① $z_t = \begin{bmatrix}(x_l - x_t)^2 \\ (y_l - y_t)^2\end{bmatrix} = h(s_t, l)$

$l_1 = [3.6, -0.25]^T$

$Q = \begin{bmatrix}0.1 & 0 \\ 0 & 0.1\end{bmatrix}$

$l_2 = [3.5, 0.1]^T$

$\bar{\mu}_t = [3.0, -0.2]^T$

$\bar{\Sigma}_t = \begin{bmatrix}2.1 & 0 \\ 0 & 0.1\end{bmatrix}$

a) Il modello sensoriale linearizzato viene calcolato mediante lo Jacobiano di $h(s_t, l)$:

$$H(s_t, l) = \begin{bmatrix}\frac{\partial h_1}{\partial x_t} & \frac{\partial h_1}{\partial y_t} \\ \frac{\partial h_2}{\partial x_t} & \frac{\partial h_2}{\partial y_t}\end{bmatrix} = \begin{bmatrix}-2(x_l - x_t) & 0 \\ 0 & -2(y_l - y_t)\end{bmatrix}$$

b) $z_t = [0.01, 0.0016]^T$

La distanza dai landmark l_1 e l_2 usando la metrica EUCLIDEA è

$$d_{e_1} = d_e(\bar{\mu}_t, l_1) = \sqrt{(3.0 - 3.6)^2 + (-0.2 + 0.25)^2} = \sqrt{0.3625} = 0.602$$

$$d_{e_2} = d_e(\bar{\mu}_t, l_2) = \sqrt{(3.0 - 3.5)^2 + (-0.2 - 0.1)^2} = \sqrt{0.34} = 0.583$$

La distanza di Mahalanobis da l_1 e l_2 viene calcolata applicando la formula

$$d_{m_1} = (z_t - h(\bar{\mu}_t, l_1))^T P_1^{-1} (z_t - h(\bar{\mu}_t, l_1)) \quad \text{con } P_1 = Q + H(\bar{\mu}_t, l_1)\bar{\Sigma}_t H(\bar{\mu}_t, l_1)^T$$

e analogamente per d_{m_2}.

$$P_1 = \begin{bmatrix}0.1 & 0 \\ 0 & 0.1\end{bmatrix} + \begin{bmatrix}-1.2 & 0 \\ 0 & 0.1\end{bmatrix}\begin{bmatrix}2.1 & 0 \\ 0 & 0.1\end{bmatrix}\begin{bmatrix}-1.2 & 0 \\ 0 & 0.1\end{bmatrix} = \begin{bmatrix}0.1 & 0 \\ 0 & 0.1\end{bmatrix} + \begin{bmatrix}-2.52 & 0 \\ 0 & 0.01\end{bmatrix}\begin{bmatrix}-1.2 & 0 \\ 0 & 0.1\end{bmatrix}$$

$$= \begin{bmatrix}0.1 & 0 \\ 0 & 0.1\end{bmatrix} + \begin{bmatrix}3.024 & 0 \\ 0 & 0.001\end{bmatrix} = \begin{bmatrix}3.124 & 0 \\ 0 & 0.101\end{bmatrix}$$

$$P_2 = \begin{bmatrix}0.1 & 0 \\ 0 & 0.1\end{bmatrix} + \begin{bmatrix}-1 & 0 \\ 0 & -0.6\end{bmatrix}\begin{bmatrix}2.1 & 0 \\ 0 & 0.1\end{bmatrix}\begin{bmatrix}-1 & 0 \\ 0 & -0.6\end{bmatrix} = \begin{bmatrix}0.1 & 0 \\ 0 & 0.1\end{bmatrix} + \begin{bmatrix}2.1 & 0 \\ 0 & 0.036\end{bmatrix} = \begin{bmatrix}2.2 & 0 \\ 0 & 0.136\end{bmatrix}$$

$$P_1^{-1} = \frac{1}{0.315524}\begin{bmatrix} 0.101 & 0 \\ 0 & 3.124 \end{bmatrix} \simeq \begin{bmatrix} 0.32 & 0 \\ 0 & 9.9 \end{bmatrix}$$

$$P_2^{-1} = \frac{1}{0.2992}\begin{bmatrix} 0.136 & 0 \\ 0 & 2.2 \end{bmatrix} = \begin{bmatrix} 0.4545 & 0 \\ 0 & 7{,}3529 \end{bmatrix}$$

