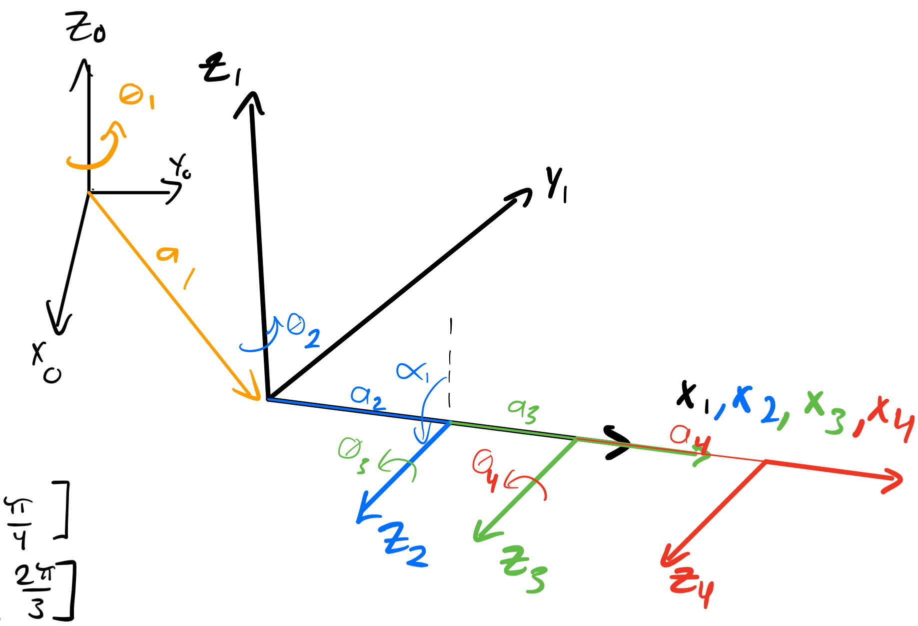
$\text{tansZ}(d_i)\ \text{rotZ}(\theta_i)\ \text{transX}(a_i)\ \text{rotX}(\alpha_i)$

DH	a	d	α	θ
1	a_1	0	0	θ_1
2	a_2	0	$\pi/2$	*
3	a_3	0	0	*
4	a_4	0	0	*



$a_2 = 40\text{mm}$ $\quad \theta_2 \in [-\frac{\pi}{4}, \frac{\pi}{4}]$

$a_3 = 80\text{mm}$ $\quad \theta_3 \in [-\frac{\pi}{3}, \frac{2\pi}{3}]$

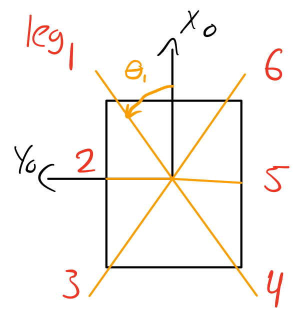
$a_4 = 125\text{mm}$ $\quad \theta_4 \in [-\frac{5\pi}{6}, \frac{\pi}{10}]$

