27/12/07
Antenes

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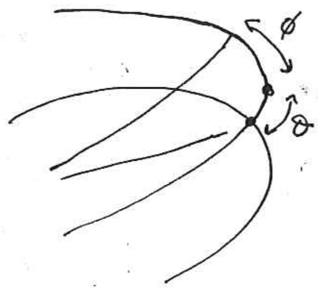
1. Introducció

$$\frac{D}{A_{ef}} = \frac{4\pi}{\lambda^2}$$



Directivitat: $D = \frac{4\pi}{\Omega_{eq}} = \frac{4\pi}{\Delta\theta_{-3dB}\,\Delta\phi_{-3dB}}$

$$\left[l_{ef}^2 = \frac{4R_r}{\eta_{=120\pi}} A_{ef} \right]$$

Distància de Fraunhoffer: $\boxed{r > \frac{2D^2}{\lambda}}$

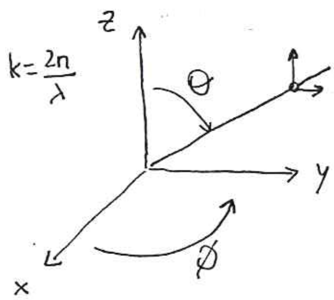
Treballem en pla llunyà.

$\vec{E} \perp \vec{H}$

$$\begin{pmatrix} \hat{r} \\ \hat{\theta} \\ \hat{\phi} \end{pmatrix} = \begin{pmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{pmatrix} \begin{pmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \end{pmatrix}$$

$$\begin{pmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \end{pmatrix} = \begin{pmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{pmatrix} \begin{pmatrix} \hat{r} \\ \hat{\theta} \\ \hat{\phi} \end{pmatrix}$$

$k = \frac{2\pi}{\lambda}$



$k\hat{r} \rightarrow$
$$\boxed{\begin{aligned} k_x &= k\sin\theta\cos\phi \\ k_y &= k\sin\theta\sin\phi \\ k_z &= k\cos\theta \end{aligned}}$$

$$\vec{A} = \frac{\mu}{4\pi}\frac{e^{-jkr}}{r}\vec{N}$$
$$\vec{N} = \int_{V'} \vec{J}(\vec{r}')\, e^{jk\hat{r}\cdot\vec{r}'}\, dv'$$

$\vec{N} \propto \vec{E}$

$$N_\theta = \vec{N}\hat{\theta} = N_x\cos\theta\cos\phi + N_y\cos\theta\sin\phi - N_z\sin\theta$$
$$N_\phi = -N_x\sin\phi + N_y\cos\phi$$

$$\vec{E}^r = j\omega\, \hat{r}\times(\hat{r}\times\vec{A})$$

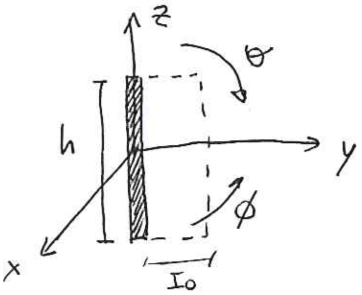
$\vec{N} = TF_{3D}(\vec{J})$

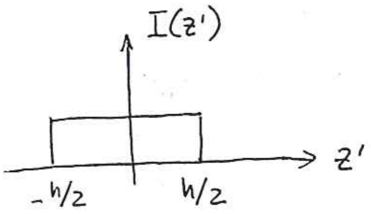
$$\boxed{E_\theta = -jk\eta\frac{e^{-jkr}}{4\pi r}N_\theta}$$

$$\vec{H}^r = -j\frac{\omega}{\eta}\left[\hat{r}\times\vec{A}\right]$$

2. Antenes bàsiques

- $\boxed{\text{Dipol elemental}}$ $(h << \lambda)$





$$I(z') = \begin{cases} 1 & |z'| \leq \frac{h}{2} \\ 0 & \text{altres} \end{cases}$$

$(N_x = N_y = 0)$ → només és ≠0 en la direcció del dipol

$N_\theta = -\sin\theta \, N_z$

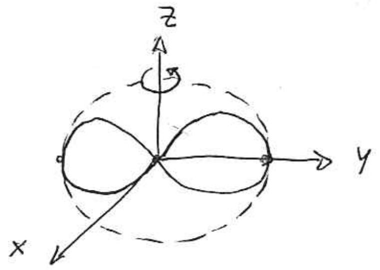
$k_z = k\cos\theta$

$$N_z = \int_{-h/2}^{h/2} I(z') \underbrace{e^{jk_z z'}}_{} dz' = \int_{-h/2}^{h/2} I(z')\, dz' = I_0 \cdot h$$

