Examination Timetabling Problem

DODM - Final Project

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Problem (1)

Aim of the problem: scheduling a set E of exams during T ordered time-slots for a set S of students.

- n_{e_1,e_2} represents the number of students enrolled in both e_1 and e_2;
- $e_1, e_2 \in E$ are considered *conflicting* if $n_{e_1,e_2} > 0$.

Given $e_1, e_2 \in E$ scheduled at distance of i time-slots, with $1 \leq i \leq 5$, a penalty is assigned and defined as follows:

$$2^{5-i}\frac{n_{e_1,e_2}}{|S|}. \tag{1}$$

Problem (2)

The Examination Timetabling Problem's goal is scheduling the exams into the available time-slots, taking into account the following imposals:

1. Each exam is scheduled exactly once during the examination period;
2. Two conflicting exams cannot be scheduled in the same time-slot;
3. The total penalty resulting from the created timetable is minimized.

Model - Inputs (1)

- Set of the **students** involved in the creation of the examination timetable:

$$S = \{s_1, s_2, \ldots, s_{|S|}\}$$

- Set of the **exams** to be scheduled:

$$E = \{e_1, e_2, \ldots, e_{|E|}\}$$

- Set of the available **time-slots**:

$$T = \{t_1, t_2, \ldots, t_{|T|}\}$$

Model - Inputs (2)

- Enrol matrix A: an $|S| \times |E|$ Boolean matrix, representing the exams to which each student is enrolled:

$$a_{s,e} = \begin{cases} 1 & \text{if student } s \text{ is enrolled in exam } e \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

- Conflict matrix C: an $|E| \times |E|$ Integer matrix.
  - $c_{i,j}$ = number of students enrolled in both the exams;
  - $c_{i,j} > 0 \iff$ exam i and exam j are conflicting.

In the implementation, C is an upper triangular matrix since it is symmetric.

Model - Variables

Aim: assigning exams to time-slots $\rightarrow$ binary variables.

Each variable is associated to a couple (e, t), where e is an exam and t is a time-slot, selected from sets E and T, respectively. The total number of variables is $|E| \times |T|$.

$$x_{e,t} = \begin{cases} 1 & \text{if exam } e \text{ is scheduled in time-slot } t \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

Model - Constraints

1. Each exam must be scheduled exactly once.
   For each exam e, the sum of the variables $x_{e,t}$ for all time-slots t must be equal to one:

$$\sum_{t \in T} x_{e,t} = 1 \qquad \forall e \in E \tag{4}$$

2. Conflicting exams cannot be scheduled in the same time-slot.
   If two exams are conflicting, they must be placed in different time-slots:

$$x_{e_i,t} + x_{e_j,t} \leq 1 \quad \forall t \in T, \ \forall e_i, e_j \in E \ s.t. \ c_{e_i,e_j} > 0. \tag{5}$$

Model - Objective Function

Goal: minimizing the total penalty of the created timetable.

$$obj = \sum_{e_i, e_j \in E} \sum_{t_m, t_n \in T : 1 \leq |t_m - t_n| \leq 5} 2^{5-|t_m - t_n|} \frac{c_{e_i,e_j}}{|S|} x_{e_i,t_m} x_{e_j,t_n} \tag{6}$$

- Only conflicting exams ($c_{e_i,e_j} > 0$), scheduled in time-slots with a distance between 1 and 5, give a contribution to the total penalty;
- Only the penalty of the couples of exams that are effectively present in the solution of the problem ($x_{e_i,t_m}, x_{e_j,t_n} > 0$) is considered.

Equity measure

There are many other different ways of measuring the goodness of a timetable: the equity measures.

Example: the total number of times students have back-to-back exams.

- Back-to-back situation: whenever a student is enrolled in exams scheduled in two consecutive time-slots.
- Lower number of back-to-back situations $\rightarrow$ more equitable timetable for students.