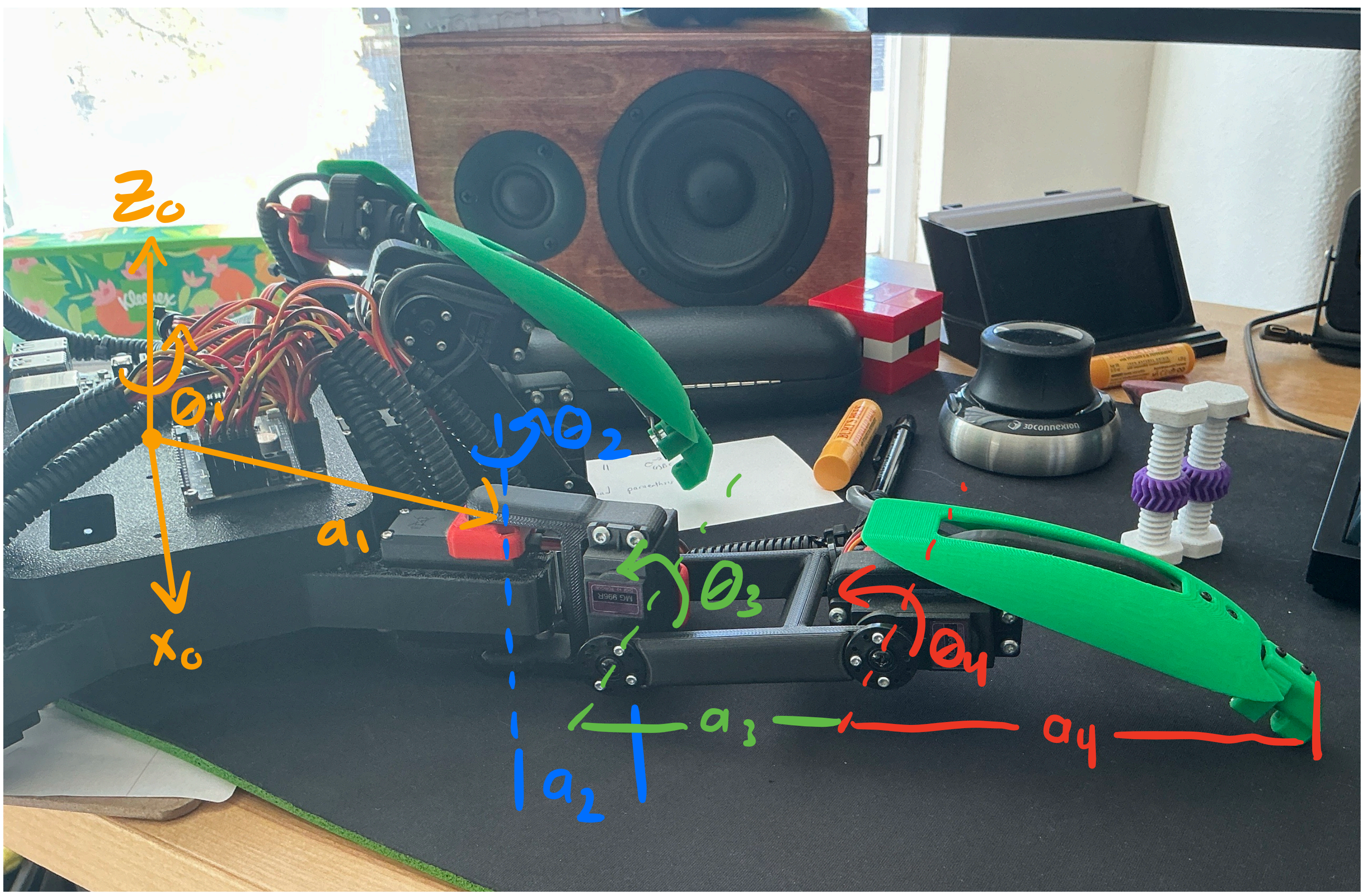
legs 1, 3, 4, 6
$a_1 = 135\text{mm}$

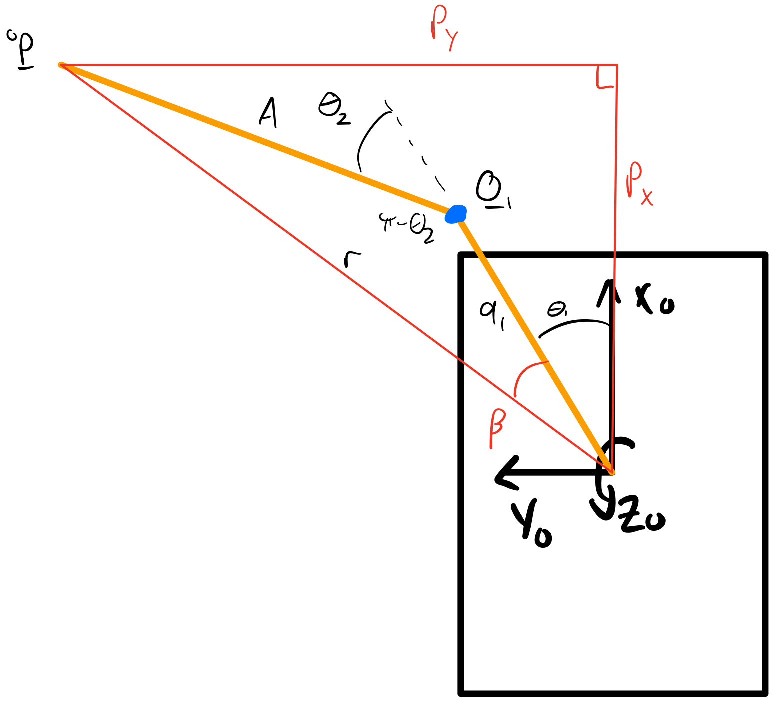
legs 2, 5
$a_1 = 80\text{mm}$

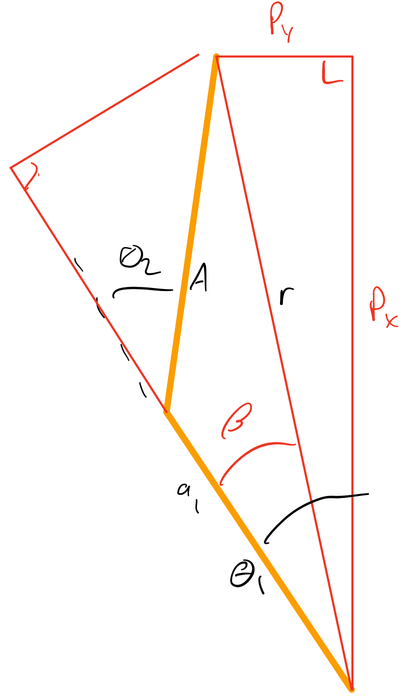
θ_1 by leg $= [30, 90, 150, 210, 270, 330]$

$= [\frac{\pi}{6}, \frac{\pi}{4}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}]$









$$r = \sqrt{{}^0p_x^2 + {}^0p_y^2}$$

