Quantum Crypto Exam

Alice has 2 EPR pairs, one on qubits q_B and q_A : $|\phi\rangle_{q_B,q_A} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ and one on qubits $q_{A'}$ and q_C : $|\phi\rangle_{q_{A'}q_C} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$.

She sends the qubit q_B to Bob and the qubit q_C to Charlie.

Alice applies CNOT then $H \otimes I$ on her qubits $q_A, q_{A'}$, then measures them in the standard basis. Let m denotes the classical result obtained from measuring q_A and n the classical result obtained from measuring q_B.

We recall that CNOT is the two qubit unitary defined on the basis states as $\forall u, v \in \{0,1\}$, $\text{CNOT}|u,v\rangle = |u, u \oplus v\rangle$ and Hadamard is the one qubit unitary defined on the basis states by $\forall u \in \{0,1\}$, $\text{H}|u\rangle = \frac{1}{\sqrt{2}} \sum_{v\in\{0,1\}} (-1)^{u.v} |v\rangle$ where $\oplus$ is the XOR and . is the product.

- **Q1** Draw a circuit representing Alice unitaries.
- **Q2** What is the initial state $|\psi\rangle$ of the system of 4 qubits $q_B, q_A, q_{A'}, q_C$? (Bob has the left qubit, Alice the two in the middle and Charlie the right one.)
- **Q3** What is the state $|\psi_1\rangle$ of the system after the CNOT is applied?
- **Q4** What is the state $|\psi_2\rangle$ of the system after the Hadamard is applied ?
- **Q5** What is the probability that Alice gets as classical outcome $(m,n) = (0,0), (0,1), (1,0)$ and $(1,1)$? In each case what is the quantum 2-qubit state $|\phi_{m,n}\rangle$ of the two qubits of Charlie and Bob when Alice gets (m,n) as classical result ?
- **Q6** Show that for any $m, n \in \{0,1\}$ there exists a one qubit unitary $U_{m,n}$ such that $|\phi_{0,0}\rangle = I \otimes U_{m,n} |\phi_{m,n}\rangle$.
- **Q7** Deduce a protocol that allows Alice who shares an EPR pair with Bob and an other EPR pair with Charlie to create an EPR pair (and therefore entanglement) between Charlie and Bob using only 2 bits of classical communication from Alice to Charlie.
- **Q8** If Charlie and Bob do not know the result of Alice's measurement, is their system entangled ?