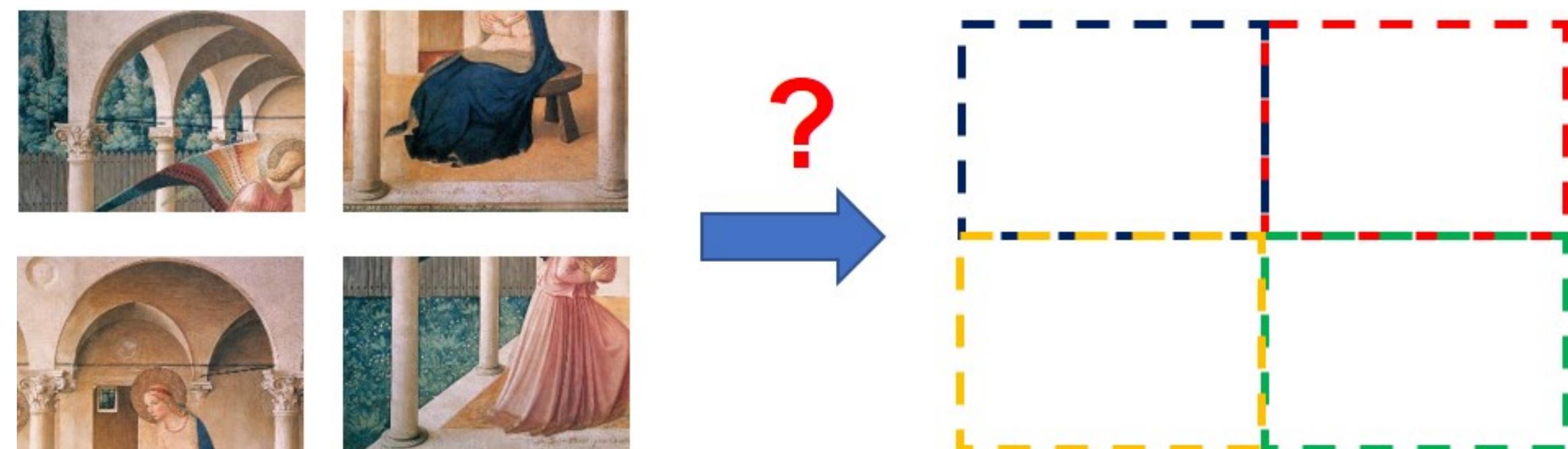




1. The Problem

Given a set of pieces (non-overlapping squares patches of an image) in a random configuration to infer the correct permutation to recover the original image.

