Problem 5

1) We define Ω as $\Omega = \{ \delta_i : \delta \text{ is a possible graph configuration} \}$

In a graph with n nodes we can have up to $\binom{n}{2} = \frac{n(n-1)}{2}$ edges. In the case of $G_{n,p}$ we have a prob. p that each edge ~~appears~~ be ~~and~~ is in the configuration and $(1-p)$ that it isn't.

So the size of Ω is $|\Omega| = 2^{\frac{n(n-1)}{2}}$ where

2 is because an edge can be present or not.

---

2) Assume a specific configuration δ_1 of the $G_{n,p}$ model in which we have m edges. So we have

m edges that occurs with probab. p

$\frac{n(n-1)}{2} - m$ edges that // // // $(1-p)$. Moreover the # of ways to build

$$\Rightarrow \Pr(\delta_1) = \binom{\frac{n(n-1)}{2}}{m} p^m \cdot (1-p)^{\frac{n(n-1)}{2} - m}$$

$\binom{\frac{n(n-1)}{2}}{m}$ = a graph with m edges in

it is multiplied bc. each edge is an indipendent event.

---

3) n is even and we want the $G_{n,p}$ have 2 cycles of $\frac{n}{2}$ edges.

Let $k = \frac{n}{2}$. We divide n ~~edges~~ nodes into 2 disjoint subsets each of size $\frac{n}{2} = k$. Each set will form a cycle.

ways to divide n ~~...~~ nodes $\Rightarrow \frac{\binom{n}{k}}{2}$ divided by 2 because the configurations ~~...~~ repeats for the 2 sets of nodes.

of ways to form a cycle on k vertices $\Rightarrow \frac{(k-1)!}{2}$
of lenght k

~~...~~ EX $\Rightarrow$ $k=3$ $\Rightarrow$ 1 cycles, the triangle

⇒ total # of ways to form 2 disjoint cycles of k $\frac{m}{2}$ edges and m nodes is

$$\frac{\binom{m}{k}}{2} \cdot \frac{(k-1)!}{2} \cdot \frac{(k-1)!}{2}$$

For the configuration G_2 we have m edges ~~that~~ to be present with prob. p and the others not with prob. $(1-p)$

$$Pr(G_2) = \left(\frac{\binom{m}{k}}{2} \frac{(k-1)!^2}{4}\right) p^m \cdot (1-p)^{\frac{m(m-1)}{2}-m}$$

4) Similar to the previous points in this case we can have ~~&~~ 2 cycles of any lenght

lenght cycle 1 ⇒ l_1 edges. lenght cycle 2 ⇒ l_2 edges. , $l_1 + l_2 = m$

⇒ # of ways to choose l_1 nodes from m $\binom{m}{l_1}$

the other $m - l_1 = l_2$ nodes will form a cycle too

⇒ # of ways to form cycles $\frac{(l_1-1)!}{2}$ $\frac{(l_2-1)!}{2}$

so the total is $\frac{(l_1-1)!}{2} \cdot \frac{(l_2-1)!}{2} \cdot \binom{m}{l_1}$

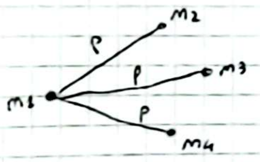
As before the edges present in the conf. have prob. p and the others $(1-p)$

$$\Rightarrow Pr(G_3) = \frac{(l_1-1)!}{2} \frac{(l_2-1)!}{2} \cdot \binom{m}{l_1} \cdot p^m \cdot (1-p)^{\frac{m(m-1)}{2}-m}$$

for $l_1, l_2 \geq 3 \wedge l_1 + l_2 = m$

5) In the $G_{n,p}$ model there are n nodes so a node n_i can connect to other $n-1$ nodes. The probab. that there is an edge bet. 2 nodes is p so

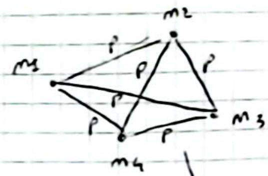
expected degree of node $\Rightarrow (n-1) \cdot p$



---

6) In the $G_{n,p}$ model the number of possible edges is

$$\binom{n}{2} = \frac{n \cdot (n-1)}{2}$$



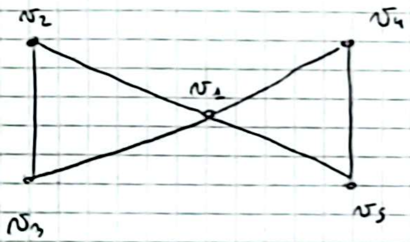
$\frac{4 \cdot 3}{2} = 6$ edges

each one can appear with pr. p so

expected # of edges $\Rightarrow \frac{n(n-1)}{2} \cdot p$

---

7)



For a papillon configuration we have 6 edges that appears with pr. p.

and 4 edges with $1-p$

(in this case $\{v_2, v_5\}, \{v_3, v_4\} \{v_2, v_4\} \{v_3, v_5\}$)

so for the conf. we have $\Rightarrow$ $Pr(\text{"papillon"}) = p^6 \cdot (1-p)^4$

but given a graph with n nodes the possible papillon configuration are $\binom{n}{5}$, so each choice of 5 nodes can be a papillon

$$\binom{n}{5} = \frac{n!}{5!(n-5)!}$$

$$Pr(\text{papillon in } G_{n,p}) = \binom{n}{5} p^6 \cdot (1-p)^4$$