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FLYING IN A SUPERFLUID: STARTING FLOW PAST AN AIRFOIL

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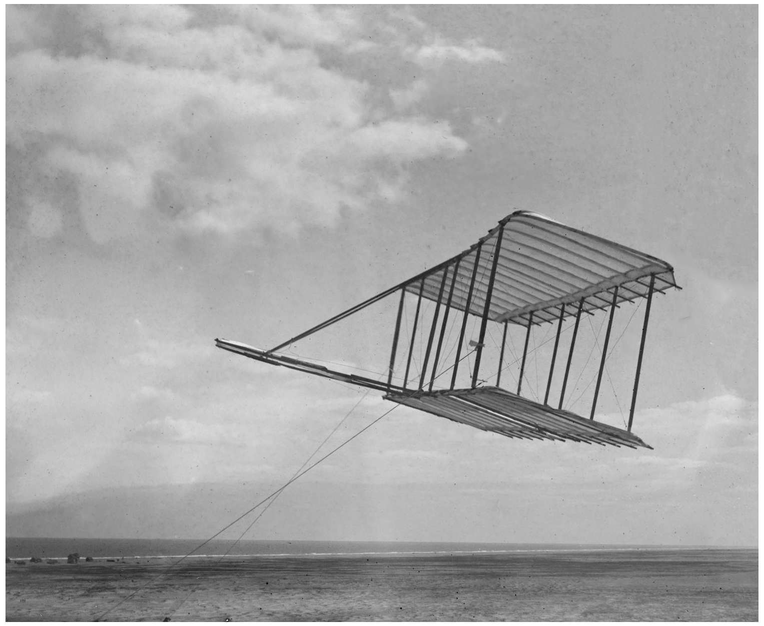
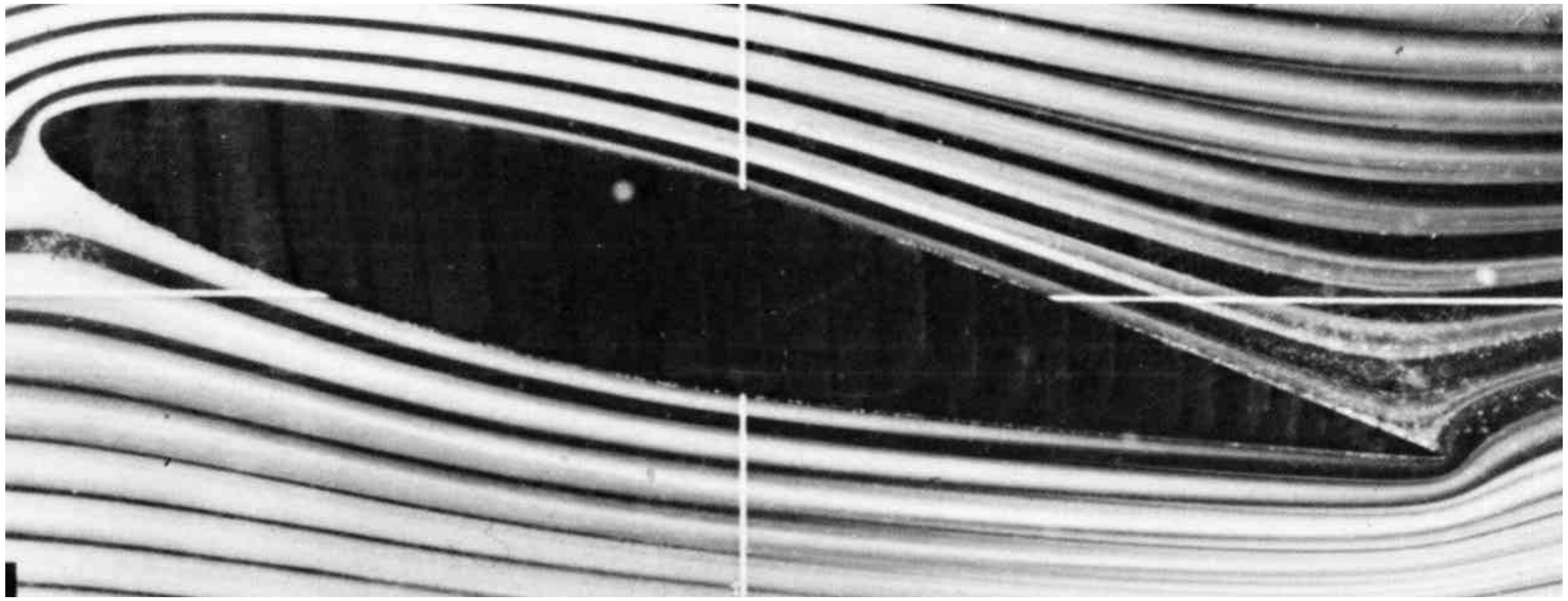
CLASSICAL THEORY OF FLIGHT (2D)



By Wright brothers - Library of Congress, Public Domain [Wikipedia]

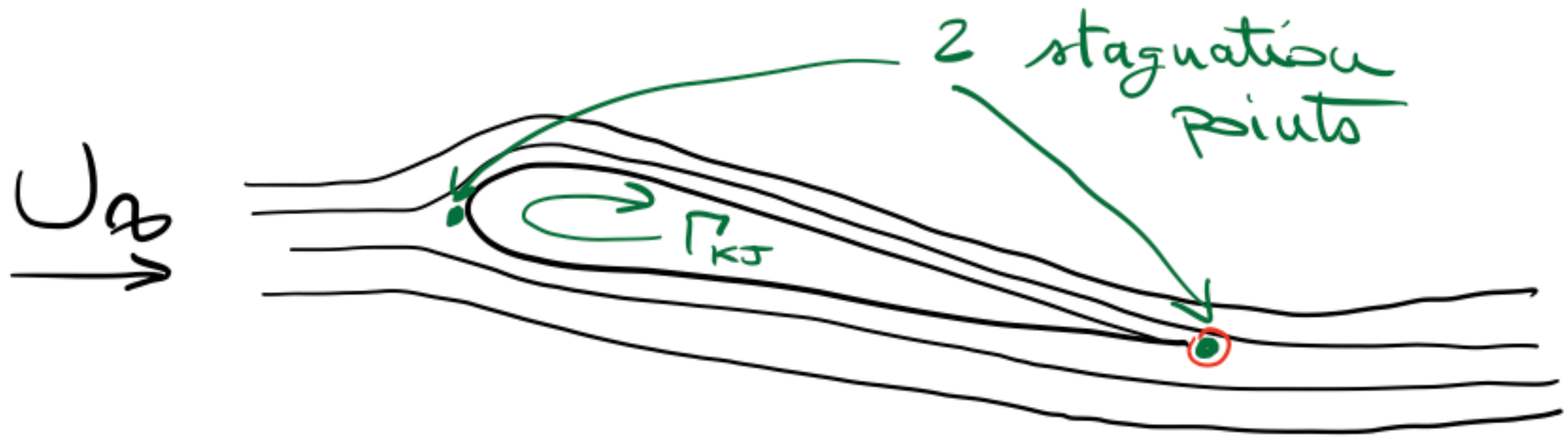


[M. Van Dyke, An Album of fluid Motion, 1982]