Challenge: To overcome the combinatorially complex of matching adjacent pieces

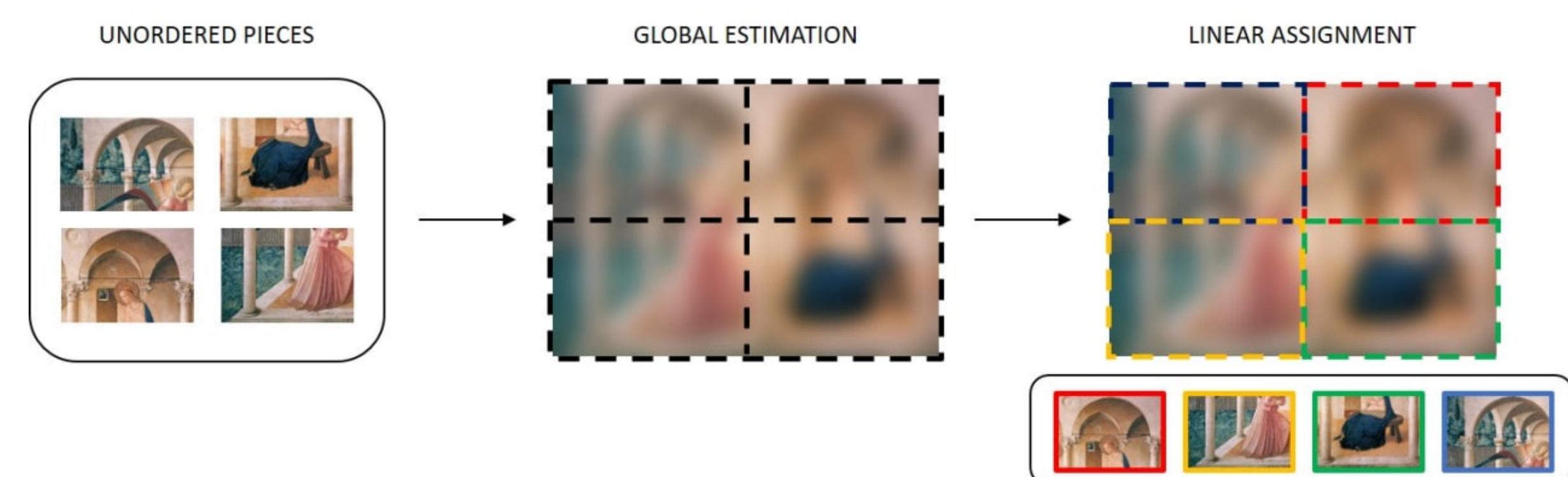


Prior work: Solving for adjacent pieces, optimization-based approaches [1] are time demanding and sensitive to initialization seed and erosion. Deep Learning approaches [7] are faster but do not generalize to multiple sizes.

2. Contribution

Exploiting advancements in Generative Adversarial Network (GAN) methods, we learn to estimate a global solution (mental image) to the problem from unordered pieces. Therefore, we frame the problem as a $R@1$ retrieval task, and then solve the linear assignment using differentiable Hungarian Attention [7].

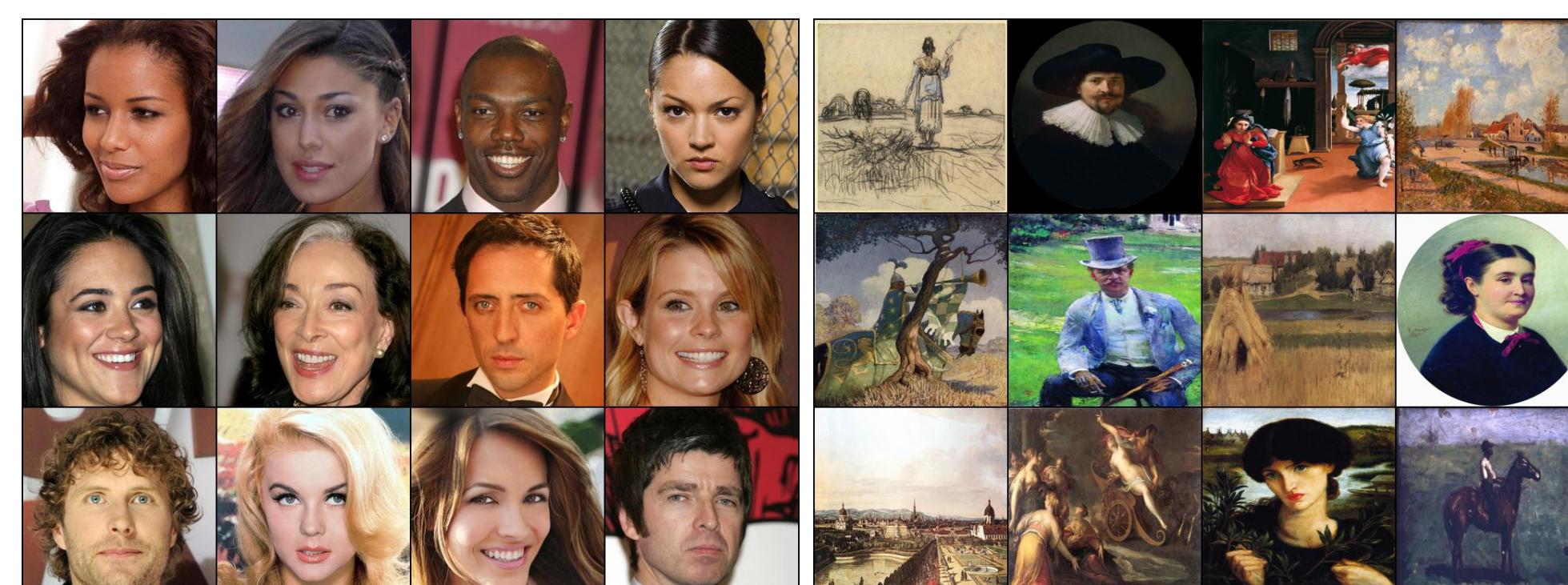
TLDR: Estimate the global solution (mental image). Match pieces against it.



