Lab 3

Monday, 28 October 2024 10:15

Probabilități condiționate

1) Dacă $A \in K$, $A \neq \emptyset$ este posibil ca $P(A)=0$?
P.p. că $\exists A \neq \emptyset$, $P(A)=0 \Rightarrow P(\bar{A})=1 \Rightarrow \bar{A}=\Omega$ ↯

$A, B \in K$, $B \neq \emptyset$

$$P(A|B)=\frac{P(A\cap B)}{P(B)}$$

Bayes (pt $A \neq \emptyset$)
OBS: $P(A|B)=P(A)$

$$P(B|A)=\frac{P(A|B)\cdot P(B)}{P(A)}$$

Poincaré

$P(A\cup B)=P(A)+P(B)-P(A\cap B)$

A, B incompatibile $\Rightarrow P(A\cup B)=P(A)+P(B)$

Seminar 3

4. A, B, X
$P(A|X)=0.22$
$P(A\cap X)=0.11$
$P(X\cap B)=0.16$
$P(B\cup X)=0.76$
$P(A)=0.31$

a) $P(B|X)=?$

$P(B|X)=\frac{P(B\cap X)}{P(X)}$

$P(X)=?$

$P(X)=\frac{P(A\cap X)}{P(A|X)}=\frac{0.11}{0.22}=\frac{1}{2}$

$P(B|X)=0.16\cdot 2=0.32$

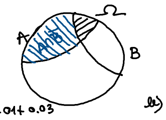
b) $P(B)=?$

$P(X\cup B)=P(X)+P(B)-P(X\cap B)$
$0.76=0.5+P(B)-0.16$
$P(B)=0.42$

$P(X)=0.5$ (din subpunctul a))
$P(B)=0.42$ (b))

d) $P(A\cup X)=?$
$P(A\cup X)=P(A)+P(X)-P(A\cap X)=0.31+0.5-0.11$

5. (Ω, K, P)
$A, B \in K$
$P(A\cap B)=0.01$
$P(A\cap \bar{B})=0.03$
$P(A|B)=0.05$



a) $P(A), P(B)=?$

$P(A\cap B)+P(A\cap\bar{B})=0.01+0.03$
disjuncte, adică incompatibile

$\Leftrightarrow P((A\cap B)\cup(A\cap\bar{B}))=0.04$
$\Leftrightarrow P(A\cap \underbrace{(B\cup\bar{B})}_{\Omega})=0.04$
$\Leftrightarrow P(A)=0.04$

$P(A|B)=\frac{P(A\cap B)}{P(B)} \Leftrightarrow P(B)=\frac{P(A\cap B)}{P(A|B)}$

$\Leftrightarrow P(B)=\frac{0.01}{0.05}=\frac{1}{5}=0.02$

b) $P(A\cup B)$, $P(\bar{A}\cap\bar{B})$

$P(A\cup B)=P(A)+P(B)-P(A\cap B)=0.24-0.01=0.23$

$P(\bar{A}\cap\bar{B})=P(\overline{A\cup B})=1-P(A\cup B)=1-0.23=0.77$

c) $P(B|A)=?$, $P(A|\bar{B})=?$

$P(B|A)=\frac{P(A|B)\cdot P(B)}{P(A)}=\frac{0.05\cdot 0.2}{0.04}=0.25$

SAU

$P(B|A)=\frac{P(B\cap A)}{P(A)}=\frac{0.01}{0.04}=0.25$

$P(A|\bar{B})=\frac{P(A\cap\bar{B})}{P(\bar{B})}=\frac{0.03}{1-0.2}=\frac{0.03}{0.8}=0.037$

d) $P(B|\bar{A})$, $(\bar{A}|\bar{B})$

$P(B|\bar{A})=\frac{P(B\cap\bar{A})}{P(\bar{A})}=$

$\underbrace{P(B\cap\bar{A})+P(B\cap A)}_{\text{incompatibile}}=P(B)$ (ac. raționament de la subpunctul a))

$P(B\cap\bar{A})=0.2-0.01=0.19$

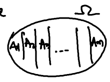
$P(B|\bar{A})=\frac{0.19}{1-0.04}=\frac{0.19}{0.96}=0.197$

