

HOMOGENEOUS DYNAMICS

DAVIDE RAVOTTI

OVERVIEW

Contents. The course consists in an introduction to the study of homogeneous flows, which are smooth flows on (quotients of) Lie groups obtained by multiplication by a 1-parameter subgroup. In particular, we will focus on the ergodic theory of Heisenberg nilflows and of geodesic and horocycle flows. We will also explore some connections with topics in number theory.

No prior knowledge in Lie group theory is assumed and all the relevant objects will be introduced during the lectures. However, some familiarity with basic notions in ergodic theory and in differential topology is recommended.

Assessment. There will be an oral examination. In order to pass the exam, the student should have developed a solid understanding and working knowledge of the material discussed in the lectures.

All the material covered in the lectures is subject of examination. The student should also be able to solve simple exercises, which will be assigned during the lectures.

Material. Lecture notes are available at davidravotti.github.io (the file will be updated throughout the semester). Additional useful material is:

- [1] M. Einsiedler, T. Ward. Ergodic Theory with a view towards Number Theory. Graduate Texts in Mathematics 259, Springer, London, 2011.
- [2] J.M. Lee Introduction to Smooth Manifolds. Graduate Texts in Mathematics 218, Springer, New York, NY, 2012.
- [3] M.B. Bekka, M. Mayer. Ergodic Theory and Topological Dynamics of Group Actions on Homogeneous Spaces. London Mathematical Society Lecture Note Series 269, Cambridge University Press, Cambridge, 2000.

LESSON REGISTER

01 Mar: Introduction. Review of notions in ergodic theory (invariant measures, ergodicity, unique ergodicity).

Lecture notes: §§1.1, 1.2, 1.3.1 (except from Proposition 1.9 and Exercise 1.10), 1.3.2, 1.3.3.

08 Mar: Review of notions in ergodic theory (weak mixing, mixing). Review of concepts in differential topology (tangent vectors, vector fields, differentials of smooth maps).

Lecture notes: §§1.1, 1.4.

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