- A many-to-one GAN Architecture for recovering a global image from its pieces.
- Dynamic size puzzle solver using Hungarian attention and contrastive loss.
- Two new large-scale puzzle solving datasets, named **PuzzleCelebA** [3] and **PuzzleWikiArts** [6], permutations are available for direct comparison.

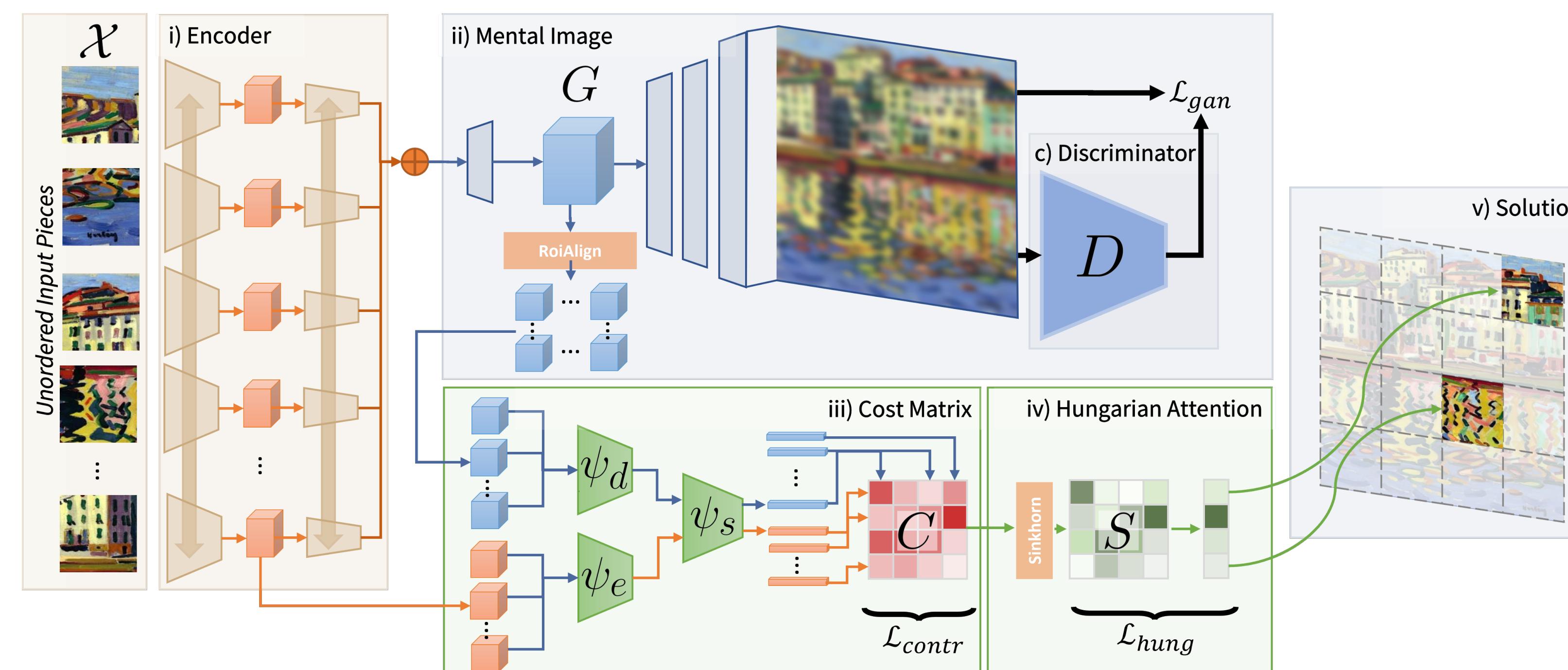
3. Dataset

The proposed benchmark builds on (left) **PuzzleCelebA** (30k images) and (right) **PuzzleWikiArts** (63k images), providing permutation of pieces at different puzzle sizes.



