0.4960	0.7043	0.5861	0.6264
NB	0.8279	0.7805	0.7434	0.7212	0.8414
B	0.8155	0.7001	0.8502	0.5879	0.7399

Notes: $\gamma_n^{j,V}$ represents the value added share for sector j in economy n. Source: WIOD 2016 Release. AG: Agriculture, MN: Manufacturing, SR: Services, NB: Non-Brown Energy, B: Brown. US: United States, EU: European Union, ROW: Rest of the World.

Table A18: Material Input Share ($\gamma_n^{j,k}$)

From (k) $\rightarrow$ To (j)	**US**	**China**	**India**	**EU**	**ROW**
AG $\rightarrow$ **AG**	0.4146	0.3585	0.4965	0.2744	0.3416
MN $\rightarrow$ **AG**	0.2652	0.5019	0.1755	0.3834	0.3462
SR $\rightarrow$ **AG**	0.3202	0.1396	0.3280	0.3422	0.3122
AG $\rightarrow$ **MN**	0.0854	0.0875	0.1062	0.0650	0.0886
MN $\rightarrow$ **MN**	0.5878	0.7389	0.5618	0.5785	0.6269
SR $\rightarrow$ **MN**	0.3268	0.1736	0.3320	0.3565	0.2844
AG $\rightarrow$ **SR**	0.0023	0.0205	0.0499	0.0052	0.0129
MN $\rightarrow$ **SR**	0.1634	0.4865	0.3603	0.1699	0.2716
SR $\rightarrow$ **SR**	0.8342	0.4930	0.5898	0.8249	0.7154
AG $\rightarrow$ **NB**	0.0001	0.0005	0.0003	0.0086	0.0008
MN $\rightarrow$ **NB**	0.0814	0.4928	0.3150	0.2351	0.3098
SR $\rightarrow$ **NB**	0.9184	0.5067	0.6847	0.7563	0.6894
AG $\rightarrow$ **B**	0.0006	0.0014	0.0005	0.0021	0.0015
MN $\rightarrow$ **B**	0.2391	0.4784	0.3199	0.2701	0.2951
SR $\rightarrow$ **B**	0.7300	0.3685	0.6043	0.6651	0.5010
NB $\rightarrow$ **B**	0.0303	0.1517	0.0753	0.0627	0.2023

Notes: $\gamma_n^{j,k}$ represents the material input share from sector k to sector j in economy n. Source: WIOD 2016 Release. AG: Agriculture, MN: Manufacturing, SR: Services, NB: Non-Brown Energy, B: Brown. US: United States, EU: European Union, ROW: Rest of the World. Disregard shares for non-employment (NE).

Table A19: Labor Input Share ($\psi_n^{j,L}$)

Sector	US	China	India	EU	ROW
AG	0.3327	0.9044	0.5378	0.5741	0.6599
MN	0.4656	0.3075	0.2641	0.5428	0.4162
SR	0.5732	0.5199	0.4919	0.5901	0.5443
NB	0.2152	0.0826	0.2546	0.1355	0.1120
B	0.0993	0.1862	0.0749	0.0834	0.1153

Notes: $\psi_n^{j,L}$ represents the labor input share for sector j in economy n. Source: WIOD 2016 Release. AG: Agriculture, MN: Manufacturing, SR: Services, NB: Non-Brown Energy, B: Brown. US: United States, EU: European Union, ROW: Rest of the World.

Table A20: Capital Input Share ($\psi_n^{j,K}$)

Sector	US	China	India	EU	ROW
AG	0.5526	0.0477	0.4456	0.3306	0.2792
MN	0.4590	0.3782	0.4508	0.3387	0.3473
SR	0.3885	0.4059	0.4532	0.3746	0.4012
NB	0.5967	0.1781	0.2698	0.3356	0.2422
B	0.4160	0.2758	0.2434	0.1707	0.4852

Notes: $\psi_n^{j,K}$ represents the capital input share for sector j in economy n. Source: WIOD 2016 Release. AG: Agriculture, MN: Manufacturing, SR: Services, NB: Non-Brown Energy, B: Brown. US: United States, EU: European Union, ROW: Rest of the World.

