Mathematics 3A03 Real Analysis I

http://www.math.mcmaster.ca/earn/3A03

2019 ASSIGNMENT 6

This assignment is **due** on **Monday 1 April 2019 at 11:25am**.
PLEASE NOTE that you must **submit online** via crowdmark.
You will receive an e-mail from crowdmark with the required link.
Do **NOT** submit a hardcopy of this assignment.

Note: Not all questions will be marked. The questions to be marked will be determined after the assignment is due.

1. Suppose f is continuous on $[a, b]$. Prove that
$$\left|\int_a^b f(x)\,dx\right| \le \int_a^b |f(x)|\,dx\,.$$

2. Prove that if $f(x) = \displaystyle\int_0^x f(t)\,dt$ then $f = 0$.

   Hint: First prove that f is differentiable and $f'(x) = f(x)$. Then consider the derivative of the function $g(x) = f(x)/e^x$.

3. Consider the sequence of functions $\{f_n\}$, where
$$f_n(x) = \frac{1}{n(1+nx^2)}\,, \qquad x \in \mathbb{R}.$$

   (a) For which $x \in \mathbb{R}$ does the series of functions $\displaystyle\sum_{n=1}^{\infty} f_n(x)$ converge pointwise?

   (b) For which $a, b \in \mathbb{R}$ $(a < b)$ does the series of functions $\displaystyle\sum_{n=1}^{\infty} f_n$ converge uniformly on $[a, b]$ to a continuous function?

   (c) For which $a, b \in \mathbb{R}$ $(a < b)$ does the series of functions $\displaystyle\sum_{n=1}^{\infty} f_n$ converge uniformly on $[a, b]$ to a differentiable function f? For such a, b, is f' necessarily the uniform limit of $\displaystyle\sum_{n=1}^{\infty} f_n'$?

   (d) Rather than closed, finite intervals $[a, b]$, consider infinite open intervals (a, ∞). Answer parts (b) and (c) again after revising them to read "For which $a \in \mathbb{R}$ does the series… converge uniformly on (a, ∞) to…".