INTRODUCTION TO (GENOMIC) ALIGNMENTS

DAVIDE BOLOGNINI

MOLECULAR MEDICINE PHD FELLOW

DEVELOPING @ HTTPS://GITHUB.COM/DAVIDEBOLO1993

DAVIDEBOLOGNINI7@GMAIL.COM

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1. BACKGROUND



Illumina Nextera DNA Flex Library Prep Kit



Invitrogen QuBit

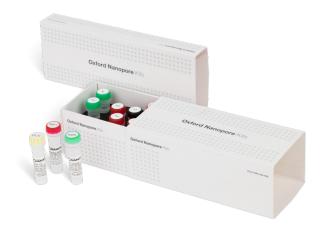


Illumina HiSeq 4000

SAMPLE PREPARATION

SAMPLE QUANTIFICATION SAMPLE SEQUENCING

SEQUENCING READS



Oxford Nanopore Technologies DNA sequencing kit



Invitrogen QuBit



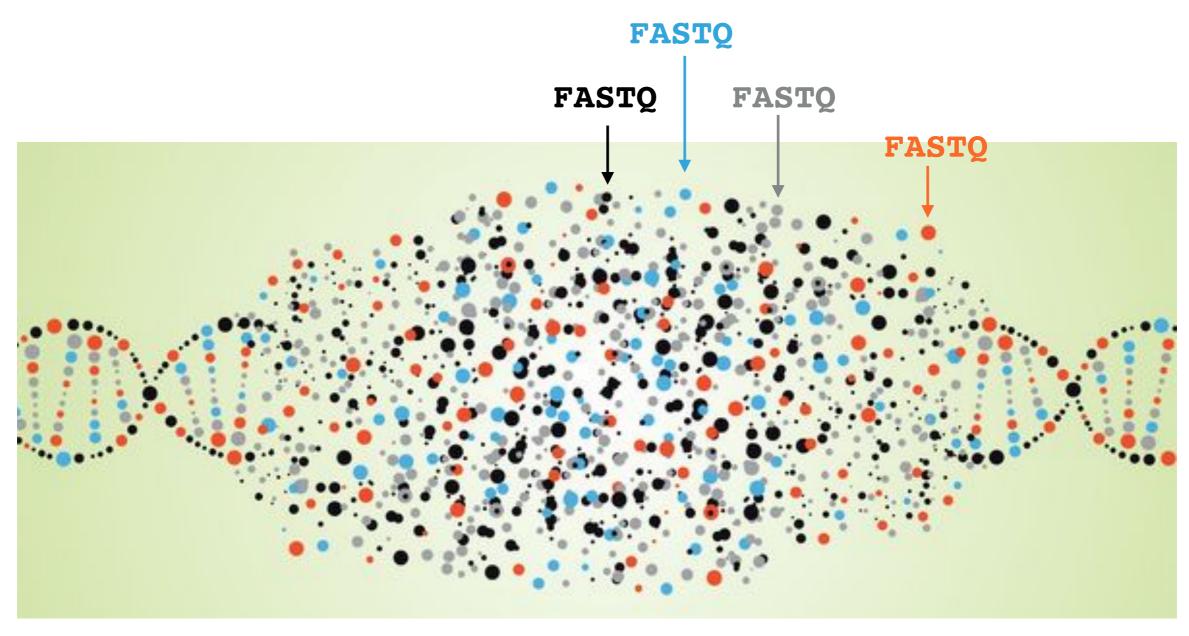
Oxford Nanopore Technologies GridION X5

READ: FASTQ

```
@SEQ_ID @ Sequence ID
TGCCCAGTGGTGGAGGTCTCAGAAATGGTATAAAAGTCAGACAAATCTTGGTTTTTGCATT Sequence
++ (Sequence ID)
!'**((((***+))%%%++)(%%%%).1***-+*''))**55CCF>>>>>CCCCCCC65 Sequence quality
```

REFERENCE: FASTA

Sequence



The genome jigsaw

7

ALIGNING READS: WHY?

Take-home message #1

Genomic alignments map the sequencing reads to the reference genome in order to identify their original genomic *loci*

READ

REFERENCE

@SEQ_ID
TGCCCAGTGGTGGAGGTCTCAGAAATGGTATAAAAGTCAGACAAATCTTGGTTTTTGCATT
+
!'**((((***+))%%%++)(%%%%).1***-+*''))**55CCF>>>>>CCCCCCC65

>chr20:17000000-17000539

ALIGNMENT

TGCCCAGTGGTGGAGGTCTCAGAAATGGTATAAAAGTCAGACAAATCTTGGTTTTTGCATT

TAGCAGTGCCCAGTGGTGGAGGTCTCAGAAATGGTATAAAAGTCAGACAAATCTTGGTTTTTGCATTCTCCAA

2. EXACT STRING MATCHING ALGORITHMS

P: word

T: There would have been a time for such a word

```
Q: How many alignments ?
A: |T| - |P| + 1
Q: How many character comparison ?
A: Range from |T| - |P| + 1 (best case) to |P| * (|T| - |P| + 1) (worst case)
Q: How this can be improved ?
A: Learn from character comparisons to skip pointless alignments
            word
             There would have been a time for such a word
                   word
                     ···→ skip
                      ----> skip
                       word
            #As 'u' not in 'word', skip the next 2 alignments
```

Boyer Moore's rules:

- 1. Test alignments from left to right, but perform character comparisons in opposite direction
- 2. Upon hitting a mismatch, skip alignments until mismatch becomes a match or P moves past the mismatch (aka, BAD CHARACTER RULE)
- 3. Let t be a substring of T matched by a substring of P. Upon hitting a mismatch, skip alignments until there are no mismatches between P and t/a prefix of P matches a suffix of T or P moves past t (aka, GOOD CHARACTER RULE)

T: G C T T C T G C T A C C T T T T G C G C G C G C G C G A A



T: G C T T C T G C T A C C T T T T G C G C G C G C G C G A A

P: 2 skips C C T T T T G C

T: G C T T C T G C T A C C T T T T G C G C G C G C G C G A A

P: CCTTTTGC

T: G C T T C T G C T A C C T T T T G C G C G C G C G C G A A

P:

6 skips

C C T T T T G C

T: CGTGCCTACTTACTTACGGAA
P: CTTACTTAC

T: CGTGCCTACTTACTTACTTACTTACGGAA

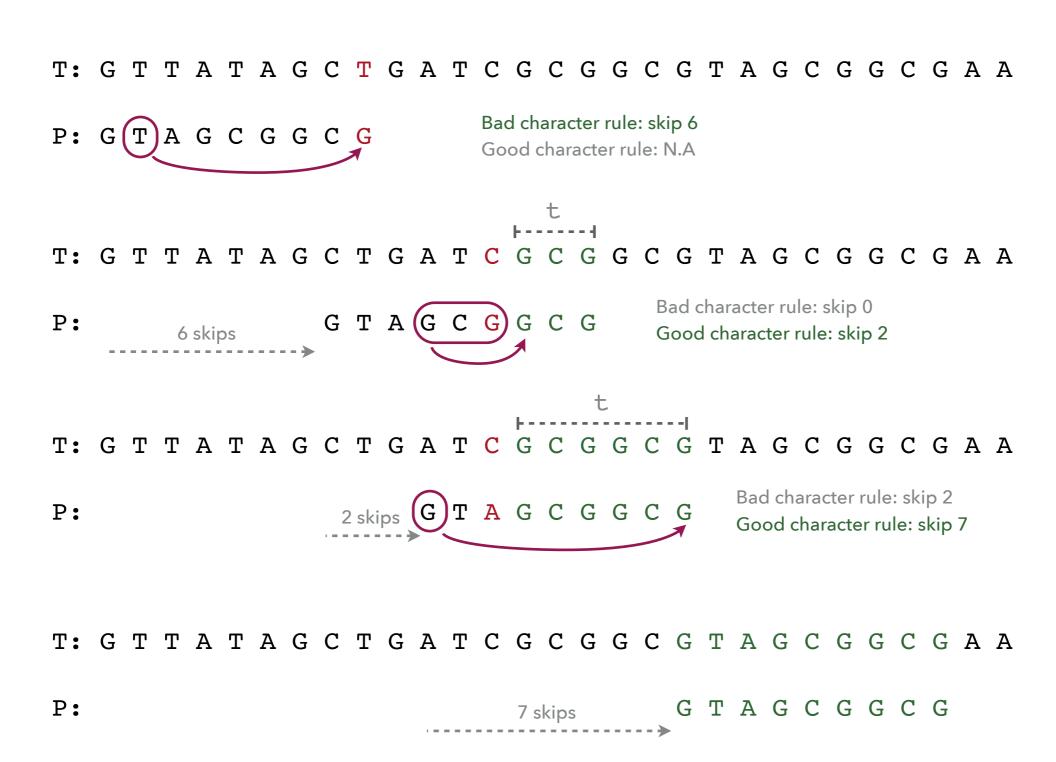
P: 3 skips C T T A C T T A C

T: CGTGCCTACTTACTTACTTACGCAA

P: CTTACTTAC

T: CGTGCCTACTTACTTACTTACCTACGCGAA

P: CTTACTAC



			P		
		T	С	G	C
	A	0	1	2	3
Σ	С	0		0	_
	G	0	1	_	0
,		_	0	1	2

Bad character rule: lookup table for P

T: A A T C A A T A G C

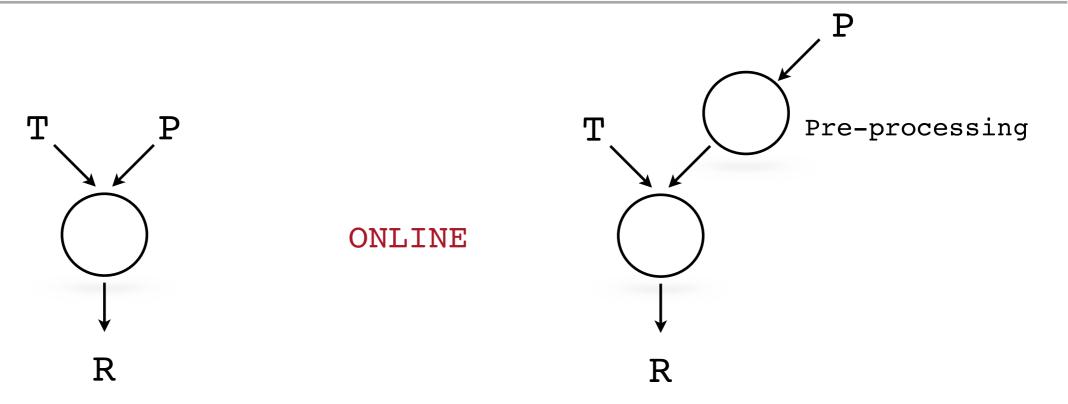
T: C G C A T G C G A G

P: TCGC

P: TC GC

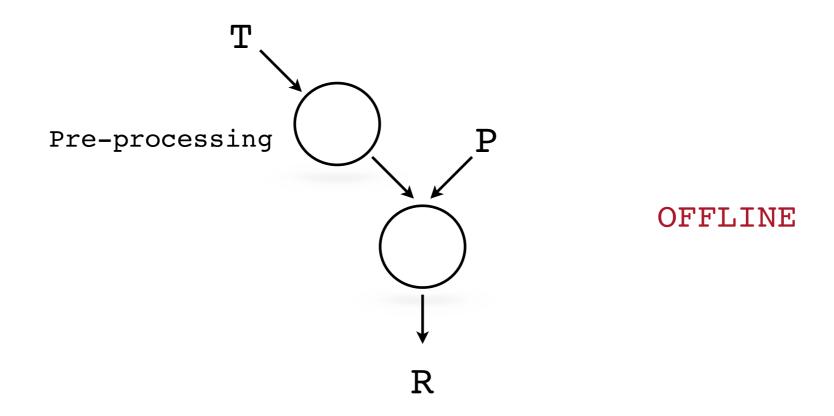
Q: Any benefits?

