Laborator 8

luni, 18 noiembrie 2024 10:25

variabilă aleatoare: funcție care atribuie tuturor valorilor din Ω o valoare

1. $X: \begin{pmatrix} -2 & -1 & 0 & 1 & 2 \\ 3p & 4p & 2p & p & p \end{pmatrix}$

a) $p = ?$

C. E: $p > 0$

$3p + 4p + 2p + p + p = 1$; $p = \frac{1}{11}$

$X = \begin{pmatrix} -2 & -1 & 0 & 1 & 2 \\ \frac{3}{11} & \frac{4}{11} & \frac{2}{11} & \frac{1}{11} & \frac{1}{11} \end{pmatrix}$

c) $Y = 16X - 23$ $E(Y), E(Z)$

$Z = 3X - 2$

$E(X) = \frac{-6-4+0+1+2}{11} = -\frac{7}{11}$

$E(Y) = 16 \cdot \left(-\frac{7}{11}\right) - 23 = \frac{-112}{11} - 23 = \frac{-112-253}{11} = \frac{-365}{11}$

Var(x) $= E((X-\lambda)^2) = E(X^2) - E(\lambda)$ (variație)

$X^2 \sim \begin{pmatrix} 4 & 1 & 0 & 1 & 4 \\ \frac{3}{11} & \frac{4}{11} & \frac{2}{11} & \frac{1}{11} & \frac{1}{11} \end{pmatrix} = \begin{pmatrix} 0 & 1 & 4 \\ \frac{2}{11} & \frac{5}{11} & \frac{4}{11} \end{pmatrix}$

$E(X^2) = 0 \cdot \frac{2}{11} + 1 \cdot \frac{5}{11} + 4 \cdot \frac{4}{11} = \frac{21}{11}$

$Var(X) = \frac{21}{11} - \left(-\frac{7}{11}\right)^2 = \frac{11 \cdot 21 + 49}{121} = \frac{182}{121}$

$E(X) = \frac{-7}{11}$

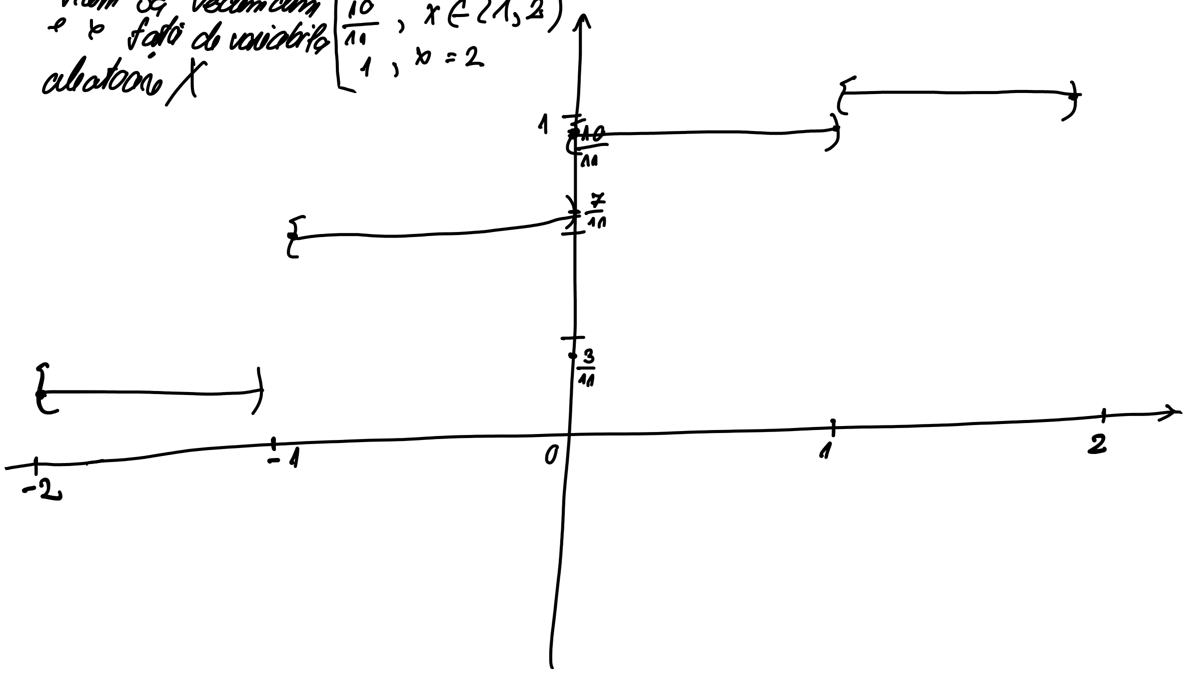
$Var(X) = \frac{182}{121}$

$E(16X - 23) = 16E(X) - 23 = 16 \cdot \left(\frac{-7}{11}\right) - 23 = \frac{-112}{11} - 23 = \frac{-112-253}{11} = \frac{-365}{11}$

