Math 3A03 - Tutorial 9 Questions - Winter 2019

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Problem 1. *Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function with the property that*

$$\lim_{x \to -\infty} f(x) = \lim_{x \to \infty} f(x) = 0. \tag{1}$$

Show that f has either an absolute maximum or an absolute minimum but not necessary both.

Problem 2. *Let f be a continuous, one-to-one function defined on the interval $[a, b]$ with $f(a) < f(b)$. Show that, for all $x, y \in [a, b]$, if $x < y$ then $f(x) < f(y)$.*

Problem 3. *Let $f : [0, 1] \to \mathbb{R}$ be a continuous function that is differentiable on $(0, 1)$ and with $f(0) = 0$ and $f(1) = 1$. Show there must exist distinct numbers ξ_1 and ξ_2 in that interval such that*

$$f'(\xi_1) f'(\xi_2) = 1.$$