Probabilité et Simulation

PolyTech INFO4 (Grenoble) – TD

Last updated: 2025-09-12

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References

- Fundamentals: [1] B. Jourdain, *Probabilités et statistique pour l'ingénieur*. 2018.
- Further reading: [2] P. Billingsley, *Probability and Measure*. John Wiley & Sons, 2012.

Websites

- CM: https://github.com/jonatha-anselmi/INFO4-PS
- TD: https://github.com/davidelegacci/probasim24

Contents

1. Theory recap 2025-09-12 - Probability on finite set

1.1. Probability space

- *Random process*: one random outcome out of finitely many
- *Sample space* $\Omega =$ finite set of all possible outcomes ω
- *Probability* on $\Omega =$ set of weights $\mathbb{P}(\omega) \in \mathbb{R}$ on each $\omega \in \Omega$ such that
  - $\mathbb{P}(\omega) > 0 \forall \omega \in \Omega$
  - $\sum_{\omega \in \Omega} \mathbb{P}(\omega) = 1$

- *Event* $A \subseteq \Omega =$ subset of the sample space

$$\mathbb{P}(A) = \sum_{\omega \in A} \mathbb{P}(\omega) \tag{1}$$

- *Complementary event* $A^c = \Omega / A$ (pronounced "*not* A")
- "A *and* B" $= A \cap B$
- "A *or* B" $= A \cup B$

$$\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cap B) \tag{2}$$

- Probability of complementary event

$$\Omega = A^c \sqcup A \Rightarrow 1 = \mathbb{P}(A^c) + \mathbb{P}(A) \tag{3}$$

- *Indicator function* of event $A \subseteq \Omega$

$$\mathbb{1}_A : \Omega \to \mathbb{R}, \quad \mathbb{1}_A(\omega) = \begin{cases} 1 \text{ if } \omega \in A \\ 0 \text{ if } \omega \notin A \end{cases} \tag{4}$$

- Denoted by $|S|$ the *cardinality* of a set S
- Denoted by S^n the *cartesian product* of S with itself n times

$$S^n = S \times S \times ... \times S = \{(s_1, ..., s_n) : s_i \in S \text{ for all } i = 1, ..., n\} \tag{5}$$

- *Cardinality of cartesian product*

$$|S^n| = |S|^n \tag{6}$$

1.2. Uniform probability

- *Every outcome* $\omega \in \Omega$ *has the same weight*

$$\mathbb{P}(\omega) = \frac{1}{|\Omega|} \tag{7}$$

- *Uniform probability of the event* $A \subseteq \Omega$

$$\mathbb{P}(A) = \sum_{\omega \in A} \mathbb{P}(\omega) = \sum_{\omega \in A} \frac{1}{|\Omega|} = \frac{|A|}{|\Omega|} \tag{8}$$

1.3. Counting

- Number of *permutations* of k elements:
  - Number of ways to *order* k elements
  - **Only order matters**

$$P_k = k! \tag{9}$$

Permutation of 5 elements

1	5	4	3	2

- Number of *dispositions* of k elements out of n $(k \leq n)$:
  - Number of ways to *choose and order* k elements out of n
  - **Order and elements** matter
  - Number of injections : $\{1, ..., k\} \to \{1, ..., n\}$

$$D_{n,k} = \underbrace{n(n-1)...}_{k \text{ times}} = n(n-1)...(n-k+1) = \frac{n!}{(n-k)!} \tag{10}$$

Disposition of 3 elements out of 5

	2	1	3	

- Number of *combinations* of k elements out of n $(k \leq n)$:
  - Number of ways to *choose* k elements out of n
  - **Only elements** matter
  - Number of subsets of cardinality k of a set of cardinality n
  - Number of dispositions modulo number of permutations

$$C_{n,k} = \frac{D_{n,k}}{P_k} = \frac{n!}{k!(n-k)!} = \binom{n}{k} = \text{choose}(n, k) \tag{11}$$

Combination of 3 elements out of 5

		X	X	X

- *Binomial theorem*

$$(x+y)^n = \sum_{j=0}^{n} \binom{n}{j} x^{n-j} y^j \tag{12}$$

Bibliography

[1] B. Jourdain, *Probabilités et statistique pour l'ingénieur*. 2018.

[2] P. Billingsley, *Probability and Measure*. John Wiley & Sons, 2012.