$$\nabla\cdot\vec{f}=\left(\frac{\partial}{\partial x},\frac{\partial}{\partial y},\frac{\partial}{\partial z}\right)\cdot\left(r^\alpha x, r^\alpha y, r^\alpha z\right) \qquad r=\sqrt{x^2+y^2+z^2}$$

$$\frac{\partial r^\alpha x}{\partial x}=\frac{\partial\, x(x^2+y^2+z^2)^{\alpha/2}}{\partial x}=(x^2+y^2+z^2)^{\frac{\alpha}{2}}+x\,\frac{\partial (x^2+y^2+z^2)^{\alpha/2}}{\partial x}=r^\alpha+x\frac{\alpha}{2}(x^2+y^2+z^2)^{\frac{\alpha}{2}-1}2x=$$

$$=r^\alpha+\alpha x^2\frac{r^\alpha}{r^2} \qquad \nabla\cdot\vec{f}=3r^\alpha+\alpha\,\frac{\cancel{(x^2+y^2+z^2)}\,r^\alpha}{\cancel{r^2}}=r^\alpha(3+\alpha)$$

$$\nabla^2\equiv\nabla\cdot\nabla \qquad \nabla^2 f=\frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial y^2} \qquad \begin{cases}x=r\cos\theta\\ y=r\sin\theta\end{cases} \Rightarrow \begin{cases}r=\sqrt{x^2+y^2}\\ \theta=\arctan\frac{y}{x}\end{cases}$$

$$f(x,y)=f(x(r,\theta),y(r,\theta)) \qquad \frac{\partial r}{\partial x}=\cos\theta \quad \frac{\partial r}{\partial y}=\sin\theta \quad \frac{\partial\theta}{\partial x}=-\frac{\sin\theta}{r} \quad \frac{\partial\theta}{\partial y}=\frac{\cos\theta}{r}$$

$$\frac{\partial f}{\partial x}=\frac{\partial f}{\partial r}\frac{\partial r}{\partial x}+\frac{\partial f}{\partial\theta}\frac{\partial\theta}{\partial x}=\cos\theta\frac{\partial f}{\partial r}-\frac{\sin\theta}{r}\frac{\partial f}{\partial\theta} \qquad \frac{\partial f}{\partial y}=\frac{\partial f}{\partial r}\frac{\partial r}{\partial y}+\frac{\partial f}{\partial\theta}\frac{\partial\theta}{\partial y}=\sin\theta\frac{\partial f}{\partial r}+\frac{\cos\theta}{r}\frac{\partial f}{\partial\theta}$$

$$\frac{\partial^2 f}{\partial x^2}=\frac{\partial}{\partial r}\left(\frac{\partial f}{\partial x}\right)\frac{\partial r}{\partial x}+\frac{\partial}{\partial\theta}\left(\frac{\partial f}{\partial x}\right)\frac{\partial\theta}{\partial x}=\cos^2\theta\frac{\partial^2 f}{\partial r^2}+\frac{2\sin\theta\cos\theta}{r^2}\frac{\partial f}{\partial\theta}-\frac{2\sin\theta\cos\theta}{r}\frac{\partial^2 f}{\partial r\partial\theta}+\frac{\sin^2\theta}{r}\frac{\partial f}{\partial r}+\frac{\sin^2\theta}{r^2}\frac{\partial^2 f}{\partial\theta^2}$$

$$\frac{\partial^2 f}{\partial y^2}=\sin^2\theta\frac{\partial^2 f}{\partial r^2}-\frac{2\sin\theta\cos\theta}{r^2}\frac{\partial f}{\partial\theta}+\frac{2\sin\theta\cos\theta}{r}\frac{\partial f}{\partial\theta}+\frac{\cos^2\theta}{r}\frac{\partial f}{\partial r}+\frac{\cos^2\theta}{r^2}\frac{\partial^2 f}{\partial\theta^2}$$

$$\Rightarrow \nabla^2 f=\frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial y^2}=\frac{1}{r}\frac{\partial f}{\partial r}+\frac{\partial^2 f}{\partial r^2}+\frac{1}{r^2}\frac{\partial^2 f}{\partial\theta^2}$$