The total number of back-to-backs is given by:

$$b2b = \sum_{t=1}^{|T|-1} \sum_{e_i, e_j \in E} c_{e_i,e_j}(x_{e_i,t}x_{e_j,t+1} + x_{e_j,t}x_{e_i,t+1}), \tag{7}$$

First additional restriction

Change the constraints that impose that no conflicting exams can be scheduled in the same time slot. Instead, impose that at most 3 conflicting pairs can be scheduled in the same time slot.

$$\sum_{e_i, e_j \in E : c_{e_i, e_j} > 0} x_{e_i,t} x_{e_j,t} \leq 3 \quad \forall t \in T. \tag{8}$$

Second additional restriction (1)

At most 3 consecutive time slots can have conflicting exams.
It is necessary to add additional binary variables z_t, defined through a set of constraints:

$$z_t = \begin{cases} 1 & \text{if } t \text{ contains at least a pair of conflicting exams} \\ 0 & \text{otherwise} \end{cases} \tag{9}$$

$$\sum_{e_i, e_j \in E : c_{e_i, e_j} > 0} x_{e_i,t} x_{e_j,t} \leq M z_t \quad \forall t \in T, \tag{10}$$

$$z_t \leq \sum_{e_i, e_j \in E : c_{e_i, e_j} > 0} x_{e_i,t} x_{e_j,t} \quad \forall t \in T, \tag{11}$$

where M is a large enough number.

Second additional restriction (2)

Whenever 3 consecutive time-slots have conflicting exams, two constraints are imposed to limit the presence of conflicting exams in the following and in the previous time-slots:

$$5 - \sum_{i=0}^{2} z_{t+i} \geq 3z_{t+3} \quad \forall t \in \{1, ..., |T| - 3\}, \tag{12}$$

$$5 - \sum_{i=0}^{2} z_{t+i} \geq 3z_{t-1} \quad \forall t \in \{2, ..., |T| - 2\}. \tag{13}$$

Third additional restriction (1)

If two consecutive time slots contain conflicting exams, then no conflicting exam can be scheduled in the next 3 time slots.

Another binary variable is added:

$$y_{e_i,e_j,t} = \begin{cases} 1 & \text{if } e_i \text{ and } e_j \text{ are scheduled in } t \text{ and } t+1 \\ 0 & \text{otherwise} \end{cases} \tag{14}$$

The values of y_t are imposed with this constraint:

$$y_{e_i,e_j,t} = x_{e_i,t}x_{e_j,t+1} + x_{e_j,t}x_{e_i,t+1}, \tag{15}$$

which guarantees that $y_{e_i,e_j,t}$ is 0 if the right-hand side sum is 0, and 1 if the sum is 1.

Third additional restriction (2)

The following expression represents the restriction on the three consecutive time-slots:

$$\sum_{e \in E^*} \sum_{k=t+2}^{t+5} x_{e,k} y_{e_i,e_j,t} = 0, \tag{16}$$

where $E^* = \{e \in E : e \neq e_i \text{ and } e \neq e_j \text{ and } (c_{e,e_i} > 0 \text{ or } c_{e,e_j} > 0)\}$ and $t \in \{1, \ldots, |T| - 5\}, \forall e_i, e_j \in E \ s.t. \ c_{i,j} > 0$.

E^* allows to select all the exams that are different to e_i and e_j, and that are in conflict with at least one exam between e_i and e_j.

Bonus profit

Include a bonus profit each time no conflicting exams are scheduled for 6 consecutive time slots.

$$6 - \sum_{i=0}^{5} z_{t+i} \geq 6b_t \quad \forall t \in \{1, ..., |T| - 5\}, \tag{17}$$

$$b_t \geq 1 - \sum_{i=0}^{5} z_{t+i} \quad \forall t \in \{1, ..., |T| - 5\}, \tag{18}$$

where b_t is a variable which is equal to 1 if the bonus is assigned to time-slot t and 0 otherwise.
After defining it, b_t should be integrated with the objective function.

Results - Basic Model

Results with time-limit of 1000 seconds:

instance	*obj*	benchmark
test	3.375	3.375
instance01	157.357	157.033
instance02	42.527	34.709
instance03	46.338	32.627
instance04	14.223	7.717
instance05	18.945	12.901
instance06	6.535	3.045
instance07	11.492	10.050
instance08	27.597	24.769
instance09	16.429	9.818
instance10	8.883	3.707
instance11	9.657	4.395

Results - Equity Measure

Results obtained from the implementation of the basic model, with the substitution of the penalty objective function with the back-to-back objective function. The time limit was still fixed to 1000 seconds.

instance	$b2b$	obj
test	0	3.375
instance01	3021	163.358
instance02	1315	48.358
instance03	1208	46.925

Results - Additional Restriction

Results for the test and for instance01 achieved in the model composed of the original penalty objective function, plus the additional restrictions already described.

instance	advanced	basic
test	3.375	3.375
instance01	193.817	157.357