Math 3A03 - Tutorial 7 Questions - Winter 2019

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Problem 1. *Prove that* $\lim\limits_{x \to a} x^3 = a^3$ *for every* $a \in \mathbb{R}$.

Problem 2. *Let*

$$f(x) = \begin{cases} \frac{1}{q} & p, q \in \mathbb{N}, \quad \gcd(p, q) = 1 \\ 0 & x \notin \mathbb{Q} \\ 0 & x = 0 \end{cases}.$$

For which $y \in \mathbb{R}$ *does the limit exist? Where is this function continuous?*

Problem 3. *Which of the following are uniformly continuous:*

1. $f(x) = x^3$ *with* $x \in \mathbb{R}$

2. $f(x) = x^3$ *with* $x \in [0, 3]$

3. $f(x) = \frac{1}{x}$ *with* $x \in (0, 1)$

4. $f(x) = \frac{1}{x}$ *with* $x \in [1, \infty)$

5. $f(x) = \sin\left(\frac{1}{x}\right)$ *with* $x \in (0, 1)$

Problem 4. *Suppose that* $f(x)$ *and* $g(x)$ *are continuous on* $\mathbb{R}$, *with* $f(x) = g(x)$ *on E, a dense subset of* $\mathbb{R}$. *Prove that* $f(x) = g(x)$ *on* $\mathbb{R}$.