A New Compression Technique for Repetitive Tries

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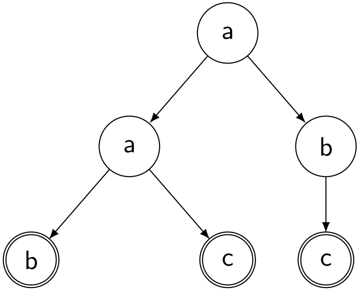
Introduction and Motivation

Motivation

- Tries are fundamental data structures for representing large sets of strings.
- Efficient for prefix-based queries, but often very large in memory.
- Typical applications:
  - Autocomplete and predictive text
  - Spell checking
  - IP routing
  - Bioinformatics (DNA pattern matching)
- Objective: reduce memory footprint while keeping efficient queries.

Example: Trie Representation



Each path from the root node a to a leaf represents one string in the language $\{aab, aac, abc\}$.

Challenges

- Tries can contain large, identical subtrees.
- Standard compression (e.g., XBWT, LZ77, top tree) does not exploit structural repetitions.

Fundamental trade-off between compression and indexability:

- **Full Indexability, No Compression:** the input trie itself can be used as an index. It offers no compression, as even highly repetitive subtrees are stored explicitly
- **Full Compression, Difficult Indexing:** maximum compression but very poor indexability.

Goal: find the sweet spot between these two extremes.

Theoretical Background

Tries, DFAs and Minimization

- A trie can be seen as an **acyclic DFA**.
- Minimizing a DFA merges **Myhill–Nerode equivalent states**.
- Revuz' algorithm allows linear-time minimization for acyclic DFAs.

Example: reduction of an acyclic DFA by merging equivalent subtrees.

XBWT (Extended Burrows–Wheeler Transform)

- Extends the classic BWT from strings to labeled trees.
- Encodes the tree as two arrays:
  - S_α: node labels
  - S_{last}: structure bits
- Enables compressed storage and prefix queries.

The children of each node form a contiguous block in S_α.

Proposed Compression Scheme

Motivation for a New Approach

- Full minimization $\Rightarrow$ excellent compression but unindexable.
- Uncompressed trie $\Rightarrow$ fully indexable but memory-hungry.

Our approach:

Partial Minimization

Produce a smaller automaton that remains indexable by allowing partial merging of equivalent subtrees.

Key concept: **p–sortable automata**.

p–Sortable Automata

p–sortable Automaton

Let $\mathcal{A} = (Q, \Sigma, \delta, q_0, F)$ be a finite-state automaton. We call $\mathcal{A}$ *p–sortable* if there exists a co–lexicographic order $\leq$ on Q such that Q can be partitioned into p chains $\{Q_i\}_{i=1}^{p}$, where each $(Q_i, \leq)$ is totally ordered.

Let $\mathcal{A}$ be a p–sortable automaton. There exists a compressed data structure for A that supports subpath queries on a query word α of length m in $O(mp^2 \log\log(p|\Sigma|))$ time. The space required is:

- $\log(|\Sigma|) + \log p + 2$ bits per edge if A is a DFA.
- $\log(|\Sigma|) + 2\log p + 2$ bits per edge if A is an NFA.

The Intuition

- Parameter p interpolates between the two extremes:
  - $p = 1$: Wheeler graph $\Rightarrow$ maximum indexability, less compression.
  - $p \to \infty$: more compression, less indexability.
- Increasing p improves compression but complicates indexing.

Example of a 2–sortable automaton: partial merging with preserved order.

Compression Pipeline

1. Sort nodes by co–lexicographic order.
2. Partition nodes into p subsequences respecting the order.
3. Merge consecutive Myhill–Nerode equivalent states within each partition.
4. Construct the resulting compressed p–sortable automaton.

String Partitioning Problem

Consider a sting s.

Run

Maximal contiguous subsequence of identical symbols within s.

String Partitioning Problem

Partition s into p subsequences minimizing number of runs.

Example: String Partitioning Problem

$$s = 2213122152$$

of length 10. The number of runs in S is 8, given by the decomposition:

$$(22)\,(1)\,(3)\,(1)\,(22)\,(1)\,(5)\,(2)$$

A possible partition is:

- $I_1 = \{3, 5, 8, 9\}$
- $I_2 = \{1, 2, 4, 6, 7, 10\}$

This yields the following subsequences:

- $S[I_1] = 1115 \Rightarrow 2$ runs: (111), (5)
- $S[I_2] = 223222 \Rightarrow 3$ runs: (22), (3), (222)

Total runs: $2 + 3 = 5 \Rightarrow$ reduction from the original 8 runs in S.

