Technical University of Munich
Coding for Communications and Data Storage
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Coding Theory for Storage and Networks

Lab 3: Interleaved Reed–Solomon Codes

Solve the tasks in *IRS.sage*. Complete the lines with ? markers in comment or the noted blocks.

Problem 1: Homogeneous IRS Code

Let $\mathcal{C}_1$ denote an IRS code consisting of $l = 2$ Reed–Solomon codes $\mathcal{RS}(15, 8)$ over $\mathbb{F}_{2^4}$. Assume that the codewords of $\mathcal{C}_1$ were transmitted over a bursty channel and we therefore choose a common error-locator polynomial for both Reed–Solomon (RS) codes in the decoding process.

1. Calculate the decoding radius $t_{max,1}$ of $\mathcal{C}_1$ for collaborative IRS decoding.
2. In the file *IRS.sage*, 4 random information vectors **info_1**x are given, compute the corresponding codewords **c_1**x.
   2 error vectors occur and the resulting received words are given **y_1**x. Compute the corresponding syndromes **s_1**x.
3. Implement the function *irs_decoding()* that returns the error positions given the syndromes.
   Hint: construct and solve the linear system of equations for finding the error locator polynomial. Find the roots of the error locator polynomial to find the error locations.

Problem 2: Heterogeneous IRS Code

We extend $\mathcal{C}_1$ to a *heterogeneous* IRS code $\mathcal{C}_2$ of interleaving order $\ell = 3$ by adding an RS code over $\mathbb{F}_{2^4}$ of length $n = 15$ and dimension $k_2 = 6$. We assume one common error-locator polynomial for all three RS codes.

1. What is the decoding radius $t_{max,2}$ of $\mathcal{C}_2$?
2. Calculate the syndromes $\mathbf{s}_{11}$, $\mathbf{s}_{12}$, $\mathbf{s}_2$ corresponding to the new error vector given in the file *IRS.sage*.
3. Use the function *irs_decoding()* to find the error locations.
4. Do we need the syndromes of all three RS codes for the decoding?

Problem 3: Virtual Interleaving (Power Decoding)

In order to decode the $\mathcal{RS}(15,2)$ code over $\mathbb{F}_{2^4}$ beyond half the minimum distance, we extend it virtually to a heterogeneous IRS code.

1. Calculate the maximal decoding radii $t_{max}^{(i)}$ for virtual interleaving orders $i = 2, \ldots, 6$. Let $t_{max} = \max_{i=2,\ldots,6} t_{max}^{(i)}$. Determine the minimum i_{max} such that t_{max} can be achieved.

2. Implement the power decoding algorithm in the function *Power_dec()*. This algorithm consists of the following steps:

   - Calculate the element-wise powers of the received word up to interleaving order i_{max} to obtain $\mathbf{y}_i$, for $i = 2, \ldots, i_{max}$;
   - Calculate the dimensions k_i, $i = 2, \ldots, i_{max}$ of the virtual codes;
   - Calculate the corresponding syndromes $\mathbf{s}_i$, $i = 1, \ldots, i_{max}$, where $\mathbf{s}_1$ denotes the syndrome of the original received word;
   - Calculate the error positions by solving a system of equations.
     Hint: use the function irs_decoding().

Problem 4: Decoding Failure

As discussed in the lecture, the virtual interleaving principle can results in a decoding failure.

1. Run power decoding over 1000 random error vectors of length 15 and weight t_{max} and calculate the failure probability of power decoding.
   Compare the result with the estimation from the lecture.
   Hint: adapt the function irs_decoding() *to raise an error when the decoding fails; use* try-except *to catch the errors.*