- Ideal theory: stationary flow, prediction of lift
- Viscous effects: explain generation of lift and drag effects

[D.J. Achenson, Elementary Fluid Dynamics, Oxford University Press, 1990]

Ideal theory (inviscid and incompressible): full family of stationary flows depending on $(\alpha, U_\infty, L, \Gamma)$



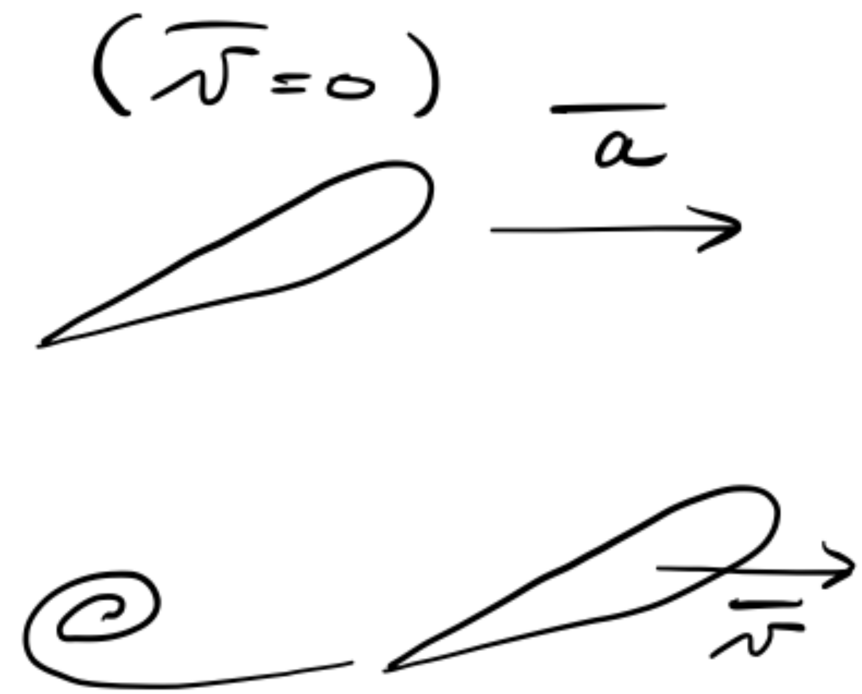
- Kutta—Joukowski (KJ) condition

$$\boxed{\Gamma_{KJ} = -\pi U_\infty L \sin\alpha}$$

- lift per unit of wingspan of $-\rho U_\infty \Gamma_{KJ}$

Viscous effects

- viscous boundary layer around the airfoil
- viscosity allows the generation of the KJ circulation around an accelerated airfoil
- drag forces arise (form drag and skin drag)



FLYING IN A SUPERFLUID

- Can an accelerated airfoil acquire circulation?
- If so, what are the admissible values of the lift for a given airfoil, angle of attack and terminal velocity?
- Does the airfoil experience any drag?

THE GROSS-PITAEVSKII MODEL

$$\imath\hbar\frac{\partial\psi}{\partial t}+\frac{\hbar^2}{2m}\nabla^2\psi-g|\psi|^2\psi-V_{ext}\psi=0$$

Madelung transformation $\psi=\sqrt{\rho}\exp(\imath\phi)$ $\boxed{\mathbf{u}=\hbar/m\nabla\phi\,,\ \rho=m|\psi|^2}$

$$\frac{\partial\rho}{\partial t}+\nabla\cdot(\rho\mathbf{u})=0$$

$$\frac{\partial\mathbf{u}}{\partial t}+(\mathbf{u}\cdot\nabla)\mathbf{u}=\nabla\left[-\frac{g}{m}\rho+\frac{1}{m}V+\frac{\hbar^2}{2m^2}\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}\right]$$

