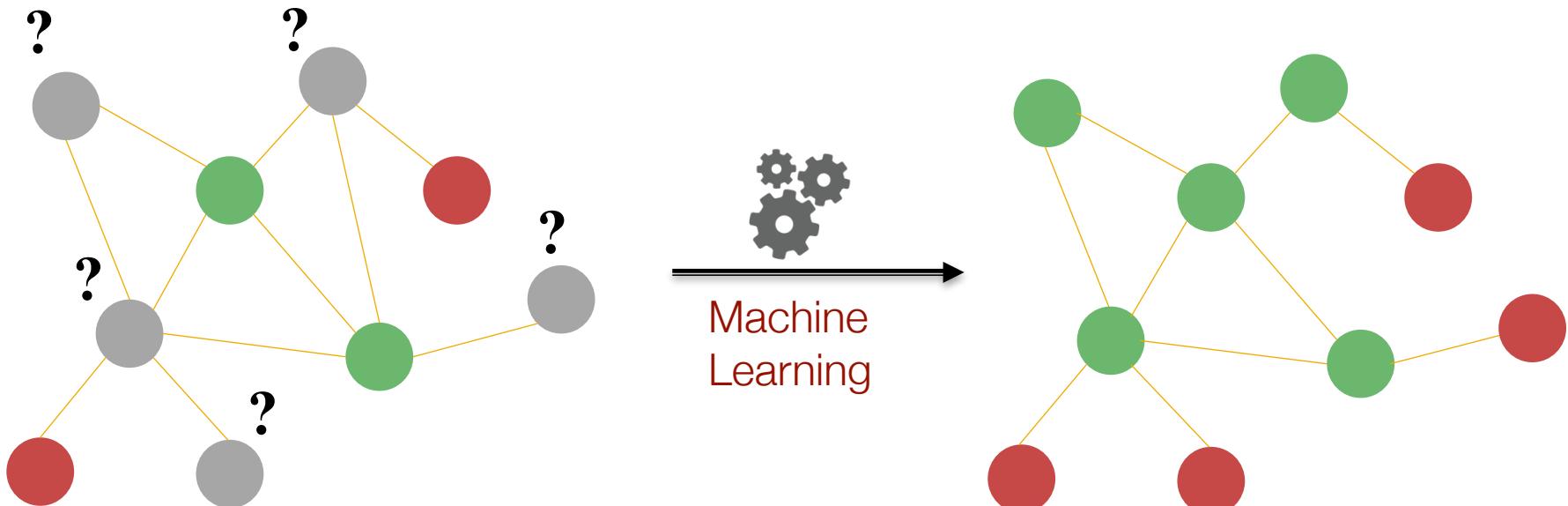


# Automatic Feature Learning In Graphs

CS224W: Analysis of Networks  
Jure Leskovec, Stanford University  
<http://cs224w.stanford.edu>



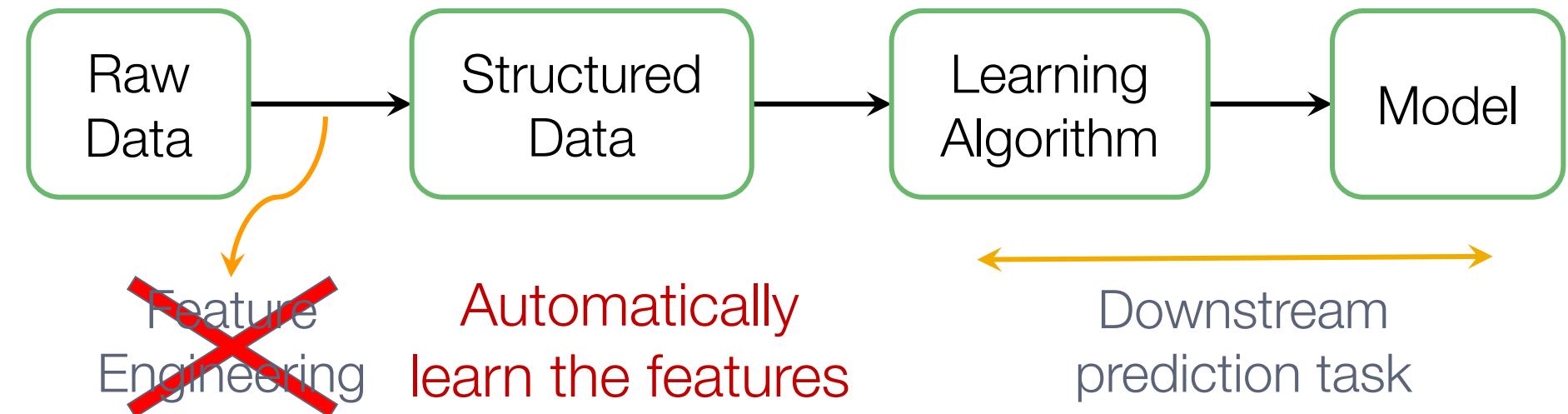
# Machine Learning in Networks



Node classification

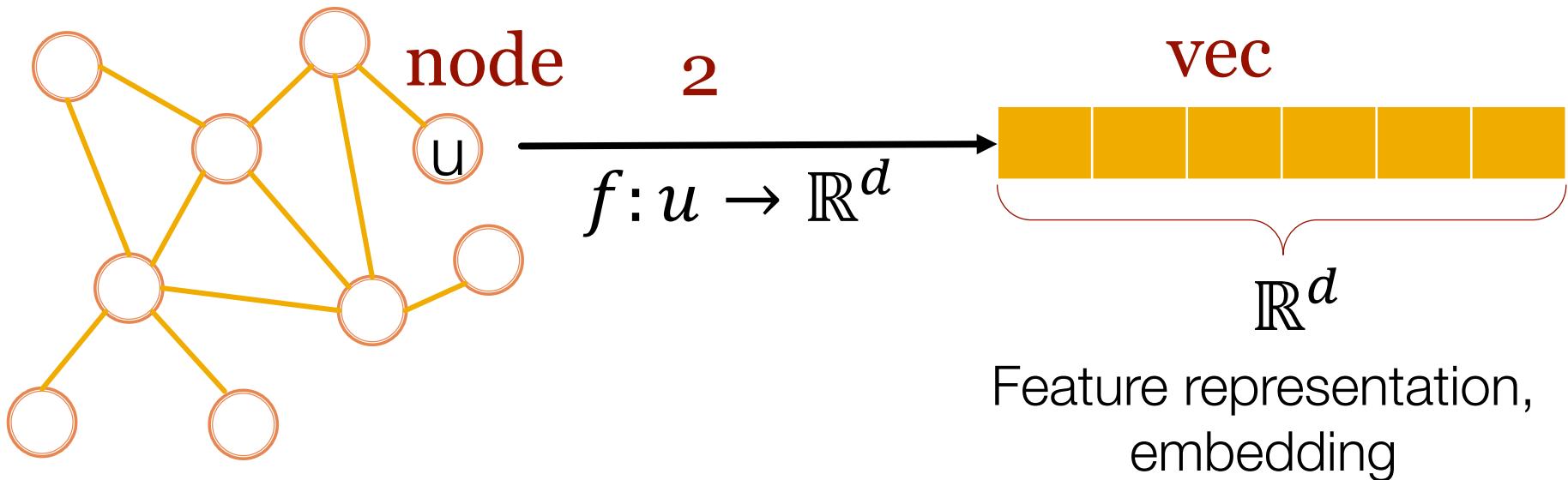
# Machine Learning Lifecycle

- (Supervised) Machine Learning Lifecycle: This feature, that feature. Every single time!



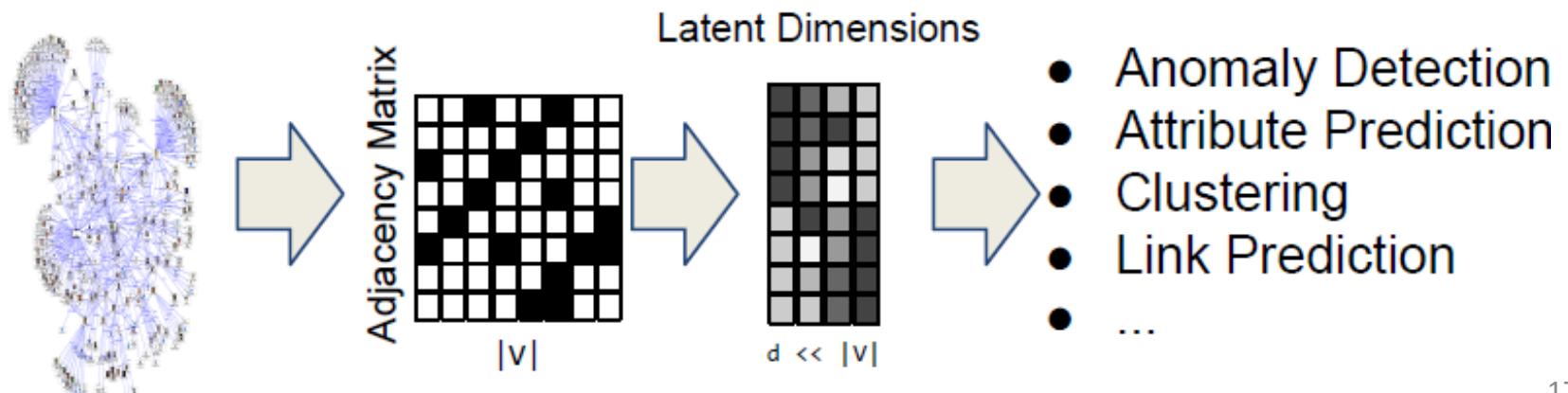
# Feature Learning in Graphs

**Goal:** Efficient task-independent feature learning  
for machine learning  
in networks!



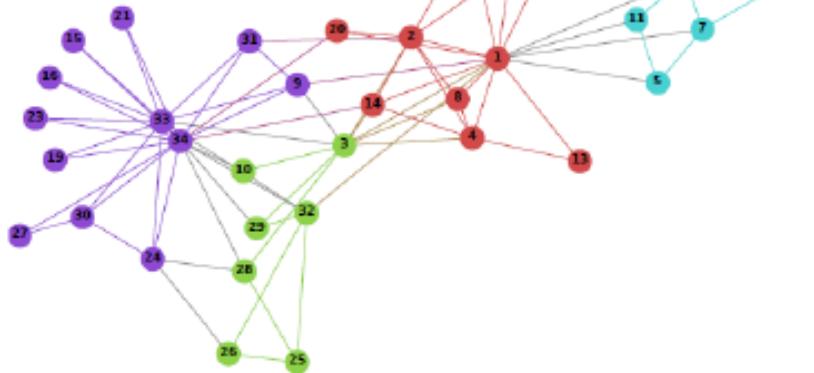
# Why network embedding?

