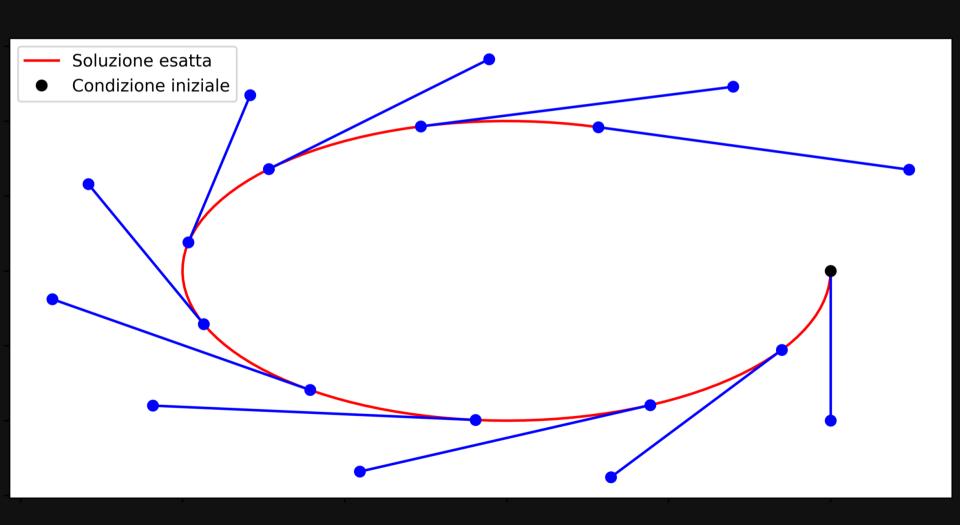
PRIMIESEMPI DI METODI AD UN PASSO

IDEA



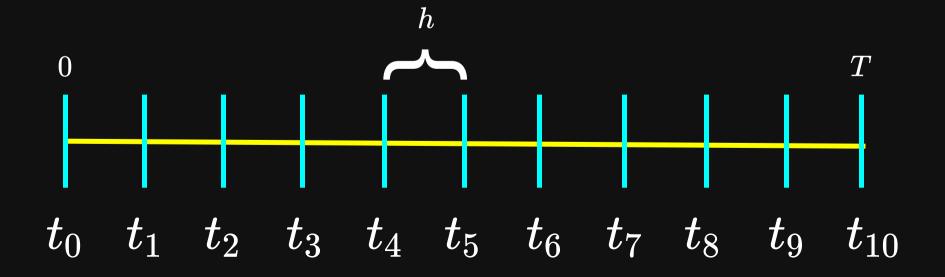
IDEA

Data l'equazione differenziale

$$\dot{x}(t)=f(x(t))\in\mathbb{R}^n$$
, con $x(0)=x_0$,

voglio approssimare la curva

 $[0,T] \in t \mapsto x(t) \in \mathbb{R}^n$ su una discretizzazione dell'intervallo [0,T]



IDEA

Useremo 2 strumenti principali:

$$x(t) = x(0) + \int_0^t f(x(s))ds$$

$$x(t+h)=x(t)+hf(x(t))+rac{h^2}{2}rac{d}{dt}f(x(t))+\mathcal{O}(h^3)$$

EULERO ESPLICITO

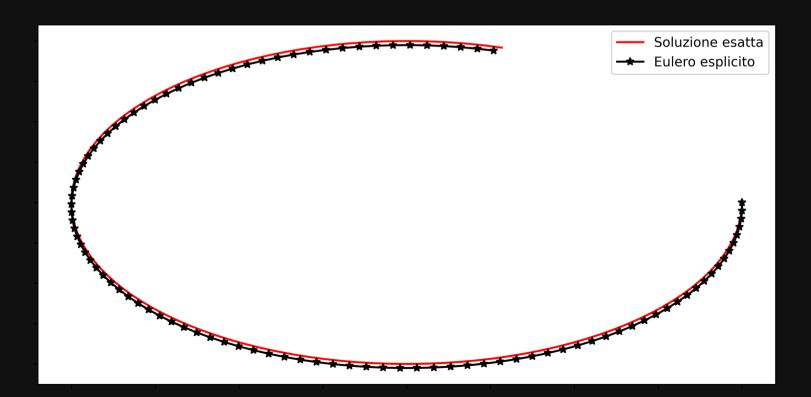
$$\dot{y}(t) = f(y(t))$$

$$egin{aligned} f(x(t)) &= x'(t) = \lim_{h o 0} rac{x(t+h) - x(t)}{h} \ &\Longrightarrow f(x_n) pprox rac{x_{n+1} - x_n}{h} \end{aligned}$$

$$y(t+h) pprox y(t) + hf(y(t))$$

EULERO ESPLICITO

$$egin{aligned} x_0 &= x(0) \ x_{n+1} &= x_n + hf(x_n) \end{aligned}$$



EULERO IMPLICITO

$$egin{aligned} f(x(t)) &= x'(t) = \lim_{h o 0} rac{x(t) - x(t-h)}{h} \ \implies f(x_n) pprox rac{x_n - x_{n-1}}{h} \end{aligned}$$

$$x_{n+1} = x_n + hf(x_{n+1})$$

EULERO IMPLICITO

$$egin{aligned} f(x(t)) &= x'(t) = \lim_{h o 0} rac{x(t) - x(t-h)}{h} \ \implies f(x_n) pprox rac{x_n - x_{n-1}}{h} \end{aligned}$$

$$x_{n+1} = x_n + hf(x_{n+1})$$

PUNTO MEDIO IMPLICITO

$$f\left(y\left(t+rac{h}{2}
ight)
ight)=y'\left(t+rac{h}{2}
ight)pproxrac{y(t+h)-y(t)}{h}$$

$$y\left(t+rac{h}{2}
ight)pproxrac{1}{2}(y(t)+y(t+h))$$

$$y_{n+1} = y_n + hf\left(rac{y_n + y_{n+1}}{2}
ight)$$

PUNTO MEDIO IMPLICITO

