

Problem 4

Consider p_A = "page chosen by Ann", p_G = "page chosen by Giulio".

We know that $p_A \neq p_G$.

Assume that N is the total # of pages in the book.

We take $M = \frac{N}{2}$ if N is even, $M = \frac{N+1}{2}$ if it is odd.

Let's assume N even so $M = \frac{N}{2}$.

Now we can ask both Ann and Giulio their page with probability $\frac{1}{2}$.

Assume that we choose Giulio which reveals his p_G .

~~We have to choose the correct page~~ The strategy is:

Case 1, $(p_A \leq p_G)$ so Ann has earlier page.

If $p_G > M$ then we have to choose Ann for earlier page, guessing ~~incorrectly~~ correctly.

If $p_G \leq M$ then we choose Giulio incorrectly.

$$\Pr(p_G > M) = \frac{N-M}{N-1} = \frac{N/2}{N-1}, \quad N-1 \text{ because } p_G \text{ is selected}$$

from all ~~the~~ N pages except p_A that for hypothesis is different.

Case 2 ($p_A > p_G$)

If $p_G > M$ we have to choose Ann, guessing incorrectly.

If $p_G \leq M$ we have to choose Giulio, guessing correctly.

$$\Pr(p_G \leq M) = \frac{N-M}{N-1} = \frac{N/2}{N-1}$$

So the overall probability of the game is, ~~for guessing correctly~~

$$\Pr(\text{game}) = \frac{1}{2} \left(\frac{N/2}{N-1} \right) + \frac{1}{2} \left(\frac{N/2}{N-1} \right) = \frac{N/2}{N-1} = \frac{N}{2(N-1)}$$

that is bigger than $\frac{1}{2}$.