- We map each node in a network into a low-dimensional space
  - Distributed representation for nodes
  - Similarity between nodes indicates link strength
  - Encode network information and generate node representation

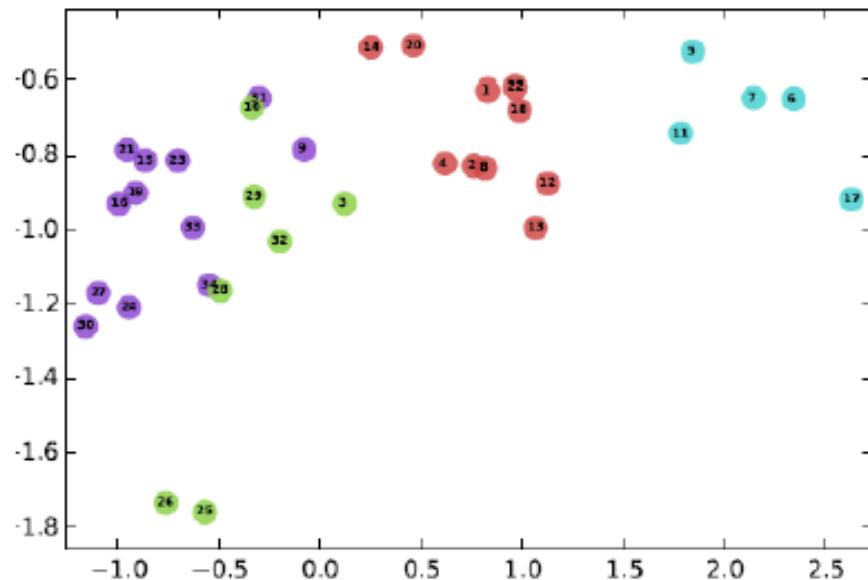


# Example

- Zachary's Karate Club network:



Input

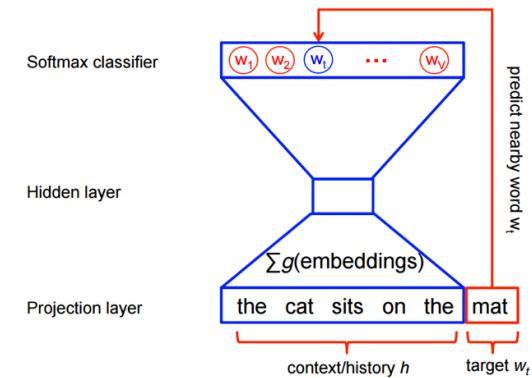
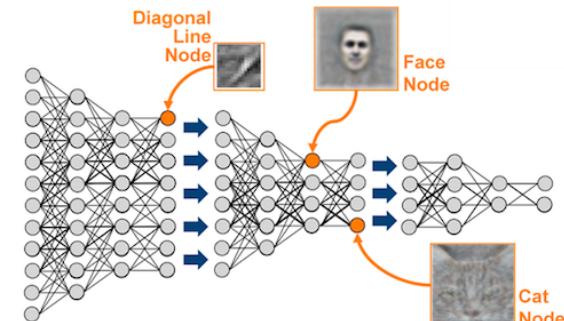


Output

# Why Is It Hard?

Graph representation learning is hard:

- Images are fixed size
  - Convolutions (CNNs)
- Text is linear
  - Sliding window (word2vec)
- Graphs are neither of these!
  - Node numbering is arbitrary  
(node isomorphism problem)
  - Much more complicated structure



# **node2vec:** Random Walk Based (Unsupervised) Feature Learning

[node2vec: Scalable Feature Learning for Networks](#)  
A. Grover, J. Leskovec. KDD 2016.

# Overview of node2vec

- **Goal:** Embed nodes with similar network neighborhoods close in the feature space.
- We frame this goal as prediction-task independent maximum likelihood optimization problem.
- Key observation: Flexible notion of network neighborhood  $N_S(u)$  of node u leads to rich features.
- Develop biased 2<sup>nd</sup> order random walk procedure S to generate network neighborhood  $N_S(u)$  of node u.

# Unsupervised Feature Learning

- **Intuition:** Find embedding of nodes to  $d$ -dimensions that preserves similarity
- **Idea:** Learn node embedding such that **nearby** nodes are close together
- Given a node  $u$ , how do we define nearby nodes?
  - $N_S(u)$  ... neighbourhood of  $u$  obtained by some strategy  $S$

# Feature learning as optimization

- Given  $G = (V, E)$ ,
- Our goal is to learn a mapping  $f: u \rightarrow \mathbb{R}^d$ .
- Log-likelihood objective:  
$$\max_f \sum_{u \in V} \log \Pr(N_S(u) | f(u))$$
  - where  $N_S(u)$  is neighborhood of node  $u$ .
- Given node  $u$ , we want to learn feature representations predictive of nodes in its neighborhood  $N_S(u)$ .

# Feature learning as optimization

$$\max_f \sum_{u \in V} \log \Pr(N_S(u) | f(u))$$

- **Assumption:** Conditional likelihood factorizes over the set of neighbors.

$$\log \Pr(N_S(u | f(u)) = \sum_{n_i \in N_S(u)} \log \Pr(f(n_i) | f(u))$$

- Softmax parametrization:

$$\Pr(f(n_i) | f(u)) = \frac{\exp(f(n_i) \cdot f(u))}{\sum_{v \in V} \exp(f(v) \cdot f(u)))}$$

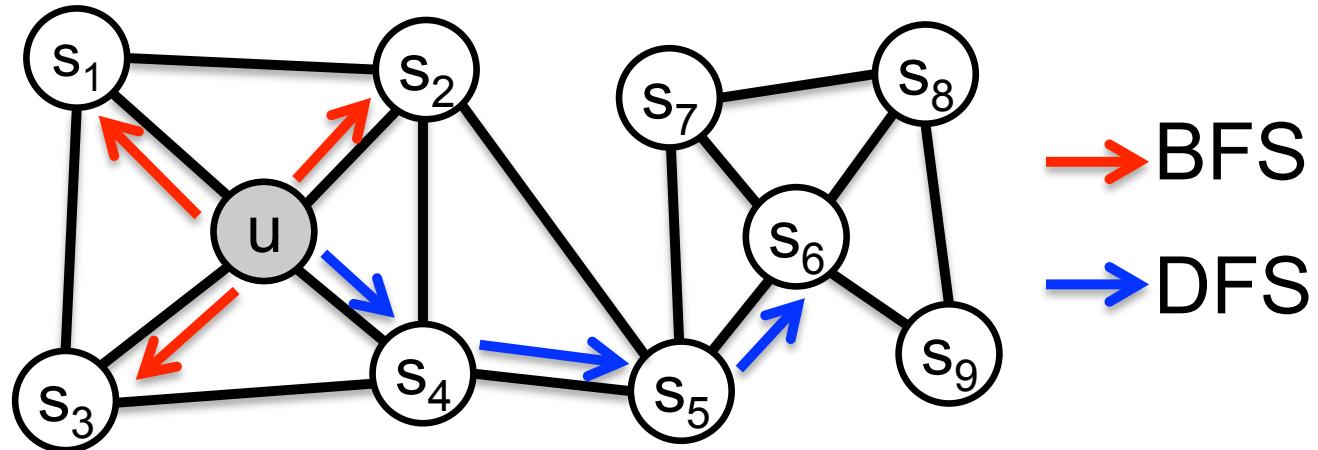
# Negative Sampling

$$\max_f \sum_{u \in V} \sum_{n \in N_S(u)} \log \frac{\exp(f(n_i) \cdot f(u))}{\sum_{v \in V} \exp(f(v) \cdot f(u)))}$$

- Maximize the objective using Stochastic Gradient descent with **negative sampling**.
  - Computing the **summation** is expensive
  - **Idea:** Just sample a couple of “negative nodes”
  - This means at each iteration only embeddings of a few nodes will be updated at a time
  - Much faster training of embeddings

# How to determine $N_S(u)$

Two classic strategies to define a neighborhood  $N_S(u)$  of a given node  $u$ :



