

Mathematics 3A03 Real Analysis I
<http://www.math.mcmaster.ca/earn/3A03>
2019 ASSIGNMENT 5

This assignment is **due** on **Monday 25 March 2019 at 11:25am**.
PLEASE NOTE that you must **submit online** via [crowdmark](#).
You will receive an e-mail from [crowdmark](#) with the required link.
Do **NOT** submit a hardcopy of this assignment.

Note: Not all questions will be marked. The questions to be marked will be determined after the assignment is due.

1. Classify the discontinuities of the rational function

$$f(x) = \begin{cases} \frac{x+1}{x^2-1}, & x \neq \pm 1, \\ c_1, & x = 1, \\ c_2, & x = -1. \end{cases}$$

Note: See the textbook (TBB, §5.9.1, p. 331) for the definitions of removable, jump and essential discontinuities.

2. Suppose that f is a function on a closed domain D , and let $E = f(D)$ be the range of f . Prove that f is continuous on D if and only if the inverse image of every closed set is closed.

Note: The inverse image of a set A is the set of all points in the domain of f that are mapped into A , i.e., $f^{-1}(A) = \{x \in D : f(x) \in A\}$.

Note: Problem 1(b) on 2016 Assignment 5 showed that a continuous function does not necessarily map closed sets to closed sets.

3. Suppose f and g are continuous on $[a, b]$ and differentiable on (a, b) . Prove that there is some $x \in (a, b)$ such that

$$[f(b) - f(a)]g'(x) = [g(b) - g(a)]f'(x).$$

Hint: Construct a function $h(x)$ to which you can apply Rolle's Theorem.