1. (a)
$$1 = \int_0^1 dx \int_0^x dy A(x+y) = A\int_0^1 dx \frac{3}{2}x^2 = \frac{A}{2} \Longrightarrow A = 2.$$
$$m_{r,s} = E[X^r Y^s] = \int_0^1 dx \int_0^x dy x^r y^s 2(x+y) = 2\int_0^1 dx x^{r+s+2}(\frac{1}{s+1}+\frac{1}{s+2}) = \frac{2(2s+3)}{(r+s+3)(s+1)(s+2)}.$$

(b) $m_X = m_{1,0} = 3/4$, $m_Y = m_{0,1} = 5/12$, $E[X^2] = m_{2,0} = 3/5$, $E[Y^2] = m_{0,2} = 7/30$, $E[XY] = m_{1,1} = 1/3$, $\sigma_X^2 = 3/5 - (3/4)^2 = 3/80$, $\sigma_Y^2 = 7/30 - (5/12)^2 = 43/720$, $C[X,Y] = 1/3 - (3/4)(5/12) = 1/48$,
$$\rho_{XY} = \frac{C[X,Y]}{\sigma_X \sigma_Y} = \frac{1/48}{\sqrt{3/80}\sqrt{43/720}} = \frac{5}{\sqrt{129}} = 0.44$$

(c) $c(x) = E[Y|x] = \int_{-\infty}^{\infty} y f(y|x) dy$. $f(x) = \int_0^x 2(x+y)dy = 3x^2$ si $x \in [0,1]$, $f(y|x) = \frac{f(x,y)}{f(x)} = \frac{2(x+y)}{3x^2}$ si $y \in [0,x]$.
$$c(x) = \int_0^x y\frac{2(x+y)}{3x^2} dy = \frac{5}{9}x.$$

2. Definim $Z = X_{n+1} - X_1 - X_2 + \cdots - X_n$. Així, volem $P(Z > 0) = 1 - F_Z(0)$. Z és normal amb paràmetres $m_Z = E[X_{n+1}] - E[X_1] - E[X_2] + \cdots - E[X_n] = 1 - n$ i $\sigma_Z^2 = \sigma_{X_{n+1}}^2 + \sigma_{X_1}^2 + \sigma_{X_2}^2 + \cdots + \sigma_{X_n}^2 = (n+1)2$. Llavors
$$P(Z > 0) = 1 - \frac{1}{2}(1 + \mathrm{erf}(\frac{0-(1-n)}{\sqrt{2}\sqrt{2(n+1)}})) = \frac{1}{2}(1 - \mathrm{erf}(\frac{n-1}{2\sqrt{n+1}})).$$

3. (a)
$$P(Y > X) = 1 - P(Y < X) = 1 - \int_{-\infty}^{\infty} dx \int_{-\infty}^{x} dy f_X(x) f_Y(y)$$
$$= 1 - \int_{-\infty}^{\infty} dx f_X(x) F_Y(x) = 1 - E[F_Y(X)].$$

(b)
$$1 = P(Y > X) + P(X < Y) = 1 - E[F_Y(X)] + 1 - E[F_X(Y)]$$
$$\Longrightarrow E[F_Y(X)] + E[F_X(Y)] = 1 \Longrightarrow E[Z] = 1$$

(c) $Z = X - Y = X + (-Y)$. Llavors $f_Z = f_X * f_{-Y}$. Com $f_{-Y}(y) = f_Y(-y)$
$$f_Z(z) = \int_{-\infty}^{\infty} f_X(x) f_Y(x-z).$$
Si X i Y són idèntiques, $f_Z(-z) = f_Z(z)$. Per $z > 0$
$$f_Z(z) = \int_z^{\infty} \lambda e^{-\lambda x} \lambda e^{-\lambda(x-z)} = \frac{\lambda}{2} e^{-\lambda z}.$$
Així, $f_Z(z) = \frac{\lambda}{2} e^{-\lambda |z|}$, $z \in R$.