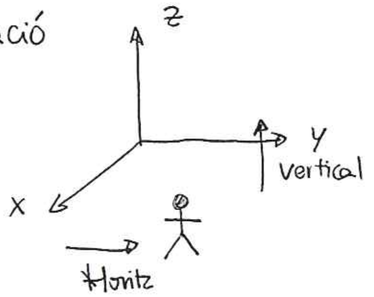
$$\frac{2\pi}{\lambda} z' \xrightarrow[z' << \lambda]{h << \lambda} \emptyset$$

$$\boxed{E_\theta = +jk\eta \frac{e^{-jkr}}{4\pi r} \sin\theta \, I_0 h}$$

Diagrama $\frac{|E|}{|E_{max}|} \Longrightarrow \sin\theta$



→ Polarització



$E = (\hat{x} + j\hat{y}) E_0$ circular

$E = (a\hat{x} + jb\hat{y}) E_0$ elíptica

$$R_r = \frac{P_r}{|I_0|^2}$$

↳ corrent d'alimentació

$$P_r = \int_0^{2\pi}\int_0^{\pi} S(\theta,\phi) \underbrace{r^2 \sin\theta \, d\theta \, d\phi}_{d\Omega}$$

$$S(\theta,\phi) = \frac{|E(\theta,\phi)|^2}{\eta}$$

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$$P_r = \int_0^{2\pi}\int_0^{\pi} \left(\frac{k\eta \sin\theta\, I_0 h}{4\pi r}\right)^2 \frac{1}{\eta} r^2 \sin\theta \, d\theta\, d\phi =$$

$$= \int_0^{2\pi}\int_0^{\pi} \sin^3\theta \, d\theta \, d\phi = cte \int_0^{\pi} \sin\theta \,(1-\cos^2\theta)\, d\theta = \dots = \frac{4}{3} \cdot cte$$

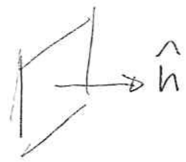
$$\boxed{R_r = 80\pi^2 \left(\frac{h}{\lambda}\right)^2}$$

$$D = \frac{\mathcal{S}_{max}}{P_r / 4\pi r^2} = \boxed{1{,}5}$$

Longitud efectiva

$$\boxed{l_{ef} = h}$$

- **Espira elemental** (dimensions $<< \lambda$)



$$\vec{N} \simeq jk\, \vec{m} \times \hat{r}$$

moment dipolar magnètic $\vec{m} = I \cdot Area \cdot \hat{n}$

$$E_\theta = -jk\eta \frac{e^{-jkr}}{4\pi r} N_\theta$$

$$E_\phi = -jk\eta \frac{e^{-jkr}}{4\pi r} N_\phi$$

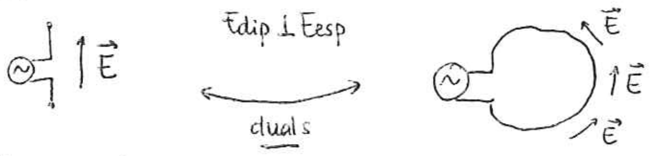
$$E_r = 0$$

$$t(\theta,\phi) = \frac{|\vec{E}(\theta,\phi)|^2}{|\vec{E}_{max}|^2}$$

Diagrama de potència

Diagrama de camp

$$\frac{|\vec{E}(\theta,\phi)|}{|\vec{E}_{max}|}$$

dipol $<< \lambda$		espira $<< \lambda$
	$E_{dip} \perp E_{esp}$ duals	
$\vec{E} = E_\theta \hat{\theta}$		$\vec{H} = H_\theta \hat{\theta}$
$\vec{H} = H_\phi \hat{\phi}$		$\vec{E} = E_\phi \hat{\phi}$
circuit obert		curtcircuit
$X_{in} < 0$		$X_{in} > 0$
capacitiu		inductiva

D = 1,5 (pels 2)

$R_r = 20\pi^2 (ka)^4$ perímetro

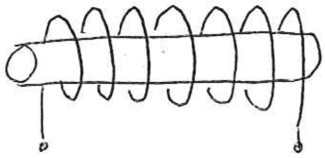
$R_r = 20\pi^2 \left(\frac{c}{\lambda}\right)^4$

$c = 2\pi a$

$k = \frac{2\pi}{\lambda}$

$R_{espira} << R_r$ dipol elemental

Bobines



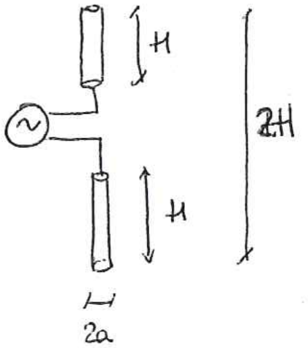
N voltes

$\vec{E} \sim \sin\theta$

$$\left[\mu_{ef} = \frac{\mu_r}{1 + D(\mu_r + 1)}\right]$$

$$\underline{R_{r\,N\,espires}} = \underline{N^2 \underbrace{\mu_{ef}^2}_{\sim 1000} R_{r_1\,espira}}$$