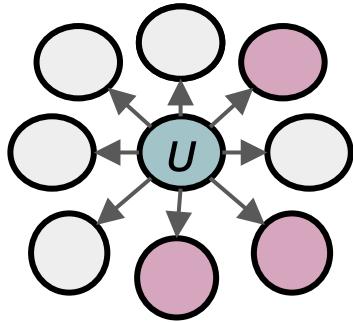
$$N_{BFS}(u) = \{ s_1, s_2, s_3 \}$$

Local microscopic view

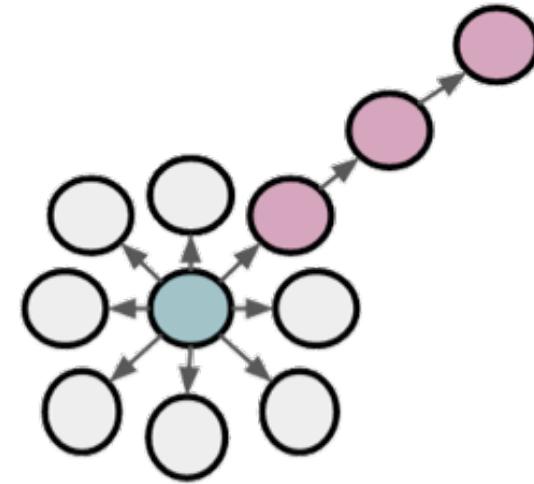
$$N_{DFS}(u) = \{ s_4, s_5, s_6 \}$$

Global macroscopic view

# BFS vs. DFS



**BFS:**  
Micro-view of  
neighbourhood



**DFS:**  
Macro-view of  
neighbourhood

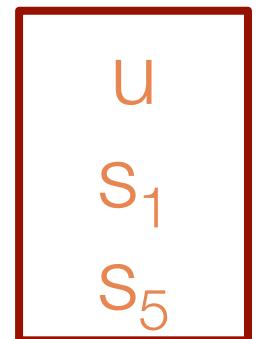
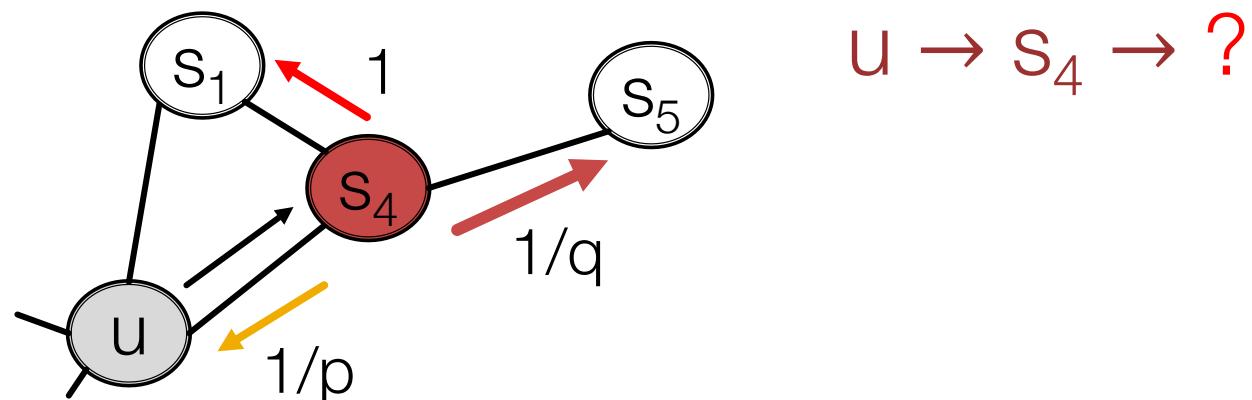
# Interpolating BFS and DFS

Biased random walk  $S$  that given a node  $u$  generates neighborhood  $N_S(u)$

- Two parameters:
  - Return parameter  $p$ :
    - Return back to the previous node
  - In-out parameter  $q$ :
    - Moving outwards (DFS) vs. inwards (BFS)

# Biased Random Walks

$N_S(u)$ : Biased 2<sup>nd</sup>-order random walks explore network neighborhoods:



- BFS-like: low value of  $p$
- DFS-like: low value of  $q$

$p, q$  can learned in a semi-supervised way

# node2vec algorithm

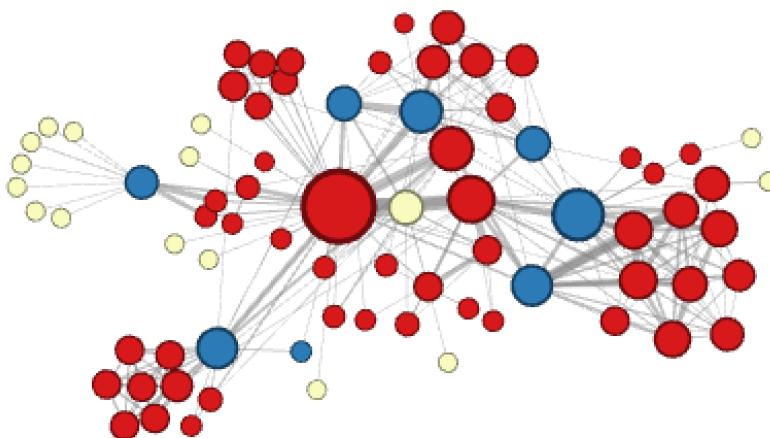
- 1) Compute random walk probs.
- 2) Simulate  $r$  random walks of length  $l$  starting from each node  $u$
- 3) Optimize the node2vec objective using Stochastic Gradient Descent

Linear-time complexity.

All 3 steps are individually parallelizable

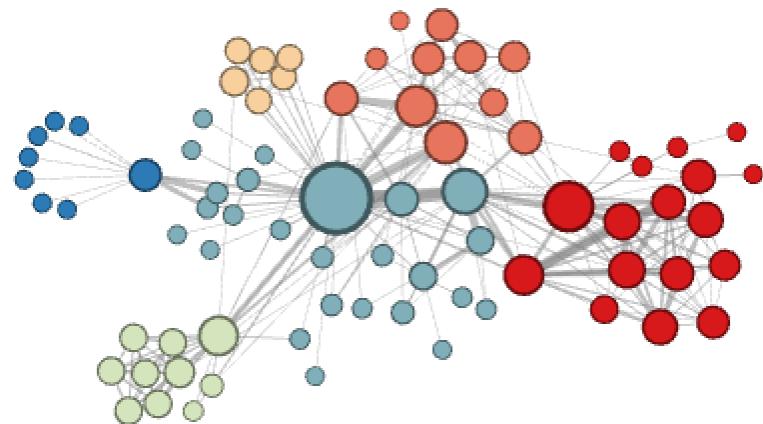
# Experiments: Micro vs. Macro

Interactions of characters in a novel:



$$p=1, q=2$$

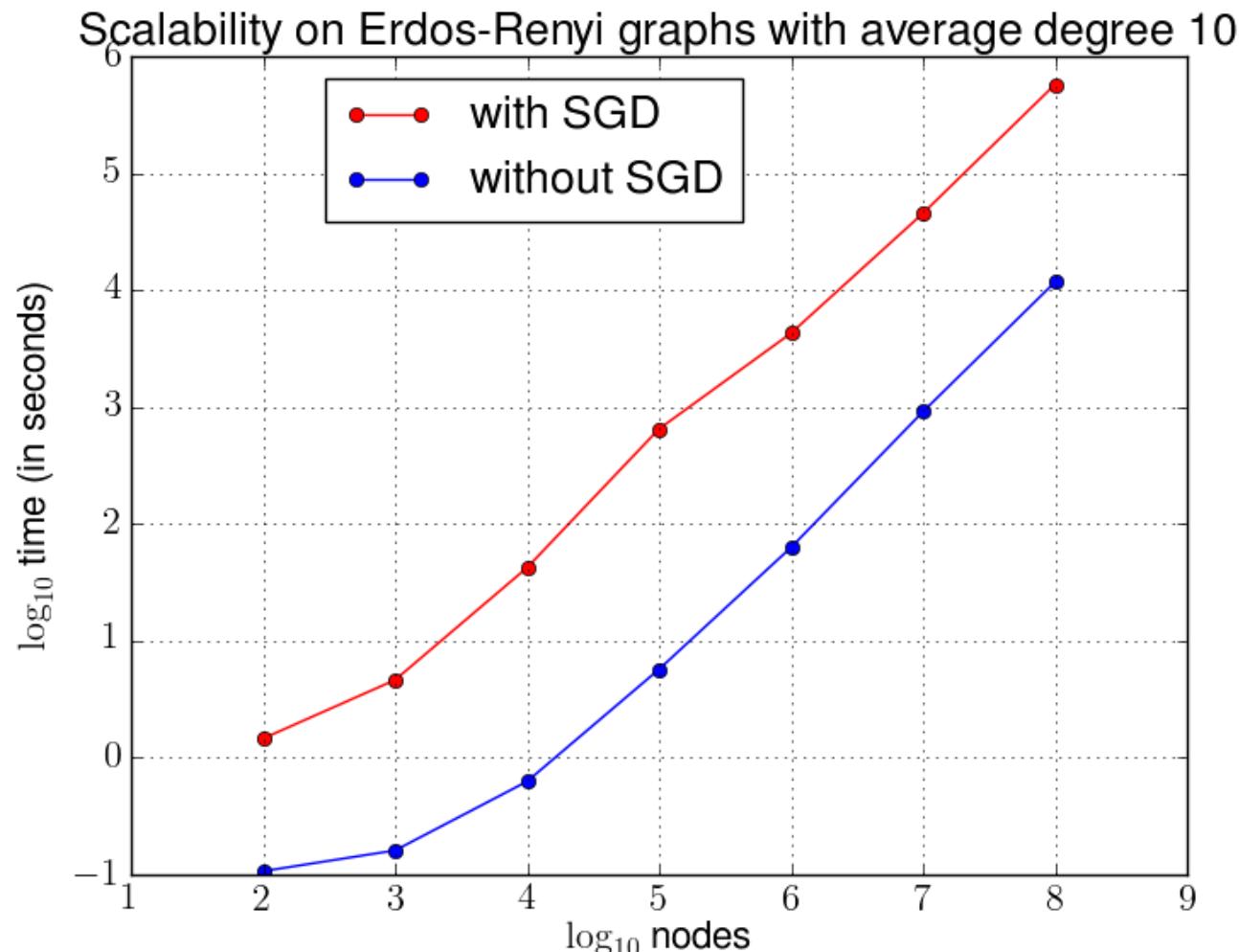
Microscopic view of the network neighbourhood



