# Math 3A03 - Tutorial 3 Questions - Winter 2019

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**Problem 1.** *Prove, using the algebra of limits,* $\lim_{n\to\infty}\left(\frac{1}{n^3}+\frac{n}{2(n+3)}\right)=\frac{1}{2}$.

**Problem 2.** *Suppose that* $a_n \geq 0$*. Prove if* $\lim_{n\to\infty} a_n \to L$ *then* $L \geq 0$ *and* $\lim_{n\to\infty} \sqrt{a_n} \to \sqrt{L}$.

**Problem 3.** *Let* $F_1 = 1, F_2 = 1, F_n = F_{n-1} + F_{n-2}$,

*a) Prove* $F_n$ *diverges as* $n \to \infty$.

*b) How can you modify the above to show* $F_n$ *diverges to* $\infty$*?*

**Problem 4.** *Let* $x_n \to \infty$ *(i.e. diverges to* $\infty$*), then* $s_n = \frac{x_n}{x_n+1}$ *converges to 1 as* $n \to \infty$.

**Problem 5.** *Show that the definition of the limit of a sequence is equivalent to the following slight modification (note this isn't the one from your assignment!):*

*A sequence* $\{s_n\}$ *converges to* $L$ *if given any* $\epsilon > 0$ *there is some real number* $K$ *such that if* $n \geq K$ *then* $|s_n - L| < \epsilon$.

**Problem 6.** *Show that "the set* $E$ *is dense in* $\mathbb{R}$*" is equivalent to "given any* $y \in \mathbb{R}$ *there exists a sequence of points* $x_n \in E$ *such that* $\lim_{n\to\infty} x_n = y$*".*