A: Overall work reduced via reuse



Brute Force String Matching

Boyer-Moore String Matching

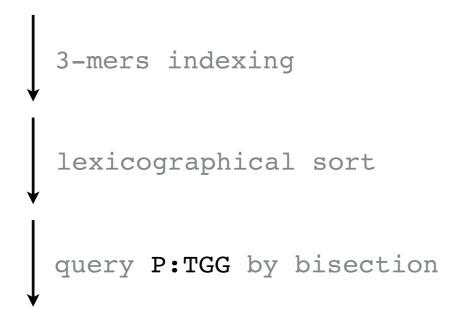


Index of a book for terms 'memory' and 'mind'



Index of DNA string T for 5-mers substrings

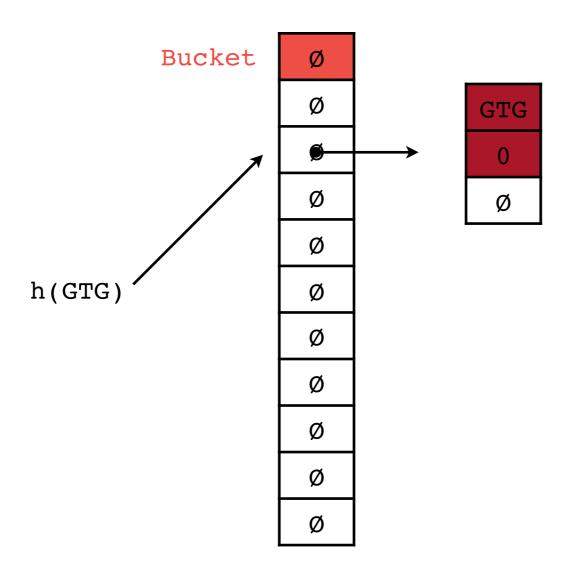
T: G T G C G T G T G G G G

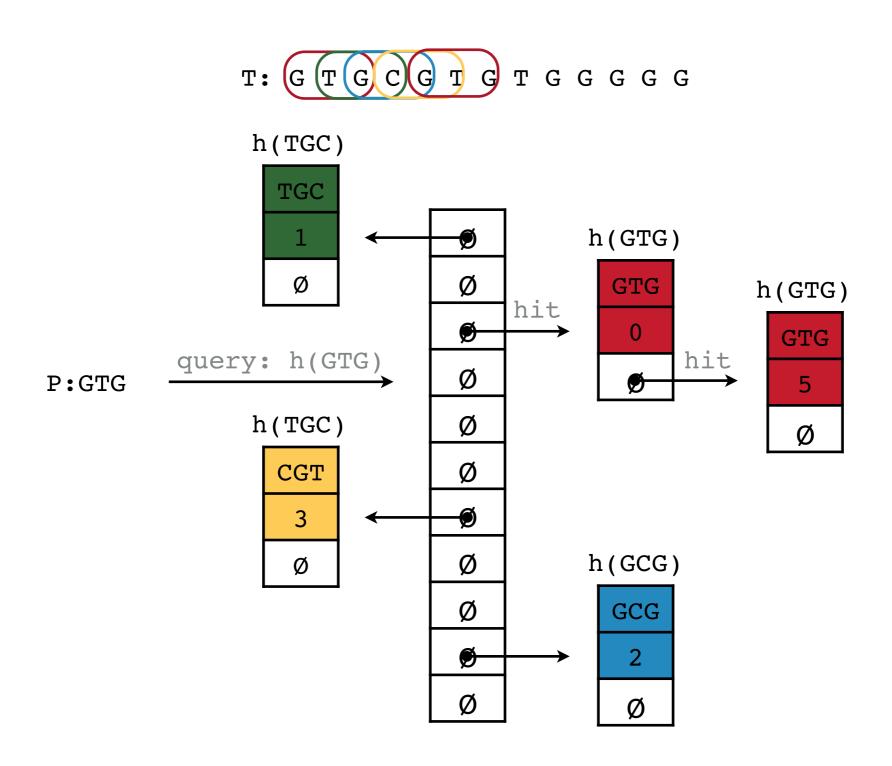


CGT	3		CGT	3					
GCG	2	TGG > GTG query				GCG	2		
GGG	8,9,10		GGG	8,9,10	TGG = TGG				
GTG	0,4,6		GTG	0,4,6	→	7			
TGC	1		TGC	1	hit				
TGG	7		TGG	7					
TGT	5		TGT	5	•				

20







3. A BIOINFORMATIC EXAMPLE

```
In [1]: !wget --no-check https://d28rh4a8wg0iu5.cloudfront.net/ads1/data/phix.fa
        --2019-10-16 16:17:36-- https://d28rh4a8wq0iu5.cloudfront.net/ads1/data/phix.fa
        Risoluzione di d28rh4a8wq0iu5.cloudfront.net (d28rh4a8wq0iu5.cloudfront.net)... 143.204.10.105, 143.204.10.216, 143.2
        04.10.21, ...
        Connessione a d28rh4a8wq0iu5.cloudfront.net (d28rh4a8wq0iu5.cloudfront.net) | 143.204.10.105 | :443... connesso.
        Richiesta HTTP inviata, in attesa di risposta... 200 OK
        Lunghezza: 5528 (5,4K) [application/octet-stream]
        Salvataggio in: "phix.fa"
        phix.fa
                                                      5,40K --.-KB/s
                           100%[=========>]
                                                                           in 0s
        2019-10-16 16:17:37 (195 MB/s) - "phix.fa" salvato [5528/5528]
In [2]: def ReadGenome(filename):
           *Open and read a FASTA, concatenating different sequences into a single one
           ∍genome=''
         with open(filename, 'r') as f:
           → for line in f:
           → → → if line[0] != '>':
           # # genome+=line.rstrip()
           *return genome
In [3]:
        genome=ReadGenome('phix.fa')
In [4]: len(genome)
Out[4]: 5386
```

```
In [1]: !wget --no-check https://d28rh4a8wq0iu5.cloudfront.net/ads1/data/ERR266411_1.first1000.fastq
        --2019-10-16 16:31:07-- https://d28rh4a8wq0iu5.cloudfront.net/ads1/data/ERR266411_1.first1000.fastq
        Risoluzione di d28rh4a8wq0iu5.cloudfront.net (d28rh4a8wq0iu5.cloudfront.net)... 143.204.10.21, 143.204.10.216, 143.20
        4.10.34, ...
        Connessione a d28rh4a8wq0iu5.cloudfront.net (d28rh4a8wq0iu5.cloudfront.net) | 143.204.10.21 | :443... connesso.
        Richiesta HTTP inviata, in attesa di risposta... 200 OK
        Lunghezza: 254384 (248K) [audio/mpeg]
        Salvataggio in: "ERR266411 1.first1000.fastq"
        ERR266411 1.first10 100%[===========] 248,42K
                                                                777KB/s
                                                                            in 0,3s
        2019-10-16 16:31:08 (777 KB/s) - "ERR266411 1.first1000.fastq" salvato [254384/254384]
In [2]: def ReadFastq(filename):
                 . . .
                Open and read a 4-line FASTQ. Discard first/third/fourth line for each sequence
                sequences=[]
                with open(filename, 'r') as f:
                        while True:
                                f.readline()
                                 seq=f.readline().rstrip()
                                 if seq == '':
                                        break
                                 sequences.append(seq)
                                f.readline()
                                f.readline()
                return sequences
In [3]: reads=ReadFastq('ERR266411_1.first1000.fastq')
In [4]: len(reads)
Out[4]: 1000
In [6]: len(reads[0])
Out[6]: 100
```