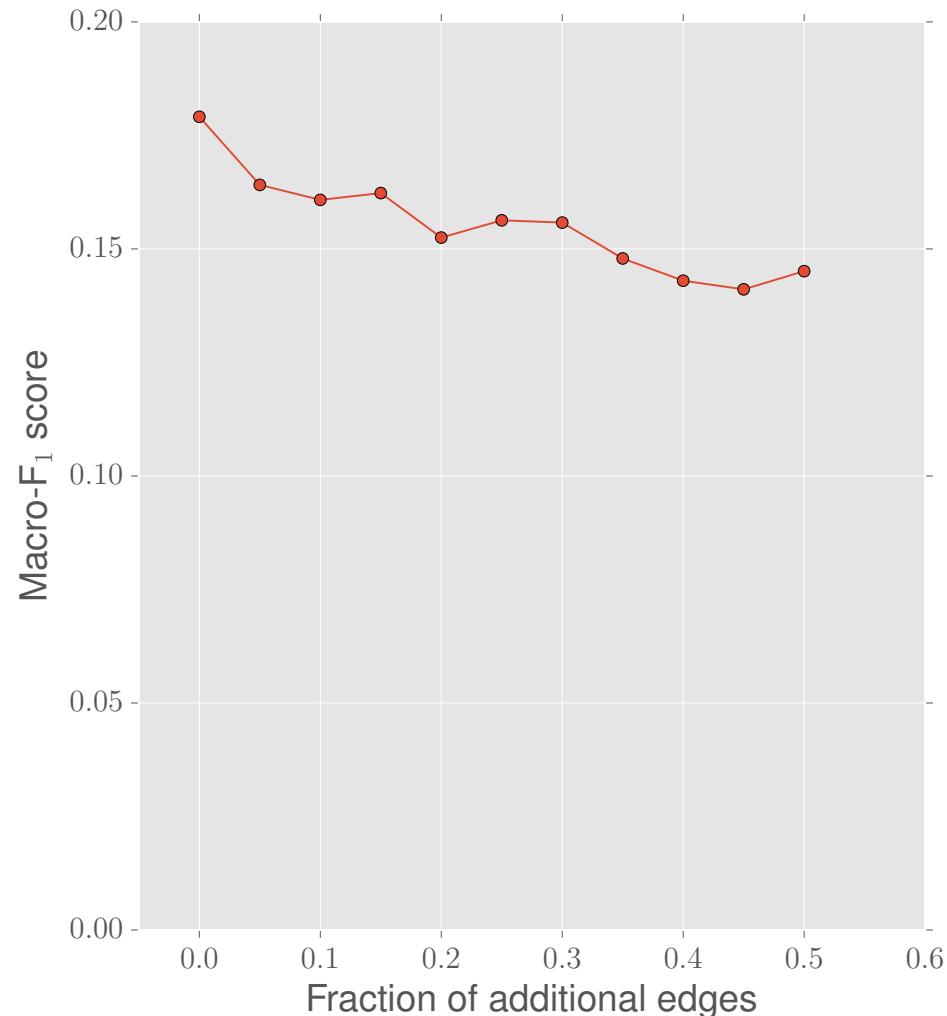
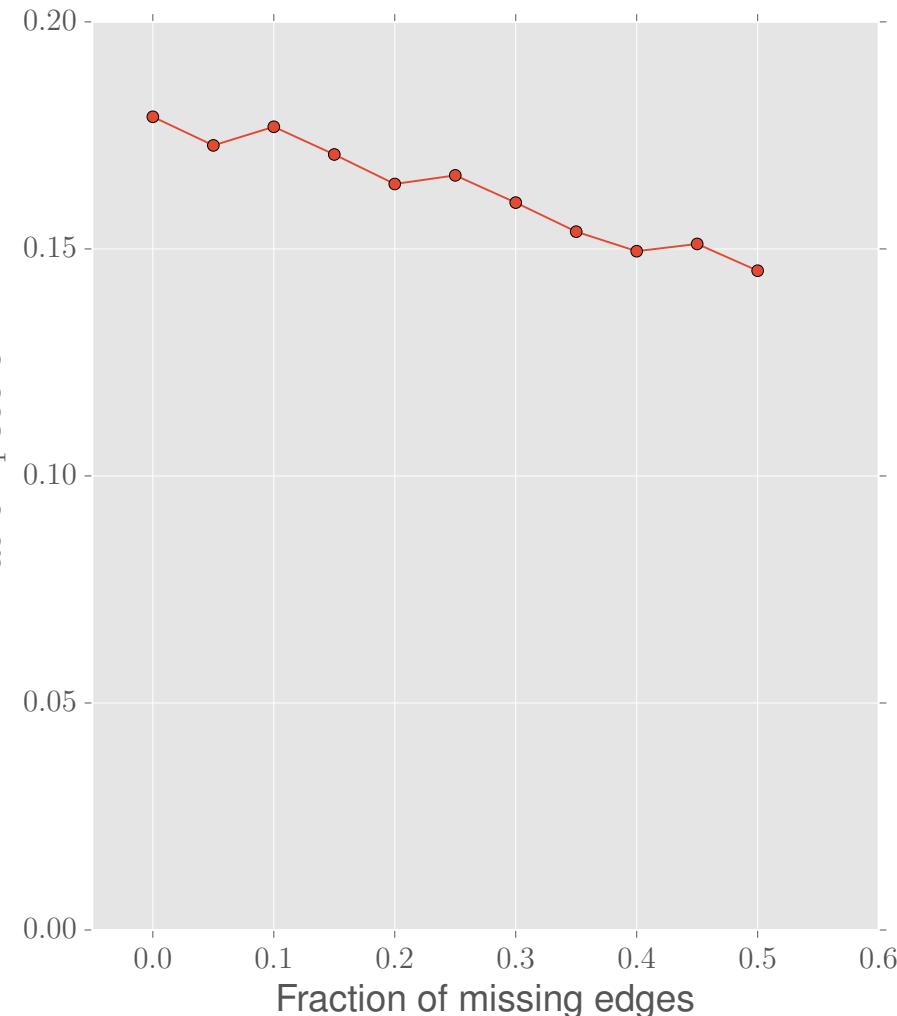
$$p=1, q=0.5$$

Macroscopic view of the network neighbourhood

# Scalability of node2vec



# Incomplete Network Data (PPI)



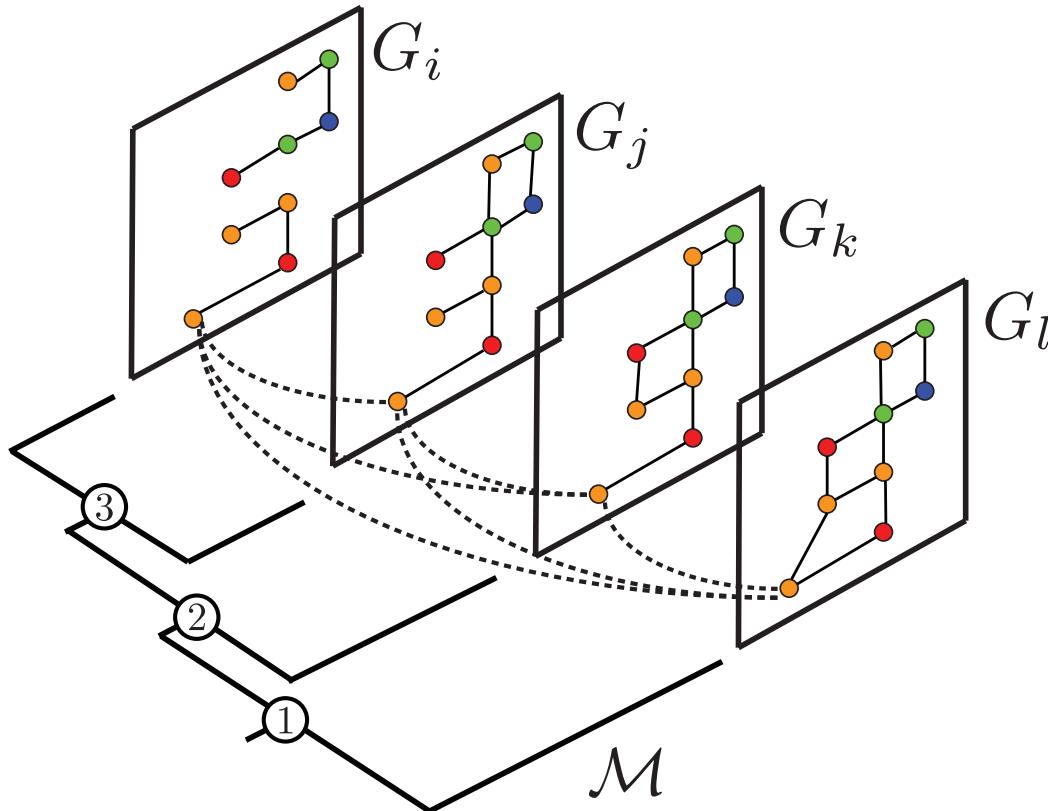
# node2vec: Discussion

General-purpose feature learning in networks:

- An explicit locality preserving objective for feature learning.
- Biased random walks capture diversity of network patterns.
- Scalable and robust algorithm with excellent empirical performance.
- Future extensions would involve designing random walk strategies entailed to network with specific structure such as heterogeneous networks and signed networks.

