Math 3A03 - Tutorial 2 Questions - Winter 2019

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January 21/23, 2019

Problem 1. *Use the induction principle to prove that* $\displaystyle\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$
for any $n \in \mathbb{N}$.

Problem 2. *Show that the set* $P := \left\{ \sqrt{2}q : q \in \mathbb{Q} \right\}$ *is dense in* $\mathbb{R}$.

Problem 3. *If a set* E *is dense in* $\mathbb{R}$, *what can you conclude about*

(a) a set A, *such that* $E \subset A$?

(b) the set $\mathbb{R} \backslash E$?

(c) the set $E \cap F$, *where* F *is also dense in* $\mathbb{R}$?

Problem 4. *Let* $I := (a, b) \subseteq \mathbb{R}$ *be an interval with* $b \in \mathbb{R}$, $a < b$, *and* $E \subseteq I$ *be a dense subset of* I.

(a) Show that $\sup(E) = \sup(I) = b$.

(b) Prove that $\sqrt{5}$ *is the* $\sup\left([0, \sqrt{5})\right)$.

(c) Prove that $\sqrt{5}$ *is the* $\sup\left([0, \sqrt{5}] \cap \mathbb{Q}\right)$.

Problem 5. *Prove, using the definition of the limit* $\displaystyle\lim_{n \to \infty} \frac{\cos(n)}{\sqrt{n^5 + n^2 + 3}} = 0$.