Problem 2

1) The sample space consists in all the pairs (gender, day of week) that can represent ~~the~~ the two child

$$\Omega = \{((B, \text{Monday})(B, \text{Tuesday})), ((B, \text{Monday})(G, \text{Tuesday})), \ldots$$
$$\ldots ((G, \text{Sunday})(G, \text{Sunday}))\}$$

Since there are 2 child and 7 days we have for 1 child 14 possible outcomes and for 2 childs 196
So the probability of each simple event in the sample space is

$$P(\text{event}) = \frac{1}{196}$$

2)
event A = "1 child is a girl"
event B = "Both children are girls"

We need to calculate the conditional probability

$$P(B|A) = \frac{P(B \cap A)}{P(A)}$$

In this case the sample space is $\Omega = \{(G,G)(G,B)(B,G)(B,B)\}$

$$P(B|A) = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$$

3)
~~An~~ event A = "girl born on Sunday"
event B = "both children are girls"

$$P(B|A) = \frac{P(B \cap A)}{P(A)}$$

$$P(A) = 1 - \frac{13 \cdot 13}{196} = \frac{27}{196}$$

for each child there are 14 possible outcome and if we exclude A we have 13 outc. for each child

$P(B \cap A) = \left(\frac{1}{14} \cdot \frac{1}{2}\right) \cdot 2 = \frac{1}{14}$

$\frac{1}{14}$ is the prob. that 1° child is born on Sunday, $\frac{1}{2}$ the prob. that other child is also a girl born on any other day. This can happen for 1° or 2° child.

$$P(B \mid A) = \frac{\frac{1}{14}}{\frac{27}{196}} = \frac{1}{14} \cdot \frac{196}{27} = \frac{14}{27} \simeq 0,518$$

Problem 3

The sample space is composed by $\Omega = \{DM, \neg DM, T_{pos}, T_{neg}\}$
We can have 4 possible outcomes:

(DM, T_{pos}), $(\neg DM, T_{pos})$, $(\neg DM, T_{neg})$, (DM, T_{neg})

We want to find

$$P(DM \mid T_{pos}) = \frac{P(DM \cap T_{pos})}{P(T_{pos})} = \frac{P(T_{pos} \mid DM) \cdot P(DM)}{P(T_{pos})}$$

where $P(T_{pos})$ is the prob of having a positive test

$$P(T_{pos}) = P(T_{pos} \mid DM) \cdot P(DM) + P(T_{pos} \mid \neg DM) \cdot P(\neg DM)$$

we have all these probability with the problem.

$$P(T_{pos}) = (0,999 \cdot 0,00001) + (0,001 \cdot 0,99999) = 0,00100998$$

$$\Rightarrow P(DM \mid T_{pos}) = \frac{0,999 \cdot 0,00001}{0,00100998} \approx 0,00989 \simeq 1\%$$