4. Architecture

Unordered pieces (\mathcal{X}) are independently encoded and then pooled to produce a latent vector, from which the global solution (mental image) is estimated. After RoiAlign, generator intermediate features are used as global representation slots to be matched against. A cost matrix evaluates the piece to global slot similarity. Hungarian attention solves for the final permutation.



5. Key blocks

Many-to-one GAN: unordered pieces are encoded independently. The set of encodings is merged using average pooling to generate a single encoding vector. Then, a decoder projects back to the image space trained using adversarial and reconstruction loss for Multi-Scale Gradients [2].

Cost matrix and contrastive loss: the similarity matrix is computed as dot product of all possible piece-slot pairs. A contrastive loss enforces the feature space to have similar embeddings for piece-slot correct pairs while pushing apart non-corresponding pairs:

$$\mathcal{L}_{contr} = -\mathbb{E}_i \left[\log \frac{\exp(\psi_s^i \cdot \psi_s^j / \tau)}{\exp(\psi_s^i \cdot \psi_s^j / \tau) + \sum_{k \neq j} \exp(\psi_s^i \cdot \psi_s^k / \tau)} \right] \quad (1)$$

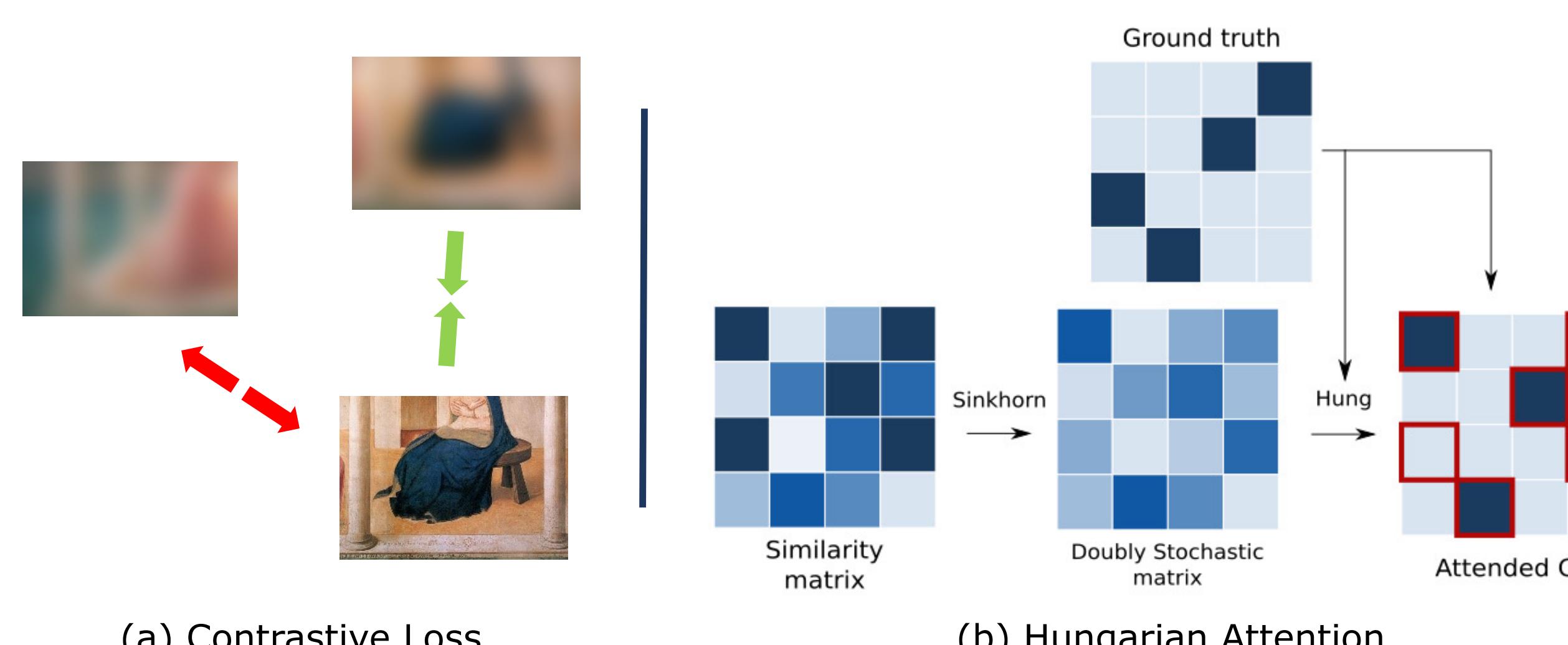
with ψ_s^i and ψ_s^j embeddings of considered piece i and j its corresponding slot.