# OhmNet: Extension to Hierarchical Networks

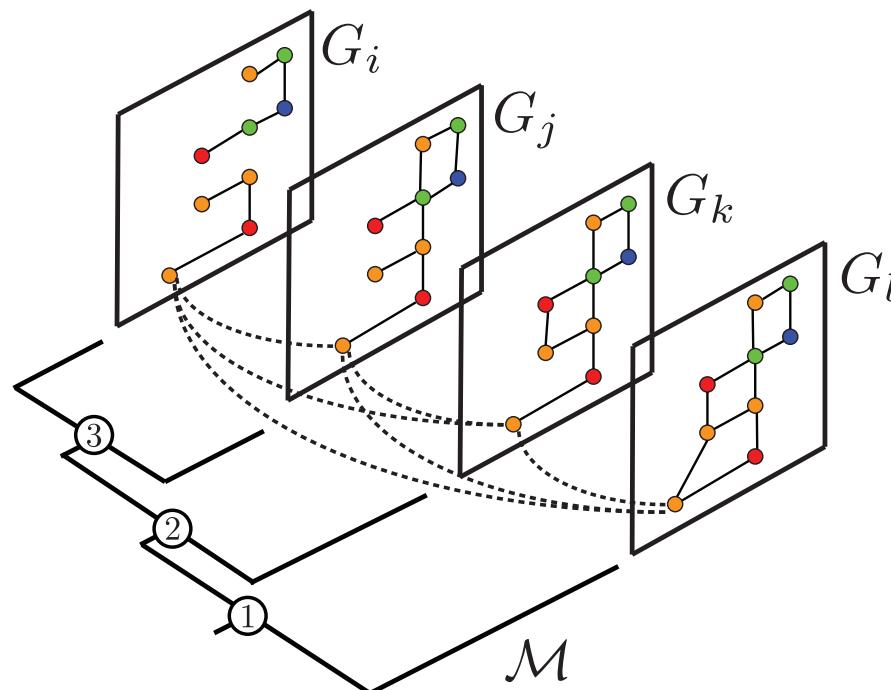
# Multilayer Networks



Let's generalize node2vec to  
multilayer networks!

# Multi-Layer Networks

- Each network is a layer  $G_i = (V_i, E_i)$
- Similarities between layers are given in hierarchy  $\mathcal{M}$ , map  $\pi$  encodes parent-child relationships

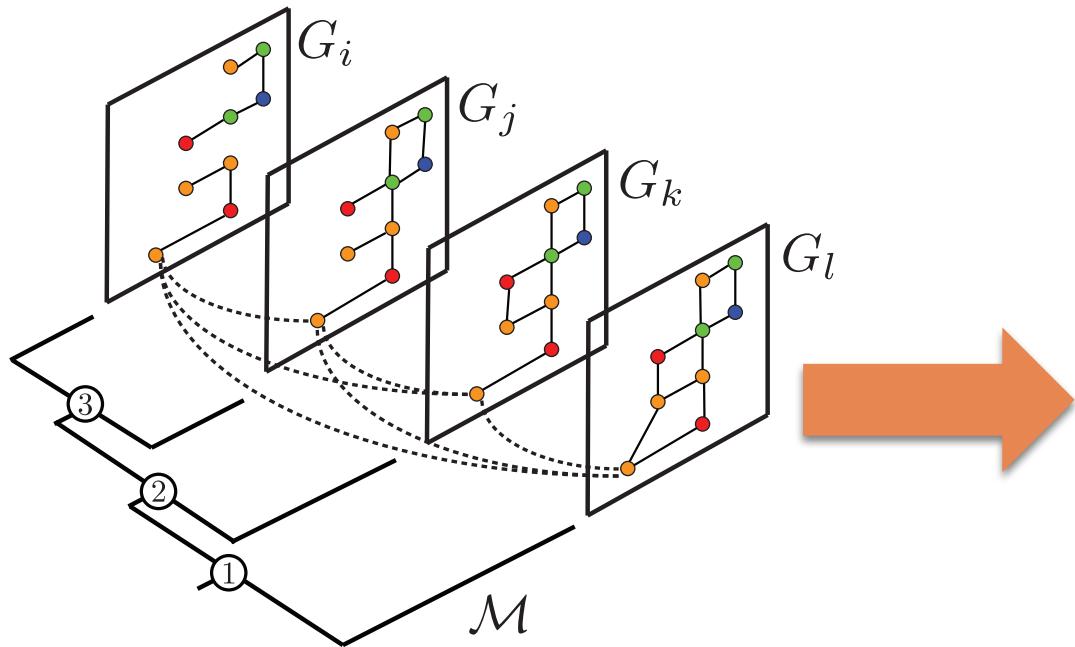


# The Approach

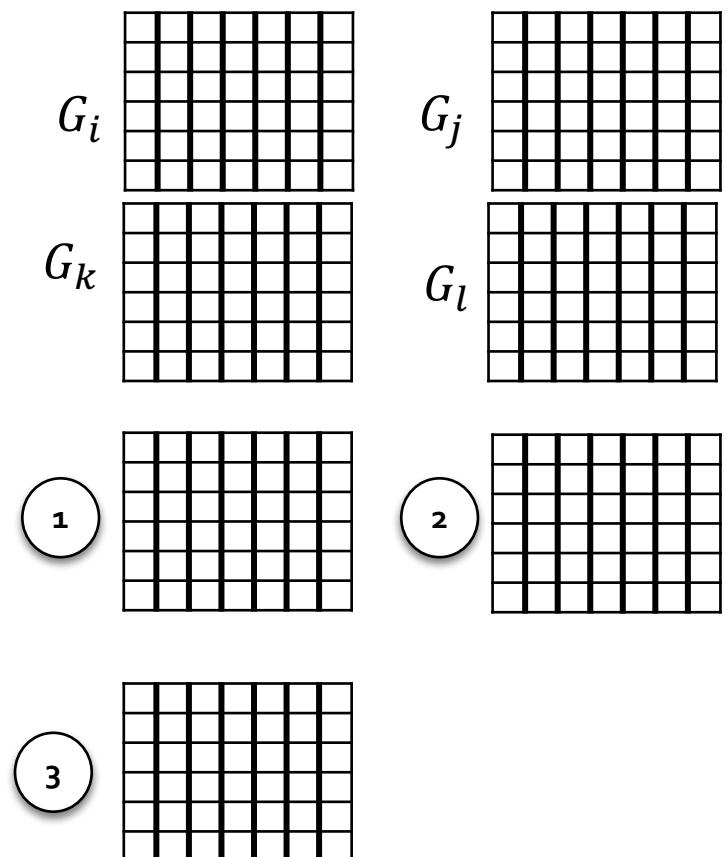
- Computational framework that learns features of every node and at every scale based on:
  - Edges within each layer
  - Inter-layer relationships between nodes active on different layers

# OhmNet

Input

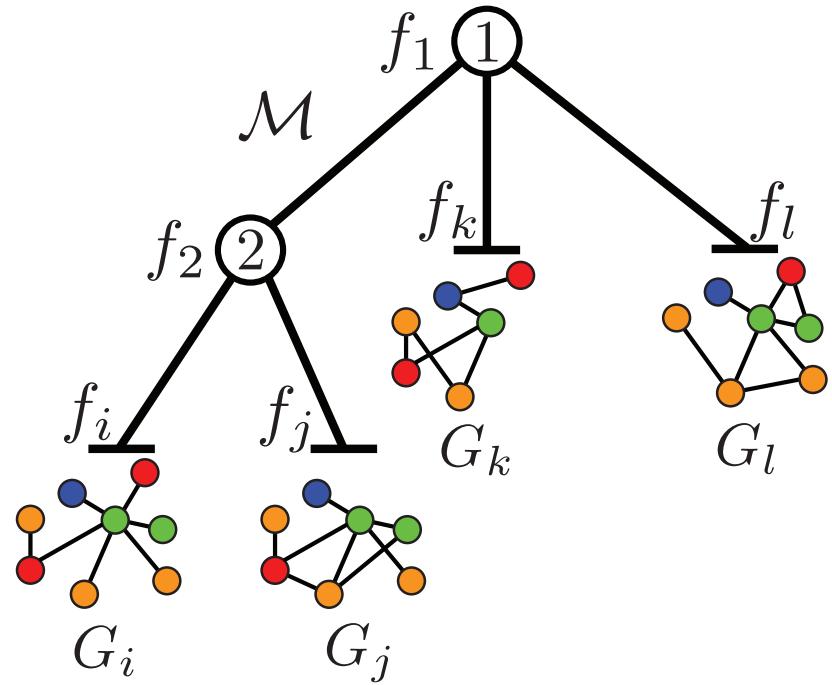


Output: embeddings of nodes in layers as well as internal levels of the hierarchy



# OhmNet

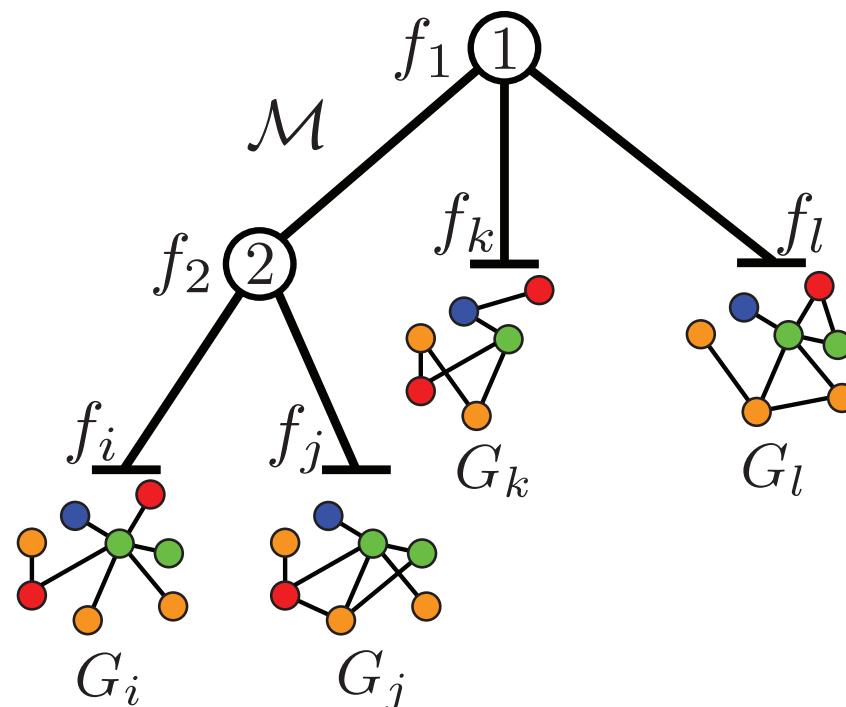
- **OhmNet**: Given layers  $G_i$  and hierarchy  $M$ , learn node features captured by functions  $f_i$
- Functions  $f_i$  embed every node in a  $d$ -dimensional feature space



