

CS 559 HW1

1.

Utility of winning given that you bet is final amount in bank for win (W) minus the original amount (B)

$$U(\text{win} \mid \text{bet}) = W - B$$

Utility of losing given that you bet is final amount in bank for loss (L) minus the original amount (B)

$$U(\text{loss} \mid \text{bet}) = L - B$$

So net utility is shown below

$$U(\text{bet}) = P_W * U(\text{win}, \text{bet}) + P_L * U(\text{lose}, \text{bet})$$

$$= P_W * (W - B) + (1 - P_W) * (L - B)$$

One must maximize utility such that $U(\text{bet}) > 0$ so one is very likely to win

$$U(\text{bet}) > 0 \rightarrow P_W * (W - B) + (1 - P_W) * (L - B) > 0$$

$$P_W * W - P_W * B + L - B - P_W * L + P_W * B > 0$$

$$P_W * W - P_W * L + L - B > 0$$

$$P_W * W - P_W * L > B - L$$

$$P_W * (W - L) > B - L$$

$$P_W > (B - L) / (W - L)$$

So as long as probability is greater than $(B - L) / (W - L)$ one should bet

2.

For the green wallet, the probability to pull a dime followed by two pennies = $(4/10) * (6/9) * (5/8) = 1/6$

For the black wallet, the probability pulled a dime followed by two pennies = $(2/10) * (8/9) * (7/8) = 7/45$

Given that the probability to use green wallet is $4/5$ and black wallet is $1/5$ from past experience

For the green wallet, the P=

$$(4/5 * 1/6) / (4/5 * 1/6) + (1/5 * 7/45) = 2/15 / (2/15 + 7/225) = 2/15 / 37/225 = 30/37 = .8108$$

For the black wallet, the P=

$$(1/5 * 7/45) / (4/5 * 1/6) + (1/5 * 7/45) = 7/225 / (2/15 + 7/225) = 7/225 / 37/225 = 7/37 = .1891$$

Given that $P(\text{error} | \text{wallet used}) = \min (P(\text{green, coins}) , P(\text{black, coin}))$

Probability that you are wrong is the probably you pick black walleyed .1891

3.

We know that sum of independent Gaussian is $Z = X+Y \rightarrow N(\text{Mean } x+\text{Mean } y, \text{Variance } x + \text{Variance } y)$

Using the mean and instance of both function the expected mean should be

$$N1*\text{Mean1}+N2*\text{Mean2}/\text{Net } N = (1*2000+4*1000) / (2000+1000) = 2$$

The combine variance should be $\text{sum}(\text{all value} - 2)^2 / 3000$

The combine variance can be expressed by $n1*\text{variance1}^2+n2*\text{variance2}^2+n1*(\text{mean1}-\text{mean3})^2+n2*(\text{mean2}-\text{mean3})^2$

$$\text{Resulting in } 8000+9000+2000+4000/3000 = 7.66666$$

Pretty close to the result of python