given bulk density ρ_0

$\xi=\sqrt{\hbar^2/(2mg\rho_0)}$

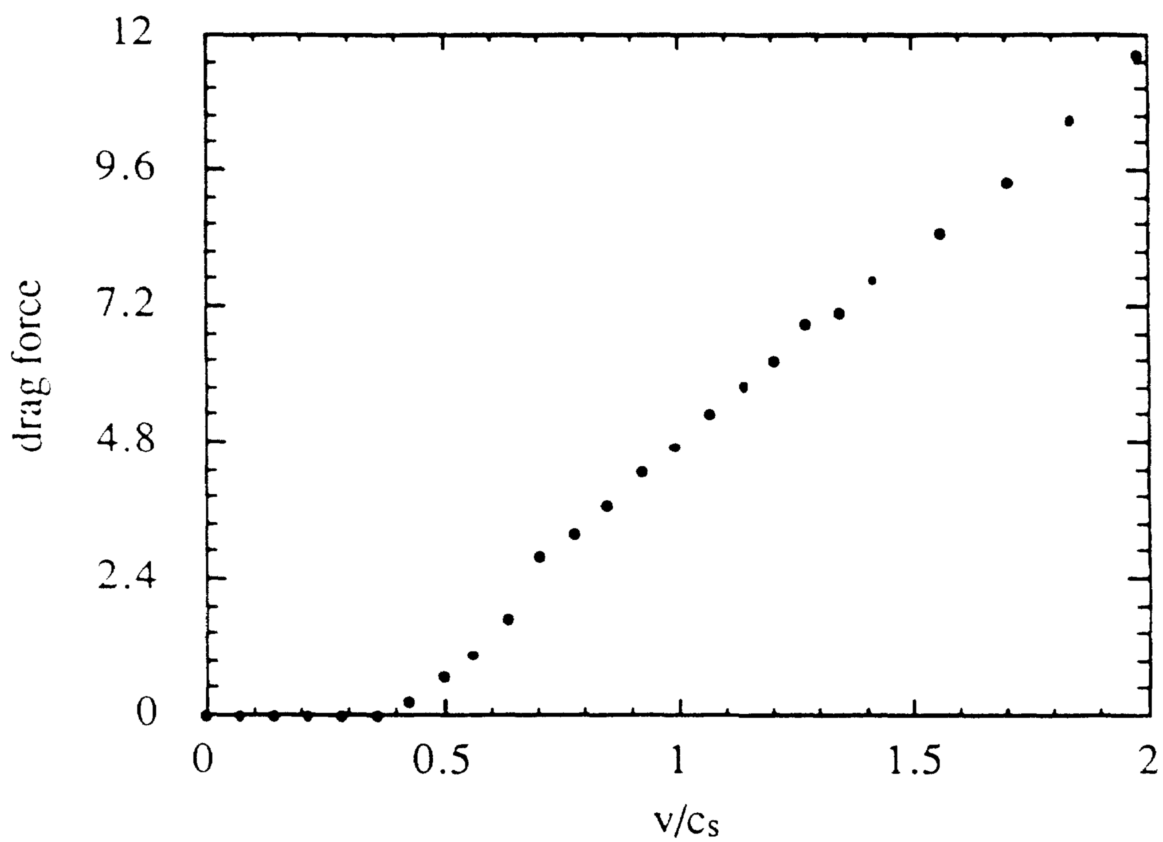
$c=\sqrt{g\rho_0/m}$

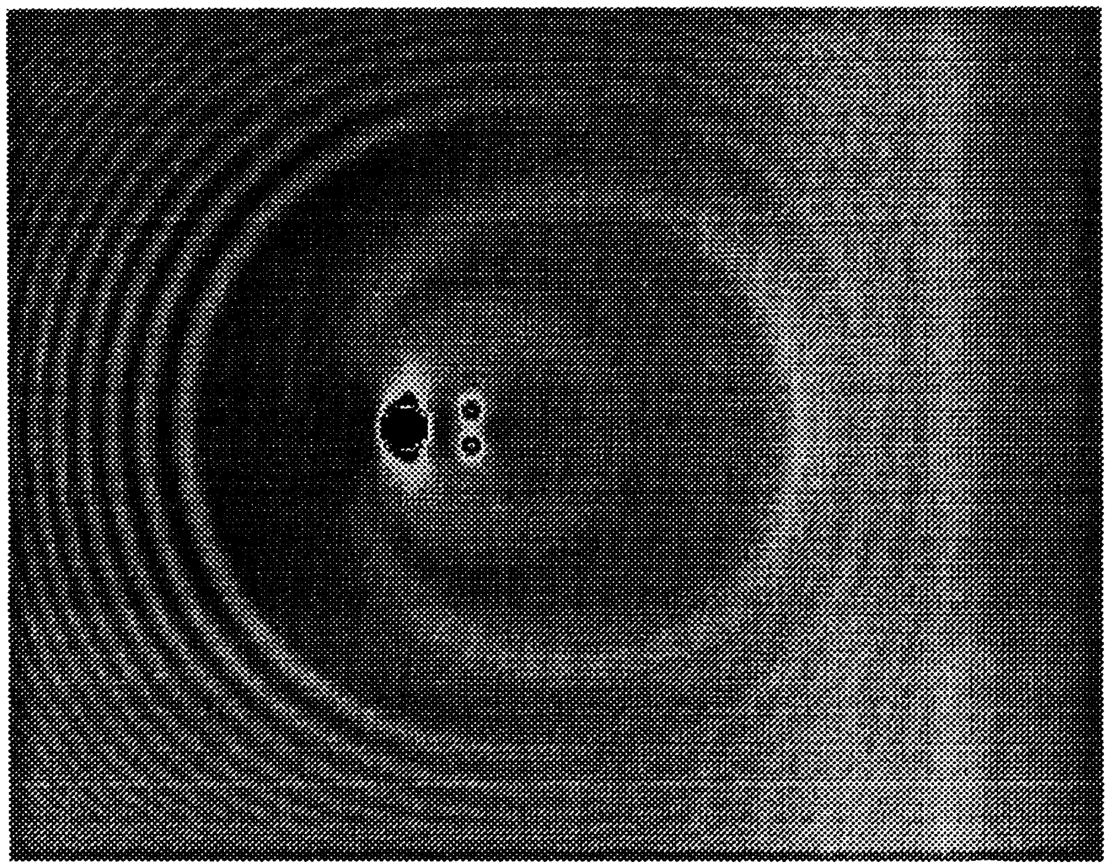
- inviscid, compressible, and irrotational fluid
- vortices are topological defects of quantum of circulation $\kappa=h/m$
- airfoil is modelled using a moving external potential V_{ext} whose intensity is much larger than the chemical potential $\mu=g\rho_0$

EXTERNAL POTENTIAL CYLINDER MOVING IN GP

An external potential moving in a superfluid may cause the flow to break the Landau's critical velocity (sound speed in GP), and generate excitations (travelling waves, solitons, vortices) and cause dissipation

<u>2d cylinder</u>



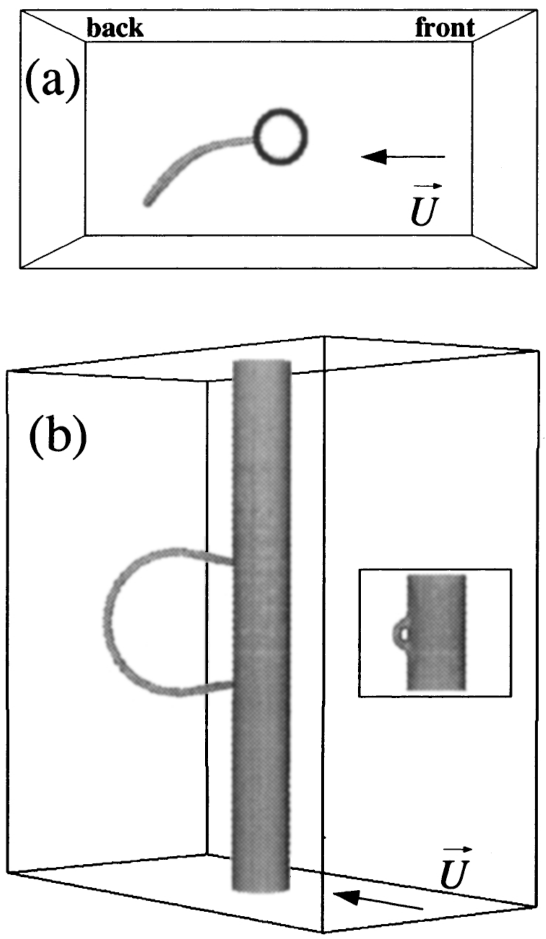


[Frisch et al., PRL 69, 1644 (1992)]

EXTERNAL POTENTIAL MOVING IN GP

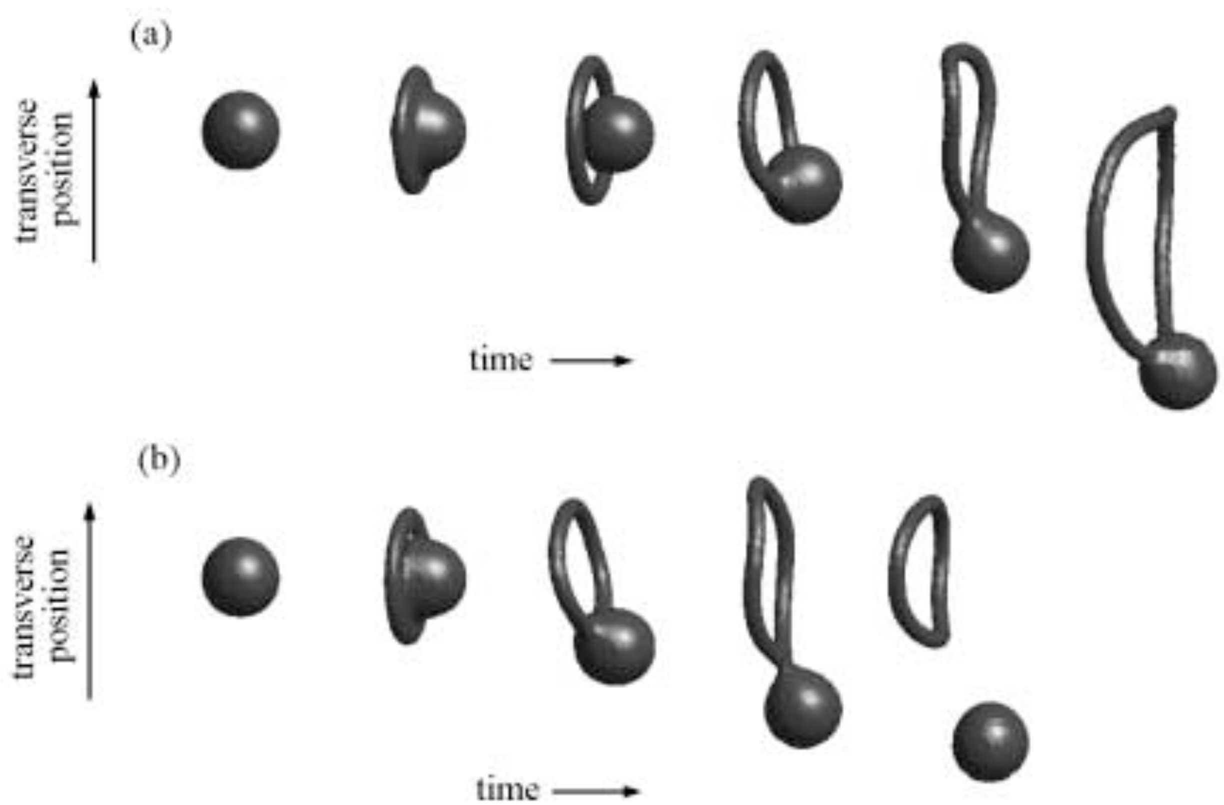
An external potential moving in a superfluid may cause the flow to break the Landau's critical velocity (sound speed in GP), and generate excitations (travelling waves, solitons, vortices) and cause dissipation

