McMaster University

Mathematics and Statistics

$$\int_M d\omega = \int_{\partial M} \omega$$

Mathematics 3F03
Advanced Differential Equations

Instructor: David Earn

Lecture 29
Invariant Sets
Wednesday 13 November 2013

Announcements

- **Assignment 4:**
  - Due this Friday 15 Nov 2013, 1:30pm.
- **Assignment 5:**
  - Due NEXT Friday 22 Nov 2013, 1:30pm.
  - To be posted by the end of the week.

Equilibria

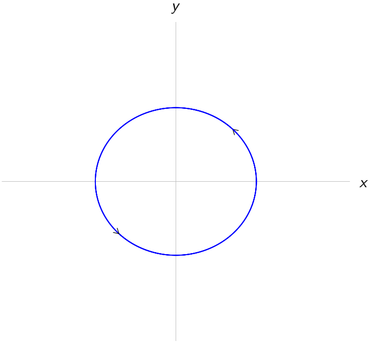
- Single points that are fixed for all time.

Periodic Orbits

- A set $\mathcal{P} \subset \mathbb{R}^n$ is a *periodic orbit* if $\exists \tau > 0$ such that $\forall X \in \mathcal{P}$

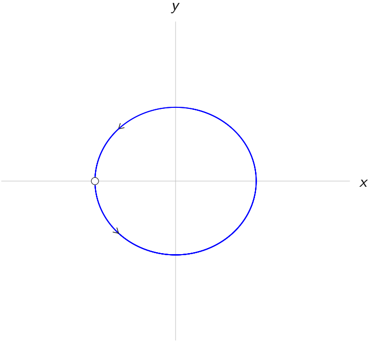
  $$\phi_{t+\tau}(X) = \phi_t(X), \quad \forall t \in \mathbb{R}.$$

- Period = mininum τ for which this is true.
- ($\tau > 0$, else it is an equilibrium)



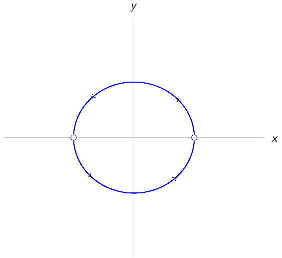
Homoclinic Orbits

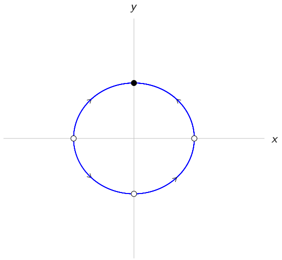
- Begin and end at an equilibrium X_*.
- A solution that tends to a given equilibrium X_* in both forward and backward time.
- $\lim_{t\to\infty} X(t) = \lim_{t\to-\infty} X(t) = X_*$.



Heteroclinic Orbits

- A solution that tends to one equilibrium X_*^1 in backward time and a different equilibrium X_*^2 in forward time.
- $\lim_{t\to-\infty} X(t) = X_*^1$.
- $\lim_{t\to\infty} X(t) = X_*^2$.
- Several heteroclinic orbits can link together to form a chain, which may or may not be closed.





Other complex invariant sets

- Arbitrarily complicated invariant sets made up of homoclinic and heteroclinic orbits can occur.
- Other "strange" sets that we'll discuss later.

- All such invariant sets can occur as *limit sets* of a nonlinear system.