7 / 1000 reads matched the genome !

```
In [10]: def BruteForce(P, T):
         _______ H T T T
          Simple implementation of Brute Force String Matching
         → assert len(P) <= len(T)</pre>
         → occurrences = []
         for i in range(len(T)-len(P)+1):

→ match = True

            # for j in range(len(P)):

→ → → if T[i+j] != P[j]:

            match = False
            <sup>⋊</sup> → wbreak

→ if match:

            woccurrences.append(i)
           →return occurrences
In [11]: total=0
         matched=0
         for read in reads:
          matches=BruteForce(read,genome)
         \rightarrowif len(matches) > 0:
          matched += 1
         ----×total+=1
         print('%d / %d reads matched the genome !' %(matched, total))
```

Q: Why this number is so small?

Al: Differences between individuals and sequenced reads

A2: Reads orientation

449 / 1000 reads matched the genome !

Q: Why this number is (still) so small?

A3: Sequencing errors

932 / 1000 reads matched the genome !

Take-home message #2

Sequencing errors occur (% is ~1% in short reads and ~10% in long reads) and exact string matching algorithms are not the method of choice for mapping

4. APPROXIMATE STRING MATCHING ALGORITHMS

Mismatch

T: GGAAAAAGAGGTAGCGGCGTTTAAACAGTAG

P: GTAACGGCG

Insertion

T: GGAAAAAGGGTAGC-GCGTTTAACAGTAG

P: GTAGCGGCG

Deletion

T: GGAAAAAGGGTAGCGGCGTTTAAACAGTAG

 $P: \qquad \qquad G T - G C G G C G$

Given T and P, with |T| = |P|, the Hamming Distance is the minimum #substitutions required to turn one into the other

T: GGAAAAAGAGGTAGCGGCGTTTAAACAGTAG

P: G G T T A A A G A G G G A G C G G C G T T T T A C A G T A G

Hamming distance = 4

Given T and P, the Edit Distance (aka, Levenshtein Distance) is the minimum #edits (substitutions, insertions or deletions) required to turn one into the other

T: GGAAAAAGAGGTAGCGGCGTTTAAACAGTAG

P: G G T A A A A G A G G T A G C G G C G T T T A C A G T A G

Edit distance = 2

Q: What is the relationship between Hamming and Edit Distance if |T| = |P|?

T: G C G T A T G C G G C T A A C G C

P: G C T A T G C G G C T A T A C G C



T: G C G T A T G C G G C T A A C G C T: G C G T A T G C G G C T A - A C G C

P: G C T A T G C G G C T A T A C G C P: G C - T A T G C G G C T A T A C G C

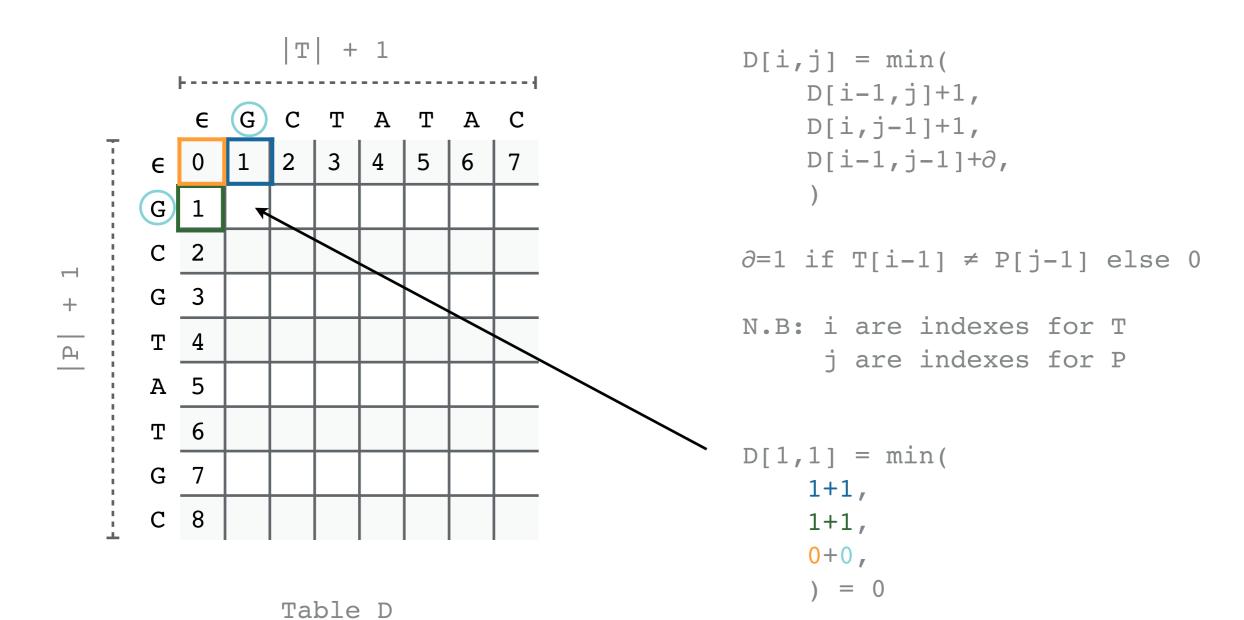
Hamming distance = 10 Edit distance = 2

A: Edit Distance (P,T) <= Hamming Distance (P,T)

Q: What is the minimum Edit Distance if $|P| \neq |T|$?