<u>3d cylinder</u>



[Nore et al., PRL 84, 2191 (2000)]

<u>3d sphere</u>

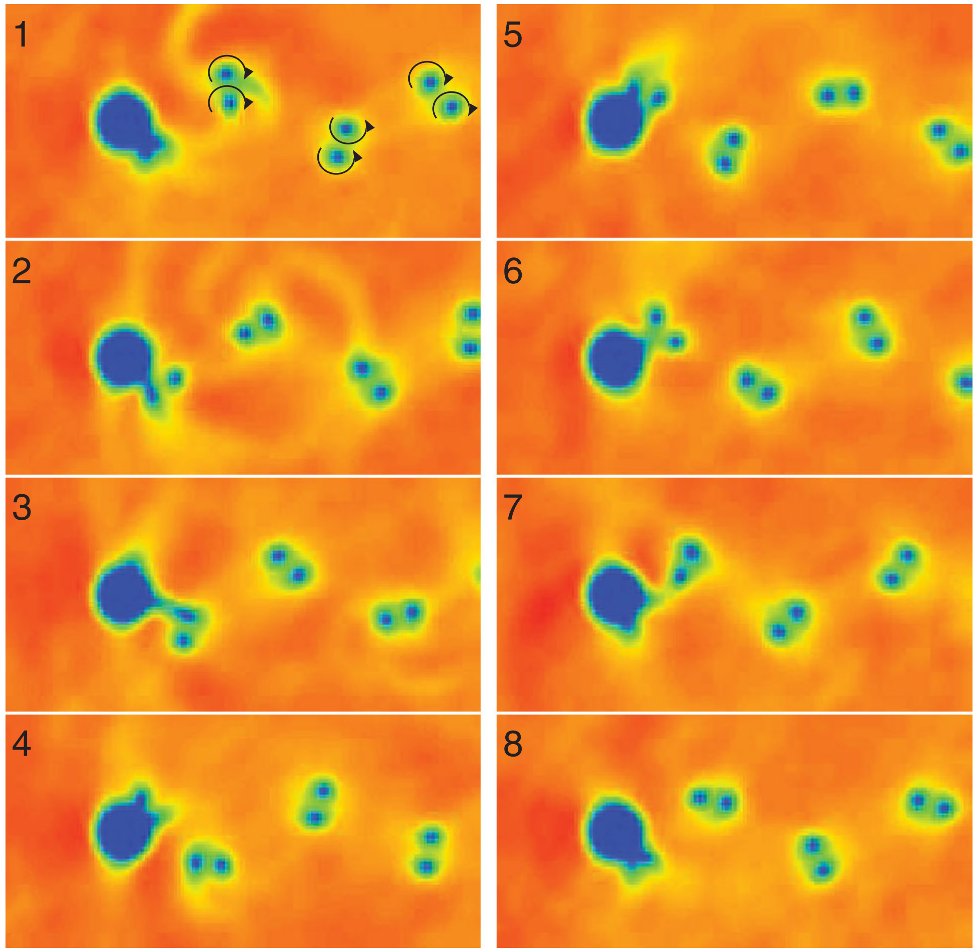


[Winiecki & Adams, Europhys. Lett. 52, 257-263 (2000)]

EXTERNAL POTENTIAL MOVING IN GP

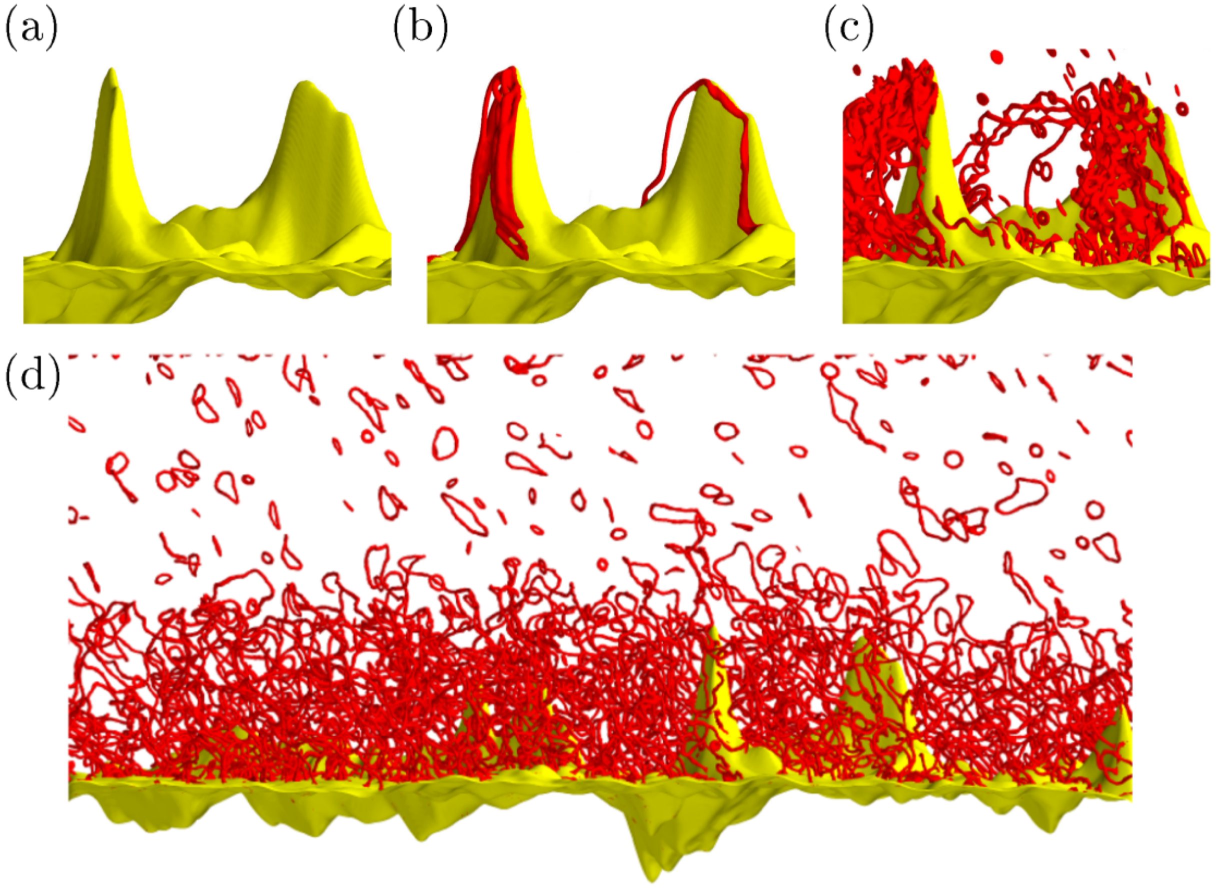
Some dynamical effects are very similar to the classical viscous ones