Table A21: Energy Input Share ($\psi_n^{j,E}$)

Sector	US	China	India	EU	ROW
AG	0.1147	0.0478	0.0166	0.0952	0.0608
MN	0.0754	0.3143	0.2851	0.1185	0.2364
SR	0.0383	0.0743	0.0549	0.0352	0.0546
NB	0.1882	0.7393	0.4755	0.5289	0.6457

Notes: $\psi_n^{j,E}$ represents the energy input share for sector j in economy n. Source: WIOD 2016 Release. AG: Agriculture, MN: Manufacturing, SR: Services, NB: Non-Brown Energy, B: Brown. US: United States, EU: European Union, ROW: Rest of the World. Disregard energy input shares for sector NE and sector G.

Table A22: Fossil Fuel Input Share ($\psi_n^{B,F}$)

	US	**China**	**India**	**EU**	**ROW**
$\psi_n^{B,F}$	0.4847	0.5380	0.6816	0.7459	0.3995

Notes: $\psi_n^{B,F}$ represents the fossil fuel input share in economy n. Source: WIOD 2016 Release. US: United States, EU: European Union, ROW: Rest of the World.

Table A23: Import Taxes ($e^j_{nm,0}$)

From \ To	**US**	**China**	**India**	**EU**	**ROW**
			Agriculture (AG)		
US	1.0000	1.0476	1.0375	1.0460	1.0315
China	1.1267	1.0000	1.1186	1.1267	1.1415
India	1.3798	1.3174	1.0000	1.3798	1.3013
EU	1.1422	1.1467	1.1332	1.0000	1.0541
ROW	1.1519	1.1578	1.1597	1.1436	1.1545
			Manufacturing (MN)		
US	1.0000	1.0445	1.0313	1.0443	1.0314
China	1.0692	1.0000	1.0638	1.0692	1.1014
India	1.1179	1.1113	1.0000	1.1179	1.0864
EU	1.0480	1.0482	1.0329	1.0000	1.0121
ROW	1.0860	1.0873	1.0859	1.0740	1.0870
			Services (SR)		
US	1.0000	1.0000	1.0000	1.0000	1.0000
China	1.0000	1.0000	1.0000	1.0000	1.0000
India	1.0000	1.0000	1.0000	1.0000	1.0000
EU	1.0000	1.0000	1.0000	1.0000	1.0000
ROW	1.0000	1.0000	1.0000	1.0000	1.0000
			Non-Brown Energy (NB)		
US	1.0000	1.0159	1.0033	1.0159	1.0089
China	1.0678	1.0000	1.0616	1.0678	1.0822
India	1.1025	1.0988	1.0000	1.1025	1.0754
EU	1.0210	1.0210	1.0059	1.0000	1.0048
ROW	1.0594	1.0590	1.0581	1.0506	1.0596
			Brown (B)		
US	1.0000	1.0135	1.0020	1.0135	1.0069
China	1.0542	1.0000	1.0513	1.0542	1.0644
India	1.0820	1.0727	1.0000	1.0820	1.0598
EU	1.0166	1.0166	1.0166	1.0000	1.0039
ROW	1.0669	1.0672	1.0660	1.0576	1.0670

Notes: Trade taxes ($e^j_{nm,0}$) are estimated based on the MAcMap-HS6 dataset (Guimbard et al. (2012)) for the year 2019. Due to the unavailability of information for the services sector, trade taxes for this sector are imputed as 1. Source: MAcMap-HS6 (Market Access Map HS6).

Table A24: Exogenous Trade Costs ($\varrho^j_{nm,0}$) - Agriculture, Manufacturing, Services

From \ To	**US**	**China**	**India**	**EU**	**ROW**
			Agriculture (AG)		
US	1.0000	3.5076	2.8975	2.6888	1.4766
China	2.4547	1.0000	3.6859	3.6420	1.6869
India	3.1896	3.8452	1.0000	3.8841	1.8247
EU	2.3837	2.8882	2.7970	1.0000	1.4708
ROW	2.2668	2.8426	3.0167	2.3440	1.0000
			Manufacturing (MN)		
US	1.0000	1.5562	2.6075	1.6877	1.2952
China	2.6936	1.0000	3.8920	2.2264	1.5460
India	2.1878	1.7732	1.0000	1.7996	1.4172
EU	2.0853	1.7659	2.8148	1.0000	1.4741
ROW	1.7958	1.5828	2.6152	1.5765	1.0000
			Services (SR)		
US	1.0000	4.4265	5.4879	3.0378	2.3623
China	3.7658	1.0000	4.7236	2.9896	2.4606
India	2.7947	3.8830	1.0000	2.6310	2.2056
EU	2.4649	3.4372	4.7347	1.0000	2.0299
ROW	2.3324	2.6026	3.2605	2.1337	1.0000

Notes: Exogenous trade costs ($\varrho^j_{nm,0}$) are estimated using a fixed effects regression specification (**??**), which is conditional on the trade elasticity σ, the bilateral trade shares ($\pi^j_{nm,t}$), and trade taxes ($e^j_{nm,t}$). Bilateral trade shares for the years 2000 to 2014 are computed using the WIOD 2016 Release. Tri-annual trade taxes ($e^j_{nm,t}$) for the years 2001 to 2014 are computed using Guimbard et al. (2012). Missing values for years between two observed years are imputed based on the assumption that trade taxes did not vary during the intervening period. The estimation assumes that exogenous trade costs did not vary from 2000 to 2014.

