Begin with:

$$v(x,t) = v_0(x) + u(t)w(x) \tag{1}$$

Require a fixed point at $v(x,t)\Big|_{x=v_ct} = v_c$ so that

$$v_c = v_0(x) + u(t)w(x)\Big|_{x=v_ct} = v_0(v_ct) + u(t)w(v_ct) \quad .$$

Now we take $\frac{d}{dt}$ of both sides:

$$0 = \left(\frac{\delta v_0(x)}{\delta x}\dot{x} + \dot{u}(t)w(x) + u(t)\frac{\delta w(x)}{\delta x}\dot{x}\right)\Big|_{x=v_ct}$$

$$0 = v_c\left(\frac{\delta v_0(x)}{\delta x} + u(t)\frac{\delta w(x)}{\delta x}\right)\Big|_{x=v_ct} + \dot{u}(t)w(v_ct) \tag{2}$$

Separately, we take $\frac{\delta}{\delta x}$ of (1) and evaluate at $x = v_ct$, yielding

$$\frac{\delta v(x,t)}{\delta x}\Big|_{x=v_ct} = \left(\frac{\delta v_0(x)}{\delta x} + u(t)\frac{\delta w(x)}{\delta x}\right)\Big|_{x=v_ct} \quad . \tag{3}$$

For the fixed velocity attractor to be stable, we require

$$\frac{\delta v(x,t)}{\delta x}\Big|_{x=v_ct} < 0 \quad . \tag{4}$$

Combining (2) and (3) yields

$$-v_c\frac{\delta v(x,t)}{\delta x}\Big|_{x=v_ct} = \dot{u}(t)w(v_ct) \quad .$$

We require that $v_c > 0$ and use (4) to determine:

$$\dot{u}(t)w(v_ct) > 0$$

which can always be made true with choice of $w(x)$

$$w(x) \propto \dot{u}\left(\frac{x}{v_c}\right)$$