<u>Von Karman vortex sheet</u>



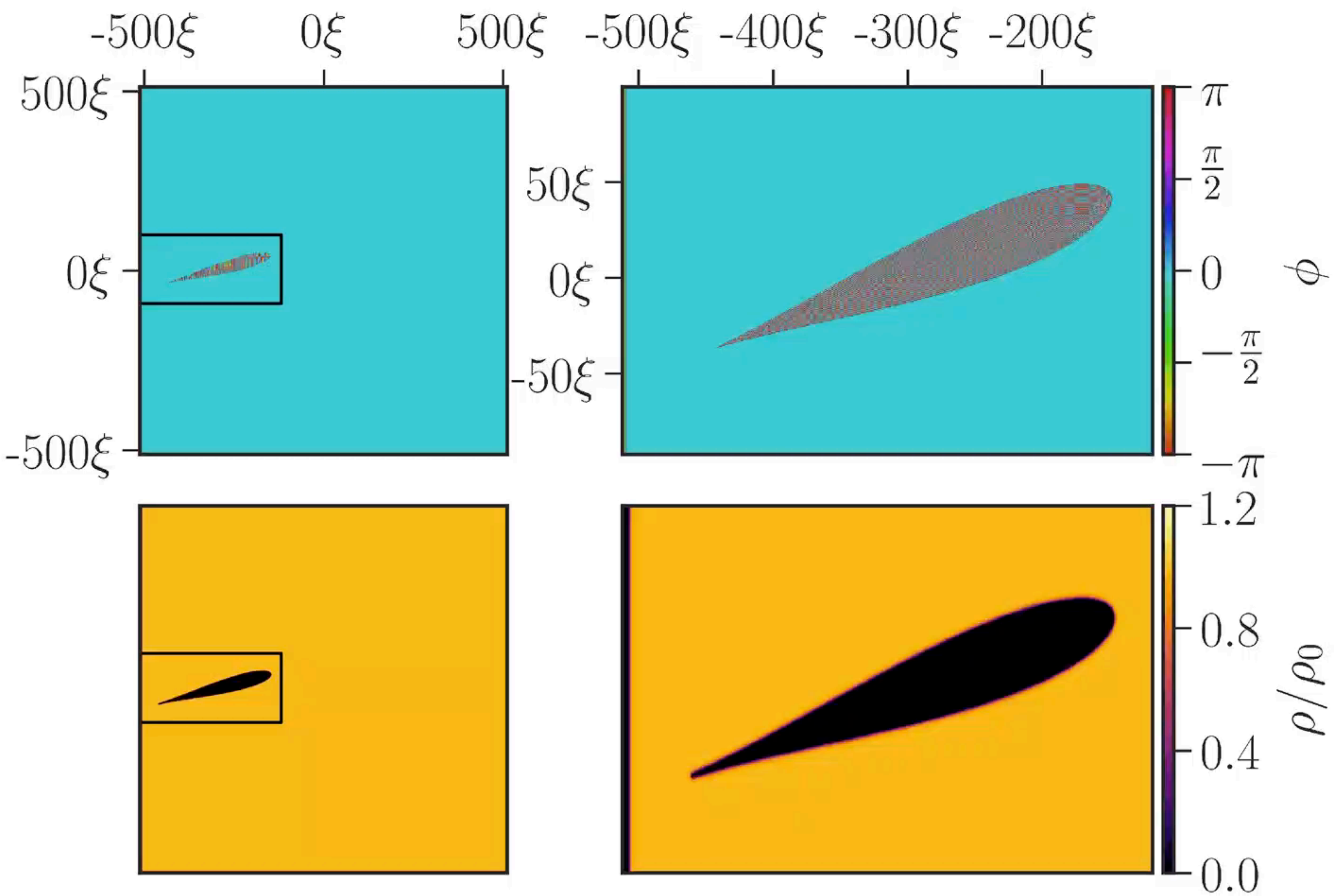
[Sasaki et al., PRL 104, 150404 (2010)]

<u>Boundary layer</u>



[Stagg et al., PRL 118, 135301 (2017)]

A TYPICAL SIMULATION



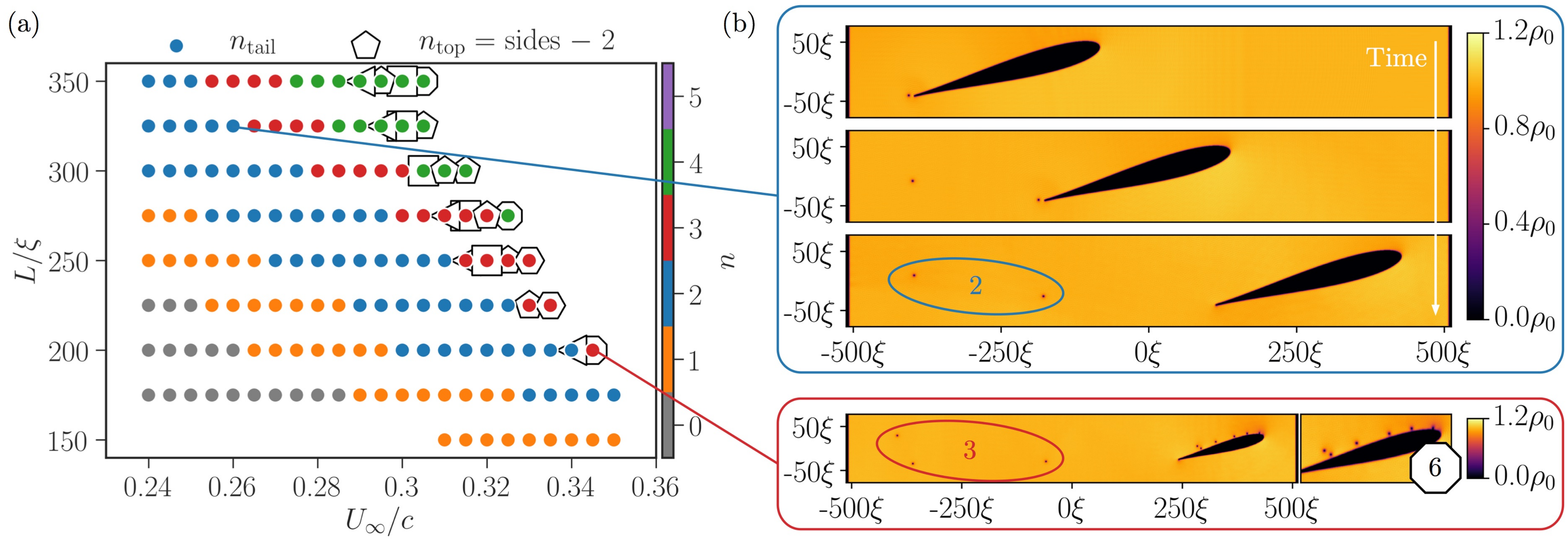
Top: evolution of the phase field.

