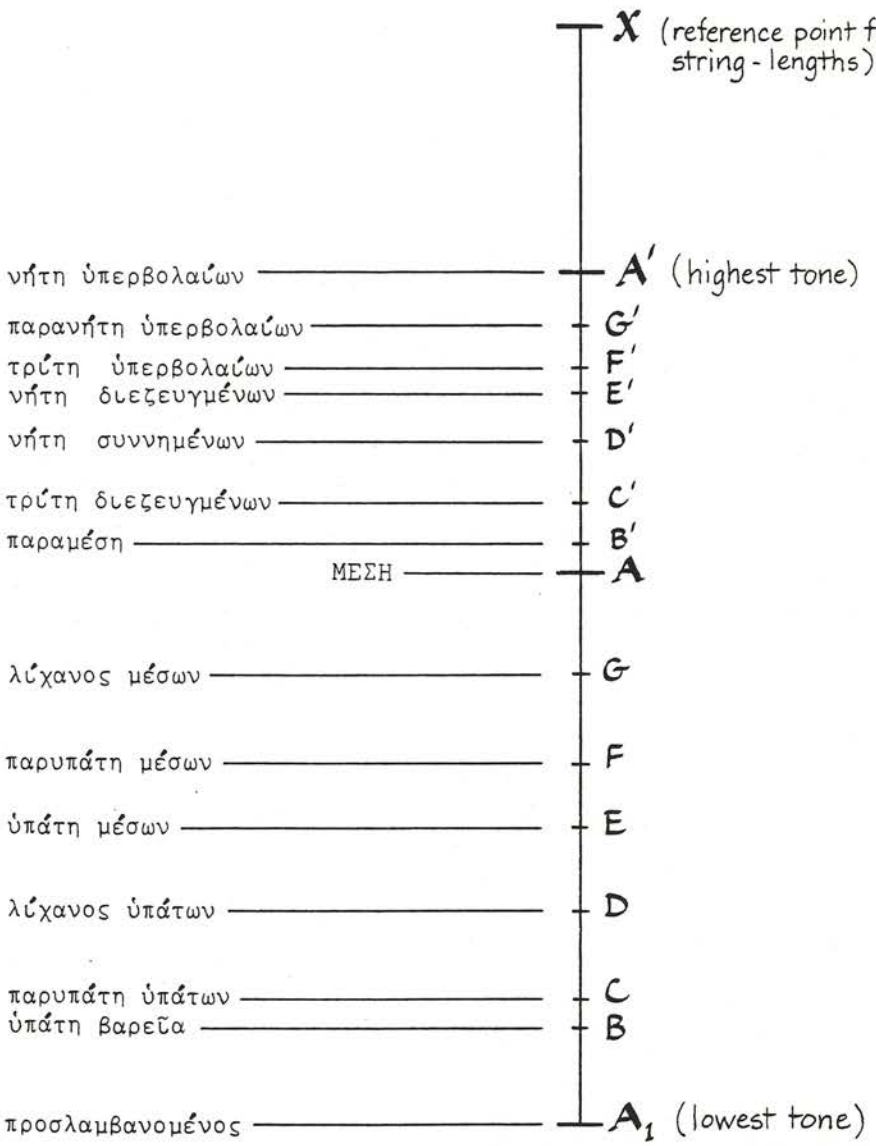
iii. Sectio Canonis: Cutting a String in Pythagorean Tuning

Euclid's book, the Sectio Canonis, ends with two propositions which build two octaves of the diatonic scale. Euclid's scale is Pythagorean. It is the cosmic scale of the Timaeus, the scale in which there are only whole tones of 9:8 and semitones of 256:243.