A multi-layer network with four layers and a two-level hierarchy  $M$

# Features in Multi-Layer Network

- Given: Layers  $\{G_i\}$ , hierarchy  $\mathcal{M}$ 
  - Layers  $\{G_i\}_{i=1..T}$  are in leaves of  $\mathcal{M}$
- Goal: Learn functions:  $f_i: V_i \rightarrow \mathbb{R}^d$



# Features in Multi-Layer Network

- Approach has two components:
  - **Per-layer objectives:** Nodes with similar network neighborhoods in each layer are embedded close together
  - **Hierarchical dependency objectives:** Nodes in nearby layers in hierarchy are encouraged to share similar features

# Per-Layer Objective: node2vec

- **Intuition:** For each layer, find a mapping of nodes to  $d$ -dimensions that preserves node similarity
- **Approach:** Similarity of nodes  $u$  and  $v$  is defined based on similarity of their network neighborhoods
- Given node  $u$  in layer  $i$  we define nearby nodes  $N_i(u)$  based on random walks starting at node  $u$

# Per-Layer Objective: node2vec

- Given node  $u$  in layer  $i$ , learn  $u$ 's representation such that it predicts nearby nodes  $N_i(u)$ :

$$\omega_i(u) = \log Pr(N_i(u) | f_i(u))$$

- Given  $T$  layers, maximize:

$$\Omega_i = \sum_{u \in V_i} \omega_i(u), \quad \text{for } i = 1, 2, \dots, T$$

- Notice:** Nodes in different networks representing the same entity have different features

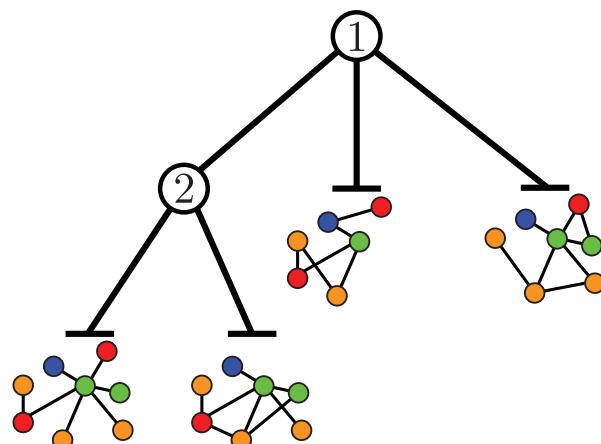
# Interdependent Layers

- So far, we did not consider hierarchy  $\mathcal{M}$
- Node representations in different layers are learned independently of each other

How to model dependencies between layers when learning node features?

# Hierarchical regularization

- We use regularization to share information across the hierarchy
  - We want to enforce similarity between feature representations of networks that are located nearby in the hierarchy



# Interdependent Layers

- Given node  $u$ , learn  $u$ 's representation in layer  $i$  to be close to  $u$ 's representation in parent  $\pi(i)$ :

$$c_i(u) = \frac{1}{2} \|f_i(u) - f_{\pi(i)}(u)\|_2^2$$

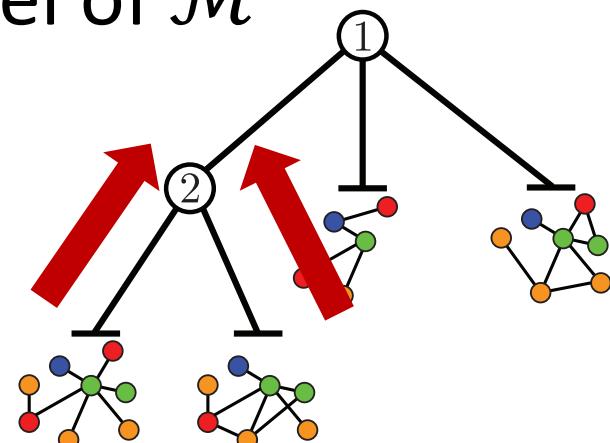
- Multi-scale:** Repeat at every level of  $\mathcal{M}$

$$C_i = \sum_{u \in L_i} c_i(u)$$

$L_i$  has all layers appearing in sub-hierarchy rooted at  $i$

12/4/17

Jure Leskovec, Stanford CS224W: Analysis of Networks, <http://cs224w.stanford.edu>



# Implications

- Nodes in different layers representing the same entity have the same features in hierarchy ancestors
- We learn feature representations at multiple scales:
  - features of nodes in the layers
  - features of nodes in non-leaves in the hierarchy
- This model is more efficient than the fully pairwise model, where dependencies between layers are modeled by pairwise comparisons of nodes across all pairs of layers

# OhmNet: Final Model

Learning node features in multi-layer networks

Solve maximum likelihood problem:

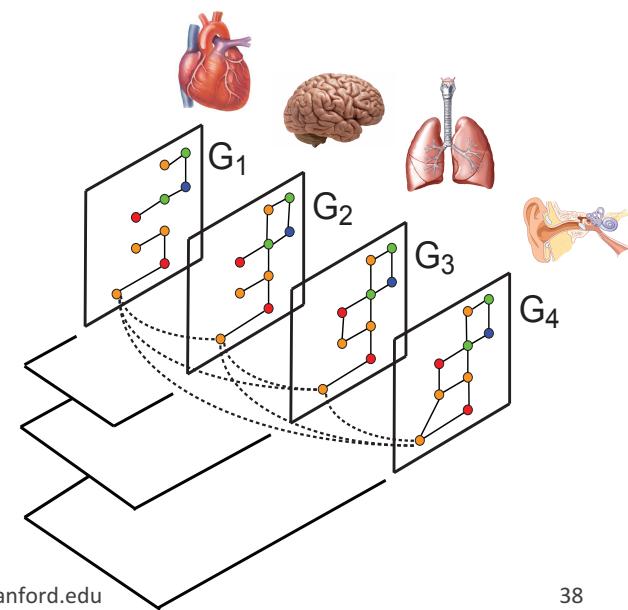
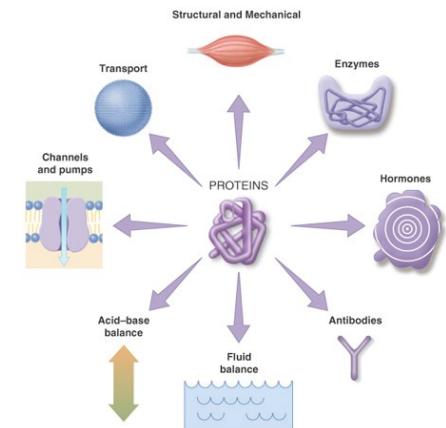
$$\max_{f_1, f_2, \dots, f_M} \left[ \sum_{i \in \mathcal{T}} \Omega_i - \lambda \sum_{j \in \mathcal{M}} C_j \right].$$

Per-layer  
network  
objectives

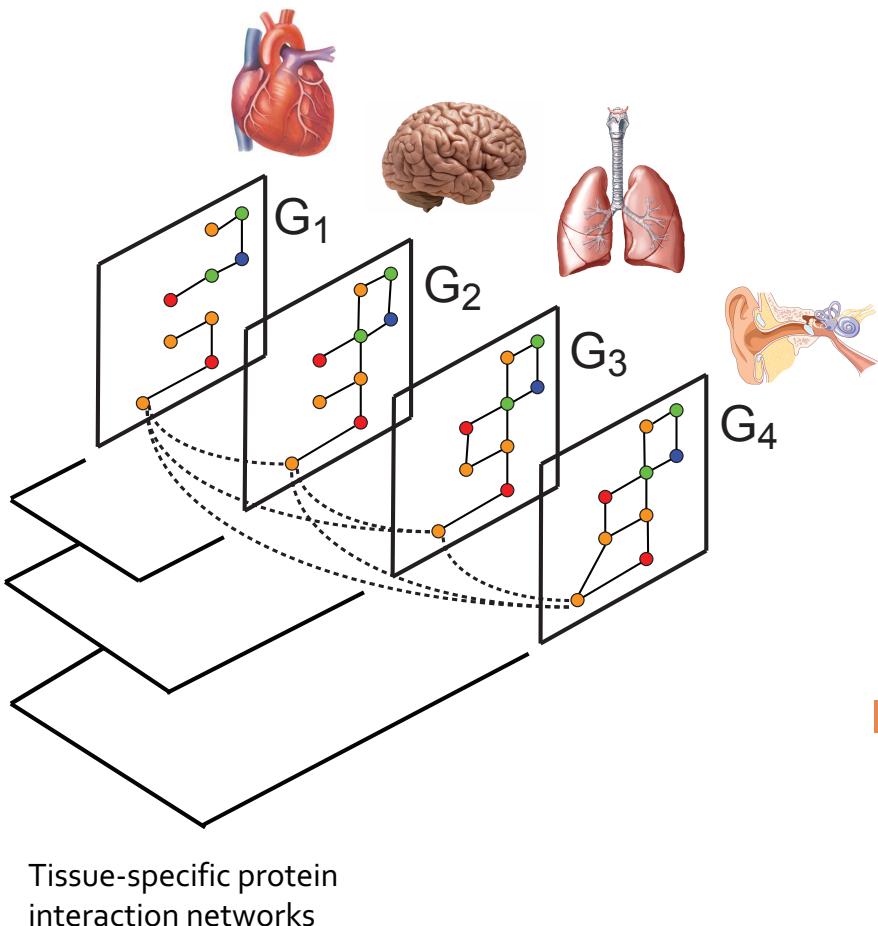
Hierarchical  
dependency  
objectives

# Application: Protein function

- Proteins are worker molecules
  - Understanding protein function has great biomedical and pharmaceutical implications
- Function of proteins depends on their tissue context  
[Greene et al., Nat Genet '15]