- Antenes cilíndriques (dipol)

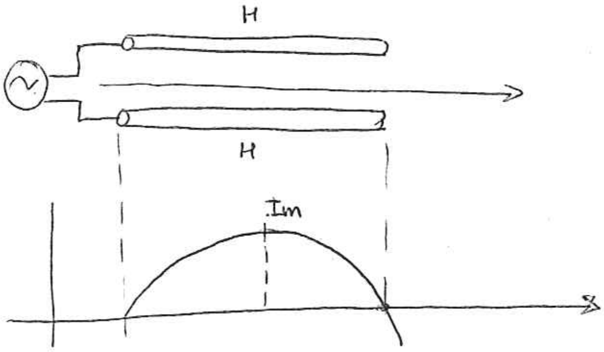


$a << H$

$a << \lambda$

$H \propto \lambda$

$I(z') = I_m \sin k(H - |z'|)$



$$\vec{N} = \int_{-H}^{H} \hat{z}\, I(z')\, e^{jk_z z'} dz$$

$$\boxed{l_{ef} \text{ dipol en } \frac{\lambda}{2} = \frac{\lambda}{\pi}}$$

$\rightarrow$ Dipol en z's:

$$N_z = \hat{z} \cdot 2k\, I_m \frac{\cos(k_z H) - \cos kH}{k^2 - k_z^2}$$

En x's:

$$N_x = \hat{x} \cdot 2k\, I_m \frac{\cos(k_x H) - \cos kH}{k^2 - k_x^2}$$

En y's:

$$N_y = \hat{y}\ 2k\, I_m \frac{\cos(k_y H) - \cos kH}{k^2 - k_y^2}$$

$$\vec{N} = \vec{N}_\theta \hat{\theta} = -\sin\theta \, N_z$$ $N_\theta = -\sin\theta \, N_z$

$$\vec{E}_\theta = jk\eta \frac{e^{-jkr}}{4\pi r} \sin\theta \, N_z$$

dipolo $\lambda/2 \to H = \frac{\lambda}{4} \Rightarrow \vec{N}_z = 2I_m \frac{\cos(\frac{\pi}{2}\cos\theta)}{k\,\mathrm{sen}^2\theta}$

$\cos\frac{\pi}{2}$

$$\vec{N}_z = \frac{2\cancel{k} I_m [\cos(k\cos\theta H) - \cos kH]}{\underbrace{k^2 - \underbrace{k^2\cos^2\theta}}_{k^{\cancel{2}}(\underbrace{1-\cos^2\theta}_{\mathrm{sen}^2\theta})}} = 2 I_m \frac{\cos(kH\cos\theta) - \cos kH}{k \cdot \mathrm{sen}^2\theta}$$

$$E_\theta = jk\eta \frac{e^{-jkr}}{4\pi r} \sin\theta \, I$$

$$\boxed{\vec{E}_\theta = j\,60\, I_m \frac{e^{-jkr}}{r} \frac{\cos(kH\cos\theta) - \cos(kH)}{\sin\theta}}$$ (en z)

En x:

$$\vec{N} = \hat{x} \cdot 2k I_m \frac{\cos(kH\sin\theta\cos\phi) - \cos kH}{k^2(1 - \sin^2\theta\cos^2\phi)}$$

En y:

$$\vec{N} = \hat{y}\, 2k I_m \frac{\cos(kH\sin\theta\sin\phi) - \cos kH}{k^2(1 - \sin^2\theta\sin^2\phi)}$$

Directivitat

$$D = \frac{P_{max}}{P_r/4\pi r}$$

$\frac{2H}{\lambda} \to 0 \longrightarrow \boxed{D = 1,5}$

$\Rightarrow 2H = \lambda/2 \longrightarrow \boxed{D = 1,64}$

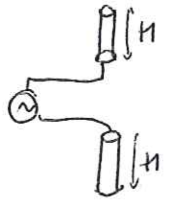
$2H = \frac{5\lambda}{4} \longrightarrow \boxed{D = 3,33} \Rightarrow$ màxima?

