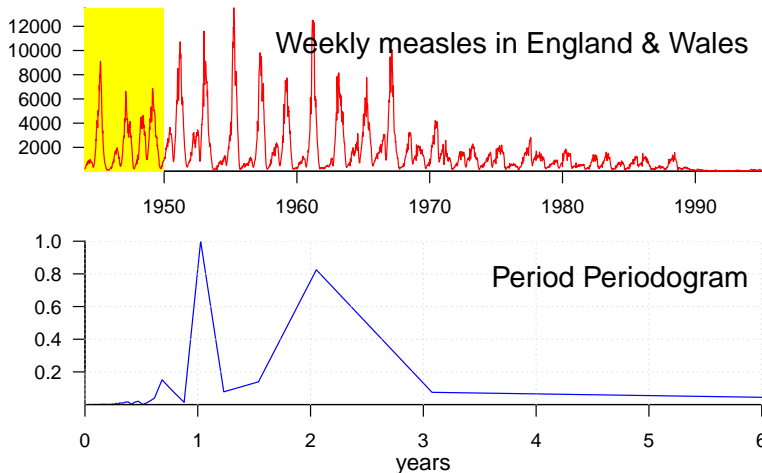


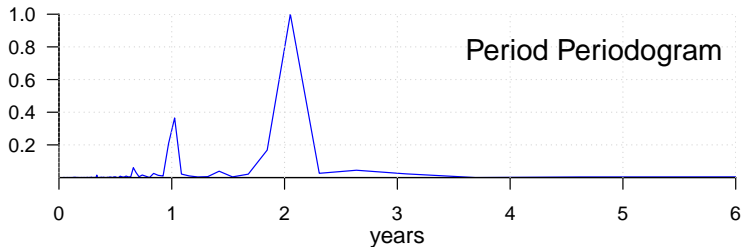
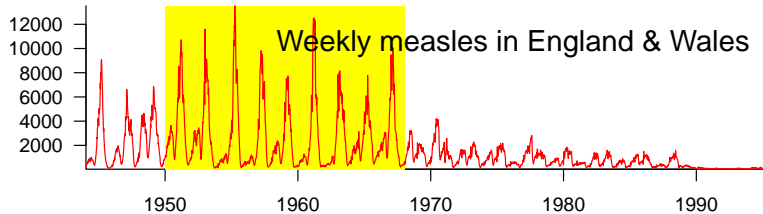


# Mechanistic Modelling of Recurrent Epidemics

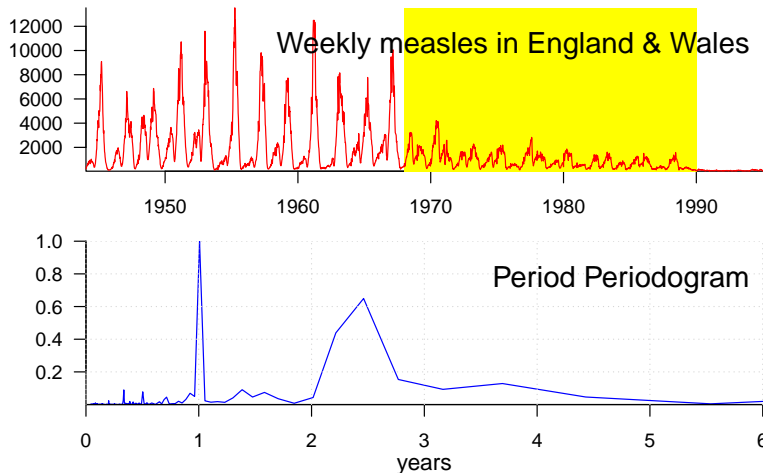
# What causes changes in frequency content over time?



