RWTH Aachen
Lehrstuhl für Informatik 9
Prof. Dr. T. Seidl

December 14th, 2015

Exercise 8 for the lecture
Data Mining Algorithms
WS 2015/2016

Hand in your solutions on December 21st <u>before</u> the lecture. The tutorial for this exercise will be held on January 8th.

Exercise 8.1) Lagrange multiplier method — voluntary (not relevant for exam)

Consider the following problem: In order to minimize the resource consumption, a car manufacturer considers how to optimize the side lengths of the (cuboid-formed) gas tank such that its volume is maximal for a given surface area. Find such dimensions (width, height, depth) of an optimal gas tank, such that the total surface area is 2400 cm^2.

a) Define the optimization criterion and the constrains for solving the problem
b) Solve the defined optimization problem using Lagrangian multipliers method

Exercise 8.2) Support Vector Machines and Kernels — 7 points

Consider the following training data:

x_1= (2,3), x_2= (3,2), x_3= (4,4), x_4= (4,2) → belong to class A, $y_A = \{-1\}$
x_5= (6,4), x_6= (6,3), x_7= (7,2), x_8= (8,3) → belong to class B, $y_B = \{+1\}$

a) Plot the points and specify which of the points should be identified as support vectors.
b) Draw the maximum margin line which separates the classes (you don't have to do any computations). Write down the normalized *normal vector* $w' \in \mathbb{R}^2$ of the separating line and the *constant term* $b \in \mathbb{R}$.
c) Consider the decision rule: $h_1(x) = \langle w', x \rangle + b$. Explain how this equation classifies points on either side of a line. Determine the class for the points x_9= (3, 4) and x_{10}= (7, 4).
d) Assume that after solving the dual optimization problem you obtain the support vectors you have specified in (a). The weights $\alpha_i$are zero for all x_i which are not support vectors. The m support vectors have equal weights α_i such that $\sum_{i=1}^{m} \alpha_i = 1$.
    a. Determine the value of the parameter vector w'
    b. Assume that the constant term b is equal to the constant term from b), determine the class for the points x_9= (3,4) and x_{10}= (7,4).
e) Consider the polynomial kernel function:
$$K: \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}, (x, y) \mapsto (\, x^T y + \gamma)^p, \text{with } p = 2, \gamma = 1.$$
Furthermore let
$$\varphi: \mathbb{R}^2 \mapsto \mathbb{R}^6, x \mapsto \left(1, \sqrt{2}\, x_1, \sqrt{2}\, x_2, x_1^2\,, x_2^2\,, \sqrt{2}\, x_1\, x_2\right).$$
Show that $K(x, y) = \langle \varphi(x), \varphi(y) \rangle$.

Exercise 8.3) Decision Tree Learning **5 = 2,5+2,5 points**

If you are still looking for Christmas presents, the decision tree generated based on the following training data might help a little: delete more columns!!

gift ID	gender of the recipient	useful	beautiful	self-made	eatable	liked by the recipient
1	male	yes	yes	no	yes	yes
2	male	yes	yes	yes	no	yes
3	male	yes	no	yes	no	yes
4	male	yes	yes	no	no	yes
5	male	no	yes	yes	yes	yes
6	male	no	little	no	yes	yes
7	male	no	no	yes	no	no
8	male	no	yes	no	no	no
9	female	no	yes	no	yes	yes
10	female	no	yes	no	no	yes
11	female	no	little	yes	no	yes
12	female	no	no	no	no	yes
13	female	no	little	yes	no	yes
14	female	no	little	yes	no	yes
15	female	no	little	no	yes	no
16	female	no	little	no	no	no

a) Generate the decision tree by using the Gini-index.
b) Generate the decision tree by using the Information Gain.