Bottom: evolution of the superfluid density field.

- The airfoil moves initially with constant acceleration until it reaches a terminal velocity $U_\infty = 0.29c$
- The airfoil's length is $L = 325\xi$ and angle of attack $\alpha = \pi/12$
- Confining potential at the end of the computational box

EXPLORATION OF THE PARAMETERS SPACE

- We vary both the airfoil length and terminal velocity
- The airfoil shape ($\lambda = 0.1$) and angle of attack $\alpha = \pi/12$ are constant



Left: number of vortices produced at the trailing edge. Vortices produced at the top are highlighted with a polygon. Right: two simulation examples, the latter with the detachment of the boundary layer causing a stall condition.

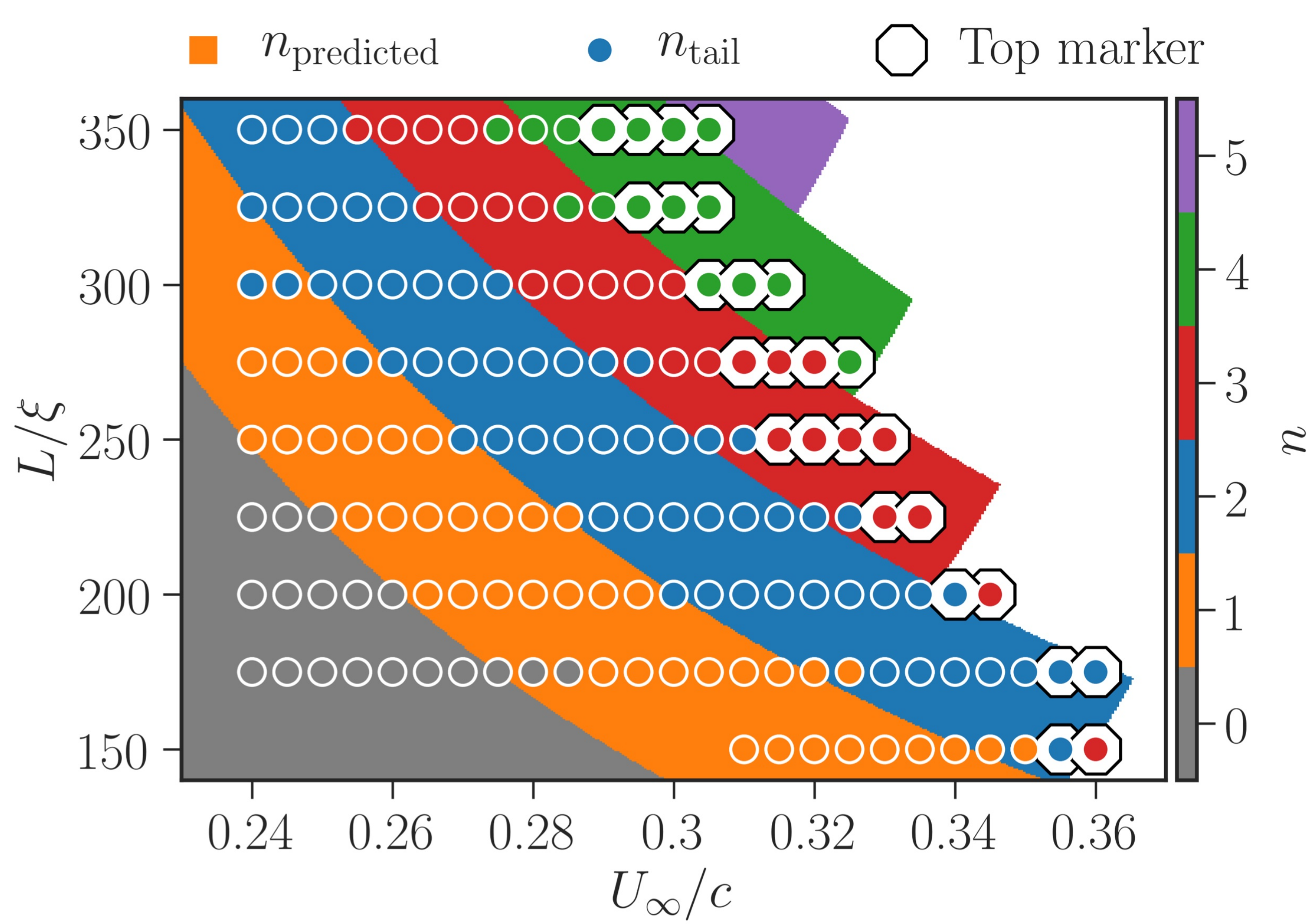
HOW TO PREDICT THE NUMBER OF VORTICES GENERATED?

Introducing a dispersive boundary layer with thickness $r = C\xi$

$$\boxed{C \leq \frac{3}{8}\frac{L}{\xi}\left(\frac{U_\infty}{c}\right)^2 \sin^2(\alpha)\left(1 - \frac{\Gamma}{\Gamma_{KJ}}\right)^2}$$

