

Math 3A03 - Tutorial 1 Questions - Winter 2019

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Problem 1. *Prove that $2\sqrt{7}$ is irrational.*

Problem 2. *Let $a, b \in \mathbb{R}$. Prove that if $0 \leq a < b$ then $\sqrt{a} < \sqrt{b}$.*

Problem 3. *Find the sup, inf, max, min of the following sets. Mark as DNE if it does not exist.*

(a) $S = \{q : q \in \mathbb{Q} \text{ and } 0 \leq q \leq \sqrt{2}\} = \mathbb{Q} \cap [0, \sqrt{2}]$
(Note: you won't be able to write a proof of the sup yet)

(b) $S = \{\sin(\frac{\pi}{2n}) : n \in \mathbb{N}\}$

(c) $S = (0, 3)$

(d) $S = \{\frac{1}{F_n} : n \in \mathbb{N}, F_N \text{ is the } n^{\text{th}} \text{ Fibonacci number}\}$ (Note: the Fibonacci numbers are given by $F_1 = 1, F_2 = 1, F_n = F_{n-1} + F_{n-2}$)

Problem 4. *Suppose that $|x - x_0| < \min\left(\frac{|x_0|}{2}, \varepsilon \frac{|x_0|^2}{2}\right)$, and $x_0 \neq 0$ prove that*

$$\left| \frac{1}{x} - \frac{1}{x_0} \right| < \varepsilon$$