$$d_{m_1} = \left(\begin{bmatrix} 0.01 \\ 0.0016 \end{bmatrix} - \begin{bmatrix} (3.6-3.0)^2 \\ (-0.25+0.2)^2 \end{bmatrix}\right)^T \cdot \begin{bmatrix} 0.32 & 0 \\ 0 & 9.9 \end{bmatrix} \cdot \left(\begin{bmatrix} 0.01 \\ 0.0016 \end{bmatrix} - \begin{bmatrix} (3.6-3.0)^2 \\ (-0.25+0.2)^2 \end{bmatrix}\right) =$$

$$= \begin{bmatrix} -0.35 \\ -0.0009 \end{bmatrix}^T \begin{bmatrix} 0.32 & 0 \\ 0 & 9.9 \end{bmatrix} \begin{bmatrix} -0.35 \\ -0.0009 \end{bmatrix} \simeq \begin{bmatrix} -0.112 & -0{,}00891 \end{bmatrix} \begin{bmatrix} -0.35 \\ -0.0009 \end{bmatrix} = 0{,}0392$$

$$d_{m_2} = \left(\begin{bmatrix} 0.01 \\ 0.0016 \end{bmatrix} - \begin{bmatrix} (3.5-3.0)^2 \\ (0.1+0.2)^2 \end{bmatrix}\right)^T \cdot \begin{bmatrix} 0.4545 & 0 \\ 0 & 7.3529 \end{bmatrix} \cdot \left(\begin{bmatrix} 0.01 \\ 0.0016 \end{bmatrix} - \begin{bmatrix} (3.5-3.0)^2 \\ (0.1+0.2)^2 \end{bmatrix}\right) =$$

$$= \begin{bmatrix} -0.24 \\ -0.0884 \end{bmatrix}^T \begin{bmatrix} 0.4545 & 0 \\ 0 & 7.3529 \end{bmatrix} \begin{bmatrix} -0.24 \\ -0.0884 \end{bmatrix} = \begin{bmatrix} -0.10908 & -0.65 \end{bmatrix} \begin{bmatrix} -0.24 \\ -0.0884 \end{bmatrix} \simeq 0.0836$$

c) Considero come distanza quella di Mahalanobis, pertanto il landmark più vicino è $l_1 = [3.6 \;,\; -0.25]^T$ e $H_1 = \begin{bmatrix} -1.2 & 0 \\ 0 & 0.1 \end{bmatrix}$

$$K_t = \bar{\Sigma}_t H_1^T \left(H_1 \bar{\Sigma}_t H_1^T + Q_t\right)^{-1} = \begin{bmatrix} 2.1 & 0 \\ 0 & 0.1 \end{bmatrix}\begin{bmatrix} -1.2 & 0 \\ 0 & 0.1 \end{bmatrix} \cdot \left(\begin{bmatrix} -1.2 & 0 \\ 0 & 0.1 \end{bmatrix}\begin{bmatrix} 2.1 & 0 \\ 0 & 0.1 \end{bmatrix}\begin{bmatrix} -1.2 & 0 \\ 0 & 0.1 \end{bmatrix} + \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}\right)^{-1} =$$

$$= \begin{bmatrix} -2.52 & 0 \\ 0 & 0.01 \end{bmatrix} \cdot \left(\begin{bmatrix} 3.024 & 0 \\ 0 & 0.001 \end{bmatrix} + \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}\right)^{-1} = \begin{bmatrix} -2.52 & 0 \\ 0 & 0.1 \end{bmatrix} \cdot \frac{1}{0.315524}\begin{bmatrix} 0.101 & 0 \\ 0 & 3.124 \end{bmatrix} =$$

$$= \begin{bmatrix} -2.52 & 0 \\ 0 & 0.1 \end{bmatrix}\begin{bmatrix} 0.32 & 0 \\ 0 & 9.9 \end{bmatrix} = \begin{bmatrix} -0.8064 & 0 \\ 0 & 0.99 \end{bmatrix}$$

$$\Sigma_t = (I - K_t H_1)\bar{\Sigma}_t = \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -0.8064 & 0 \\ 0 & 0.99 \end{bmatrix}\begin{bmatrix} -1.2 & 0 \\ 0 & 0.1 \end{bmatrix}\right)\begin{bmatrix} 2.1 & 0 \\ 0 & 0.1 \end{bmatrix} =$$

$$= \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.96768 & 0 \\ 0 & 0.099 \end{bmatrix}\right)\begin{bmatrix} 2.1 & 0 \\ 0 & 0.1 \end{bmatrix} = \begin{bmatrix} 0.03232 & 0 \\ 0 & 0.901 \end{bmatrix}\begin{bmatrix} 2.1 & 0 \\ 0 & 0.1 \end{bmatrix} = \begin{bmatrix} 0.067872 & 0 \\ 0 & 0.0901 \end{bmatrix}$$

