Technical University of Munich
Coding for Communications and Data Storage
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Coding Theory for Storage and Networks
Summer 2022
Lab 4: VT Codes

Solve the tasks in *VT.sage*. Complete the lines with ? markers in comment or the noted blocks.

Problem 1: VT Decoding

Varshamov-Tenengolts (VT) codes have been shown to correct a single deletion *or* insertion error. In this task, we develop a binary VT decoder that corrects a single deletion or insertion error. We use a code of length $n = 15$ and checksum $a = 0$, i.e.

$$\mathcal{VT}_0(n) = \left\{ \mathbf{c} = (c_0, c_1, \ldots, c_{n-1}) : \sum_{i=0}^{n-1} (i+1)c_i \equiv 0 \bmod (n+1) \right\}.$$

1. Consider the codeword $\mathbf{c} = (0,1,1,0,0,0,0,1,0,1,0,1,1,0,0)$. Check, if it is a valid codeword by computing its checksum.

2. Implement a *VT_decoder(r, n)* that can correct a single deletion or insertion.

3. (a) A random deletion occurs to the codeword $\mathbf{c}$ resulting in $\mathbf{r}_{del} \in \mathbb{F}_2^{n-1}$. Correct the error using your *VT_decoder()*.

   (b) A random insertion occurs to the codeword $\mathbf{c}$ resulting in $\mathbf{r}_{ins} \in \mathbb{F}_2^{n+1}$. Correct the error using your *VT_decoder()*.