A: Edit Distance (P,T) >= ||T| - |P||

T: G C T A T A C 0 1 2 3 4 5 6 P: G C G T A T G C 0 1 2 3 4 5 6 7



T: G C T A T A C

P: G C G T A T G C

	\in	G	C	Т	A	Т	A	C
\in	0	1	2	3	4	5	6	7
G	1	0	1	2	3	4	5	6
С	2	1	0	1	2	3	4	5
G	3	2	1	1	2	3	4	5
Т	4	3	2	1	2	2	3	4
A	5	4	3	2	1	2	2	3
\mathbf{T}	6	5	4	3	2	1	2	3
G	7	6	5	4	3	2	2	3
С	8	7	6	5	4	3	3	2
								↑

Edit Distance between T and P

T: TATTGGCTATACGGTT T: TATTGGC-TATACGGTT

P: G C G T A T G C
P: G C G T A T G C

Apply the Edit Distance formula

Initialize first row with 0s

	\	\e	Т	Α	Т	Т	G	G	С	Т	A	Т	A	С	G	G	Т	Т
	E	9/	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
M	G	1	1	1	1	1	0	0	1	1	1	1	1	1	0	0	1	1
M	С	2	2	2	2	2	1	1	0	1	2	2	2	1	1	1	1	2
I	G	3	3	3	3	3	2	1	1	1	2	3	3	2	1	1	2	2
M	Т	4	3	4	3	3	3	2	2	1	2	2	3	3	2	2	1	2
M	A	5	4	3	4	4	4	3	3	2	1	2	2	3	3	3	2	2
M	Т	6	5	4	3	4	5	4	4	3	2	1	2	3	4	4	3	2
S	G	7	6	5	4	4	4	5	5	4	3	2	2	3	3	4	4	3
M	С	8	7	6	5	5	5	5	5	5	4	3	3	2	3	4	5	4

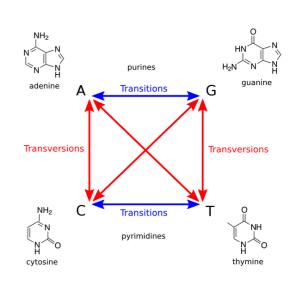
Trace back the matrix using the minimum parental value

Find the minimum value in the last row

Edit Distance-based approximate string matching penalize different kind of edits we may find with the same amount: that is, there is no difference between a substitution and a deletion or different kind of substitutions

Q: Is this correct?

	AFR		AMR		i i	AS	1	EUR	SAS		
Samples	•	661	347 7.6			504		503	489		
Mean coverage		8.2			7.7			7.4	8.0		
	Var. sites	Singletons									
SNPs	4.31M	14.5k	3.64M	12.0k	3.55M	14.8k	3.53M	11.4k	3.60M	14.4k	
Indels	625k	-	557k	-	546k	-	546k	-	556k	-	
Large deletions	1.1k	5	949	5	940	7	939	5	947	5	
CNVs	170	1	153	1	158	1	157	1	165	1	
MEI (Alu)	1.03k	0	845	0	899	1	919	0	889	0	
MEI (L1)	138	0	118	0	130	0	123	0	123	0	
MEI (SVA)	52	0	44	0	56	0	53	0	44	0	
MEI (MT)	5	0	5	0	4	0	4	0	4	0	
Inversions	12	0	9	0	10	0	9	0	11	0	



Transition/Transversion ~2.1

A global reference for human genetic variation

A: No. We need something similar to the Edit Distance-based approximate string matching but with personalized penalties (that is, give an higher penalty to deletions rather than to substitutions or an higher penalty to transversions rather than to transitions). Global Alignment (aka, Needleman-Wunsch algorithm) and Local Alignment (aka, Smith-Waterman algorithm) serve the purpose

	A	С	G	Т	_
A	0	4	2	4	8
С	4	0	4	2	8
G	2	4	0	4	8
Т	4	2	4	0	8
-	8	8	8	8	

Penalties:

- 0 for matches
- 8 for gaps
- 2 for transitions
- 4 for transversions

T: T A T G T C A T G C 0 1 2 3 4 5 6 7 8 9
P: T A C G T C A G G 0 1 2 3 4 5 6 7 8

Initialize first row and first column with gap penalties

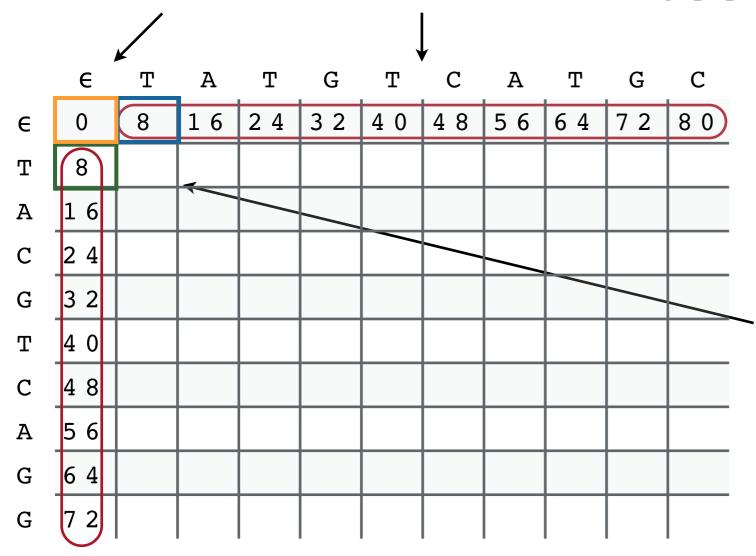


Table D

```
A 0 4 2 4 8
C 4 0 4 2 8
G 2 4 0 4 8
T 4 2 4 0 8
- 8 8 8 8
```

ACGT-

Apply the formula:

```
D[i,j] = min(
    D[i-1,j]+p(T[i-1],'-'),
    D[i,j-1]+p('-',P[j-1]),
    D[i-1,j-1]+p(T[i-1],P[j-1),
    )
```