$$\underline{A} = {}^0\underline{P} - a_1 \begin{bmatrix} c\theta_1 & -s\theta_1 \\ s\theta_1 & c\theta_1 \end{bmatrix}$$

$$A = \|\underline{A}\|$$

$$\beta = \operatorname{atan2}({}^0P_y, {}^0P_x)$$

$$r^2 = a_1^2 + A^2 - 2a_1 A \cos(\pi - \theta_2)$$

$$r^2 = a_1^2 + A^2 + 2a_1 A \cos\theta_2$$

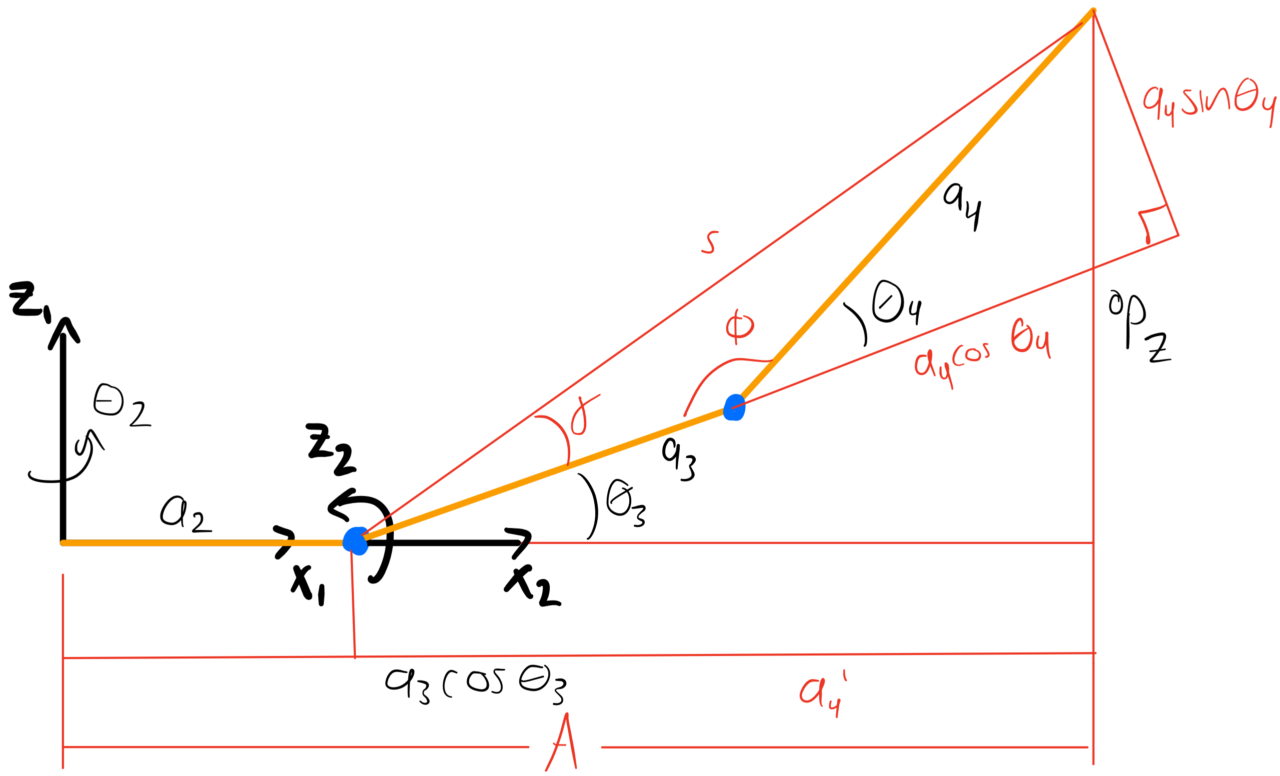
$$\cos\theta_2 = \frac{r^2 - a_1^2 - A^2}{2a_1 A}$$

