CONDITIONAL PERMUTATION

When conditions are applied to the way things arranged, it then happen that permutation is said to be conditional.

Example

Find the number of ways; the letters of the word ABUJA can be permutated

(a) If the two As must always be apart

(b) If the two As must always be together

Solution

(a) Omitting out the two As, the letter BUJ can be arranged in 3! ways with each of these ways, the first A can be inserted in any one of 4 places

B U J

When this is done, the second A can be inserted in 3 ways, in which it will not come next to the first A but also the two As cannot be distinguished.

Therefore the number of arrangement is $\frac{3! \times 4 \times 3}{2!} = 36$

(b) Taking the two As one object the letter of the word can be arranged in 3! Ways

*B * U * J *

As can occupy any of the asterisk positions in 4 ways. Hence the numbers of arrangement is 3! × 4 = 24 ways

Or

If the 2As must always be together, we take them as one then we have (AA) BUJ. The number of permutation of these letters is 4! = 24 ways

Example

1. Find the number of ways, the letters of the word ABAKALIKI can be arranged

(a) If the three As must always be together

(b) If the three As must always be apart

Solution

(a) Taking the three As one object, the letter of the B K L I K I can be arranged in $\frac{6!}{2!2!}$

* B * K * L * I * K * I *

AAA can occupy any of the asterisk position in 7 ways. Hence the number of arrangements is $\frac{61}{2!2!} \times 7 = 1260$

(b) Omitting out the three As, the letter BKLIKI can be arranged in $\frac{6!}{2!2!}$

With each of these ways, the first A can be inserted in any one 7 places

B K L I L K I

The second A can be inserted in 6 ways and the third A can be inserted in 5 ways, in which they will not come next to each other, but also the three As cannot be distinguished.

Therefore the number of arrangement is $\frac{6!}{2!2!} \times \frac{7 \times 6 \times 5}{3!} = 6300$

Example : How many number greater than 200 can be formed using the digit 1,2,3,4,5, if no digit may be repeated.

Solution

The numbers formed using all 5 digits will be greater than 200.

The 5 – digits numbers can be arranged in ${}^{5}P_{5} = 5!$ Ways

Also if we form a four digit number beginning with 5,4,3,2,1, i.e. the first digit can be 5 ways and remaining 3 – digit chosenfrom 4 can be arranged in ${}^{4}p_{3}$

The arrangement for 4 – digits numbers = $5\times {}^{4}P_{3}$ ways next we can form 3 – digits numbers greater than 200 using 5,4,3,2, to begin the number to be formed.

i.e. the first digit can be chosen 4 ways and the remaining 2 – digit chosen from 4 can be arranged ${}^{4}P_{2}$

Total arrangement for the 3 – digit number = $4 \times {}^{4}P_{2}$

Hence the total arrangement = $5! + 5 \times {}^{4}P_{3} + 4 \times {}^{4}P_{2} = 288$

Example

How many four and five digit numbers can be formed using the digit 2, 3, 4, 5 6 (no repetition)? How many will be greater than300? How many will be even numbers.

Solution

(a) Numbers of 4 – digits = ${}^{5}P_{4} = 5\times4\times3\times2 = 120$

Numbers of 5 – digits = ${}^{5}P_{5} = 5! = 5\times4\times3\times2\times1 = 120$

Number of 4 digits and 5 – digits number = 120 + 120 = 240

(b) Number of 3 – digit greater than 300 are those that begin with 3,4,5,6 = $4 \times {}^4P_2$ = 48

Numbers that will be greater than 300 are number of 3 – digits, 4 – digits and 5 – digits

= 120 + 120+ 48 = 288

(c) If all the five digits are used and the number is to be even, the last (right hand) digits must be either 2,4,6 which gives 3 choices, when this is done, the remaining 4 digits from 4 can be arranged ${}^4P_4$

Total arrangement for 5 – digit will be $3 \times {}^4P_4$

If four digits is formed, and the number is even, the last (right hand) digits must be either 2,4,6 which also gives 3 choices, the remaining 3 – digits from ${}^4P_3$

Total arrangement for 4 – digit will be $3 \times {}^4P_3$

If three digits are formed and the number will be even. The last (right hand) digits must be either 2,4,6 which will give 3 choices, the remaining 2 digits from 4.

Total arrangement for 3 – digit = $3 \times {}^4P_2$

Total number possible that would be even = $3\times {}^4P_4 + 3 \times {}^4P_3 + 3 \times {}^4P_4$ = 72 + 72 + 36

= 180

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