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Coding for Communications and Data Storage
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Coding Theory for Storage and Networks

Lab 3: Streaming Codes for Low-Latency Network Communication

Problem 1: Empirical Performance of Streaming Codes Over the Gilbert-Elliot (GE) Channel

The goal of this problem is to evaluate the empirical performance of streaming codes over the GE channel (Figure 1). Throughout this problem, we consider streaming codes obtained via horizontal interleaving of (n, k) block codes. The two performance metrics to be evaluated are the block erasure probability (BEP) and the packet loss probability (PLP). The empirical BEP is defined as the ratio of the total number of blocks (of size n) that have at least one lost information packet to the total number of transmitted blocks. The empirical PLP is defined as the ratio of total number of lost information packets to the total number of transmitted information packets.

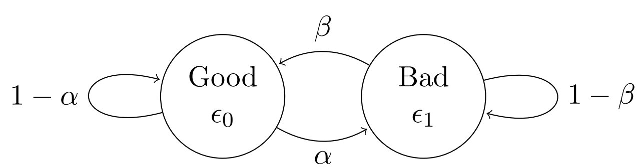

Figure 1: 2-state Markov chain illustration of the Gilbert-Elliot (GE) channel.

1. Implement the function *GE(...)* that generates a realization of the GE channel of length ℓ for given channel parameters $\alpha, \beta, \epsilon_0, \epsilon_1$.

Let T denote the maximum decoding delay of a code. Consider the horizontally interleaved versions of the following codes: *(i)* $(n = T+1, k = n-N)$ MDS code; *(ii)* $(n = k+B, k = T)$ Martinian-Trott code; and *(iii)* $(n = k + B, k = T - N + 1)$ Domanovitz *et al.* code.

2. Implement a function *MT(...)* that outputs the generator matrix of an (n, k) Martinian-Trott block code based on given code parameters T, B, and N, where $B < T$. *Hint: You may use the provided function in Sage to find a generator polynomial for a cyclic code.*

3. Implement a function *Domanovitz(...)* that outputs the generator matrix of a random (n,k) Domanovitz block code in $\mathbb{F}_{q^2}$ based on given code parameters T, B, N, and q. *Hint: Generate a random matrix whose structure is consistent with the generator matrix of a Domanovitz code (see Figure 2), and check if the sub-matrices G_1 and G_2 satisfy the MDS property. Keep generating such random matrices until a valid generator matrix is found.*

$$\mathbf{G} = \begin{bmatrix} 1 & X & \cdots & X & 0 & 0 & 0 & \cdots & 0 & \alpha & \cdots & 0 \\ 0 & 1 & X & \cdots & X & 0 & 0 & \cdots & 0 & 0 & \ddots & 0 \\ 0 & 0 & \ddots & \ddots & \ddots & \ddots & 0 & \cdots & 0 & 0 & \cdots & \alpha \\ \vdots & \vdots & & 1 & \ddots & \ddots & \ddots & & \vdots & X & \cdots & X \\ \vdots & \vdots & & & \ddots & X & \cdots & X & 0 & \vdots & & \vdots \\ 0 & 0 & \cdots & \cdots & 0 & 1 & X & \cdots & X & X & \cdots & X \end{bmatrix} \quad \underbrace{\qquad\qquad}_{k}\ \underbrace{\qquad}_{N-1}\ \underbrace{\qquad}_{B-N+1} \cdot \quad \mathbf{G} = \begin{bmatrix} 1 & X & \cdots & X & 0 & 0 & 0 & \cdots & 0 & \alpha & \cdots & 0 \\ 0 & 1 & X & \cdots & X & 0 & 0 & \cdots & 0 & 0 & \ddots & 0 \\ 0 & 0 & \ddots & \ddots & \ddots & \ddots & 0 & \cdots & 0 & 0 & \cdots & \alpha \\ \vdots & \vdots & & 1 & \ddots & \ddots & \ddots & & \vdots & X & \cdots & X \\ \vdots & \vdots & & & \ddots & X & \cdots & X & 0 & \vdots & & \vdots \\ 0 & 0 & \cdots & \cdots & 0 & 1 & X & \cdots & X & X & \cdots & X \end{bmatrix}.$$

$\mathbf{G}_1$ $\mathbf{G}_3$ $\mathbf{G}_2$

Figure 2: Generator matrix structure of (n,k) Domanovitz code. The $k \times (k+N-1)$ matrix G_1 and the $(k-(B-N+1)) \times (n-(B-N+1))$ matrix G_2 are generator matrices of MDS codes in $\mathbb{F}_q$. The matrix $G_3 = \alpha I_{B-N+1}$, where I denotes an identity matrix and $\alpha \in \mathbb{F}_{q^2} \setminus \mathbb{F}_q$.

Consider the four constructions of streaming codes in Table 1. Next, you shall simulate the empirical BEP and PLP of the codes in Table 1 over the GE channel. The GE channel length you consider should be at least $\ell = n \times 10^3$, i.e., 10^3 adjacent blocks each of size n. *Hint: In order to get smoother plots, you may want to perform* 10 *independent simulations where in each simulation the empirical BEP or PLP is considered over a GE channel with length $\ell = n \times 10^3$, and then aggregate the results by averaging over the* 10 *simulations.*

	(B,N)	T	(n,k)	q
MDS Code	$(7,7)$	14	$(15,8)$	17
Martinian-Trott Code	$(12,1)$	14	$(26,14)$	2
Domanovitz Code 1	$(11,2)$	14	$(24,13)$	29
Domanovitz Code 2	$(11,3)$	14	$(23,12)$	29

Table 1: Streaming Codes with maximum delay $T = 14$.

4. Plot the BEP of the codes in Table 1 as a function of $\epsilon_0 \in [10^{-3}, 10^{-2}]$, for: i) $\alpha = 1 \times 10^{-2}$, $\beta = 0.3$, $\epsilon_1 = 1$; ii) $\alpha = 1 \times 10^{-2}$, $\beta = 0.5$, $\epsilon_1 = 1$; and iii) $\alpha = 8 \times 10^{-2}$, $\beta = 0.5$, $\epsilon_1 = 1$. You shall show a total of 3 graphs.

5. Plot the PLP of the codes in Table 1 for the same settings mentioned in the part 4. *Hint: A source symbol u_i can be decoded iff the i-th canonical basis vector is in the column span of the punctured generator matrix (i.e. the generator matrix with the columns corresponding to erasures removed).*

6. Interpret the results in parts 4 and 5.