$$\mu_t = \bar{\mu}_t + K_t\left(z_t - h_1(\bar{\mu}_t, l_1)\right) = [3.0, -0.2]^T + \begin{bmatrix} -0.8064 & 0 \\ 0 & 0.99 \end{bmatrix}\left(\begin{bmatrix} 0.01 \\ 0.0016 \end{bmatrix} - \begin{bmatrix} (3.6-3.0)^2 \\ (-0.25+0.2)^2 \end{bmatrix}\right) =$$

$$= \begin{bmatrix} 3.0 \\ -0.2 \end{bmatrix} + \begin{bmatrix} -0.8064 & 0 \\ 0 & 0.99 \end{bmatrix}\begin{bmatrix} -0.35 \\ -0.0009 \end{bmatrix} = \begin{bmatrix} 3.0 \\ -0.2 \end{bmatrix} + \begin{bmatrix} 0.28224 \\ 0.00089 \end{bmatrix} = \begin{bmatrix} 3.28224 \\ -0.19911 \end{bmatrix}$$

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2) $z_t = h(s_t, l) = \frac{1}{2}\left[(x_l - x_t)^2 + (y_l - y_t)^2\right]$

$Q = 0.1$

$\bar{\mu}_t = [1.5, 0.0]^T$

$\bar{\Sigma}_t = \begin{bmatrix} 0.8 & 0 \\ 0 & 0.8 \end{bmatrix}$

a) Il modello sensoriale linearizzato sarà

$$H(s_t, l) = \left[\frac{\partial h(s_t, l)}{\partial x_t} \quad \frac{\partial h(s_t, l)}{\partial y_t}\right] = \left[x_t - x_l \quad y_t - y_l\right]$$

b) $z_t = 0.7$

$l_1 = [2.0 \quad 0.0]^T$

$$K_t = \bar{\Sigma}_t H(\bar{\mu}_t, l_1)^T \left(H(\bar{\mu}_t, l_1)\bar{\Sigma}_t H(\bar{\mu}_t, l_1)^T + Q_t\right)^{-1} =$$

$$= \begin{bmatrix} 0.8 & 0 \\ 0 & 0.8 \end{bmatrix}\begin{bmatrix} -0.5 \\ 0.0 \end{bmatrix} \cdot \left(\begin{bmatrix} -0.5 & 0 \end{bmatrix}\begin{bmatrix} 0.8 & 0 \\ 0 & 0.8 \end{bmatrix}\begin{bmatrix} -0.5 \\ 0 \end{bmatrix} + 0.1\right)^{-1} =$$

$$= \begin{bmatrix} -0.4 \\ 0 \end{bmatrix} \cdot (0.2 + 0.1)^{-1} = \begin{bmatrix} -0.4 \\ 0 \end{bmatrix} \cdot 3.33 = \begin{bmatrix} -1.332 \\ 0 \end{bmatrix}$$

$$\mu_t = \bar{\mu}_t + K_t\left(z_t - h(\bar{\mu}_t, l_1)\right) = \begin{bmatrix} 1.5 \\ 0 \end{bmatrix} + \begin{bmatrix} -1.332 \\ 0 \end{bmatrix}\left(0.7 - \frac{1}{2}\left[(2.0-1.5)^2 + (0.0-0.0)^2\right]\right) =$$

$$= \begin{bmatrix} 1.5 \\ 0 \end{bmatrix} + \begin{bmatrix} -1.332 \\ 0 \end{bmatrix}(0.7 - 0.125) = \begin{bmatrix} 1.5 \\ 0 \end{bmatrix} + \begin{bmatrix} -0.7659 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.7341 \\ 0 \end{bmatrix}$$

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$$\Sigma_t = \left(I - K_t H(\bar{\mu}_t, l_1)\right)\bar{\Sigma}_t = \left(\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} - \begin{bmatrix}-1.332\\ 0\end{bmatrix}\begin{bmatrix}-0.5 & 0\end{bmatrix}\right)\begin{bmatrix}0.8 & 0\\ 0 & 0.8\end{bmatrix} =$$

$$= \left(\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} - \begin{bmatrix}0.666 & 0\\ 0 & 0\end{bmatrix}\right)\begin{bmatrix}0.8 & 0\\ 0 & 0.8\end{bmatrix} = \begin{bmatrix}0.334 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}0.8 & 0\\ 0 & 0.8\end{bmatrix} = \begin{bmatrix}0.2672 & 0\\ 0 & 0.8\end{bmatrix}$$

c) Si nota che lungo la direzione y la stima era corretta e non vi è stata correzione.