Hungarian attention (HA): enables supervised learning of optimum assignments. The Sinkhorn normalization relaxes the cost matrix C to a doubly stochastic matrix S . Here, both correct and misplaced pieces are attended:

$$\mathbf{Z} = OR(\text{Hung}(\mathbf{S}), \mathbf{S}^G). \quad (2)$$

Binary cross-entropy loss with respect to the ground-truth \mathbf{S}^G assignment matrix is attended through the mask \mathbf{Z} :

$$\mathcal{L}_{hung} = \sum_{i,j \in [n]} \mathbf{Z}_{ij} \left(\mathbf{S}_{ij}^G \log \mathbf{S}_{ij} + (1 - \mathbf{S}_{ij}^G) \log (1 - \mathbf{S}_{ij}) \right), \quad (3)$$

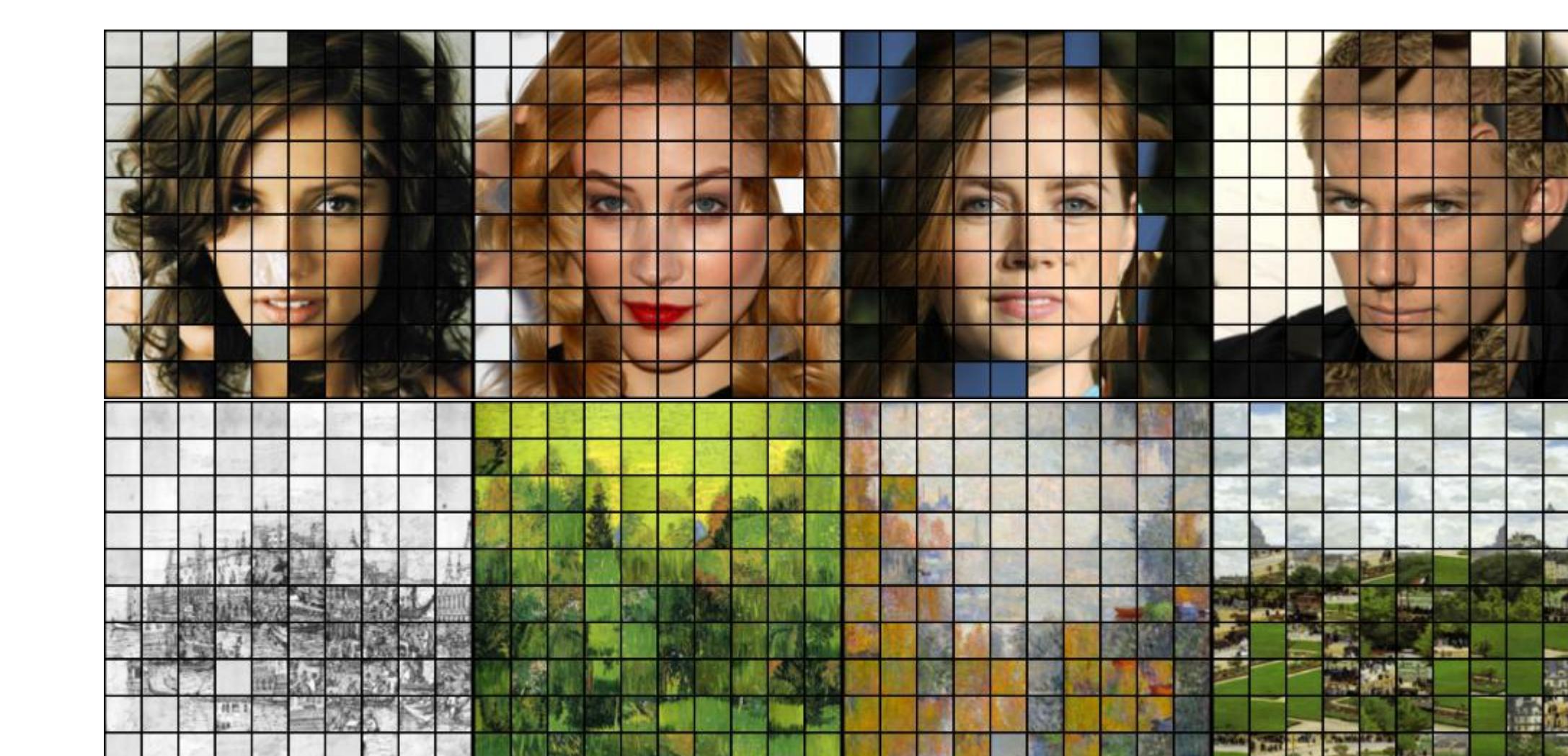


6. Results

Our method handles different sizes while performing on par with deep learning single-size approaches in terms of direct comparison accuracy.

Dataset	PuzzleCelebA				PuzzleWikiArts			
	6x6	8x8	10x10	12x12	6x6	8x8	10x10	12x12
Paikin and Tal [4]	99.12	98.67	98.39	96.51	98.03	97.35	95.31	90.52
Pomeranz et al. [5]	84.59	79.43	74.80	66.43	79.23	72.64	67.70	62.13
Gallagher [1]	98.55	97.04	95.49	93.13	88.77	82.28	77.17	73.40
PO-LA [8]	71.96	50.12	38.05	-	12.19	5.77	3.28	-
Hung-perm	33.11	12.89	4.14	2.18	8.42	3.22	1.90	1.25
GANzzle-Single (Ours)	71.00	51.81	43.74	-	11.78	6.23	8.97	-
GANzzle (Ours)	72.18	53.26	32.84	12.94	13.48	6.93	4.10	2.58

Qualitative results of GANzzle for 10×10 on (top) **PuzzleCelebA** and (bottom) **PuzzleWikiPaintings**



Limitations emerge with challenging pieces, i.e., pieces with similar content, as they can be interchangeable, however GANzzle is able to resolve for the structure of the image.

Take home message

- It is possible to achieve state-of-the-art performances for puzzle problems of different sizes with a single trained model.