$Var(16X - 23) = 16^2 Var(X) - 23 \ldots$

b) $F(x) = P(X \le x) = \begin{cases} 0, & x < -2 \text{ (avem } -2, -1 .. 2) \\ \frac{3}{11}, & x \in [-2, -1) \\ \frac{7}{11}, & x \in [-1, 0) \\ \frac{3}{11}, & x \in [0, 1) \\ \frac{10}{11}, & x \in [1, 2) \\ 1, & x = 2 \end{cases}$

→ în drept: vrem identificarea treptelor, vrem să vedem cum e x față de variabila aleatoare X



cerință suplimentară

$X: \begin{pmatrix} -2 & -1 & 0 & 1 & 2 \\ \frac{3}{11} & \frac{4}{11} & \frac{2}{11} & \frac{1}{11} & \frac{1}{11} \end{pmatrix}$

$P(-1,5 \le x < 1,5) = \frac{4}{11} + \frac{2}{11} + \frac{1}{11} = \frac{7}{11}$

sau

$= F(1,5) - F(-1,5) - P(X = 1.5) = \frac{10}{11} - \frac{3}{11} - 0 = \frac{7}{11}$

$P(X < 0 \mid X > -2) = \frac{P((X<0) \cap (X > -2))}{P(X > -2)} = \frac{\frac{3}{11}}{1 - \underbrace{P(X \le 2)}_{F(-2)}} = \frac{1}{2}$

A B

12: lucrăm cu o monedă măsluită $\Rightarrow p = \frac{1}{3}$, nu 1/2

$X: \begin{pmatrix} 1 & 2 & .. & 10 \\ p_1 & p_2 & .. & p_{10} \end{pmatrix}$

a) $P(K) = C_{10}^{K} p^{K} \cdot (1-p)^{10-K}$

b) $E(X) = \sum_{K=0}^{m} K \cdot P_K = \sum_{K=0}^{m} C_{10}^{K} p^{K} \cdot (1-p)^{10-K} = \ldots = \frac{10}{3}$

$Var(X) = E(X^2) = (E(X))^2 = 10 \cdot \frac{1}{3} \cdot \frac{2}{3}$

4: $X: \begin{pmatrix} -2 & 3 & 4 & 6 \\ 6p & 2p & 9p & p \end{pmatrix}$

$6p + 2p + 3p + p = 12p = 1 \Rightarrow p = \frac{1}{18}$

$X: \begin{pmatrix} -2 & 3 & 4 & 6 \\ 1/3 & 1/9 & 1/2 & 1/18 \end{pmatrix}$

$\begin{cases} E(aX+b) = 57 \\ Var(aX+b) = 75 \end{cases} \Rightarrow \begin{cases} aE(X) + b = 57 \\ a^2 Var(X) = 75 \end{cases} \Rightarrow \begin{cases} 2a + b = 57 \\ a^2 \cdot \frac{25}{3} = 75 \end{cases}$; $a^2 = 9$; $a = \pm 3$, $b = \{51, 63\}$

$E(X) = -2 \cdot \frac{6}{18} + 3 \cdot \frac{2}{18} + 4 \cdot \frac{9}{18} + 6 \cdot \frac{1}{18} = -\frac{2}{3} + \frac{1}{3} + 2 + \frac{1}{3} = 2$

$E(X^2) = (-2)^2 \cdot 6/18 + 3^2 \cdot 2/18 + 4^2 \cdot 9/18 + 6^2 \cdot 1/18 = \frac{24+64+144+36}{18} = \frac{222}{18} = \frac{111}{9} = \frac{37}{3}$

$Var(X) = E(X^2) - (E(X))^2 = 37/3 - 4 = \frac{25}{3}$