N.B i are indexes of T j are indexes of P

T: TATGTCATGC
T: TATGTCATGC

P: TACGTCAGG
P: TACGTCA-GG

	Α	С	G	Т	_
A	0	4	2	4	8
C	4	0	4	2	8
G	2	4	0	4	8
Т	4	2	4	0	88
_	8	8	8	8	

		E	Т	A	Т	G	Т	С	A	Т	G	С
	€	0	8	1 6	2 4	3 2	4 0	4 8	5 6	6 4	7 2	8 0
M	Т	8	0	8	1 6	2 4	3 2	4 0	4 8	5 6	6 4	7 2
M	A	1 6	8	0	8	1 6	2 4	3 2	4 0	4 8	5 6	6 4
S	С	2 4	1 6	8	2	1 0	1 8	2 4	3 2	4 0	4 8	5 6
M	G	3 2	2 4	1 6	1 0	2	1 0	18	2 6	3 4	4 0	4 8
M	T	4 0	3 2	2 4	1 6	1 0	2	1 0	18	2 6	3 4	4 2
M	С	4 8	4 0	3 2	2 4	1 8	1 0	2	1 0	1 8	2 6	3 4
M	A	5 6	4 8	4 0	3 2	2 6	1 8	1 0	2	1 0	18	2 6
DM	G	6 4	5 6	4 8	4 0	3 2	2 6	18	1 0	6	1 0	18
M	G	7 2	6 4	5 6	4 8	4 0	3 4	2 6	18	1 2	1 0	10



Given 2 strings (T and P), Local Alignment does not try to find the alignment of P to T but try to identify the substring of T and the substring of P which are the most similar to each other

T: he will after his sour fashion tell you

P: struts and frets his hour upon the stage

T: he will after his sour fashion tell you

P: struts and frets his hour upon the stage

	A	С	G	Т	-
A	2	4	_4	4	<u>-6</u>
С	<u>4</u>	2	4	4	<u>-6</u>
G	<u>4</u>	4	2	-4	<u>-6</u>
Т	4	4	4	2	<u>-6</u>
-	<u>-6</u>	-6	<u>-6</u>	-6	

Rewards:

2 for matches

-6 for gaps

-4 for substitutions



A C G T A 2 - 4 - 4 - 4 - 6
C - 4 2 - 4 - 4 - 6
G - 4 - 4 2 - 4 - 6
T - 4 - 4 - 4 2 - 6
- - 6 - 6 - 6 - 6

Initialize first row and first column with 0s

	\downarrow														
	€	T	A	Т	Α	Т	G	С	G	G	С	G	Т	Т	T
ϵ	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
G	0														
G	0														
${f T}$	0														
A	0														
T	0														
G	0														
С	0														
Т	0														
G	0														
G	0														
С	0														
G	0														
C	0														
${f T}$	0														
A	0														
							_	1 7	_						

Table D

Apply the formula:

```
D[i,j] = max(
        D[i-1,j]+r(T[i-1],'-'),
        D[i,j-1]+r('-',P[j-1]),
        D[i-1,j-1]+r(T[i-1],P[j-1),
        0
        )
```

N.B i are indexes of T j are indexes of P

```
D[1,1] = min(0+(-6), 0+(-6), 0+(-4), 0)
0 = 0
```

T: TATATGCGGCGTTT T: TATATGC-GGCGTTT

P: G G T A T G C T G G C G C T A P: G G T A T G C T G G C G C T A

	Α	С	G	T	_	
	2					
С	_ 4	2	- 4	- 4	- 6	
G	- 4	- 4	2	- 4	- 6	
\mathbf{T}	_ 4	- 4	- 4	2	- 6	
_	- 6	- 6	- 6	- 6		

Stop to trace back when reach 0

		\in	T	Α	Т	A	Т	G	С	G	G	С	G	Т	Т	Т
	\in	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	G	0	0	l 0	0	0	0	2	0	2	2	0	2	0	0	0
	G	0	0	0	0	0	0	2	0	2	4	0	2	0	0	0
M	Т	0	2	0	2	0	2	0	0	0	0	0	0	4	2	2
M	A	0	0	4	0	4	0	0	0	0	0	0	0	0	0	0
M	Т	0	2	0	6	0	6	0	0	0	0	0	0	2	2	2
M	G	0	0	0	0	2	0	8	2	2	2	0	2	0	0	0
M	C	0	0	0	0	0	0	2	1 0	4	0	4	0	0	0	0
I	Т	0	2	0	2	0	2	0	4	6	0	0	0	2	2	2
M	G	0	0	0	0	0	0	4	0	6	8	2	2	0	0	0
M	G	0	0	0	0	0	0	2	0	2	8	4	4	0	0	0
M	C	0	0	0	0	0	0	0	4	0	2	1 0	4	0	0	0
M	G	0	0	0	0	0	0	2	0	6	2	4	12	6	0	0
	C	0	0	0	0	0	0	0	4	0	2	4	61	8	2	0
	Т	0	2	0	2	0	2	0	0	0	0	0	0	8	1 0	4
	A	0	0	4	0	4	0	0	0	0	0	0	o	2	4	6

Start to trace back from cell with the highest value

Q: Is there a unique way of performing Global and Local alignments ?

A: No. It depends on the values we use in penalty and score matrixes. Moreover, usually there is a different score for gap-opening and gap-extending (the cost for gap-extending is usually lower)

Q: What is the time complexity of dynamic programming ?

