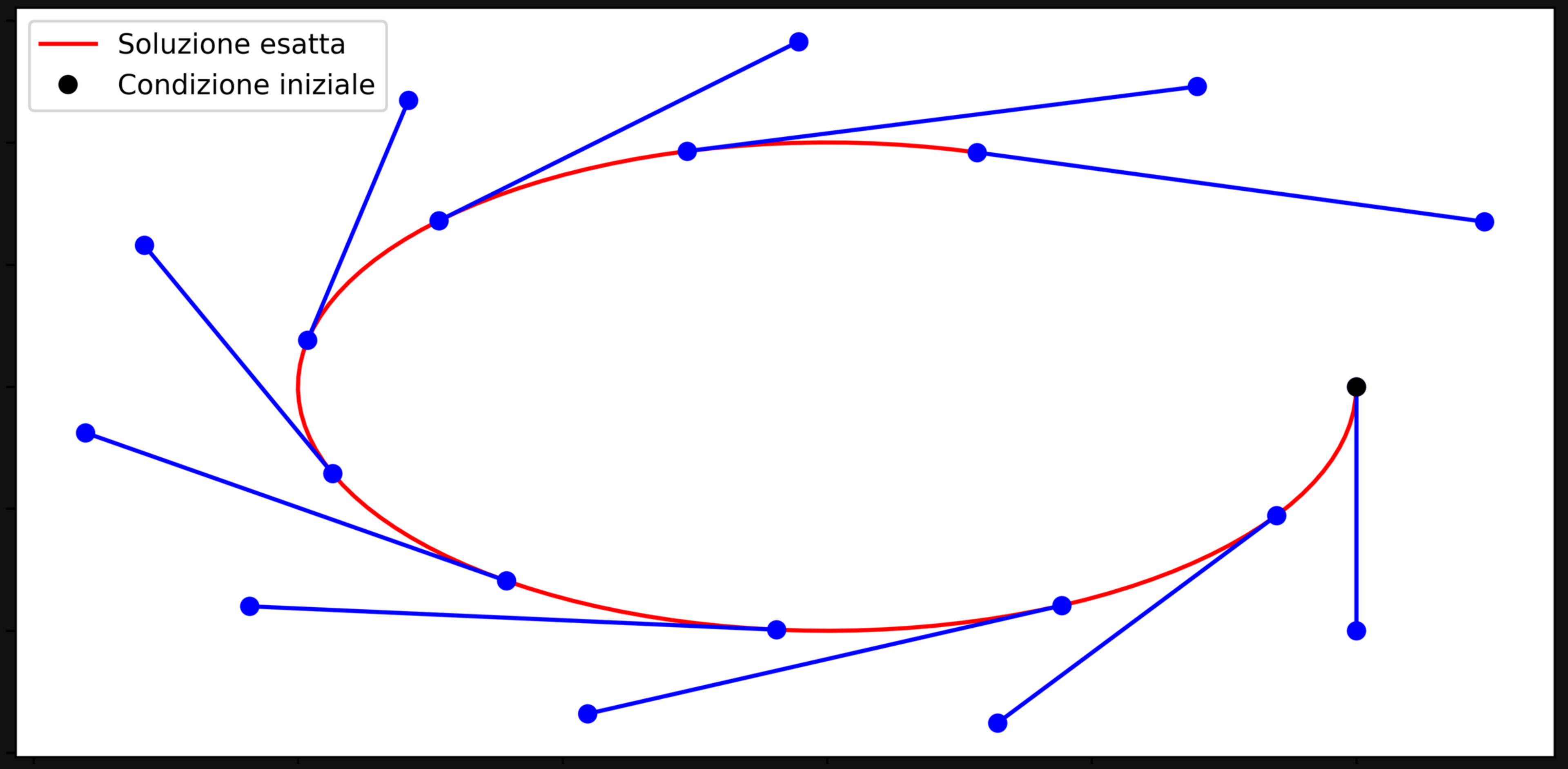
PRIMI ESEMPI DI METODI AD UN PASSO

IDEA



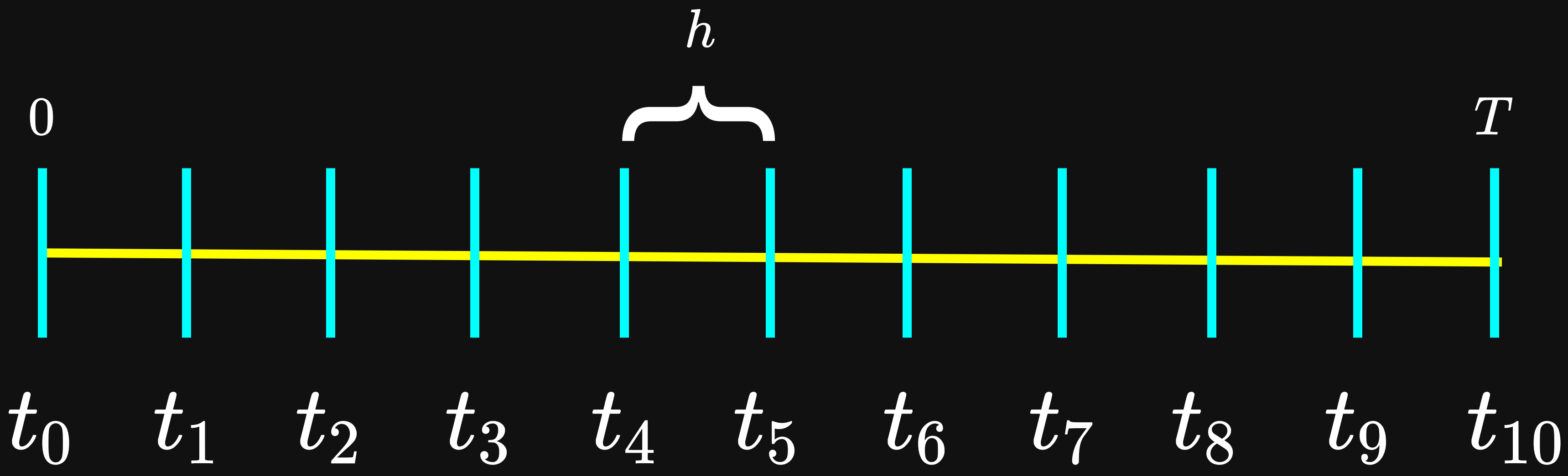
IDEA

Data l'equazione differenziale

$\dot{x}(t) = f(x(t)) \in \mathbb{R}^n$, con $x(0) = x_0$,

voglio approssimare la curva

$[0, T] \in t \mapsto x(t) \in \mathbb{R}^n$ su una discretizzazione dell'intervallo $[0, T]$



IDEA

Useremo 2 strumenti principali:

$$x(t) = x(0) + \int_0^t f(x(s))ds$$

$$x(t+h) = x(t) + hf(x(t)) + \frac{h^2}{2}\frac{d}{dt}f(x(t)) + \mathcal{O}(h^3)$$