$P(\bar{A}|\bar{B})=\frac{P(\bar{A}\cap\bar{B})}{P(\bar{B})}=\frac{P(\overline{A\cup B})}{1-P(B)}=\frac{1-0.23}{1-0.2}=\frac{0.77}{0.8}=0.96$

$A_1, A_2, \dots, A_n$
sist. complet de evenimente

$A_i \cap A_j=\emptyset \; \forall i, j$

$\bigcup_{i=1}^{n} A_i=\Omega$



$P(\bar{A}|B)=1-P(A|B)$
$P(A|\bar{B}) \neq 1-P(A|B)$

$$P(X)=\sum_{i=1}^{n} P(X|A_i)P(A_i) \quad \text{formula probabilității totale}$$

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10. F_1, F_2, F_3
30 50 20
2% 4% 5%

X - evenimentul că un calculator e defectat în perioada de garanție
A_i - evenimentul că un calculator provine de la furnizorul F_i, $i=\overline{1,3}$

$P(A_1)=\frac{30}{100}=0.3$ $\quad P(X|A_1)=0.02$
$P(A_2)=\frac{50}{100}=0.5$ $\quad P(X|A_2)=0.04$
$P(A_3)=\frac{20}{100}=0.2$ $\quad P(X|A_3)=0.05$

a) $P(X)=?$
A_1, A_2, A_3 form. sist. complet de evenimente
$P(X)=P(X|A_1)P(A_1)+P(X|A_2)\cdot P(A_2)+P(X|A_3)\cdot P(A_3)$
$=0.02\cdot 0.3+0.04\cdot 0.5+0.05\cdot 0.2$
$=0.036$

b) $P(A_2|X)=\frac{P(X|A_2)\cdot P(A_2)}{P(X)}=\frac{0.04\cdot 0.5}{0.036}=\frac{0.02}{0.036}=\frac{5}{9}=0.55$

c) $P(X|A_1\cup A_3)=\frac{P(X\cap(A_1\cup A_3))}{P(A_1\cup A_3)}=\frac{P((X\cap A_1)\cup(X\cap A_3))}{P(A_1\cup A_3)}=\frac{P(X\cap A_1)+P(X\cap A_3)}{P(A_1)+P(A_3)}=\frac{P(X|A_1)\cdot P(A_1)+P(X|A_3)\cdot P(A_3)}{P(A_1)+P(A_3)}=\frac{0.02\cdot 0.3+0.05\cdot 0.2}{0.3+0.2}=\frac{4}{125}=0.032$

(incompatibile)

d) $P(A_1\cup A_2|\bar{X})=\frac{P((A_1\cup A_2)\cap\bar{X})}{P(\bar{X})}=\frac{P((A_1\cap\bar{X})\cup(A_2\cap\bar{X}))}{1-P(X)}=\frac{P(A_1\cap\bar{X})+P(A_2\cap\bar{X})}{1-P(X)}=\frac{P(\bar{X}|A_1)\cdot P(A_1)+P(\bar{X}|A_2)\cdot P(A_2)}{1-P(X)}=\frac{(1-P(X|A_1))P(A_1)+(1-P(X|A_2))P(A_2)}{1-P(X)}=\dots$

(incompatibile, inc.)

Seminar 2

1. A_i - avem 2 aruncări i
$$P\left(\bigcup_{i=1}^{7} A_i\right)=1-P\left(\bigcap_{i=1}^{7}\bar{A_i}\right)=1-\prod_{i=1}^{7} P(\bar{A_i})=1-\left(\frac{5}{6}\right)^7$$

2. X - ev. că primul e mai mare ca al doilea
Y - "primul mai mic decât al doilea"
Z - "primul = al doilea"
X, Y, Z form. sist. complet de ev.
$\Rightarrow P(X)+P(Y)+P(Z)=1$
$P(Z)=\frac{6}{36}=\frac{1}{6}$
$P(X)=P(Y) \Rightarrow 2P(X)=\frac{5}{6}$
$\Rightarrow P(X)=\frac{5}{12}$

3. A_n - ev. că 5 apare la a n-a încercare și 5 și 8 nu apar la primele n-1 încercări
$\sum_{n=1}^{\infty} P(A_n)=\sum_{n=1}^{\infty}$