Math 3A03 - Tutorial 4 Questions - Winter 2019

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Problem 1. *Let a_n, b_n be Cauchy sequences. Prove that $a_n b_n$ is also Cauchy. If b_n were only bounded, would their product be Cauchy?*

Problem 2. *Show that $\lim\limits_{n \to \infty} \sqrt[n]{n} = 1$. Hint: Think of another sequence x_n with $\sqrt[n]{n} = 1 + x_n$ and show that x_n approaches zero by (for example) the squeeze theorem.*

Problem 3. *Let $a_1 = 2$, $a_n = \sqrt{3 + 5a_{n-1}}$, prove that a_n converges and find its limit.*

Problem 4. *Show explicitly that $s_n = \sqrt{n+1} - \sqrt{n}$ is Cauchy.*

Problem 5. *Suppose that s_n is a bounded sequence. Prove that if subsequence of s_n has a further subsequence which converges to L, then s_n converges to L.*