Dipol de $\lambda/2$

$$\boxed{Z = 73 + j\,42'5\ \Omega}$$

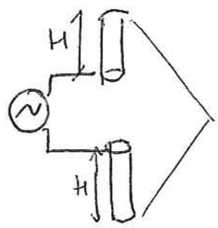
Dipol de $\lambda/2$ ressonant : $Z = 73\,\Omega$

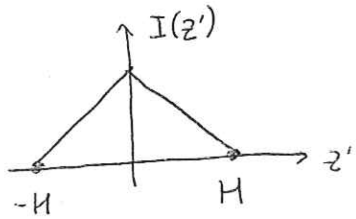
(realment és de $\lambda/2 - 5\%$
$\sim 0{,}48\lambda$)



- Dipol curt $\left(H < \frac{\lambda}{10}\right)$

$$I(z') = I_m \sin k\,(H - |z'|)$$





$$I(0) = I_m \sin kH \longrightarrow kH \to \frac{2\pi}{\lambda} H \to \emptyset$$

$$I(0) = I_m\, kH \longrightarrow I_m = \frac{I(0)}{kH}$$

$$I(z') = \frac{I(0)}{kH} \sin k(H - |z'|) \simeq \frac{I(0)}{\cancel{k}H} \cancel{k}(H - |z'|) = \boxed{I(0)\left[1 - \frac{|z'|}{H}\right]}$$

la fòrmula del corrent és triangular

$$\vec{E}_{curt} = \frac{1}{2} \vec{E}_{elemental}$$

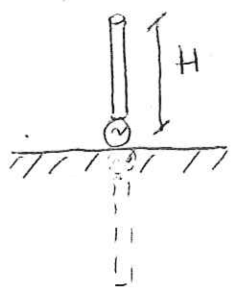
$$\text{Diagrama de radiació} = \left| \frac{\vec{E}(\theta,\phi)}{\vec{E}_{max}} \right| = \sin\theta$$

$D = 1{,}5$

→ $R_{r\,curt} = \frac{1}{4} R_{r\,elemental} = 20\pi^2 \left(\frac{2H}{\lambda}\right)^2$

→ $l_{ef\,curt} = \frac{1}{2} l_{ef\,elemental} = H$

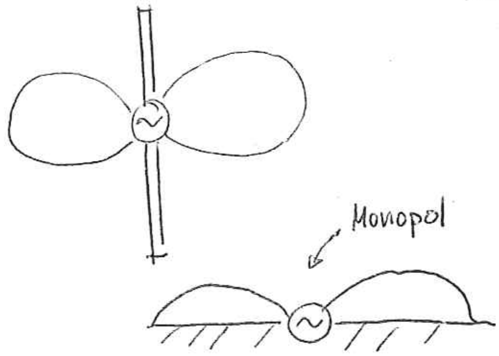
◦ Monopol



Monopol	Dipol equivalent (no l'elemental, qualsevol dels altres)
I_m	$I_d = I_m$
V_m	$V_d = 2V_m \rightarrow V_m = \frac{V_d}{2}$

$$E_m : \begin{cases} \vec{E}_{superior} \\ \vec{E}_{inferior} = \phi \end{cases}$$

$$Z_{ind} = 2\, Z_{in\ monopol} \Rightarrow Z_{monopol} = \frac{Z_{dip}}{2}$$



$$\begin{cases} R_{rd\,monopol} = \frac{1}{2} R_{rdipol} \\ P_{r\,monopol} = \frac{1}{2} P_{rdipol} \end{cases}$$

$$D_{monopol} = 2 D_{dipol}$$

$$\boxed{l_{ef\,m} = \frac{1}{2}\, l_{ef\,dipol}}$$

$$A_{ef\,m} = \frac{1}{2} A_{ef\,dipol}$$

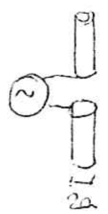
→ Mai calculem el MONOPOL !

$$\left[D = \frac{4\pi}{\lambda^2} \underline{\underline{A_{ef}}} \right]$$

→ Dipol elemental ($h \ll \lambda$)

$$\boxed{\begin{matrix} E \propto \sin\theta \\ D = 1.5 \end{matrix}}$$

→ Antena cilíndrica (dipol)



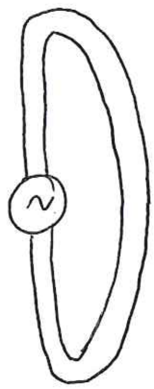
$H \propto \lambda$

Per calcular els paràmetres del monopol:

$$\text{Monopol} \xrightarrow{\text{taula monopol}} \text{Curt}$$
$$\downarrow$$
$$\text{Dipol elemental}$$

→ Espira elemental (dimensions $\ll \lambda$)

◦ Dipol doblegat



$Z_{in} = 4\, Z_{dipol}$

$\boxed{R_r = 4\, R_{r\,dipol}} \approx 300\,\Omega$

Dipol equivalent:

$\vec{E} = 2\, \vec{E}_{dipol\ equivalent}$

$l_{ef} = 2\, l_{ef\ dipol\ equivalent}$

BW és + gran

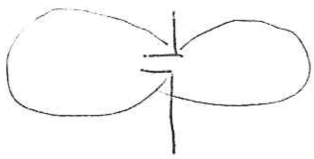
$D = D_{equivalent}$

Diagrama (forma) és igual.

