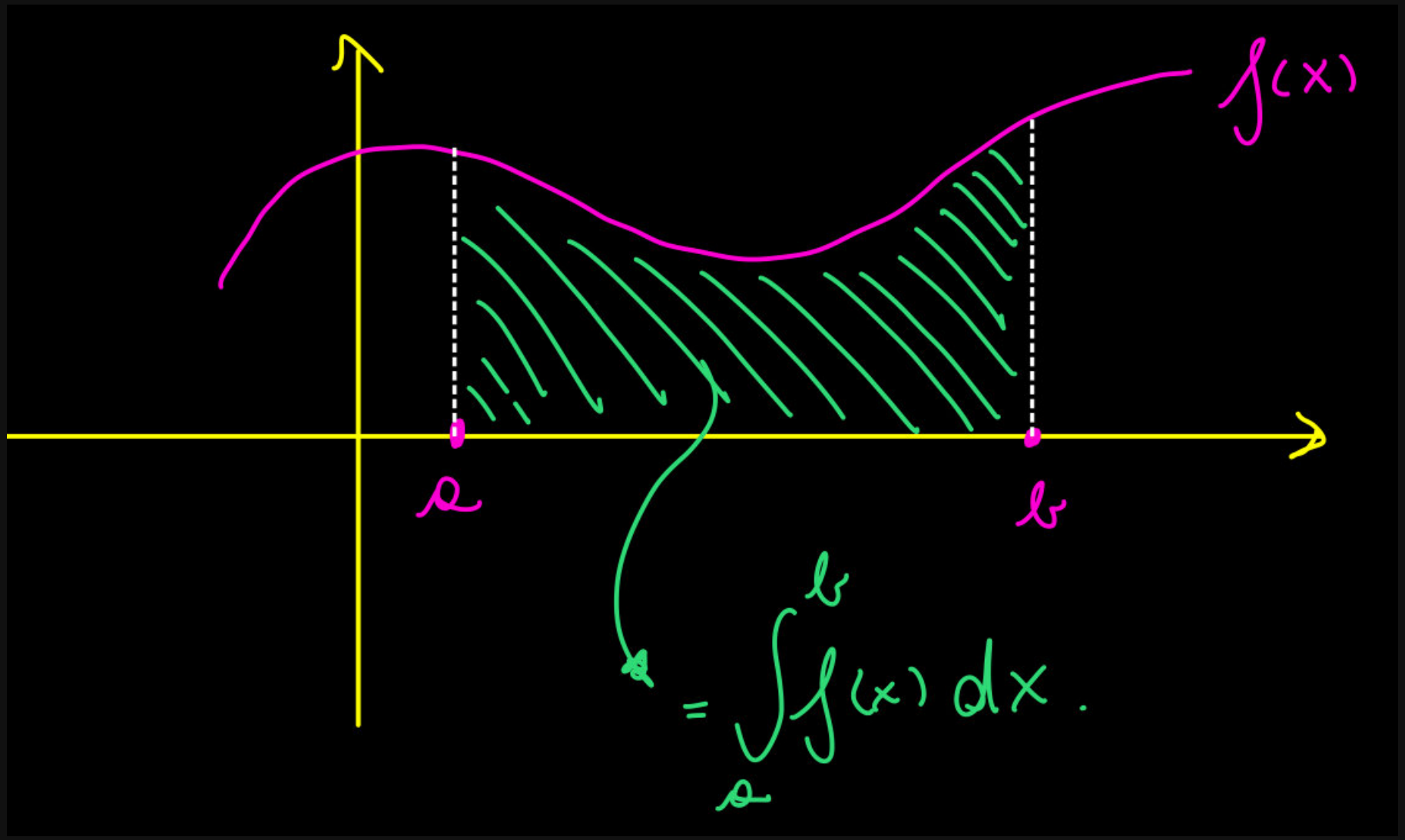
