Math 3A03 - Tutorial 8 Questions - Winter 2019

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March 11/13, 2019

Problem 1. *Let*

$$f(x) = \begin{cases} \frac{1}{q} & x = \frac{p}{q}, p, q \in \mathbb{N}, \quad \gcd(p,q) = 1 \\ 0 & x \notin \mathbb{Q} \\ 0 & x = 0 \end{cases}.$$

For which $y \in \mathbb{R}$ does the limit exist? Where is this function continuous?

Problem 2. *Which of the following are uniformly continuous:*

1. $f(x) = x^3$ *with* $x \in \mathbb{R}$
2. $f(x) = x^3$ *with* $x \in [0, 3]$
3. $f(x) = \frac{1}{x}$ *with* $x \in (0, 1)$
4. $f(x) = \frac{1}{x}$ *with* $x \in [1, \infty)$
5. $f(x) = \sin\left(\frac{1}{x}\right)$ *with* $x \in (0, 1)$

Problem 3. *Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function with the property that*

$$\lim_{x \to -\infty} f(x) = \lim_{x \to \infty} f(x) = 0. \tag{1}$$

Show that f has either an absolute maximum or an absolute minimum but not necessary both.

Problem 4. *Let f be a continuous, one-to-one function defined on the interval $[a, b]$ with $f(a) < f(b)$. Show that, for all $x, y \in [a, b]$, if $x < y$ then $f(x) < f(y)$.*