3. Teoria d'imatges i Tª Reciprocitat

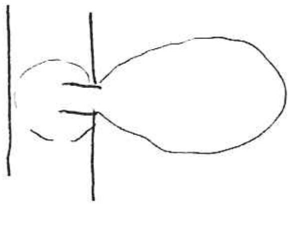
elements que hi afegim són paràsits i no estan alimentats.

$l_{paràsit} < l_{dipol}$

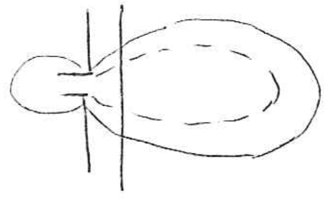


$l_{paràsit} > l_{dipol}$

paràsit reflector



paràsit director



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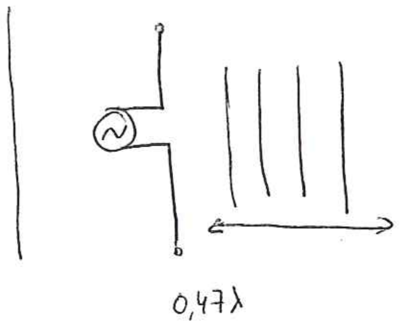
Antena Yagi



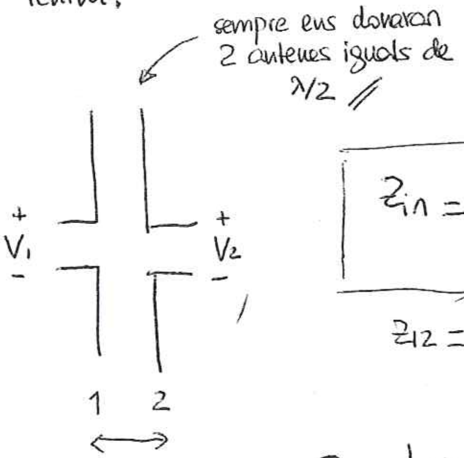
NLPS dolent
BW bo

$G_{\lambda/2} \sim \underline{5\text{-}18\,\mathrm{dB's}}$

$G \simeq D \cdot \eta_\Omega$

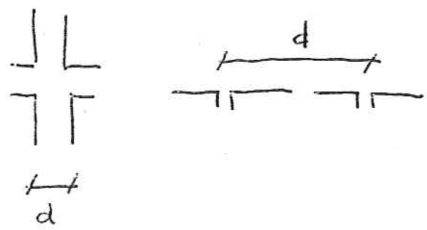
$\eta_\Omega = \dfrac{R_r}{R_r + R_\Omega}$

Tenim:



$$\boxed{Z_{in} = Z_{11} - \frac{Z_{12}\,Z_{21}}{Z_{22} + Z_2}}$$

$Z_{12} = Z_{21}$

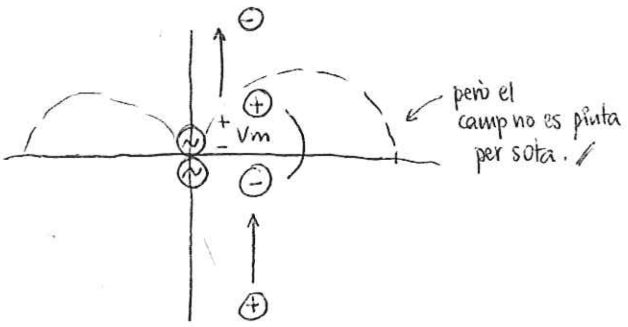


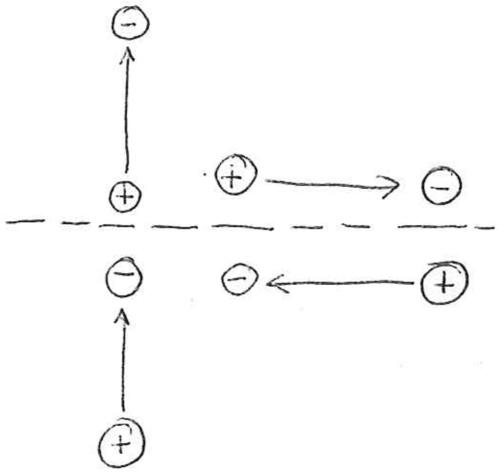
Si $d \to \infty$ $\quad Z_{IN} = Z_{11}$

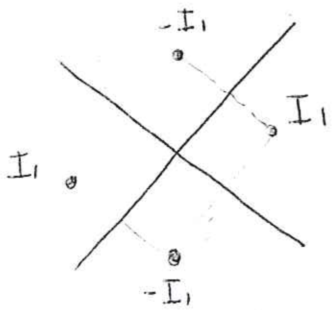
$d \to 0$ $\quad Z_{IN} = 0$

Teoria d'Imatges

Simetria però oposició de fase







(4.1) $f = 300\ \text{kHz} \longrightarrow \lambda = 1\ \text{km}$

$\left.\begin{array}{l} l_1 = 1{,}1\ m \\ l_2 = 0'8\ m \end{array}\right\}$ espira elemental