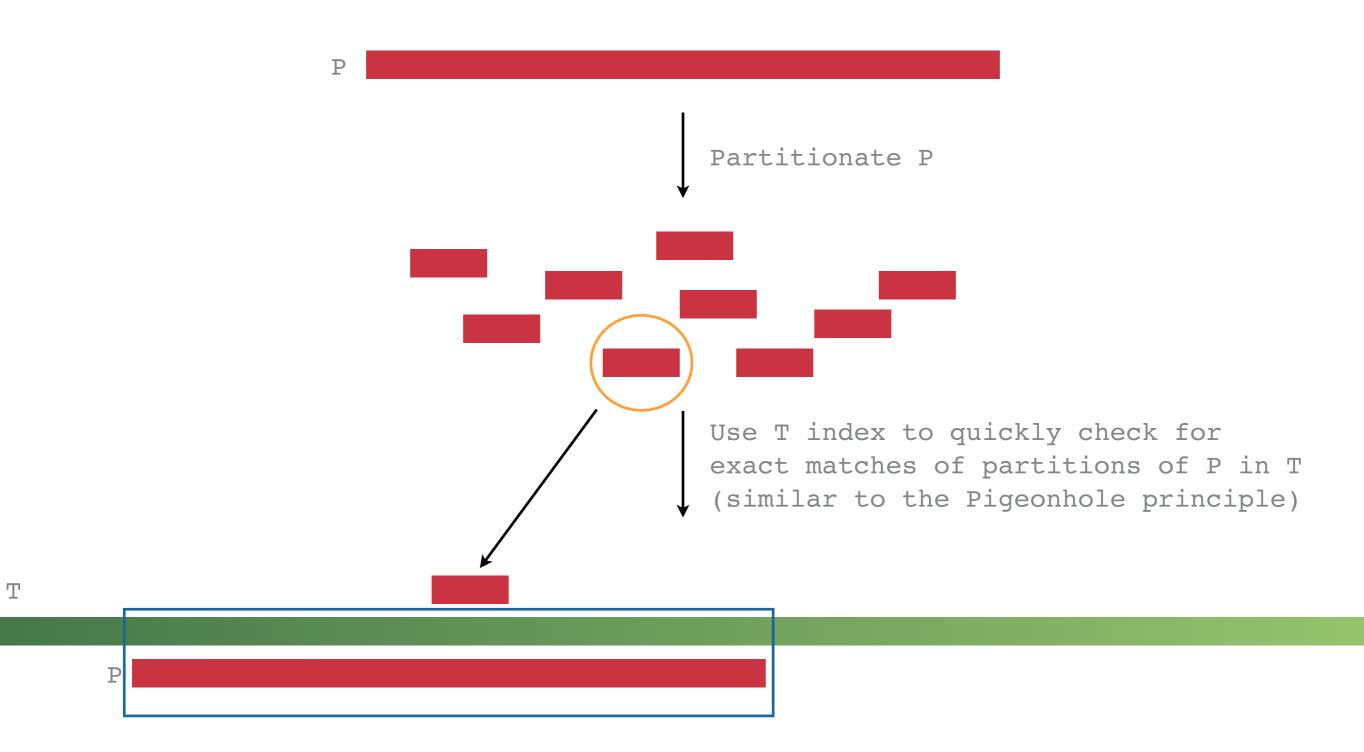
A: O(mn)

Q: What is the space complexity of dynamic programming ?

A: O(mn)

Q: Can Global and Local alignments algorithms be applied as described here ?

A: No. The human reference genome $\approx 3*10^9$ nucleotides (m). 1-week long run of an Illumina HiSeq 4000 produce $\approx 6*10^9$ reads (d), 100 nucleotides each (n). Time (and space, even if we can reduce this somehow) are then O(mnd). With 1000 processors, 3 GHz each, this will require \approx 2 years



Perform dynamic programming using P and T in the vicinity of the match

Take-home message #3

Index-assisted global and local alignment algorithms combine dynamic programming's efficacy (that is, they produce the results we want) and indexes' efficiency (that is, they do not waste too much time doing what they do) and are the method of choice for mapping the reads to the genome

5. A TRUE TO LIFE EXAMPLE

nature methods

Brief Communication | Published: 04 March 2012

Fast gapped-read alignment with Bowtie 2

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Ben Langmead  

& Steven L Salzberg

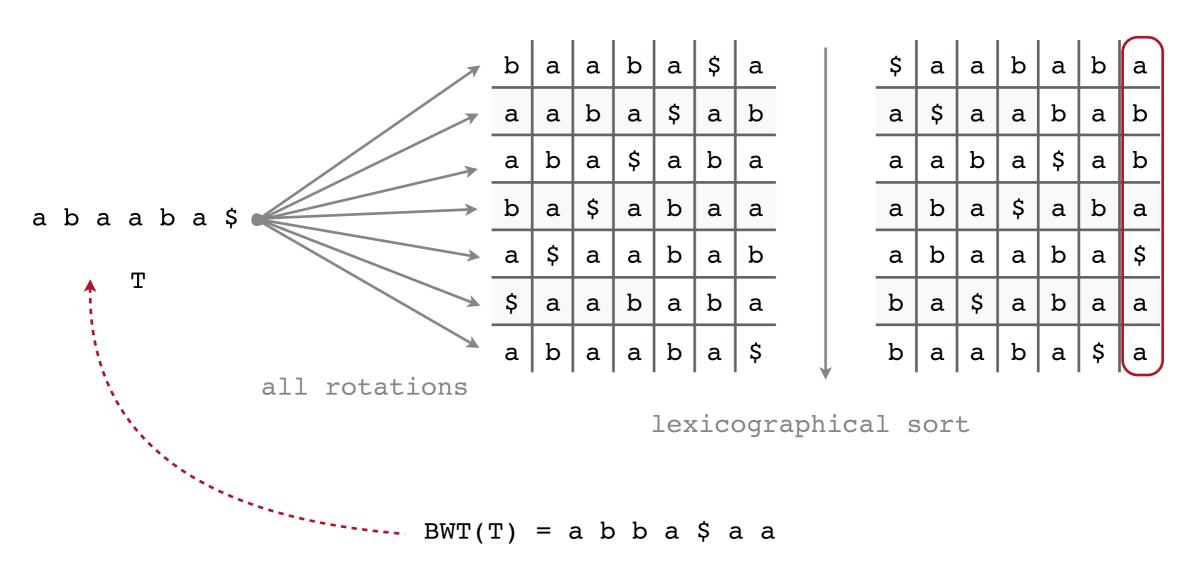
Nature Methods 9, 357–359 (2012) | Download Citation 