$$\underline{\underline{\theta_2 = 2\operatorname{atan2}\left(\sqrt{1-\cos^2\theta},\ 1+\cos\theta\right)\operatorname{sign}\left(\hat{\underline{X}}_1 \times {}^0\underline{P}\right)}}$$

$$\tan^2\left(\frac{\phi}{2}\right) = \frac{1-\cos\phi}{1+\cos\phi}$$

$$\phi = \pm 2\operatorname{atan}\left(\sqrt{1-\cos^2\phi},\ 1+\cos\phi\right)$$

there is a second solution 180° opposite, but it is not useful.



$$s = \sqrt{{}^0p_z^2 + (A - a_2)^2}$$

$$s^2 = a_3^2 + a_4^2 - 2 a_3 a_4 \cos(\pi - \theta_4)$$

$$\cos\theta_4 = \frac{s^2 - a_3^2 - a_4^2}{2 a_3 a_4}$$

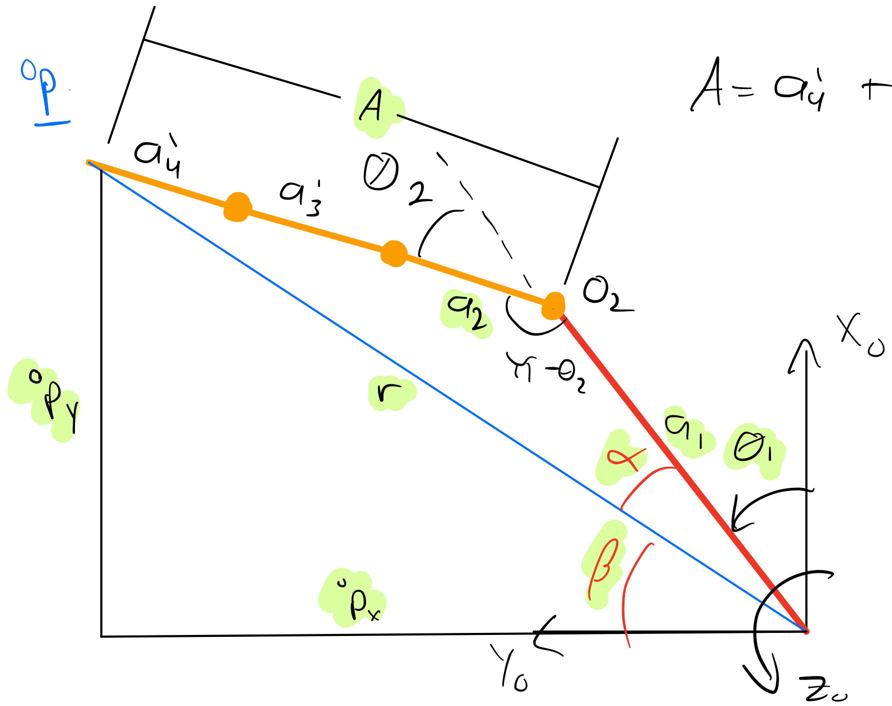
$$\underline{\underline{\theta_4 = \pm 2\,\mathrm{atan2}\left(\sqrt{1-\cos^2\theta_4},\ 1+\cos\theta_4\right)}}$$ two solutions

$$\theta_3 + \gamma = \mathrm{atan2}({}^0p_z,\ A - a_2)$$

$$\gamma = \mathrm{atan2}(a_4 \sin\theta_4,\ a_3 + a_4 \cos\theta_4)$$

$$\underline{\underline{\theta_3 = \mathrm{atan2}({}^0p_z,\ A - a_2) - \mathrm{atan2}(a_4 \sin\theta_4,\ a_3 + a_4 \cos\theta_4)}}$$