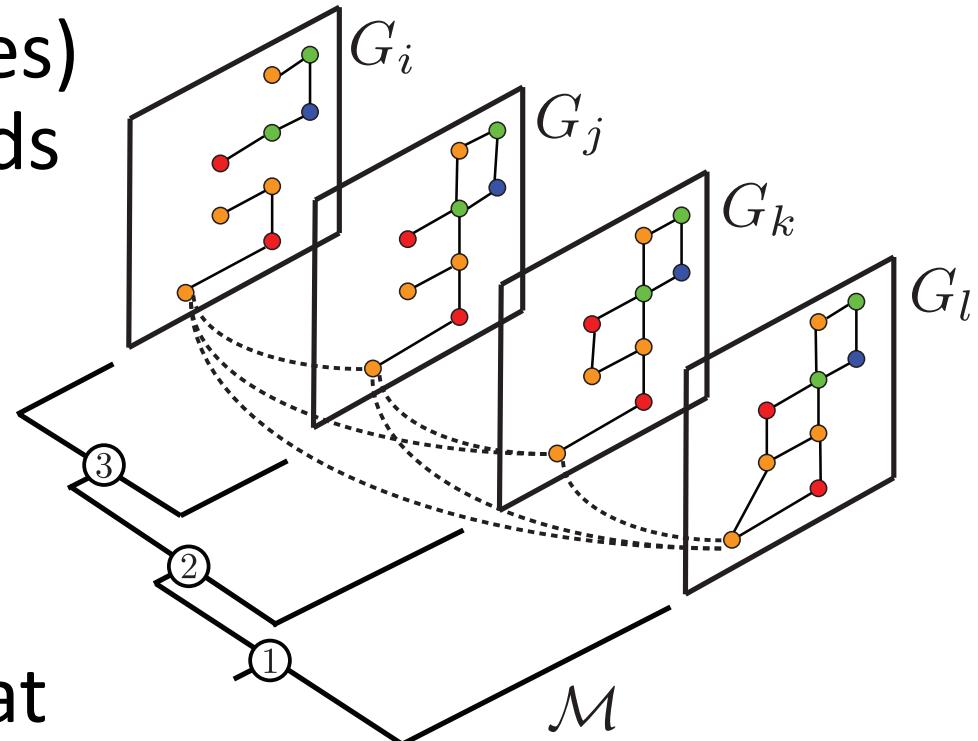
# Protein functions are tissue-specific



- The precise function of proteins depends on their tissue context (Greene et al., Nat Genet 2015)
- Diseases result from the failure of tissue-specific processes (Hu et al., Nat Rev Genet 2016)
- Current models assume that protein functions are constant across tissues

# Multi-layer tissue network

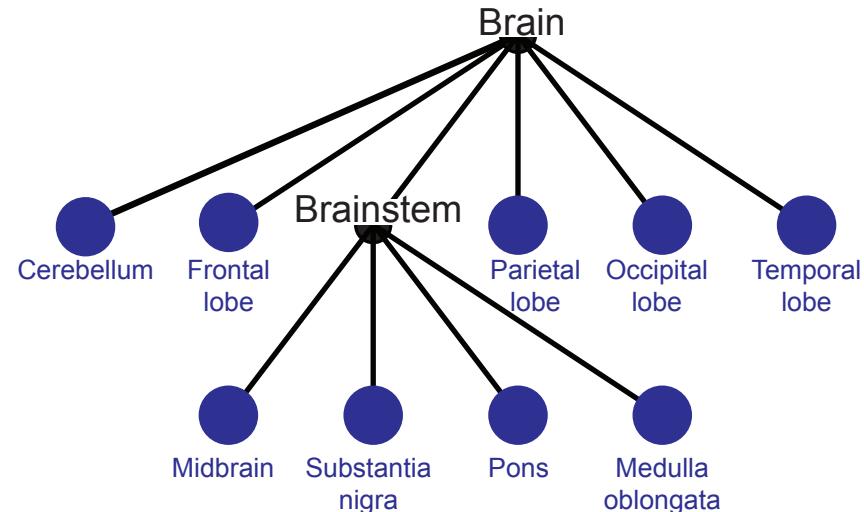
- A multi-layer tissue network has many network layers (tissues)
- Each layer corresponds to one tissue-specific protein interaction network
- Hierarchy  $\mathcal{M}$  encodes biological similarities between the tissues at multiple scales



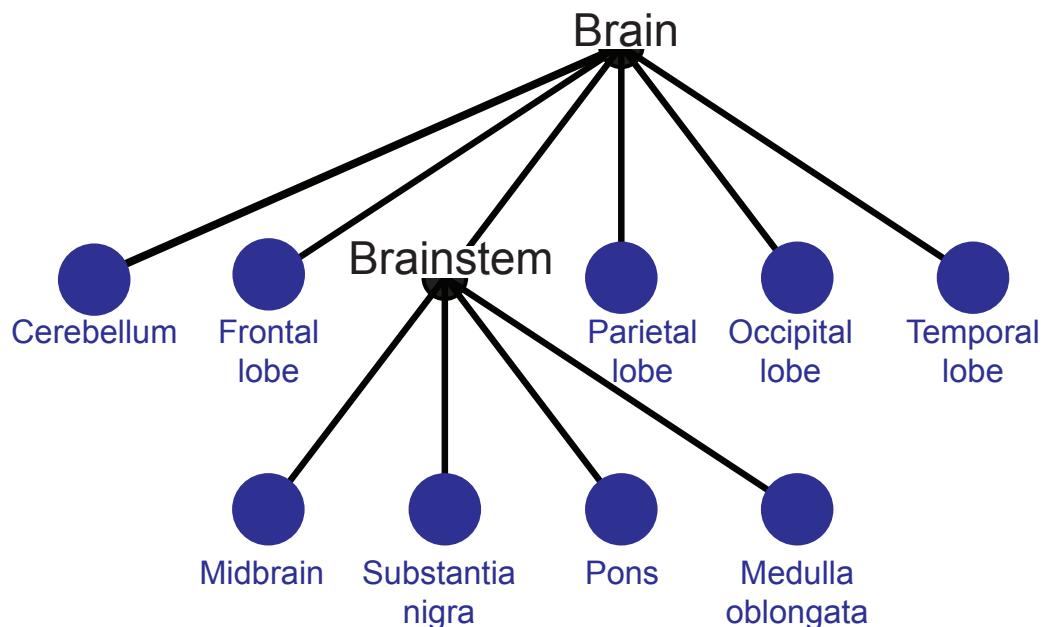
# Experiments: Biological Nets

107 genome-wide  
tissue-specific  
protein interaction  
networks

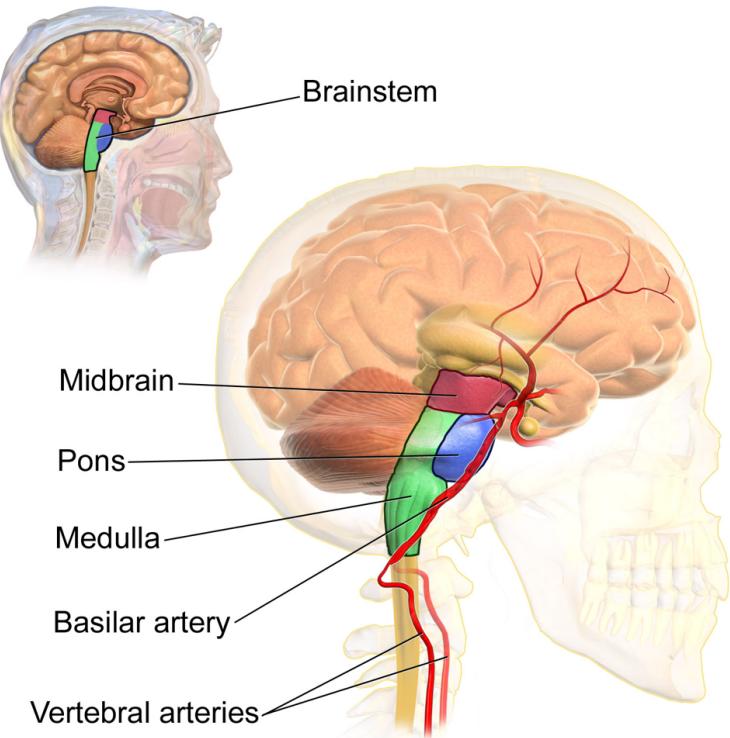
- 584 tissue-specific cellular functions
- Examples (tissue, cellular function):
  - (renal cortex, cortex development)
  - (artery, pulmonary artery morphogenesis)