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Abstract

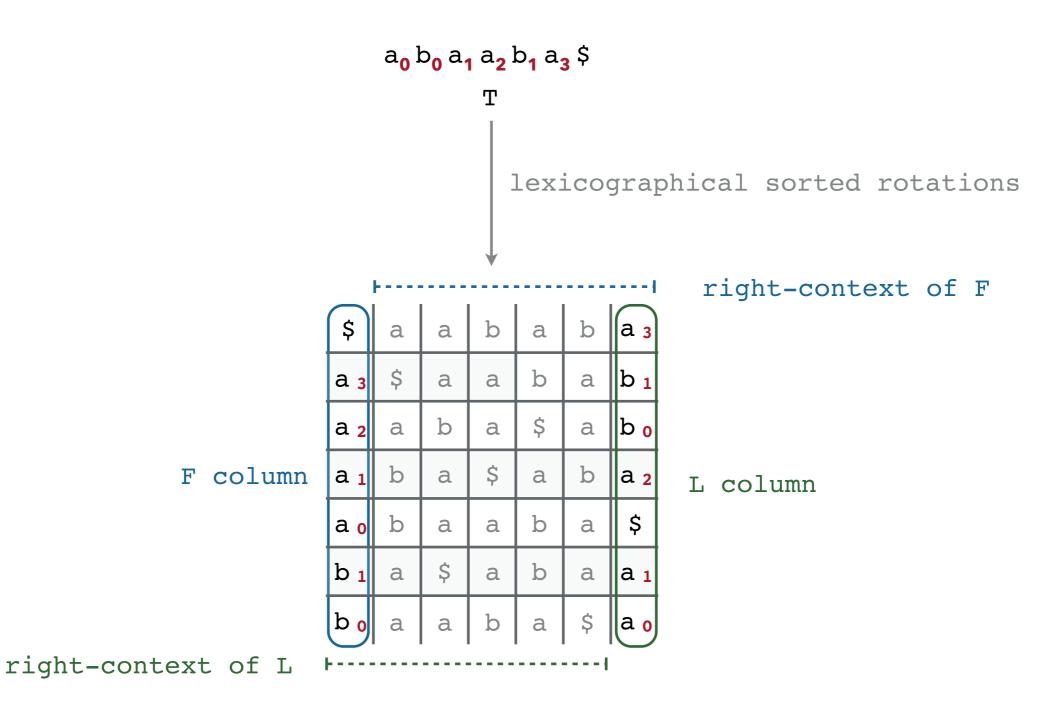
As the rate of sequencing increases, greater throughput is demanded from read aligners. The full-text minute index is often used to make alignment very fast and memory-efficient, but the approach is ill-suited to finding longer, gapped alignments. Bowtie 2 combines the strengths of the full-text minute index with the flexibility and speed of hardware-accelerated dynamic programming algorithms to achieve a combination of high speed, sensitivity and accuracy.

The Burrows-Wheeler transform (BWT) of a string T is the reversible permutation of all its characters, originally used for compression (for example, it is the core of bzip2)

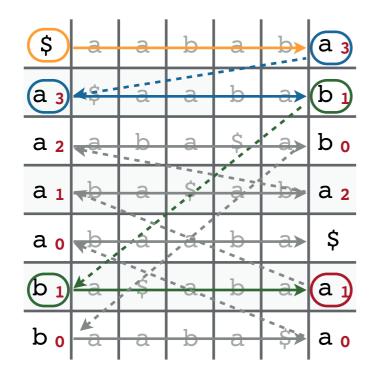


Q: How do we revert this ?

Give each character in T a rank, equal to the #times the character occurred previously in T



The ith occurrence of a character c in L and the ith occurrence of c in F have the same rank (that is, correspond to same occurrence in T). This is because they are sorted by their right-context



Start in F, first row, from \$
L contains character prior to \$: a₃

Move to a₃ in F
L contains character prior to a₃: b₁

Move to b₁ in F
L contains character prior to b₁: a₁

Continue until reversion is complete

a b a a b a \$ = T

We proceed by trying to match longer suffixes of P to BWT(T)

P: (a (b (a)

_	\$	a	a	b	a	b	a 3	
	a 3	\$	a	a	b	a	b ₁	T
	a 2	a	b	a	\$	a	b ₀	1
	a 1	b	a	\$	a	b	a 2	
11	a o	b	a	a	b	a	\$	
Ī	b 1	a	\$	a	b	a	a 1	Ī
1	b ₀	a	a	b	a	\$	a o	1

Get the range of rows beginning with a

Now seek rows starting with ba

Look at the same occurrences in F

Seek rows starting with aba

Look at the same occurrences in F

Get the range of the BWT(T) matching P

a 1	b	a	\$	a	b	a 2
a o	b	a	a	b	a	\$

We won't talk about why the FM index method is fast and how all these informations are stored in a limited space. We need just to sort out one last thing... T: a b a a b a 0 1 2 3 4 5

P: a b a

	\$	a	a	b	a	b	a
	a	\$	a	a	b	a	b
	a	a	b	a	\$	a	b
T .	a	b	a	\$	a	b	a
1	a	b	a	a	b	a	\$
	b	a	\$	a	b	a	a
,	b	a	a	b	a	\$	a

*

	6	\$	a	a	b	a	b	a
	5	a	\$	a	a	b	a	b
	2	a	a	b	a	\$	a	b
Ī	3	a	b	a	\$	a	b	a
i	0	a	b	a	a	b	a	\$
	4	b	a	\$	a	b	a	a
	1	b	a	a	b	a	\$	a

BWT(T)

SA(T)

Suffix arrays store every possible suffix of T and its offset but this require space (~12 Gb for the Human Genome, while an FM index is just ~1.5 Gb)

T: a b a a b a 0 1 2 3 4 5

P: a b a

	\$	a	a	b	a	b	a 3		6	
	a 3	\$	a	a	b	a	b ₁			
	(a 2)	2	b	a	Ş	a	b ₀	\rightarrow	2	
Ī	(a 1)	b	a	\$	a		a 2	→		X
1	(a o	b	a	a	b	a	\$	\rightarrow	0	
	b 1	a	\$	a	b	a	a 1		4	
	b ₀	a	a	b	a	\$	a o			

Check if index lookup succeed

If it succeeds, keep the index

If not, apply LF mapping

If it succeeds, keep the index

Continue until all the indexes are found

BWT(T)

We got 1 hit at index 0 and one at index 2 + 1

6. TEACHING MATERIAL

- 1. Email me @:
 davidebolognini7@gmail.com
- 2. Get slides (.key + .pdf) from GitHub:
 https://github.com/davidebolo1993/Classes

THAT'ALL, FOLKS!