where $\Gamma = n\kappa$, with $n \in \mathbb{N}$
and Γ_{KJ} is the KJ condition

best fit gives $C \approx 0.55$



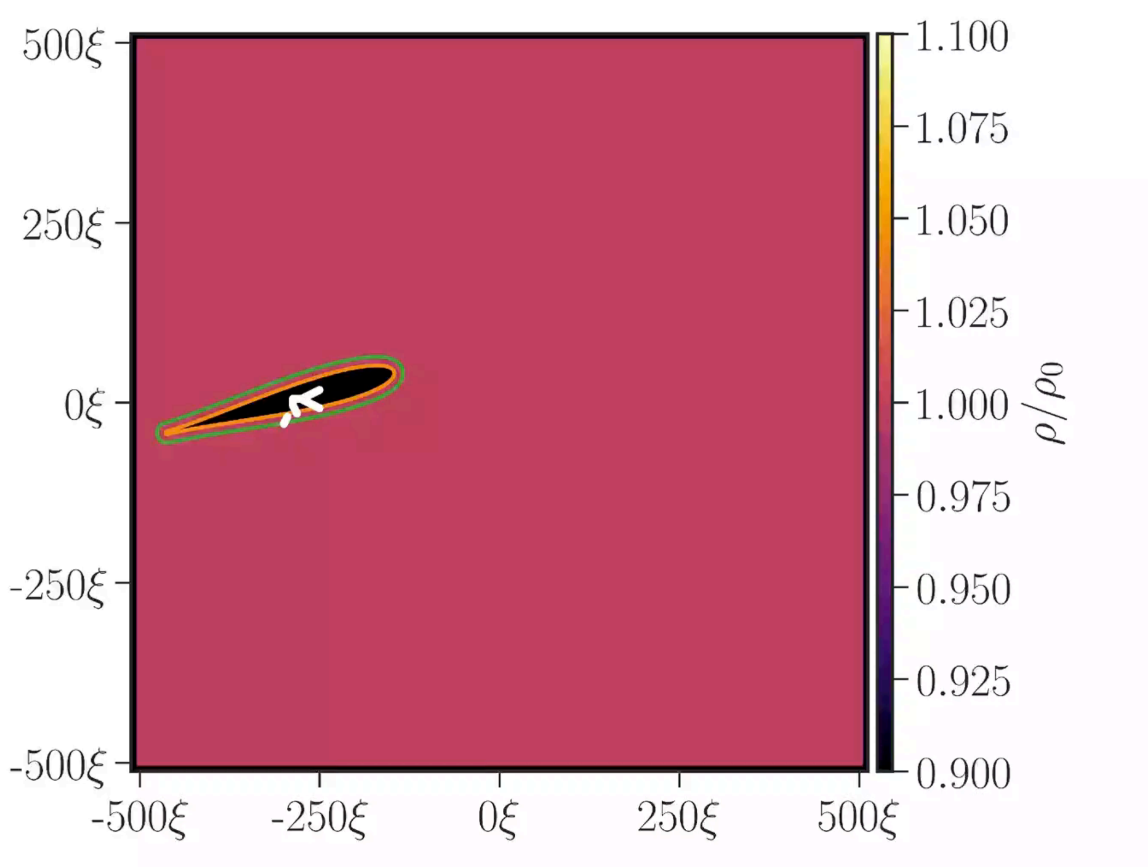
Number of vortices generated depending on the speed and length parameters. The curves indicate the phenomenological prediction. The white area indicate the stalling behaviour.

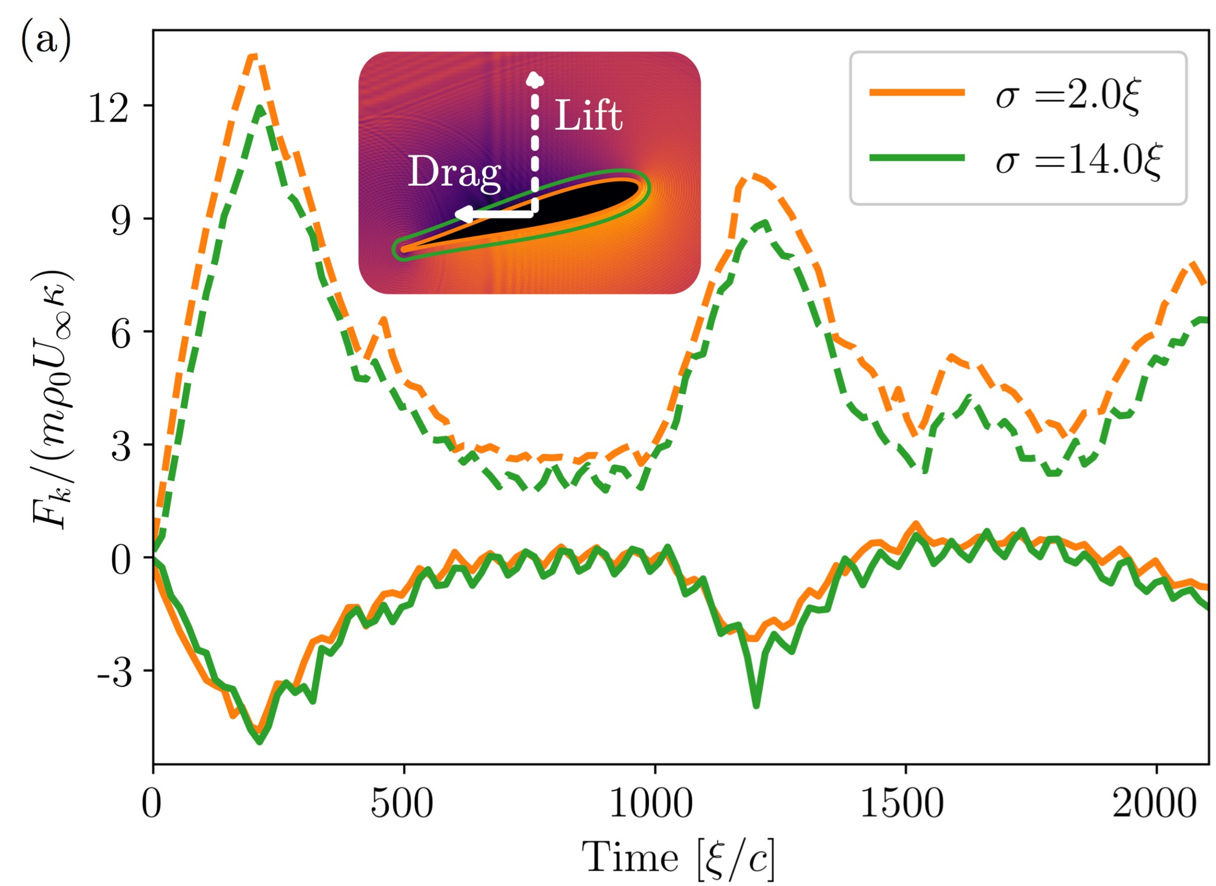
ABOUT LIFT AND DRAG

Lift and drag is obtained from the stress-energy tensor

$$F_k = -\oint_{\mathscr{C}} T_{jk}\, n_j\, d\ell\,, \quad \text{where} \quad T_{jk} = m\rho u_j u_k + \frac{1}{2}\delta_{jk} g\rho^2 - \frac{\hbar^2}{4m}\rho\partial_j\partial_k \ln\rho$$