Table A25: Exogenous Trade Costs ($\varrho^j_{nm,0}$) - Non-Brown Energy, Brown

From \ To	US	China	India	EU	ROW
Non-Brown Energy (NB)					
US	1.0000	3.8274	62.3328	3.0329	2.3645
China	8.3960	1.0000	118.3053	4.3355	3.3871
India	9.4558	3.3788	1.0000	3.2306	2.2408
EU	3.8112	3.8161	43.0767	1.0000	1.8855
ROW	5.2842	4.3795	41.5787	2.9739	1.0000
Brown (B)					
US	1.0000	3.4274	4.0656	2.4224	1.3113
China	3.0898	1.0000	2.5712	3.5095	1.3407
India	2.5363	2.4576	1.0000	2.5766	1.0333
EU	1.9383	2.7517	2.8199	1.0000	1.0466
ROW	2.0465	2.6036	2.7203	2.1445	1.0000

Notes: Exogenous trade costs ($\varrho^j_{nm,0}$) are estimated using a fixed effects regression specification (**??**), which is conditional on the trade elasticity σ, the bilateral trade shares ($\pi^j_{nm,t}$), and trade taxes ($e^j_{nm,t}$). Bilateral trade shares for the years 2000 to 2014 are computed using the WIOD 2016 Release. Tri-annual trade taxes ($e^j_{nm,t}$) for the years 2001 to 2014 are computed using Guimbard et al. (2012). Missing values for years between two observed years are imputed based on the assumption that trade taxes did not vary during the intervening period. The estimation assumes that exogenous trade costs did not vary from 2000 to 2014.

Table A26: Inter-Sectoral Labor Mobility Costs $\left(\frac{\ln(1-\kappa_n^{j,k})}{\rho_L}\right)$ for US, China, and India

From \ To	**NE**	**AG**	**MN**	**SR**	**NB**	**B**
			United States (US)			
NE	0	-2.46	-2.12	-1.46	-2.84	-4.32
AG	-2.79	0	-4.46	-2.13	-5.50	-6.04
MN	-3.22	-5.57	0	-1.82	-3.49	-5.55
SR	-3.81	-6.28	-4.89	0	-5.29	-6.64
NB	-3.73	-6.45	-3.44	-2.26	0	-5.97
B	-3.51	-5.97	-4.00	-2.14	-4.07	0
			China			
NE	0	-1.02	-1.95	-1.13	-3.84	-2.74
AG	-1.02	0	-2.68	-2.32	-4.19	-3.57
MN	-1.95	-2.68	0	-2.09	-3.21	-3.35
SR	-1.13	-2.32	-2.09	0	-3.07	-3.30
NB	-3.84	-4.19	-3.21	-3.07	0	-5.96
B	-2.74	-3.57	-3.35	-3.30	-5.96	0
			India			
NE	0	-0.49	-3.19	-2.41	-4.76	-5.25
AG	-5.35	0	-5.02	-4.28	-8.90	-8.90
MN	-3.19	-5.02	0	-3.58	-4.61	-5.56
SR	-2.41	-4.28	-3.58	0	-3.45	-4.97
NB	-4.76	-8.90	-4.61	-5.55	0	-5.55
B	-5.25	-8.90	-5.56	-4.97	-5.55	0

Notes: Inter-sectoral labor mobility costs ($\frac{\ln(1-\kappa_n^{j,k})}{\rho_L}$) are estimated using the Poisson Pseudo Maximum Likelihood (PPML) method. Sources: Current Population Survey, China Family Panel Studies, Periodic Labour Force Survey, Continuous Labour Force Survey, Permanent Household Survey, The Household, Income and Labour Dynamics in Australia, Korean Labor & Income Panel Study, Labour Force Survey.