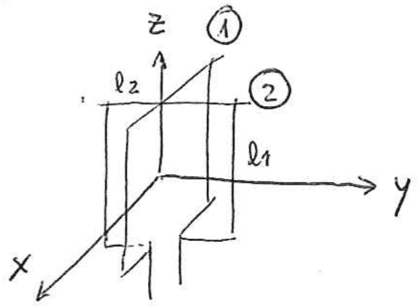


Diagrama de Radiació

Area = $S = l_1 \cdot l_2 = \underline{1{,}1 \cdot 0{,}8}$

$$\left[\begin{array}{l} \vec{N} = jk\,\vec{m} \times \hat{r} \\ \vec{m} = I \cdot \underbrace{Area}_{S} \cdot \hat{n} \end{array}\right]$$

Espira 1 (z-x) $\rightarrow \vec{N}_1 = jkIS\,\underline{\hat{y}} \times \hat{r}$

Espira 2 (z-y) $\rightarrow \vec{N}_2 = jkIS\,\underline{\hat{x}} \times \hat{r}$

$$\vec{N} = \vec{N}_1 + \vec{N}_2 = jkIS\left((\hat{x}+\hat{y}) \times \hat{r} \right)$$

$$(\hat{x}+\hat{y}) \times \hat{r} = \begin{pmatrix} \hat{r} & \hat{\theta} & \hat{\phi} \\ \sin\theta(\cos\phi+\sin\phi) & \cos\theta(\cos\phi+\sin\phi) & \cos\phi - \sin\phi \\ 1 & 0 & 0 \end{pmatrix}$$

$$\vec{N} = \text{ctes.}\left[(\cos\phi - \sin\phi)\hat{\theta} - \cos\theta(\cos\phi + \sin\phi)\hat{\phi} \right]$$

$$\vec{N}_\theta = jkSI(\cos\phi - \sin\phi)$$

$$\vec{N}_\phi = -jkSI\cos\theta(\cos\phi + \sin\phi)$$

Diagrama de rad de potencia

$$t(\theta,\phi) = \frac{|E(\theta,\phi)|^2}{|E_{max}|^2} \propto \frac{|N(\theta,\phi)|^2}{|N_{max}|^2} = \frac{|N_\theta|^2 + |N_\phi|^2}{|N_{max}|^2}$$

$$|N_\theta|^2 = k^2 S^2 I^2 (\cos\phi - \sin\phi)^2$$

$$|N_\phi|^2 = \underbrace{k^2 S^2 I^2}_{cte} \cos^2\theta\,(\cos\phi + \sin\theta)^2$$

$$|N(\theta,\phi)|^2 = cte\left[(1 - 2\cos\phi\sin\phi) + \cos^2\theta\,(1 + 2\cos\phi\sin\phi) \right] =$$

$$= cte\left[(1 - \sin 2\phi) + \cos^2\theta\,(1 + \sin 2\phi) \right] =$$

$$= \left[(\cos^2\theta + 1) + (\cos^2\theta - 1)\sin 2\phi \right] = 2 \qquad (\theta = 0, \pi)$$

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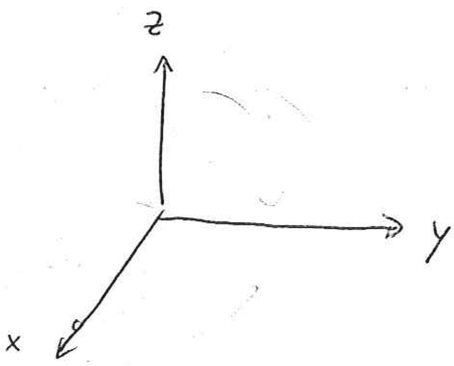
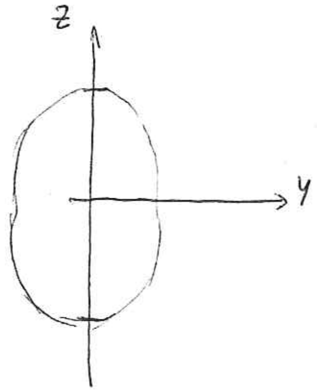
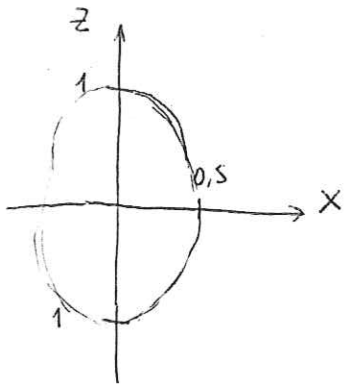
$$t \propto \frac{2 - \sin^2\theta\,(1+\sin 2\phi)}{2}$$

