CALCOLARE LE SEGUENTI DERIVATE

1. $4x \operatorname{sen} 3x$
2. $\log e^{3x} - \cos x$
3. $\operatorname{arctg}(\cos x)$
4. $\dfrac{e^x + 1}{\cos 3x}$
5. $5x^2 \cdot \log \dfrac{1}{x}$
6. $\operatorname{sen}^2 x \cdot \cos x$
7. $\dfrac{2x - \log 5x}{e^x}$
8. $\dfrac{2\sqrt{x}}{e^x}$
9. $\dfrac{e^{2x} - \log 5}{\log 2}$

CALCOLARE I SEGUENTI INTEGRALI

10. $\int 3x^2 \cdot e^x \, dx$
11. $\int 5x^2 \cdot e^{x^3} \, dx$
12. $\int x \operatorname{sen} x \, dx$
13. $\int \operatorname{sen} x \cos^2 x \, dx$
14. $\int \dfrac{1}{4 + x^2} \, dx$
15. $\int \dfrac{1}{x^2 - 5x + 6} \, dx$
16. $\int \dfrac{1}{x^2 - 4x + 3} \, dx$
17. $\int \dfrac{3x}{x + 5} \, dx$
18. $\int \dfrac{9x^2 - 6x}{3x - 1} \, dx$
19. $\int \dfrac{x}{x^2 + 1} \, dx$
20. $\int \left(6x - \operatorname{sen}^2 x + \dfrac{1}{\sqrt{4 - x^2}} + \dfrac{1}{\sqrt{3x + 5}} \right) dx$

SOLUZIONI

1. $D' = 4\operatorname{sen}3x + 4x\cos 3x \cdot 3 = 4(\operatorname{sen}3x + 3x\cos 3x)$

2. $D' = \dfrac{1}{e^{3x}} \cdot e^{3x} \cdot 3 + \operatorname{sen}x = 3 + \operatorname{sen}x$

3. $D' = \dfrac{1}{1+\cos^2 x} \cdot (-\operatorname{sen}x) = -\dfrac{\operatorname{sen}x}{1+\cos^2 x}$

4. $D' = \dfrac{e^x \cos 3x - (e^x+1)\cdot(-\operatorname{sen}3x)\cdot 3}{\cos^2 3x} = \dfrac{e^x \cos 3x + 3\operatorname{sen}3x(e^x+1)}{\cos^2 3x}$

5. $D' = 10x \cdot \log\frac{1}{x} + 5x^2 \cdot \frac{1}{\frac{1}{x}} \cdot \left(-\frac{1}{x^2}\right) = 10x \log\frac{1}{x} - 5x$

6. $D' = 2\operatorname{sen}x \cdot \cos x \cdot \cos x + \operatorname{sen}^2 x(-\operatorname{sen}x) = 2\operatorname{sen}x\cos^2 x - \operatorname{sen}^3 x =$
$= 2\operatorname{sen}x(1-\operatorname{sen}^2 x) - \operatorname{sen}^3 x = 2\operatorname{sen}x - 3\operatorname{sen}^3 x$

7. $D' = \dfrac{\left(2 - \frac{1}{5x}\cdot 5\right)e^x - (2x - \log 5x)e^x}{e^{2x}} = \dfrac{e^x\left(2 - \frac{1}{x} - 2x + \log 5x\right)}{e^{2x}} = \dfrac{2 - \frac{1}{x} - 2x + \log 5x}{e^x}$

8. $D' = \dfrac{2\cdot\frac{1}{2\sqrt{x}}\cdot e^x - 2\sqrt{x}\,e^x}{e^{2x}} = \dfrac{\frac{1}{\sqrt{x}} - 2\sqrt{x}}{e^x}$

9. $D' = \dfrac{1}{\log 2} \cdot e^{2x} \cdot 2 = \dfrac{2e^{2x}}{\log 2}$

!! ATTENZIONE: $\log 5$ e $\log 2$ sono delle costanti (numeri)!!