Table A27: Inter-Sectoral Labor Mobility Costs $\left(\frac{\ln(1-\kappa_n^{j,k})}{\rho_L}\right)$ for EU and ROW

From \ To	NE	AG	MN	SR	NB	B
European Union (EU)						
NE	0	-4.87	-5.60	-2.80	-6.61	-9.28
AG	-4.24	0	-7.32	-5.20	-9.28	-9.28
MN	-3.55	-7.32	0	-3.68	-7.26	-8.90
SR	-2.80	-8.67	-8.28	0	-9.28	-8.90
NB	-5.34	-9.28	-8.51	-5.94	0	-9.28
B	-4.94	-9.28	-8.90	-4.45	-9.28	0
Rest of the World (ROW)						
NE	0	-3.38	-2.07	-1.75	-3.98	-4.81
AG	-4.65	0	-4.52	-3.08	-6.94	-7.10
MN	-4.25	-5.65	0	-2.38	-4.59	-5.84
SR	-1.75	-6.09	-4.38	0	-6.04	-7.05
NB	-4.52	-6.55	-3.37	-2.75	0	-5.65
B	-3.92	-4.88	-2.88	-2.00	-4.13	0

Notes: Inter-sectoral labor mobility costs ($\frac{\ln(1-\kappa_n^{j,k})}{\rho_L}$) are estimated using the Poisson Pseudo Maximum Likelihood (PPML) method. Sources: Current Population Survey, China Family Panel Studies, Periodic Labour Force Survey, Continuous Labour Force Survey, Permanent Household Survey, The Household, Income and Labour Dynamics in Australia, Korean Labor & Income Panel Study, Labour Force Survey.

Table A28: Non-Employment Income Parameter (w_n^{NE})

Economy	w_n^{NE}
United States (US)	0.16
China	0.55
India	0.24
European Union (EU)	0.57
Rest of the World (ROW)	0.45

Notes: Non-employment income (w_n^{NE}) as a share of average labor income is estimated using panel labor force surveys. The estimation involves Mincerian regression of (log) wages on sector-year fixed effects, along with education, age, and gender fixed effects, weighted by individual weights to obtain residualized sector-year-level wages. Sources: Current Population Survey, China Family Panel Studies, Periodic Labour Force Survey, Continuous Labour Force Survey, Permanent Household Survey, The Household, Income and Labour Dynamics in Australia, Korean Labor & Income Panel Study, Labour Force Survey.

Table A29: Sector-Specific Total Factor Productivity (z_n^j)

Economy	AG	MN	SR	NB	B
United States (US)	3.9533	7.2295	1.4806	0.2408	12.0749
China	0.2903	9.2016	1.4455	0.5143	13.3632
India	0.4670	6.4430	0.9737	0.2955	12.1159
European Union (EU)	2.1120	6.9764	1.2983	0.3245	12.2564
Rest of the World (ROW)	0.8215	8.7546	1.5253	0.4251	12.4059

Notes: Sector-specific total factor productivity (z_n^j) is estimated by targeting bilateral trade shares and wages across sectors and countries. Sectors: AG (Agriculture), MN (Manufacturing), SR (Services), NB (Non-Brown Energy), B (Brown). Source: WIOD 2016 Release.

Table A30: Non-Pecuniary Benefits (μ_n^j)

Economy	NE	AG	MN	SR	NB	B
United States (US)	1.0000	2.1216	1.0103	3.4218	0.5545	0.2389
China	1.0000	11.4915	5.7615	8.2755	1.5491	2.2657
India	1.0000	0.0836	0.3391	0.3032	0.0265	0.0455
European Union (EU)	1.0000	7.3734	2.0734	2.7443	1.9517	1.3907
Rest of the World (ROW)	1.0000	2.1176	0.6950	1.2364	0.3972	0.3041

Notes: Non-pecuniary benefits (μ_n^j) are matched to inter-sectoral labor mobility shares. Sectors: NE (Non-Employment), AG (Agriculture), MN (Manufacturing), SR (Services), NB (Non-Brown Energy), B (Brown). Source: Panel labor force surveys.

Table A31: Weights on Fossil Fuels $\left((\omega_n^h)^{\frac{1}{\eta_2}}\right)$ with $\eta_2 = 2.67$

Economy	Crude Oil	Natural Gas	Coal
United States (US)	0.9999	0.0021	0.0429
China	0.9945	0.0022	0.2046
India	0.9865	0.0039	0.2861
European Union (EU)	0.9854	0.0057	0.2950
Rest of the World (ROW)	0.9994	0.0033	0.0918

Notes: Weights on fossil fuels (ω_n^h) target economy-level fossil fuel production by fuel type. Fossil fuels: Crude Oil, Natural Gas, Coal. Source: Ritchie and Rosado (2017).