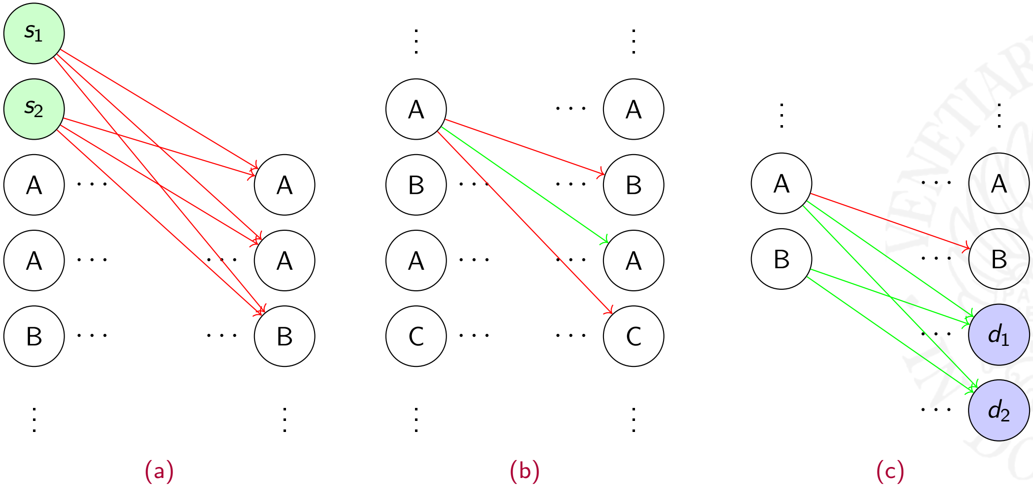
Reduction to Bipartite Graph Matching

- The String Partitioning Problem can be reduced to:

  Minimum Weight Perfect Bipartite Matching (MWPBM)

- Nodes correspond to string characters; edges encode run boundaries cost between characters.
- Allows to use efficient, well-studied algorithms to find the optimal solution.

Example: Min-Weight Bipartite Graph Matching



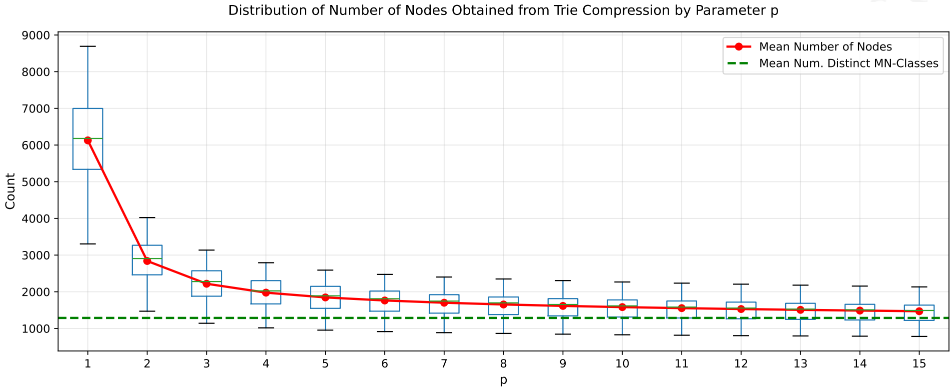
Implementation and Experiments

Experimental setup

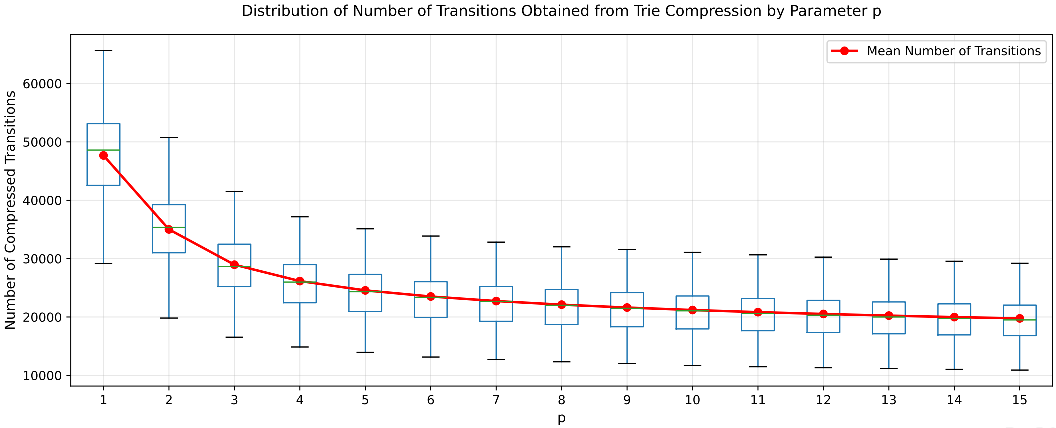
- Implemented in **C++** for performance.
- **CPU and RAM**: Apple M4 Pro, 24 GB.
- **Dataset**: synthetic generated tries to control repetitiveness.

Experimental Results

- Increasing p from 1 to 2 halves the number of states.
- Compression ratio improves rapidly, approaching the minimal number of states for larger p.



Experimental Results



Conclusions

Conclusions

- Proposed a new trie compression method based on p–sortable automata.
- Achieves:
  - Significant memory reduction on repetitive data.
  - Efficient querying capability.
  - Flexibility via parameter p.
- Opens new directions for compressed automata and string indexing.

Future Work

- **Scalability**: Investigate improvements to the proposed reduction for the String Partitioning problem to develop more scalable and efficient solutions for large-scale applications.
- **DFA Construction**: Explore methods to directly construct a p—sortable deterministic finite automaton from the pipeline, potentially by developing a pruning strategy for the output NFA.
- **DFA Minimization**: Minimize the size of the returned automaton, potentially by providing explicit guarantees of minimality of the returned p—sortable DFA.