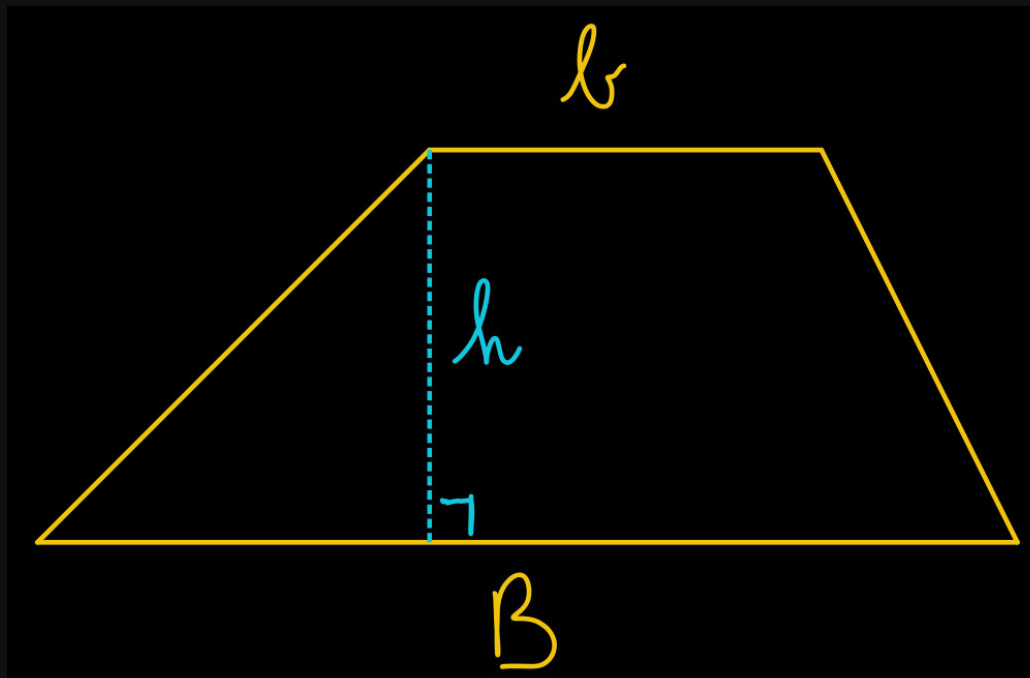
BASI DI QUADRATURA NUMERICA