Table A32: Weights on Non-Brown Energy and Brown $\left((\alpha_n^{NB})^{\frac{1}{\eta_1}}, (\alpha_n^{B})^{\frac{1}{\eta_1}}\right)$ with $\eta_1 = 2.28$

Economy	$(\alpha_n^{NB})^{\frac{1}{\eta_1}}$	$(\alpha_n^{B})^{\frac{1}{\eta_1}}$
United States (US)	0.9977	0.0999
China	0.9998	0.0318
India	0.9544	0.3651
European Union (EU)	0.9995	0.0511
Rest of the World (ROW)	0.9996	0.0453

Notes: Weights on non-brown energy (α_n^{NB}) and brown (α_n^{B}) are matched to the final expenditure on the brown and non-brown energy sectors, respectively. Source: WIOD 2016 Release.

Table A33: Environmental Damage Parameter (ξ_n)

Economy	ξ_n
United States (US)	0.015
China	0.009
India	0.025
European Union (EU)	0.021
Rest of the World (ROW)	0.029

Notes: Environmental damage parameter (ξ_n) is estimated to match the impulse response of real GDP per capita to a 1°C increase in global temperature. Source: Bilal and Känzig (2024).

A.11 Method of Simulated Moments (MSM) Procedure

A.11.1 Moments and Parameter Identification

The MSM procedure involves estimating parameters such that the moments generated by the model match the observed moments in the data. The set of parameters Θ^{MSM} includes:

- **Sector-Specific Total Factor Productivity** ($z^j_{n,0}$):
  - **Identified by**: Bilateral trade shares and sectoral output.
  - **Moments**: Bilateral trade shares $\pi^j_{nm,0}$, sectoral output $P^j_{n,0}Y^j_{n,0}$.
- **Non-Pecuniary Benefit** (μ^j_n):
  - **Identified by**: Inter-sectoral wage differences.
  - **Moments**: Wages relative to non-employment income $\frac{w^j_{n,0}}{w^{NE}_{n,0}}$.
- **Capital-Use Efficiency** ($a^j_{n,t}$):
  - **Identified by**: Real interest rates, capital allocation shares, and capital compensation.
  - **Moments**: Real Interest Rate ($R_{n,t}$), Capital stock $K^j_{n,0}$, capital allocation $\Phi^j_{n,0}$, and capital compensation $P_{n,0}\sum_{j\in\mathcal{J}} r^j_{n,0}K^j_{n,0}$.
- **Weight on Fossil Fuel** (ω^h_n):
  - **Identified by**: Fossil fuel production by type.
  - **Moments**: Production shares of different fossil fuels $f^h_{n,0}$.
- **Weight on Brown** (α^B_n):
  - **Identified by**: Brown consumption.
  - **Moments**: Consumption shares of brown and non-brown energy α^B_n.
- **Environmental Damage** (ξ_n):
  - **Identified by**: GDP response to a 1°C increase in global temperature.
  - **Moments**: GDP changes ΔGDP_n after 5 years following global temperature shocks.

A.11.2 Computation of Moments

The moments $\mathbb{M}^{\text{Data}}$ are computed from observed data as follows:

- **Trade Share** ($\pi^j_{nm,0}$): The share of total expenditure spent on products produced in a given sector of a given country. Source: WIOD 2016 Release.

- **Sectoral output and Wage** ($P^j_{n,0}Y^j_{n,0}$, $w^j_{n,0}$): Gross output and wage by sector. Source: Panel labor force surveys from eight countries (US, China, India, France, Argentina, Australia, South Korea, UK).

- **Real Interest Rate, Capital Allocation, and Compensation** ($R_{n,t}$, $K^j_{n,0}$, $\Phi^j_{n,0}$, $P_{n,0}\sum_{j\in\mathcal{J}} r^j_{n,0}K^j_{n,0}$): Real interest rate and the distribution of capital stock, allocation, and compensation across sectors. Source: International Financial Statistics (IFS) and WIOD 2016 Release.

- **Fossil Fuel Production Share** ($f^h_{n,0}$): The proportion of each type of fossil fuel produced in different countries. Source: Ritchie and Rosado (2017).

- **Energy Consumption** (α^B_n): The ratio of brown to non-brown energy consumption. Source: WIOD 2016 Release.