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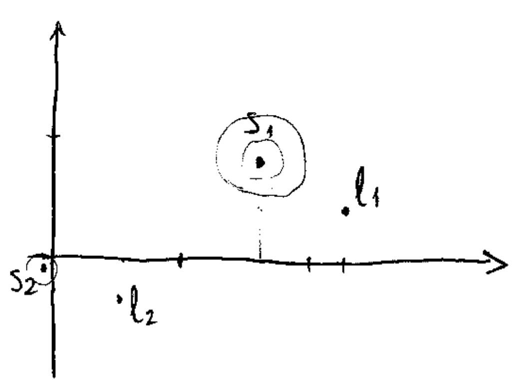
(3) a) Il criterio di massima verosimiglianza afferma che la stima $\hat{x}_{ML}$ che meglio approssima lo stato reale x è quella che massimizza $p(z_t|x_t)$, detta funzione di verosimiglianza, che indica la probabilità di osservare z_t dato lo stato x_t.

Il criterio di maximum a posteriori (MAP) afferma che la stima $\hat{x}_{MAP}$ che meglio approssima lo stato reale x è quella che massimizza

$$P(x_t|z_t) \underset{\text{Bayes}}{=} \frac{P(z|x)\cdot P(x)}{P(z)}$$

con $P(z|x)$ funzione ML, $P(x)$ distribuzione a priori e $P(x|z)$ distribuzione a posteriori.

b) Essendo $z = l - s$, posso ricavare semplicemente $s = l - z$.



Se ci fosse il valore assoluto nel modello sensoriale, gli stati stimati sarebbero 4 per ogni landmark.

c) $z_t = \begin{bmatrix}x_l - x_t\\ y_l - y_t\end{bmatrix} = h(s_t, l)$ $\quad Q = \begin{bmatrix}0.1 & 0\\ 0 & 0.1\end{bmatrix}$

$\bar{\mu}_t = [1.8, 0.7]^T$ $\quad \bar{\Sigma}_t = \begin{bmatrix}0.5 & 0\\ 0 & 0.5\end{bmatrix}$ $\quad H = \begin{bmatrix}\frac{\partial h}{\partial x_t} & \frac{\partial h}{\partial y_t}\end{bmatrix} = \begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix} = -I$

$l_1 = [2.2 \quad 0.4]^T$ $\quad z_t = [0.55 \quad -0.25]^T$

$$K_t = \bar{\Sigma}_t H_t \left(H_t \bar{\Sigma}_t H_t^T + Q_t \right)^{-1} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \left(\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} + \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix} \right)^{-1} =$$

$$= \begin{bmatrix} -0.5 & 0 \\ 0 & -0.5 \end{bmatrix} \left(\begin{bmatrix} 0.6 & 0 \\ 0 & 0.6 \end{bmatrix} \right)^{-1} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix} \cdot \frac{1}{0.36} \begin{bmatrix} 0.6 & 0 \\ 0 & 0.6 \end{bmatrix} = \begin{bmatrix} 0.835 & 0 \\ 0 & 0.835 \end{bmatrix}$$

$$\mu_t = \bar{\mu}_t + K_t \left(z_t - h(\bar{\mu}_t, l_1) \right) = \begin{bmatrix} 1.8 \\ 0.7 \end{bmatrix} + \begin{bmatrix} 0.835 & 0 \\ 0 & 0.835 \end{bmatrix} \left(\begin{bmatrix} 0.55 \\ -0.25 \end{bmatrix} - \begin{bmatrix} 0.4 \\ -0.3 \end{bmatrix} \right) =$$

$$= \begin{bmatrix} 1.8 \\ 0.7 \end{bmatrix} + \begin{bmatrix} 0.835 & 0 \\ 0 & 0.835 \end{bmatrix} \begin{bmatrix} 0.15 \\ 0.05 \end{bmatrix} = \begin{bmatrix} 1.8 \\ 0.7 \end{bmatrix} + \begin{bmatrix} 0.12525 \\ 0.04175 \end{bmatrix} = \begin{bmatrix} 1.92525 \\ 0.74175 \end{bmatrix}$$

$$\Sigma_t = \left(I - K_t H_t \right) \bar{\Sigma}_t = \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.835 & 0 \\ 0 & 0.835 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \right) \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix} =$$

$$= \begin{bmatrix} 1.835 & 0 \\ 0 & 1.835 \end{bmatrix} \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix} = \begin{bmatrix} 0.9175 & 0 \\ 0 & 0.9175 \end{bmatrix}$$