# What causes changes in frequency content over time?

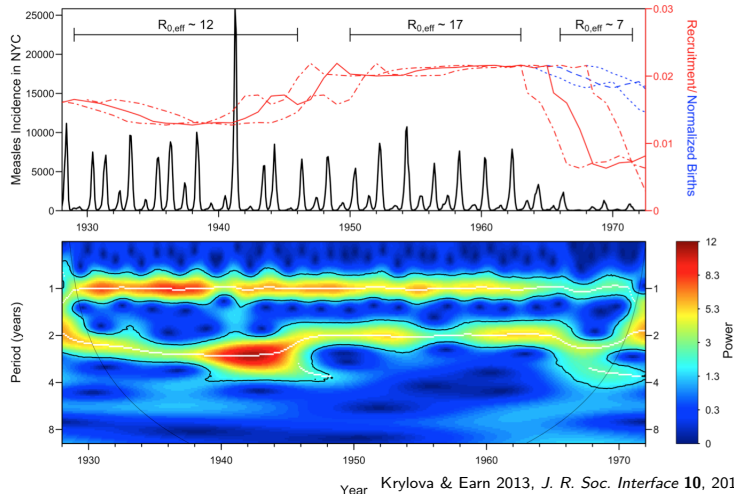


# What causes changes in frequency content over time?



# What causes changes in frequency content over time?

## Measles in New York City



# Mechanistic Epidemic Modelling: Principles

- Consider the biological mechanisms involved in disease transmission and spread
- Model mechanisms and infer their effects
- Start as simple as possible!
- Rule out simple models by comparing results with observed time series of incidence or mortality
- Add complexity one step at a time, so key mechanisms can be identified
- Ideally converge on simplest possible model that can explain observed patterns

# The SIR model: Flow Chart and Parameters



$$\frac{dS}{dt} = -\beta SI$$

$$\frac{dI}{dt} = \beta SI - \gamma I$$

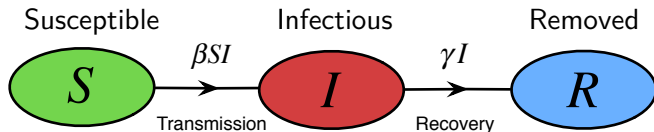
$$\frac{dR}{dt} = \gamma I$$

## ■ Parameters:

- Transmission rate  $\beta$
- Recovery rate  $\gamma$   
(or Removal rate)



# The SIR model: Derived Parameters



$$\frac{dS}{dt} = -\beta SI$$

$$\frac{dI}{dt} = \beta SI - \gamma I$$

$$\frac{dR}{dt} = \gamma I$$

## ■ Derived Parameters:

- Initial growth rate  $\beta - \gamma$
- Mean infectious period  $\frac{1}{\gamma}$
- Basic Reproduction Number

$$\mathcal{R}_0 = \frac{\beta}{\gamma}$$

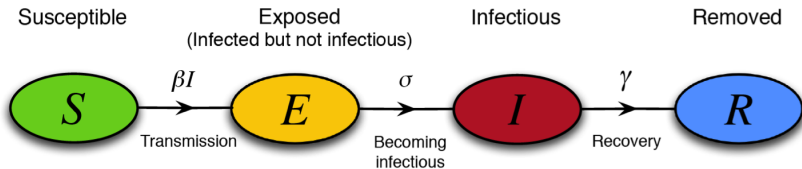
# Basic SIR Model: Important Results

- Epidemic occurs if and only if  $\mathcal{R}_0 > 1$
- Exact solution for phase portrait
- Single epidemic, then disease disappears
- Exact formula for final size as a function of  $\mathcal{R}_0$
  
- Cannot explain diseases that persist
- Cannot explain recurrent cycles of epidemics