$$\propto 1 - \frac{\sin^2\theta\,(1+\sin 2\phi)}{2}$$





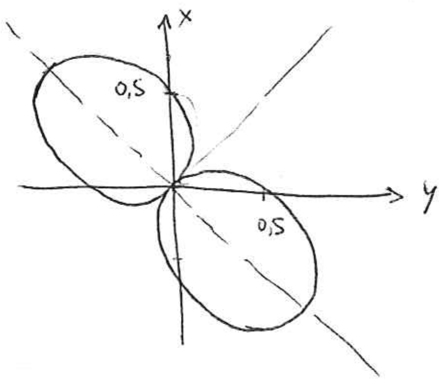


$$\phi = 0$$

$$t \propto 1 - \frac{\sin^2\theta}{2} = \frac{1+\cos^2\theta}{2}$$

X - Y

$$\theta = \frac{\pi}{2}$$



$$\frac{2-(1+\sin 2\phi)}{2} = \frac{1 - \sin 2\phi}{2}$$

$$\phi = \frac{\pi}{4} \rightarrow 0$$

$$2\phi = \frac{3\pi}{2} \rightarrow \phi = \frac{3\pi}{4} \quad \text{Máx}$$

$$D = \frac{4\pi}{\int_0^{2\pi}\int_0^{\pi} t(\theta,\phi)\sin\theta\, d\theta\, d\phi} \Rightarrow \int_0^{2\pi}\int_0^{\pi} \frac{1}{2}\left[2 - \sin^2\theta\,(1+\sin 2\phi)\right]\sin\theta\, d\theta\, d\phi$$

$$= \frac{1}{\cancel{2}} \cdot 2\pi \underbrace{\int_0^\pi \cancel{2} \sin\theta \, d\theta}_{2} - \frac{1}{2} \int_0^{2\pi} \int_0^\pi -\sin^3\theta \, (1+\underbrace{\sin 2\phi}_{\downarrow}) \, d\theta \, d\phi$$

$$= 4\pi - \frac{1}{2} 2\pi \int_0^\pi \sin^3\theta \, d\theta = 4\pi - \pi \int_0^\pi \qquad \int_0^{2\pi} \sin 2\phi = \emptyset$$

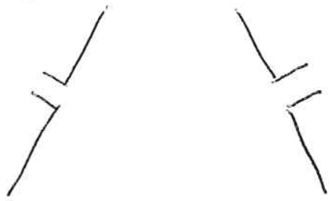
$$= 4\pi - \pi \int_0^\pi \sin\theta \, (1 - \cos^2\theta) \, d\theta$$

$$= 4\pi - \pi \left[-\cos\theta \right]_0^\pi - \pi \left[-\frac{\cos^3\theta}{3} \right]_0^\pi =$$

$$= 4\pi - 2\pi - \pi \left[\frac{1}{3} + \frac{1}{3} \right] = 4\pi - 2\pi + \frac{2\pi}{3} =$$

$$= 2\pi + \frac{2\pi}{3} = \frac{8\pi}{3} \qquad \rightarrow D = \frac{4\pi}{8\pi/3} = \frac{3}{2} = 1{,}5$$

Coef. de desacoblament de polarització



$$C_p = \frac{P_R}{P_{R\,MAX}} = \frac{V_{ca}^2}{V_{ca\,máx}^2} = \frac{|\vec{l}_{ef} \, \vec{E}^i|^2}{|\vec{l}_{ef} \, \vec{E}^i|^2_{max}}$$

$\vec{l}_{ef} = l_{ef} \cdot \hat{e}_t \rightarrow$ polarització de l'antena

$\vec{E}^i = E_0 \, \hat{e}_r \rightarrow$ polarització del camp incident

$$C_p = \frac{|\cancel{l_{ef}} \cdot \hat{e}_t \cdot \cancel{E_0} \cdot \hat{e}_r|^2}{|\cancel{l_{ef}} \cdot \underbrace{\hat{e}_t \cdot \cancel{E_0} \, \hat{e}_r}_{1}|^2_{máx}} = \boxed{|\hat{e}_r \cdot \hat{e}_t|^2}$$

$$l_{ef} = \int_{-h/2}^{h/2} \frac{I(z')}{I(0)} \, dz'$$

Per al dipol de $\frac{\lambda}{2}$ $\rightarrow$ $l_{ef} = \frac{1}{I(0)} \int_{-\lambda/4}^{\lambda/4} I(0) \cos(kz') \, dz = \frac{\lambda}{\pi}$