$\mathscr{C}$ closed path containing the airfoil

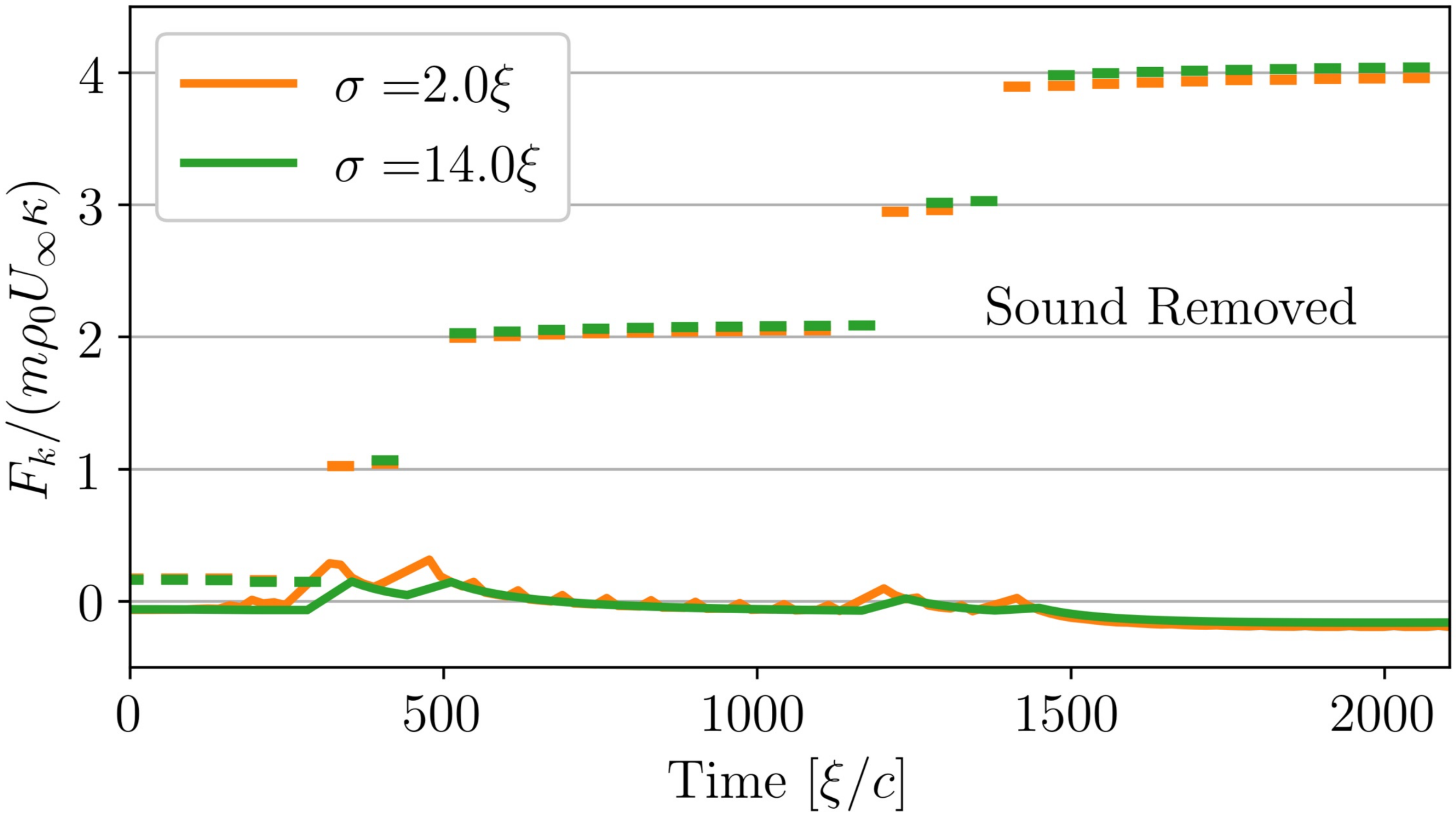




Left: video showing the sound emission during the vortex nucleation at the trailing edge. Right: rescaled lift (dashed) and drag (solid) versus time computed for different contours around the airfoil.

ABOUT LIFT AND DRAG (SOUND FILTERED)

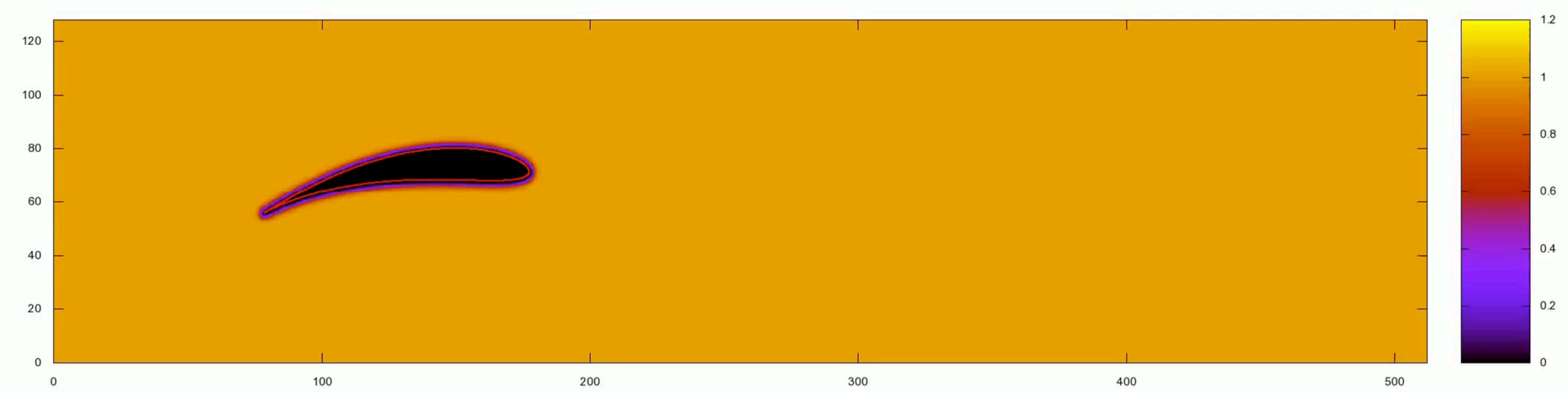
- filter the acoustic component in the velocity field
- use density field prescribed by the stationary Bernoulli equation



Rescaled lift (dashed) and drag (solid) versus time computed for different contours around the airfoil removing sound

Lift appears now quantised and drag becomes nearly zero after the vortex nucleation

CONCLUSIONS



- An airfoil moving in a superfluid can generate vortices at the trailing edge by breaking the Landau's critical speed
- To preserve the total circulation, the airfoil acquires a non-zero circulation
- This process is unsteady and generates sound
- When sound is removed (or steady regime is achieved) the airfoil experiences a quantised lift and no drag)
- If the terminal velocity of the airfoil is too high then a detachment of the boundary layer occurs (stall) and the steady regime cannot be achieved

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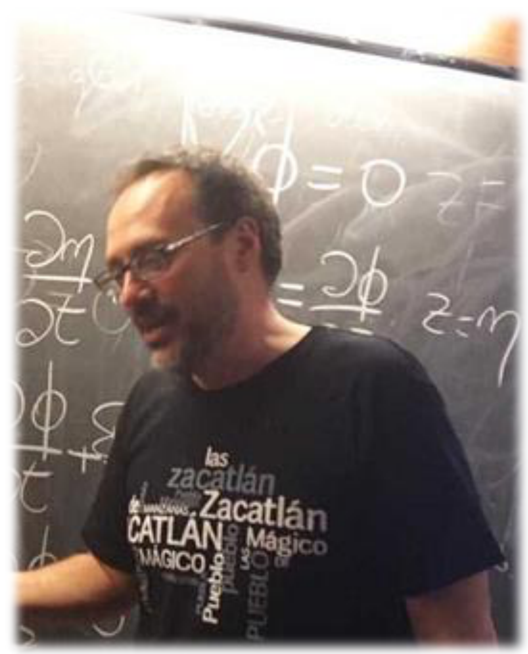
THANKS FOR YOUR ATTENTION!

Joint work with: Seth Musser, D.P., Miguel Onorato, William T.M. Irvine



Acknowledgments

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