INTUIZIONE



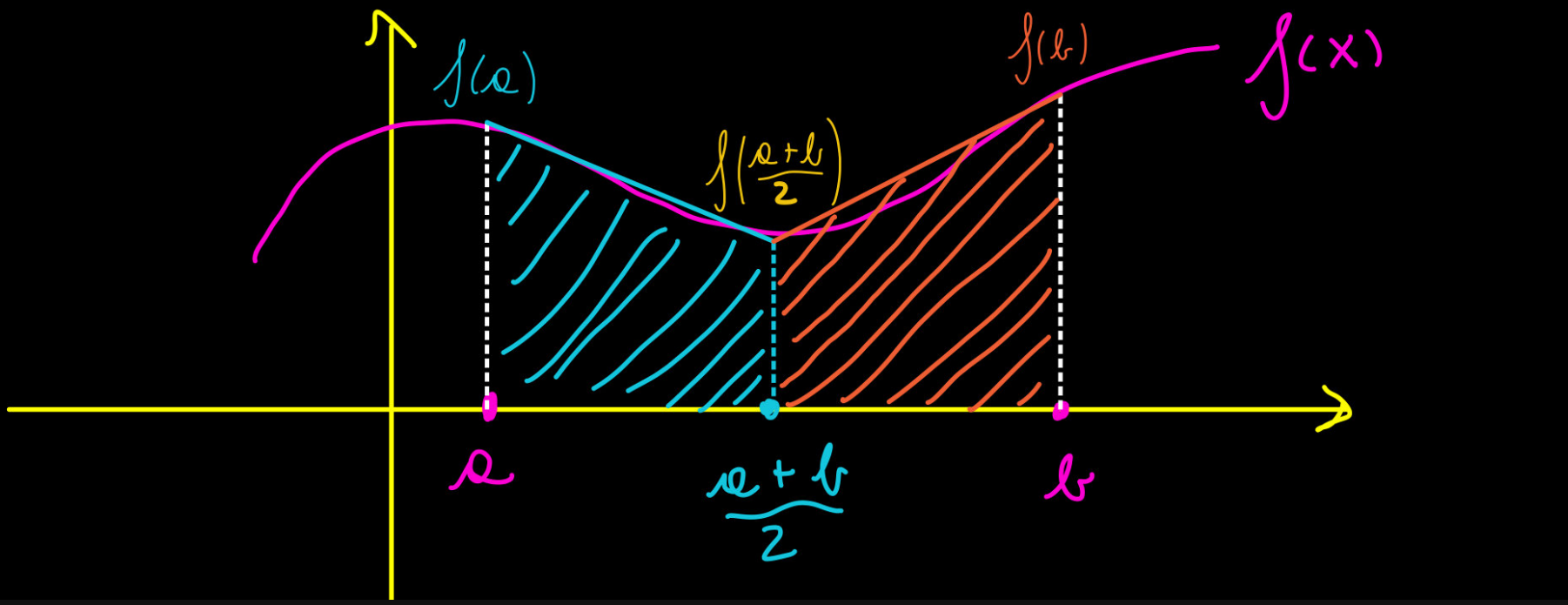
INTUIZIONE



$$A = \frac{(B + b)\,h}{2}$$

INTUIZIONE

$$\int_a^b f(x)\,dx \approx \left[f(a) + f\left(\frac{a+b}{2}\right)\right] \cdot \frac{b-a}{4} + \left[f(b) + f\left(\frac{a+b}{2}\right)\right] \cdot \frac{b-a}{4}$$



CAMBIO DI VARIABILE

$$\int_a^b f(x)dx$$

$$[-1,1] \ni t \mapsto \frac{a+b}{2} + \frac{b-a}{2}t = x(t) \in [a,b]$$

$$dx = \frac{b-a}{2}dt$$

$$\int_a^b f(x)dx = \frac{b-a}{2}\int_{-1}^1 f(t)dt$$

QUADRATURA DI GAUSS

$$\int_{-1}^{1} 1\,dx = 2 = 2 \cdot f(0)$$

$$\int_{-1}^{1} x\,dx = 0 = 1 \cdot f\left(\frac{-\sqrt{3}}{3}\right) + 1 \cdot f\left(\frac{\sqrt{3}}{3}\right)$$

$$\int_{0}^{1} x^2\,dx = \frac{2}{3}$$

$$= \frac{8}{9} f(0) + \frac{5}{9} f\left(\frac{\sqrt{3}}{\sqrt{5}}\right) + \frac{5}{9} f\left(\frac{-\sqrt{3}}{\sqrt{5}}\right)$$

QUADRATURA DI GAUSS

$$\int_a^b f(x)dx \approx \sum_{i=1}^{N_q} \omega_i f(z_i)$$

Una regola di quadratura Gaussiana a n punti, è una regola di quadratura che fornisce un risultato esatto per polinomi di grado $\leq 2n-1$ mediante un'opportuna scelta dei nodi z_i e dei pesi ω_i.

NODI DI GAUSS-LEGENDRE

Number of points, n	Points, x_i		Weights, w_i	
1	0		2	
2	$\pm\frac{1}{\sqrt{3}}$	±0.57735...	1	
3	0		$\frac{8}{9}$	0.888889...
	$\pm\sqrt{\frac{3}{5}}$	±0.774597...	$\frac{5}{9}$	0.555556...
4	$\pm\sqrt{\frac{3}{7}-\frac{2}{7}\sqrt{\frac{6}{5}}}$	±0.339981...	$\frac{18+\sqrt{30}}{36}$	0.652145...
	$\pm\sqrt{\frac{3}{7}+\frac{2}{7}\sqrt{\frac{6}{5}}}$	±0.861136...	$\frac{18-\sqrt{30}}{36}$	0.347855...
5	0		$\frac{128}{225}$	0.568889...
	$\pm\frac{1}{3}\sqrt{5-2\sqrt{\frac{10}{7}}}$	±0.538469...	$\frac{322+13\sqrt{70}}{900}$	0.478629...
	$\pm\frac{1}{3}\sqrt{5+2\sqrt{\frac{10}{7}}}$	±0.90618...	$\frac{322-13\sqrt{70}}{900}$	0.236927...