CySec Quantum Exam

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Given $\theta \in \mathbb{R}$ such that $0 < \theta < \pi/2$, we define a quantum state

$$|\psi_\theta\rangle = C(\sin(\theta)\,|11\rangle + \cos(\theta)(|01\rangle + |10\rangle))$$

with $C \in \mathbb{R}$, $C > 0$.

Q1 What is the value of C ?

Suppose that Alice have the first qubit and Bob the second of a quantum system in the state $|\psi_\theta\rangle$.

Q2 If Alice measures her qubit in the standard basis (with the measurement $\mathcal{M} = \{|0\rangle\langle 0|, |1\rangle\langle 1|\}$), what are the probabilities of each possible classical outcome A: $Pr(A = 0)$ and $Pr(A = 1)$ and what is the state of the full system after the measurement in each case.

Q3 If Alice and Bob measure both their qubit in the standard basis what is the probability that both classical outcomes are 0 : $Pr(A = 0, B = 0)$? (Where B is the classical outcome of Bob when he measures his qubit in the standard basis).

Q4 What is the state (density matrix) of the first qubit if no measurements are done ?

Q5 Let $|b_0\rangle = \sin(\theta)\,|0\rangle - \cos(\theta)\,|1\rangle$ and $|b_1\rangle = \cos(\theta)\,|0\rangle + \sin(\theta)\,|1\rangle$. Check that the measurement $\mathcal{M}' = \{|b_0\rangle\langle b_0|, |b_1\rangle\langle b_1|\}$ is a valid measurement.

Q6 If Alice measures her qubit with the measurement $\mathcal{M}'$ what are the probabilities of each possible classical outcome A': $Pr(A' = 0)$ and $Pr(A' = 1)$ and what is the state of the full system after the measurement in each case.

Let B' be the classical outcome of Bob when he measures with $\mathcal{M}'$.

Q7 What is the probability $Pr(A' = 0, B' = 0)$ that both classical outcomes are 0 when both measure their qubit with $\mathcal{M}'$?

Q8 If Alice measure in the standard basis and Bob with $\mathcal{M}'$ whats is the probability $Pr(A = 1, B' = 0)$?

Q9 If Bob measure in the standard basis and Alice with $\mathcal{M}'$ whats is the probability $Pr(A' = 0, B = 1)$?

Q10 We want to prove that the full experiment cannot be described by a classical probabilistic model in which A, A', B and B' are random variables that can be defined simultaneously. In any such model let $p_{a,a',b,b'} = Pr(A = a, A' = a', B = b, B' = b')$ be the probability that Alice and Bob outcomes are a, b if they measure in standard basis and a', b' if they measure in the basis $\mathcal{M}'$. Show that it is not possible for any such model to satisfy simulatenously $Pr(A' = 0, B' = 0) > 0$ and the properties of $Q8, Q9$ and $Q3$.