You might like to see what Euclid's scale looks like:

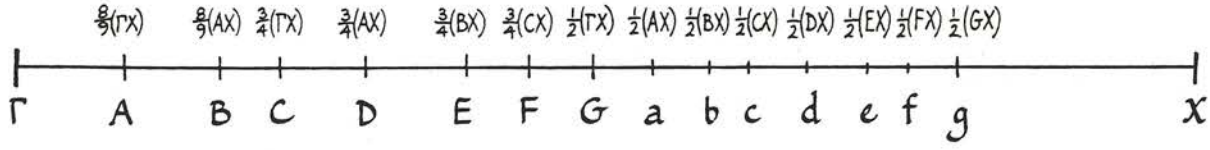


You needn't worry about the Greek names for all the tones. Note simply that the μέση or mean is the exact center of the system. I have labeled Euclid's line using letter names. As you can see, the tone of the μέση is the same for the highest and lowest tones of the scale as well. According to my lettering, Euclid's scale represents two octaves in the Greek Dorian mode.

Euclid makes it quite clear that his propositions on the scale are practical. He refers to the length of the canon or measuring rod and to the length of the string (χορδῆς). In this part of our study, we will follow Euclid's lead. We will actually construct a Pythagorean scale of two octaves on the monochord or sonometer. Instead of using Euclid's own method for dividing the string, we will employ a more direct method for achieving the same result. This method is described by the medieval musician, Odo of Cluny:

> At the first end-piece of the monochord. . . place the letter Γ, that is, a Greek G. . .Carefully divide the distance from Γ to the pointplaced at the other end into nine parts, and where the first ninth from Γ ends, write the letter A; we shall call this the first step. Then similarly, divide the distance from the first letter, A, to the end into nine, and at the first ninth, place the letter B for the second step. Then return to the beginning, divide by four from Γ , and for the third step write the letter C. From the first letter, A, divide similarly by four, and for the fourth step, write the letter D. In the same way, dividing B by four, you will find the fifth step, E. The third letter, C, likewise reveals the sixth step, F. Then return to Γ , and from it and from the other letters that follow it in order, divide the line in two parts, that is, in the middle, until, without Γ , you have fourteen or fifteen steps.
>
> When you divide the sounds in the middle, you must mark them differently. For example, when you bisect the distance from Γ , instead of Γ , write G; for A bisected, set down a second a; for B, a second b; for C, a second c . . .

Here is a picture of Odo's monochord:



X marks that end of the monochord from which measurements are taken. If we begin Odo's scale with the tone produced by the whole string, with Γ , then the two octaves center around G as μέση. The same letter name belongs to the lowest tone (Γ) and the highest tone (g) of the scale.

Exercise 14. You might want to convince yourself that Odo's monochord is truly Pythagorean. You might want to show, for example, that the interval between B and C is indeed a Pythagorean semitone whose ratio is 256:243.

Exercise 15. The sonometer consists of one or more strings stretched above a smooth "fingerboard" between a "bridge" at one end and a "nut" at the other. Beyond the nut is a tuning peg to adjust the tension of the string. Different pitches are obtained by "stopping" or pressing the string down against the fingerboard with a finger or thumb nail or the edge of a ruler and plucking the string gently near the bridge.

First slide a piece of tape under the string and smooth it out (the tape should be as long as the fingerboard). All marks should be made on this tape, not on the wood. Mark a point on this tape near the nut (90 or 81 cm from the bridge are convenient lengths) and label it Γ . Listen to the tone determined by stopping the string at Γ and determine other points as well as you can by ear that produce octaves of this tone. Try also to determine stopping points for a fifth and a fourth above Γ by ear.

Exercise 16. Tune a scale two octave's long by Odo's method. Try out some melodies with this scale, using only the positions marked out on the tape and labeled with the letter names A, B, C, etc.*

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* The description of Odo's method and the two exercises using the monochord are taken from Freshman Chorus Notes by David Stephenson. An instructive passage is quoted on p. 33 of the freshman manual: "The monochord was chiefly invented for this purpose, to be judge of Musical voices and intervals also to try whether the song be true or false furthermore, to shew haire-braind false Musitians their errors, and the way of attaining the truth. Lastly, that children which desire to learne Musicke, may have an easie meanes to it, that it may intice beginners, direct those that be forward, and make of unlearned learned." The passage is quoted from a sixteenth century Compendium of Musical Practise by Andreas Ornithoparcus, translated by John Dowland.