$$A = a_4' + a_3' + a_2$$



$$\beta = \operatorname{atan2}({}^0p_y, {}^0p_x)$$

$$A = \sqrt{r^2 + a_1^2 - 2 r a_1 \cos\alpha}$$

$$\alpha = \pi/2 - \theta_1 - \beta$$

$$r = \sqrt{{}^0p_x^2 + {}^0p_y^2}$$

$$r^2 = a_1^2 + A^2 + 2 a_1 A \cos(\cancel{\pi +}\theta_2)$$

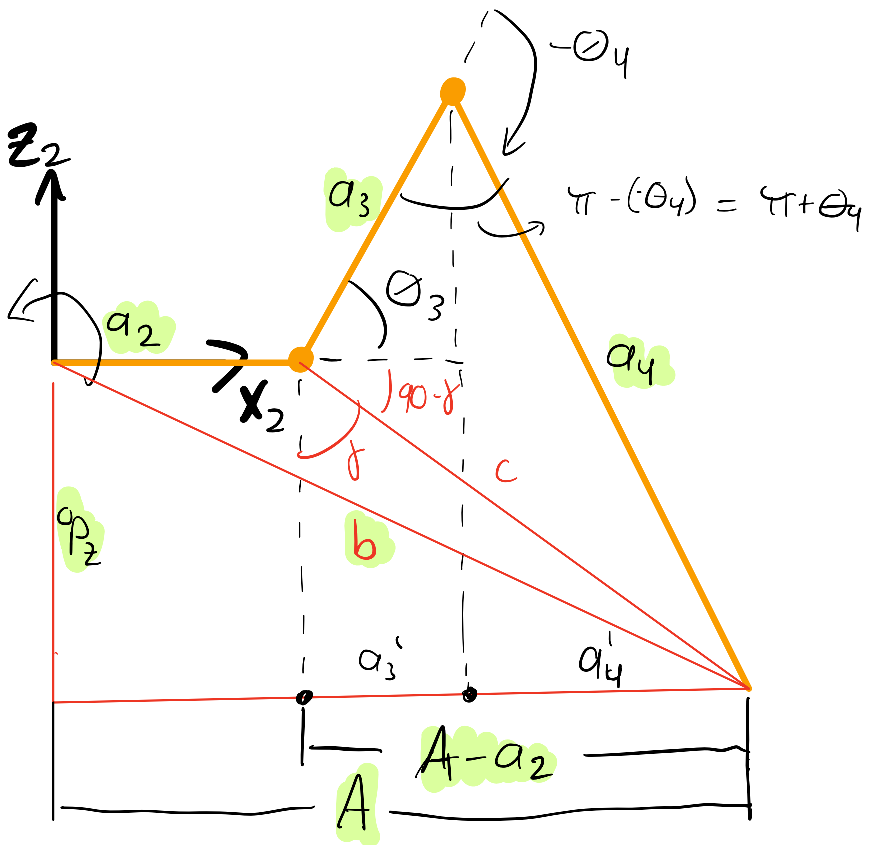
$$\cos(\pi - \theta_2) = -\cos\theta_2$$

$$\cos\theta_2 = \frac{r^2 - a_1^2 - A^2}{2 a_1 A}$$

$$\tan^2\left(\frac{\theta_2}{2}\right) = \frac{1 - \cos\theta_2}{1 + \cos\theta_2}$$

$$\theta_2 = 2\operatorname{atan2}\left(\sqrt{1 - \cos^2\theta_2},\ 1 + \cos\theta_2\right)$$

$$b = \sqrt{{}^0p_z^2 + A^2}$$



$$c = \sqrt{{}^0p_z^2 + (A - a_2)^2}$$

$$c^2 = a_3^2 + a_4^2 - 2 a_3 a_4 \cos(\pi + \theta_4)$$

$$\cos(\pi + \theta_4) = -\cos\theta_4$$

$$c^2 = a_3^2 + a_4^2 + 2 a_3 a_4 \cos\theta_4$$

$$\cos\theta_4 = \frac{c^2 - a_3^2 - a_4^2}{2 a_3 a_4}$$

$$\theta_4 = 2\,\mathrm{atan2}\left(\sqrt{1 - \cos^2\theta_4},\ 1 + \cos\theta_4\right)$$

$$\gamma = \mathrm{atan2}(A - a_2,\ {}^0p_z)$$

$$a_4^2 = a_3^2 + c^2 - 2 a_3 c \cos\left(\theta_3 + \frac{\pi}{2} - \gamma\right)$$

$$\cos\left(\theta_3 + \frac{\pi}{2} - \gamma\right) = \frac{a_3^2 + c^2 - a_4^2}{2 a_3 c}$$

$$\tan^2\left(\frac{\theta_3 + \frac{\pi}{2} - \gamma}{2}\right) = \frac{1 - \cos\left(\theta_3 + \frac{\pi}{2} - \gamma\right)}{1 + \cos\left(\theta_3 + \frac{\pi}{2} - \gamma\right)}$$

$$\theta_3 = \gamma - \frac{\pi}{2} + 2\,\mathrm{atan2}\left(\sqrt{1 - \cos^2\left(\theta_3 + \frac{\pi}{2} - \gamma\right)},\ 1 + \cos\left(\theta_3 + \frac{\pi}{2} - \gamma\right)\right)$$