10. $\int \underset{f}{3x^2}\,\underset{g'}{e^x}\,dx = 3x^2e^x - \int \underset{f'}{e^x}\cdot\underset{g}{6x}\,dx = 3x^2e^x - \left(6xe^x - \int 6e^x dx\right) =$

$= 3x^2e^x - 6xe^x + 6e^x + c = 3e^x(x^2 - 2x + 2) + c$

11. $\int 5x^2 \cdot e^{x^3} dx = \frac{15}{3} \int 3x^2 \cdot e^{x^3} = \frac{5}{3} e^{x^3} + C$

12. $\int \underset{g}{x} \underset{f'}{\operatorname{sen} x} dx = -x\cos x - \int (-\cos x) dx = -x\cos x + \operatorname{sen} x + C$

13. $\int \operatorname{sen} x \cos^2 x \, dx = -\int \cos^2 x \cdot (-\operatorname{sen} x) dx = -\frac{\cos^3 x}{3} + C$

14. $\int \frac{1}{4+x^2} dx = \int \frac{1}{4\left(1+\left(\frac{x}{2}\right)^2\right)} dx = 2 \cdot \frac{1}{4} \int \frac{1}{1+\left(\frac{x}{2}\right)^2} \cdot \frac{1}{2} dx = \frac{1}{2} \operatorname{arctg} \frac{x}{2} + C$

15. $\int \frac{1}{(x-2)(x-3)} dx \quad \frac{A}{x-2} + \frac{B}{x-3} = \frac{1}{(x-2)(x-3)} \quad \frac{Ax-3A+Bx-2B}{(x-2)(x-3)} = \frac{1}{(x-2)(x-3)}$

$\begin{cases} A+B=0 \\ -3A-2B=1 \end{cases} \quad \begin{cases} A=-B \\ 3B-2B=1 \end{cases} \quad \begin{cases} B=1 \\ A=-1 \end{cases} \quad \int \frac{-1}{x-2} dx + \int \frac{1}{x-3} dx = -\log|x-2| + \log|x-3| + C$

$= \log\left|\frac{x-3}{x-2}\right| + C$

20. $3x^2 - \frac{x - \operatorname{sen} x \cos x}{2} + \operatorname{arcsen}\left(\frac{x}{2}\right) + \frac{2}{3}\sqrt{3x+5} + C$

16. $\int \frac{1}{(x-1)(x-3)} dx = \frac{A}{x-1} + \frac{B}{x-3} = \frac{1}{(x-1)(x-3)} \quad \begin{cases} A+B=0 \\ -3A-B=1 \end{cases} \quad \begin{cases} A=-B \\ 2B=1 \end{cases} \quad \begin{cases} B=1/2 \\ A=-1/2 \end{cases}$

$= \int \frac{-1/2}{x-1} dx + \int \frac{1/2}{x-3} dx = -\frac{1}{2}\log|x-1| + \frac{1}{2}\log|x-3| + C = \frac{1}{2}\left(\log\left|\frac{x-3}{x-1}\right|\right) + C$

17. $\int \frac{3x}{x+5} dx = \quad \begin{array}{r|l} 3x & x+5 \\ \hline -3x-15 & 3 \\ \hline -15 & \end{array} \quad = \int 3 dx + \int \frac{-15}{x+5} dx = 3x - 15\log|x+5| + C$

18. $\int \frac{9x^2-6x}{3x-1} dx = \quad \begin{array}{r|l} 9x^2-6x & 3x-1 \\ \hline -9x^2+3x & 3x-1 \\ \hline -3x & \\ 3x-1 & \\ \hline " \quad -1 & \end{array} \quad = \int (3x-1) dx + \int \frac{-1}{3x-1} dx = \frac{3}{2}x^2 - x - \log|3x-1| + C$

19. $\int \frac{x}{x^2+1} dx = \frac{1}{2} \int \frac{2x}{x^2+1} dx = \frac{1}{2} \log(x^2+1) + C = \log\sqrt{x^2+1} + C$