- **GDP Response to Temperature Increase** (ΔGDP_n): The percentage change in GDP for each country due to a 1°C rise in global temperature. Source: Bilal and Känzig (2024).

A.11.3 Calibration Procedure

The MSM calibration procedure involves the following steps:

1. **Initialize Parameters**: Start with initial guesses for the parameters Θ^{MSM}.

2. **Simulate Model**: Run the dynamic general equilibrium model with the initial parameter guesses to generate simulated moments $\mathbb{M}^{\text{Model}}$.

3. **Compute Model Moments**: Calculate the model moments corresponding to the observed data moments.

4. **Minimize Error Function**: Adjust the parameters to minimize the distance between the simulated model moments and the observed data moments using the error function:

$$Q_T(\Theta) = \left[\frac{\mathbb{M}_T^{\text{Data}} - \mathbb{M}_T^{\text{Model}}(\Theta)}{\mathbb{M}_T^{\text{Data}}}\right]' I \left[\frac{\mathbb{M}_T^{\text{Data}} - \mathbb{M}_T^{\text{Model}}(\Theta)}{\mathbb{M}_T^{\text{Data}}}\right],$$

   where I is the identity matrix.

5. **Iterate**: Repeat the simulation and adjustment steps until the parameters converge to values that minimize the error function $Q_T(\Theta)$.

6. **Sequential Procedure**: The estimation of environmental damage parameters (ξ_n) is performed after the estimation of other parameters. The estimated environmental damage parameter is then fed back into the first step, and the entire procedure is repeated until convergence.

A.11.4 Environmental Damage Estimation

The environmental damage parameter (ξ_n) is estimated to match the impulse response of real GDP per capita of an economy to a 1°C increase in global temperature, as estimated by Bilal and Känzig (2024). Specifically, I simulate an unexpected increase in exogenous carbon emissions E_0^X at time 0, which results in an increase in global temperature by 1°C at time 1. This can be shown as follows, based on Equation (**??**):

$$E_0^X = \mathcal{S} \cdot 2^{\frac{\mathbb{T}(\mathcal{C}_0)}{\lambda_T}} \left(2^{\frac{1}{\lambda_T}} - 1\right). \tag{32}$$

I then match the drop in real GDP per capita of each economy, relative to the baseline simulation without the shock, for each economy after 5 years with the estimates of Bilal and Känzig (2024).

A.12 Solution Algorithm

Solution Algorithm

Initialization: Load parameters and set loop variables: $dev_\star$, $iter_\star$, $itermax_\star$, $tol_\star$, $smth_\star$

Step 1: Initialize $w^j_{nt}, r^j_{nt}, m^{jk}_{nt}, (\varphi^\star)^h_{nt}$

Production Block:

while $dev_1 > tol_1$ **and** $iter_1 < itermax_1$ **do**

  while $dev_{NN} > tol_{NN}$ **and** $iter_{NN} < itermax_{NN}$ **do**

    Step 2: Compute fossil fuel price s^h_{nt}

    Step 3: Calculate marginal costs for non-energy, non-brown energy, and brown sectors

    Step 4: Compute trade costs d^j_{nmt} and variety prices p^j_{nmt}

    Step 5: Compute sector good prices P^j_{nt} and after-tax prices $\tilde{P}^j_{nt}$

    Step 6: Compute energy price P^E_{nt}, final good price P_{nt}, and material price $P^{M,j}_{nt}$

    Step 7: Compute bilateral trade share π^j_{nmt} and capital allocation share Φ^j_{nt}

    Step 8: Compute rental rate of aggregate capital v_{nt}

    Step 9: Normalize prices, update dev_{NN}, and increment $iter_{NN}$

  end while

  Step 10: Calculate present-discounted income and capitalist consumption C^K_{nt}

  Step 11: Compute capitalist investment W_{nt} and supply of capital K^j_{nt}

  Step 12: Compute worker consumption c^j_{nt} and demand for final goods

  Step 13: Update fossil fuel extraction F_{nt} and reserves D^h_{nt}

  Labor Mobility Block:

  while $dev_m > tol_m$ **and** $iter_m < itermax_m$ **do**

    Step 14: Compute worker utility V^j_{nt} and labor transition probability $(m')^{jk}_{nt}$

    Step 15: Update dev_m, smooth m^{jk}_{nt}

  end while

  Market Clearing:

  Step 16: Check labor and capital market clearing conditions

  Step 17: Update wages w^j_{nt} and rental rates r^j_{nt}

  Step 18: Update dev_1, smooth variables, and update increment $iter_1$

end while