EULERO ESPLICITO

$$\dot{y}(t) = f(y(t))$$

$$f(x(t)) = x'(t) = \lim_{h\to 0} \frac{x(t+h)-x(t)}{h}$$

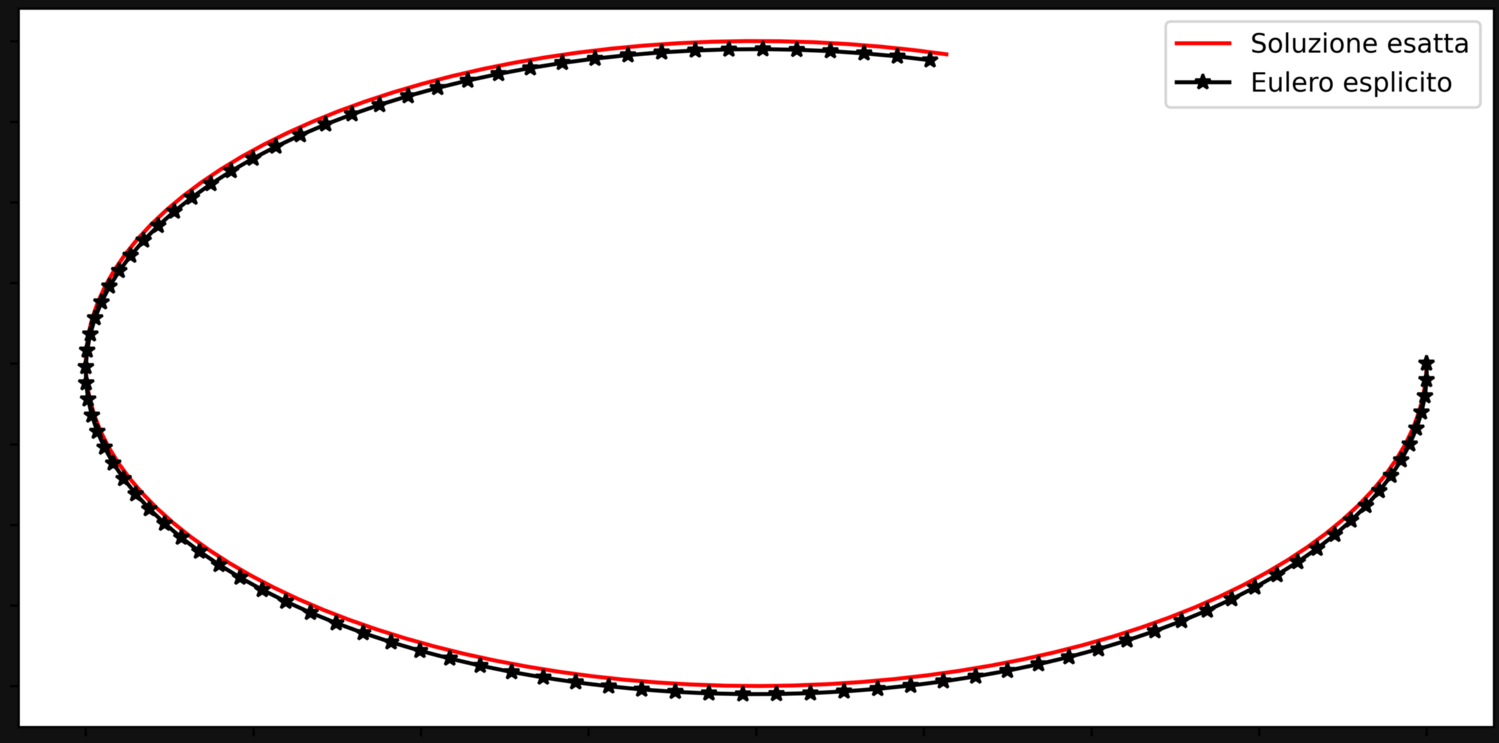
$$\implies f(x_n) \approx \frac{x_{n+1}-x_n}{h}$$

$$y(t+h) \approx y(t) + hf(y(t))$$

EULERO ESPLICITO

$$x_0 = x(0)$$

$$x_{n+1} = x_n + hf(x_n)$$



EULERO IMPLICITO

$$f(x(t)) = x'(t) = \lim_{h \to 0} \frac{x(t) - x(t-h)}{h}$$

$$\implies f(x_n) \approx \frac{x_n - x_{n-1}}{h}$$

$$x_{n+1} = x_n + hf(x_{n+1})$$

EULERO IMPLICITO

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PUNTO MEDIO IMPLICITO

$$f\left(y\left(t+\frac{h}{2}\right)\right) = y'\left(t+\frac{h}{2}\right) \approx \frac{y(t+h)-y(t)}{h}$$

$$y\left(t+\frac{h}{2}\right) \approx \frac{1}{2}(y(t)+y(t+h))$$

$$y_{n+1} = y_n + hf\left(\frac{y_n+y_{n+1}}{2}\right)$$

PUNTO MEDIO IMPLICITO