TEORIA DI BASE INTERPOLAZIONE POLINOMIALE

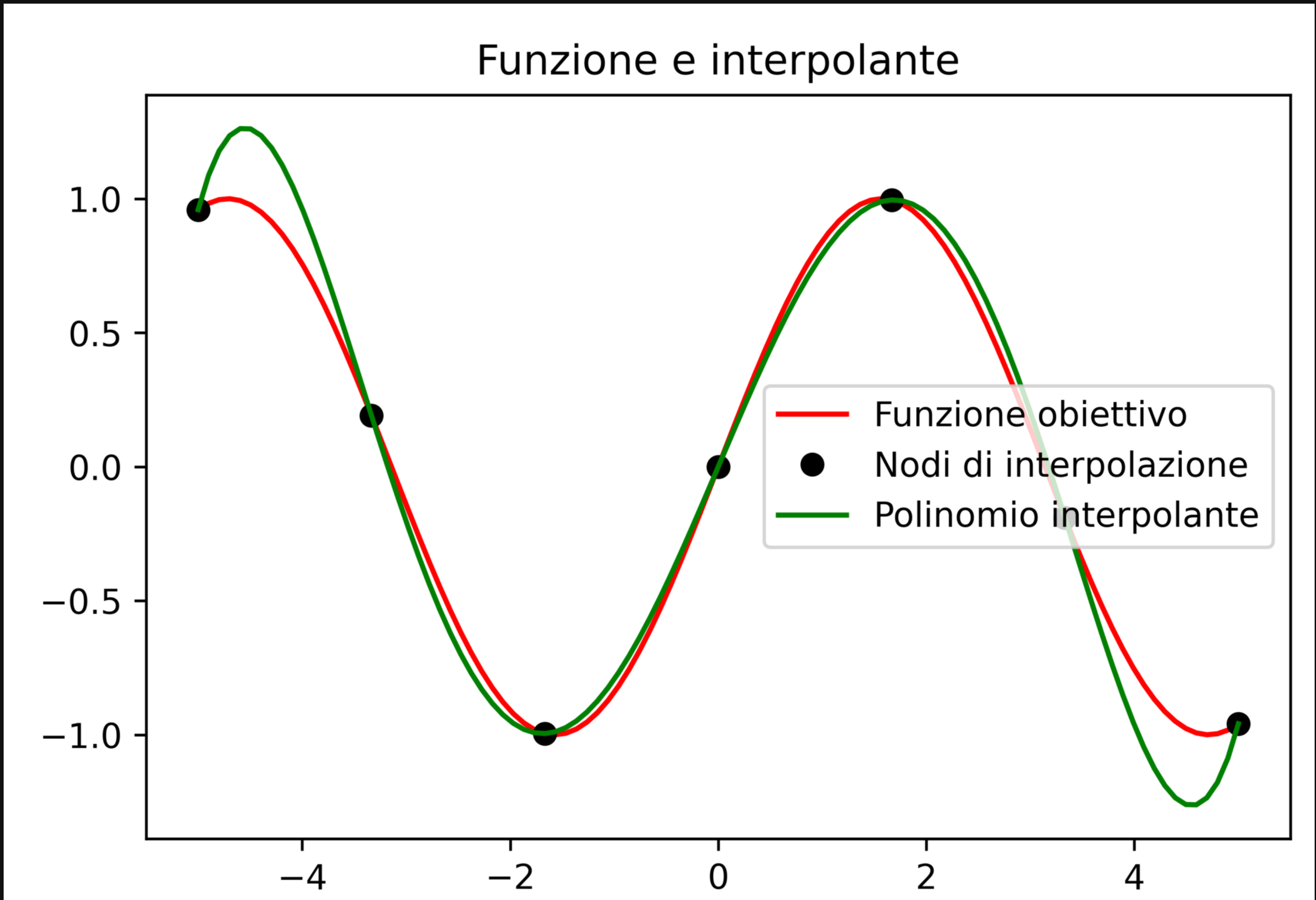
COS'É UN POLINOMIO INTERPOLANTE

$$p(x) = \sum_{i=0}^{k} a_i x^i \in \mathbb{P}^k(\mathbb{R}),$$
$$a = [a_0, ..., a_k] \in \mathbb{R}^{k+1}$$

Dati: $\{(x_i, y_i)\}_{i=0}^{k}$

Trovare: a t.c. $p(x_i) = y_i, \;\; i = 0, ..., k+1$

ESEMPIO


Funzione e interpolante
1.0
0.5
0.0
−0.5
−1.0
−4
−2
0
2
4
Funzione obiettivo
Nodi di interpolazione
Polinomio interpolante

ESEMPIO RETTA

$$\{(0,1),(1,-1)\}$$

$$p(x) = a_0 + a_1 x$$

$$p(0) = a_0 = 1$$
$$p(1) = a_0 + a_1 = -1$$

$$\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \implies p(x) = 1 - 2x$$

ESEMPIO RETTA

$$\{(0,1),(1,-1)\}$$

$$p(x) = 1 - 2x$$
$$= (1-x) - x = 1 \cdot l_0(x) - 1 \cdot l_1(x)$$

$$l_0(x) = 1 - x,\ \ l_1(x) = -x$$

GENERALIZZAZIONE

$$\{Q_0, \ldots, Q_k\} = \{(x_0, y_0), \ldots, (x_k, y_k)\}$$

POLINOMI DI LAGRANGE

$$l_i(x) = \frac{(x-x_0)\cdot(x-x_1)\cdot\ldots\cdot(x-x_{i-1})\cdot(x-x_{i+1})\cdot\ldots\cdot(x-x_k)}{(x_i-x_0)\cdot(x-x_1)\cdot\ldots\cdot(x_i-x_{i-1})\cdot(x_i-x_{i+1})\cdot\ldots\cdot(x_i-x_k)}$$

$$= \frac{\Pi_{j\neq i}(x-x_j)}{\Pi_{j\neq i}(x_i-x_j)} \in \mathbb{P}^k(\mathbb{R})$$

$$l_i(x_j) = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases} = \delta_{ij}$$

GENERALIZZAZIONE

$$\{Q_0, ..., Q_k\} = \{(x_0, y_0), ..., (x_k, y_k)\}$$

$$l_i(x) = \frac{\Pi_{j \neq i}(x - x_j)}{\Pi_{j \neq i}(x_i - x_j)} \qquad l_i(x_j) = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases}$$

POLINOMIO INTERPOLANTE DI LAGRANGE

$$p(x) = y_0 l_0(x) + ... + y_k l_k(x) \in \mathbb{P}^k(\mathbb{R})$$

$$p(x_i) = y_i$$

POLINOMIO APPROSSIMANTE

$$\{(x_i, f(x_i))\}_{i=0}^{k}, \;\; f : \mathbb{R} \to \mathbb{R}$$

$$p(x) = \sum_{i=1}^{n} y_i l_i(x)$$

$$R(x) = f(x) - p(x) = l(x)\frac{f^{k+1}(\xi)}{(k+1)!},$$

$$\xi \in (x_0, x_k), \quad l(x) = \Pi_{i=0}^{k}(x - x_i)$$