# What are we missing?



# SEIR Model: flow chart

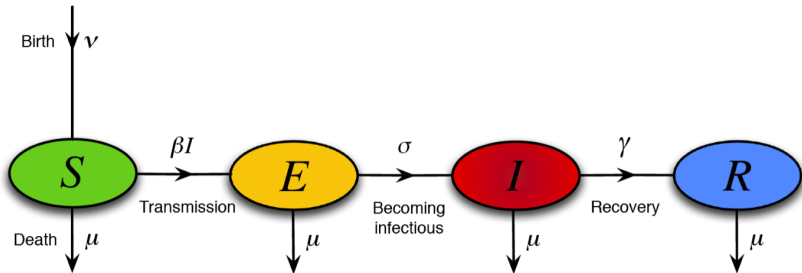


- Introduces only one new parameter ( $\sigma$ )
- Mean latent period ( $1/\sigma$ ) can often be estimated
- But... effect of inclusion of exposed class usually small

What are we **still** missing?



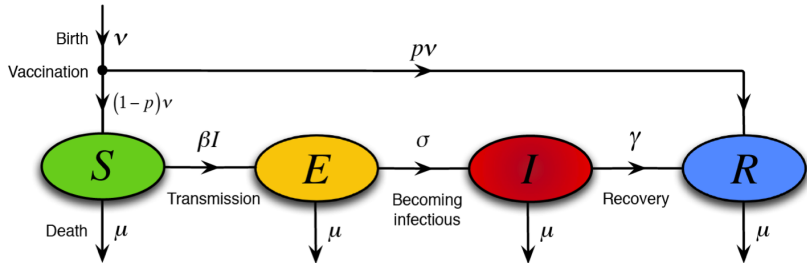
# SEIR Model with vital dynamics: flow chart



New Parameters:

- Birth rate ( $\nu$  for natality)
- Death rate ( $\mu$  for mortality)
- Mean latent period ( $1/\sigma$ )

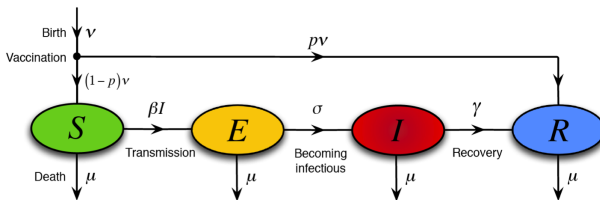
# SEIR with vital dynamics and vaccination: flow chart



New Parameters:

- Birth rate ( $\nu$  for natality)
- Death rate ( $\mu$  for mortality)
- Mean latent period ( $1/\sigma$ )
- Proportion vaccinated ( $p$ )

# SEIR with vital dynamics and vaccination: Equations



$$\frac{dS}{dt} = \nu(1 - p) - \beta SI - \mu S$$

$$\frac{dE}{dt} = \beta SI - \sigma E - \mu E$$

$$\frac{dI}{dt} = \sigma E - \gamma I - \mu I$$

$$\frac{dR}{dt} = \nu p + \gamma I - \mu R$$

- Birth rate ( $\nu$  for natality)
- Death rate ( $\mu$  for mortality)
- Proportion vaccinated ( $p$ )
- Transmission rate ( $\beta$ )
- Mean latent period ( $1/\sigma$ )
- Mean infectious period ( $1/\gamma$ )



# SEIR with vital dynamics and vaccination: Analysis

- $\mathcal{R}_0$  ?
  - Biological derivation: (assuming  $\nu = \mu$  and  $p = 0$ )
$$\mathcal{R}_0 = \beta \times \frac{\sigma}{\sigma + \mu} \times \frac{1}{\gamma + \mu} \simeq \frac{\beta}{\gamma} \quad \because \frac{1}{\mu} \gg \max\left(\frac{1}{\sigma}, \frac{1}{\gamma}\right)$$
  - Mathematical derivation:
$$\mathcal{R}_0 = 1 \text{ is stability boundary}$$
- Final size ? Not well defined (because of continuous source of new susceptibles).
- Equilibria ?
  - Disease Free Equilibrium (DFE)
  - Endemic Equilibrium (EE)
  - That's all folks.
- Periodic solutions ? No.
- What else ? Chaos?