# Brain Tissues

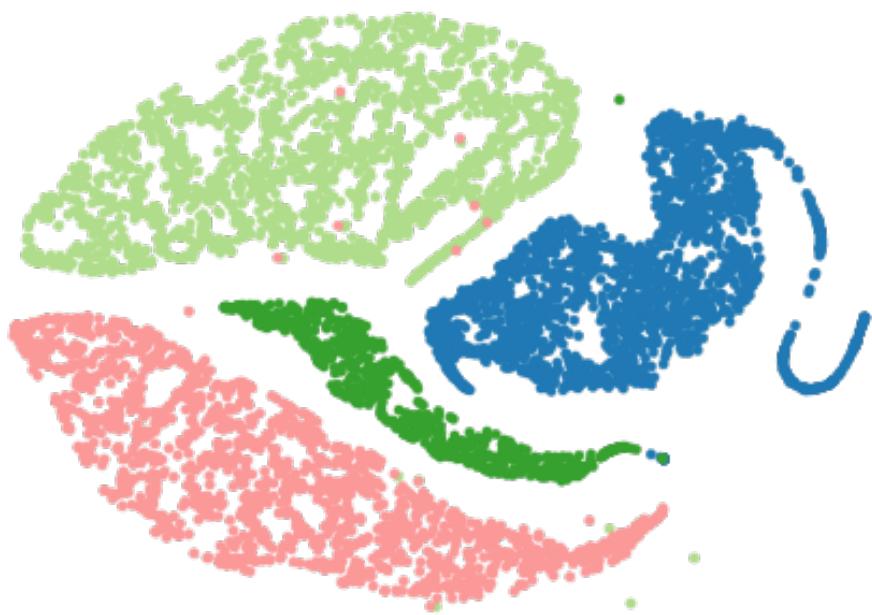


9 brain tissue PPI networks  
in two-level hierarchy



# Meaningful Node Embeddings

Brainstem

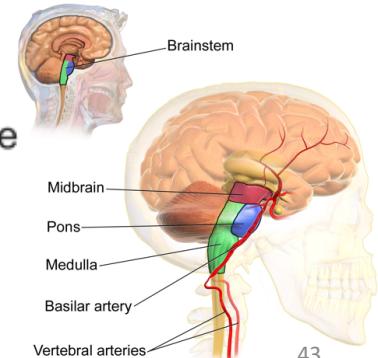


- Cerebellum
- Medulla oblongata
- Substantia nigra

Brain



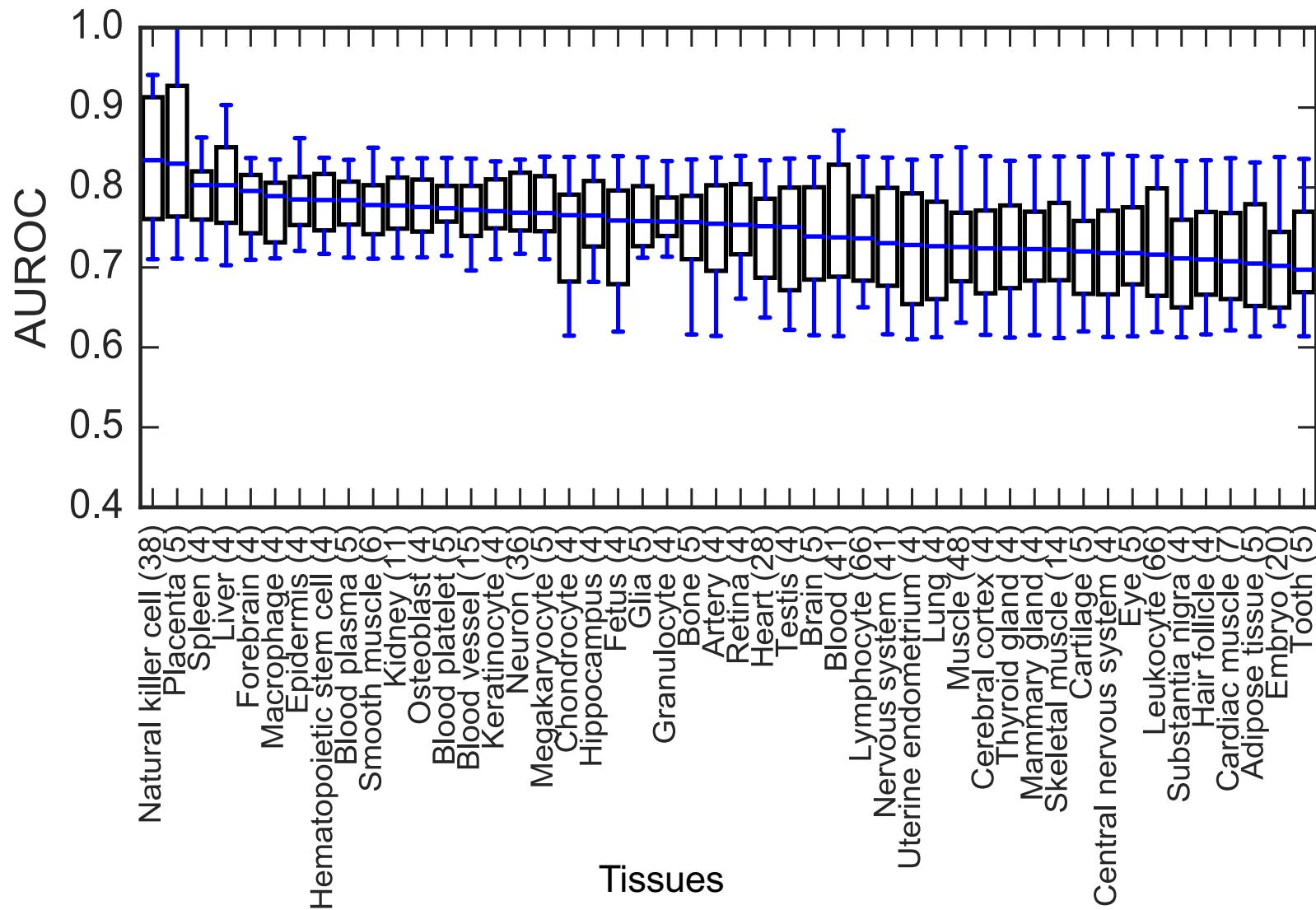
- Frontal lobe
- Temporal lobe
- Pons
- Parietal lobe
- Occipital lobe
- Midbrain



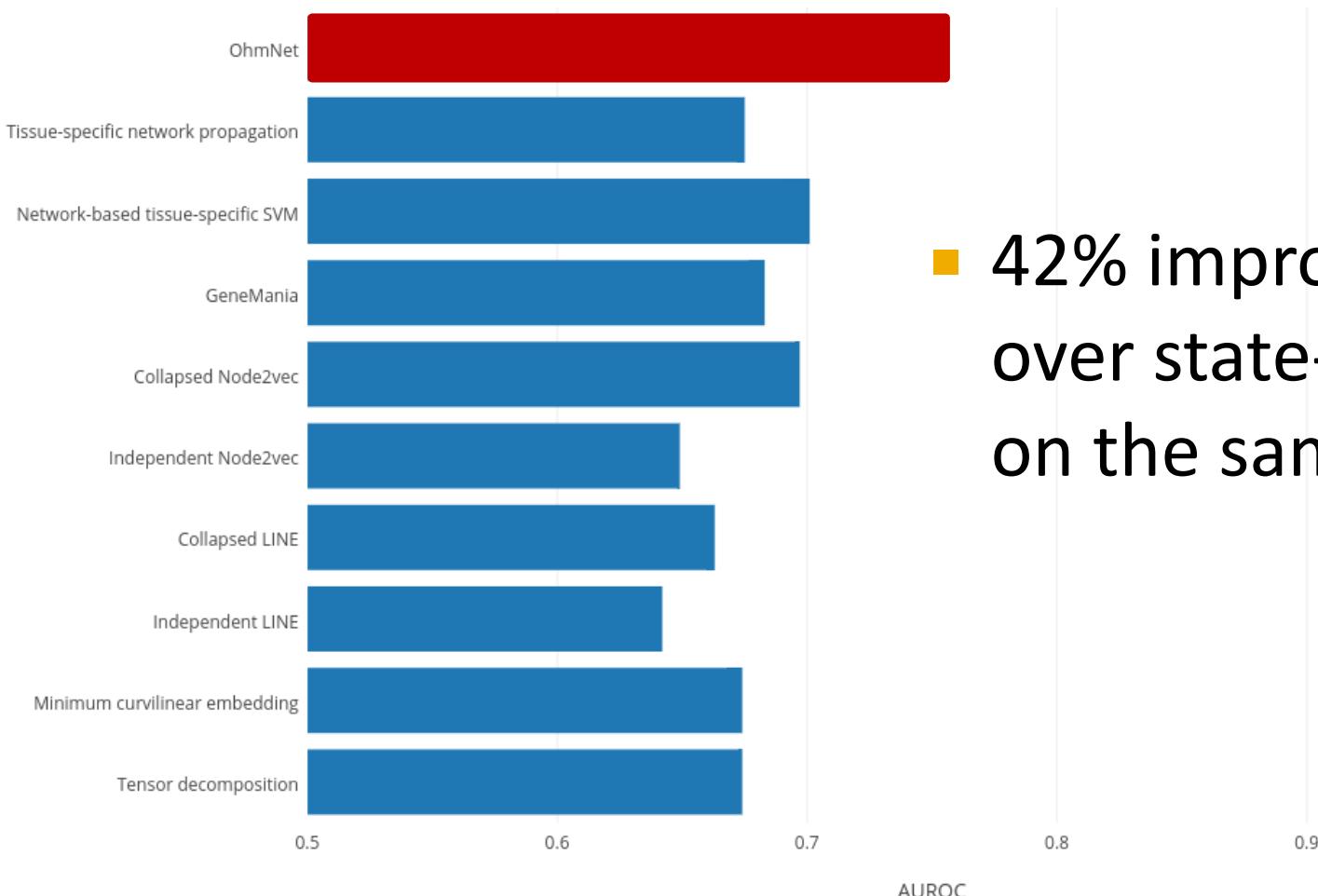
# Experimental setup

- Cellular function prediction is a multi-label node classification task
- Every node (protein) is assigned one or more labels (cellular functions)
- Setup:
  - We apply OhmNet, which for every node in every layer learns a separate feature vector in an unsupervised way.
  - For every layer and every function we then train a separate one-vs-all regularized linear classifier using the modified Huber loss
  - During the training phase, we observe only a certain fraction of proteins and all their cellular functions across the layers
  - The task is then to predict the tissue-specific functions for the remaining proteins

# Protein Function Prediction

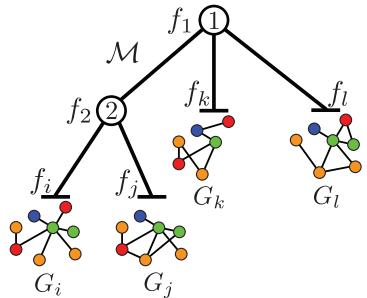


# Protein Function Prediction



- 42% improvement over state-of-the-art on the same dataset

# Transfer Learning



Transfer functions to unannotated tissues

- Task: Predict functions in target tissue without access to any annotation/label in that tissue

Target tissue	OhmNet	Tissue non-specific	Improvement
Placenta	0.758	0.684	11%
Spleen	0.779	0.712	10%
Liver	0.741	0.553	34%
Forebrain	0.755	0.632	20%
Blood plasma	0.703	0.540	40%
Smooth muscle	0.729	0.583	25%
Average	0.746	0.617	21%

Reported are AUC values