- Two benchmark datasets suitable for recent deep learning approaches are available for ease of comparison.
- Deep learning approaches are still far from optimization-based algorithms.

References

- [1] A. C. Gallagher. Jigsaw puzzles with pieces of unknown orientation. In *IEEE Conference on Computer Vision and Pattern Recognition (CVPR)*, 2012.
- [2] Animesh Karnewar and Oliver Wang. Msg-gan: Multi-scale gradients for generative adversarial networks. In *IEEE Conference on Computer Vision and Pattern Recognition (CVPR)*, 2020.
- [3] Cheng-Han Lee, Zhiwei Liu, Lingyun Wu, and Ping Luo. Maskgan: Towards diverse and interactive facial image manipulation. In *IEEE Conference on Computer Vision and Pattern Recognition (CVPR)*, 2020.
- [4] Genady Paikin and Ayelet Tal. Solving multiple square jigsaw puzzles with missing pieces. In *IEEE Conference on Computer Vision and Pattern Recognition (CVPR)*, 2015.
- [5] D. Pomeranz, M. Shemesh, and O. Ben-Shahar. A fully automated greedy square jigsaw puzzle solver. In *IEEE Conference on Computer Vision and Pattern Recognition (CVPR)*, 2011.
- [6] W. R. Tan, C. S. Chan, H. E. Aguirre, and K. Tanaka. Improved artgan for conditional synthesis of natural image and artwork. *IEEE Transactions on Image Processing*, 2019.
- [7] Tianshu Yu, Runzhong Wang, Junchi Yan, and Baoxin Li. Learning deep graph matching with channel-independent embedding and hungarian attention. In *International conference on learning representations (ICLR)*, 2019.
- [8] Yan Zhang, Jonathon Hare, and Adam Prügel-Bennett. Learning representations of sets through optimized permutations. In *International conference on learning representations (ICLR)*, 2019.

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