Problem 1

1) Ω is the set of all possible ways to arrange a deck of 52 c.

$$\Omega = \{ \delta_1, \delta_2, \dots, \delta_{52!} \}$$ where δ_i is a permutation of deck

$$Pr(\delta_i) = \frac{1}{52!}$$

2) a = "first 4 cards include at least 1 club"

We can compute $Pr(a)$ with the complementary ~~P~~ $1 - Pr(\neg a)$

$\neg a$ = "none of the first 4 c. are clubs"

we have $52 - 13 = 39$ non clubs ~~out the~~

$$Pr(\neg a) = \frac{\binom{39}{4}}{\underbrace{\binom{52}{4}}_{\text{\# of ways we can pick 4 cards in a deck of 52}}} = \frac{\frac{39!}{4!\cdot 35!}}{\frac{52!}{4!\cdot 48!}} = \frac{39\cdot 38\cdot 37\cdot 36}{4\cdot 3\cdot 2\cdot 1} \cdot \frac{4\cdot 3\cdot 2\cdot 1}{52\cdot 51\cdot 50\cdot 49} =$$

$\simeq 0{,}3038 \Rightarrow$ so $P(a) = 1 - 0{,}3038 \simeq 0{,}6962$

---

b = "first 7 cards include exactly 1 club"

as before 39 are non-clubs and 13 clubs. Over 7 card we have to choose 1 club and 6 non club

$$Pr(b) = \frac{\text{favourable outcomes}}{\text{total outcomes}} = \frac{\binom{13}{1}\cdot\binom{39}{6}}{\binom{52}{7}} =$$ ~~13·2·1 39·38·37~~

$$= \frac{13 \cdot \frac{39\cdot 38\cdot 37\cdot 36\cdot 35\cdot 34}{\cancel{6}\cdot\cancel{5}\cdot\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot\cancel{1}}}{\frac{52\cdot 51\cdot 50\cdot 49\cdot 48\cdot 47\cdot 46}{\cancel{7}\cdot\cancel{6}\cdot\cancel{5}\cdot\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot\cancel{1}}} \simeq 0{,}317$$

c = "first 3 cards of same suits"

we have 4 suits, each with 13 cards,

the # of ways we can pick 3 cards of same suit is $\binom{13}{3}$,

but we have 4 different suits so $4 \cdot \binom{13}{3}$

$$Pr(c) = \frac{4 \cdot \binom{13}{3}}{\binom{52}{3}} = \frac{4 \cdot \frac{13 \cdot 12 \cdot 11}{\cancel{3} \cdot \cancel{2} \cdot \cancel{1}}}{\frac{52 \cdot 51 \cdot 50}{\cancel{3} \cdot \cancel{2} \cdot \cancel{1}}} \simeq 0{,}0518$$

---

d = "first 3 cards are all 7"

we have 4 sevens in the deck and the # of ways to choose 3 of them is $\binom{4}{3}$

$$Pr(d) = \frac{\binom{4}{3}}{\binom{52}{3}} = \frac{4}{\frac{52 \cdot 51 \cdot 50}{3 \cdot 2 \cdot 1}} = \frac{4 \cdot 3 \cdot 2 \cdot 1}{52 \cdot 51 \cdot 50} \simeq 0{,}000181$$

---

e = "first 5 cards form a straight"

The total possible straights are 10, from A,2,3,4,5 to 10,J,Q,K,A

for each straight (5 cards) each card can be of 4 suits, so the total number of possible straights are $10 \cdot 4^5$

We have to eliminate from that the straight flush so the straights that have 5 cards with same suits. and they are $4 \times 10 = 40$

so we have

$$Pr(e) = \frac{\text{total straight}}{\text{total ways to choose 5 cards}} = \frac{(10 \cdot 4^5) - 40}{\binom{52}{5}} =$$

$$= \frac{10240 - 40}{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48} \simeq 0{,}00392$$