Mathematics 3A03 Real Analysis I

http://www.math.mcmaster.ca/earn/3A03

2019 ASSIGNMENT 5

This assignment is **due** on **Monday 25 March 2019 at 11:25am**.
PLEASE NOTE that you must **submit online** via crowdmark.
You will receive an e-mail from crowdmark with the required link.
Do **NOT** submit a hardcopy of this assignment.

Note: Not all questions will be marked. The questions to be marked will be determined after the assignment is due.

1. Classify the discontinuities of the rational function

$$f(x) = \begin{cases} \dfrac{x+1}{x^2-1}, & x \neq \pm 1, \\ c_1, & x = 1, \\ c_2, & x = -1. \end{cases}$$

   Note: See the textbook (TBB, §5.9.1, p. 331) for the definitions of removable, jump and essential discontinuities.

2. Suppose that f is a function on a closed domain D, and let $E = f(D)$ be the range of f. Prove that f is continuous on D *if and only if* the inverse image of every closed set is closed.

   Note: The inverse image of a set A is the set of all points in the domain of f that are mapped into A, *i.e.*, $f^{-1}(A) = \{x \in D : f(x) \in A\}$.

   Note: Problem 1(b) on 2016 Assignment 5 showed that a continuous function does not necessarily map closed sets to closed sets.