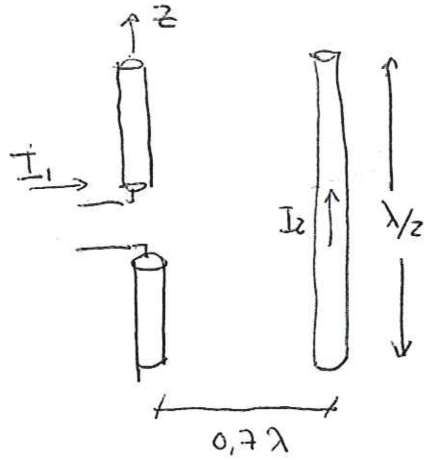
G04

27/12/07
Antenes

Problema 1

$Z_{11} = 73\Omega$

$Z_{12} = -25\Omega$



$Z_{12} = Z_{21}$

$Z_{11} = Z_{22}$ (el 2 tb és ressonant)

a) I_2/I_1

$$\hookrightarrow \boxed{\begin{aligned} V_1 &= Z_{11} I_1 + Z_{12} I_2 \\ V_2 &= Z_{21} I_1 + Z_{22} I_2 \end{aligned}} \longrightarrow Z_{21} I_1 = -Z_{22} I_2$$

$V_2 = 0$ (curtocircuitada)

$$\frac{I_2}{I_1} = \frac{-Z_{21}}{Z_{22}} = \frac{25}{73} = \boxed{0'34}$$

$$Z_{in} = \frac{V_1}{I_1} = \frac{Z_{11} I_1 + Z_{12} I_2}{I_1} = Z_{11} + Z_{12}\frac{I_2}{I_1} = 73 + 0{,}34(-25) = \boxed{64'5\Omega}$$

b) $H \propto \lambda \longrightarrow$ antena cilíndrica

un dipol situat a l'origen a l'eix z (apunts)

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = E_0 \left(1 + 0{,}34\, e^{jk_x 0{,}7\lambda}\right) = (*)$$

$$\left[\begin{aligned} E_1 &= j60 I_1 \frac{e^{-jkr}}{r} \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} \\ E_2 &= j60 I_2 \frac{e^{-jk}}{r} \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} \cdot e^{jk_x 0{,}7\lambda} \end{aligned}\right]$$

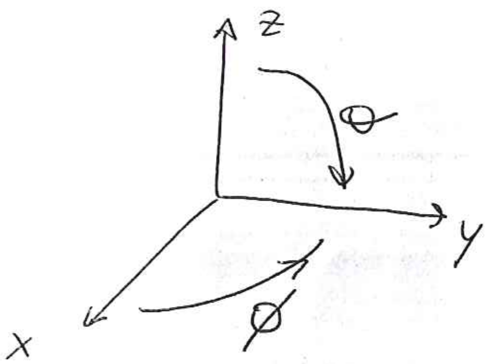
$$(*) = E_0\left(1 + 0'34 \cos(k_x\, 0{,}7\lambda) + j\, 0'34 \sin(k_x\, 0'7\lambda)\right)$$

$$E_\theta \rightarrow E_0\left(1 + 0{,}34 \cos\left(\frac{2\pi}{\lambda} 0{,}7\lambda \sin\theta\cos\phi\right) + j\, 0{,}34 \sin\left(\frac{2\pi}{\lambda} 0{,}7\lambda \sin\theta \sin\phi\right)\right)$$

$$t(\theta,\phi) = \frac{|E|^2}{|E_{max}|^2} = \frac{E_0^2\left[(1 + 0{,}34\cos k_x d)^2 + (0{,}34 \sin k_x d)^2\right]}{E_{max}^2} =$$

$$= \left[\left(1 + 0'34\cos(kd\sin\theta\cos\phi)\right)^2 + \left(0{,}34\sin(kd\sin\theta\cos\phi)\right)^2\right] \frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta}$$

$$\left.\begin{array}{l}\theta = \frac{\pi}{2} \\ \phi = \pi/2\end{array}\right\} (1+0{,}34)^2 = \underline{\underline{1{,}34^2}}$$

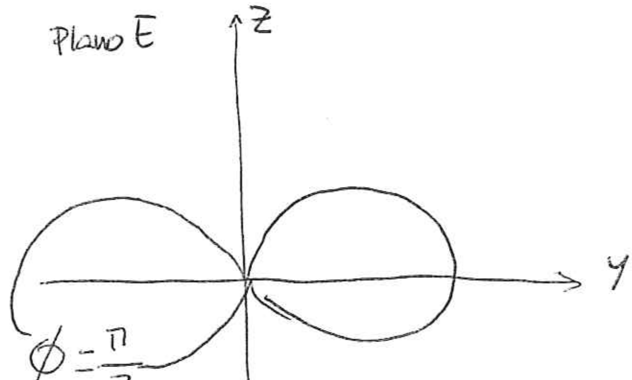


$\boxed{\text{Pla } \vec{E}}$ $(\cancel{x}\ z - y)$

$\underline{\underline{\text{intensitat}}}$ — $\underline{\underline{\text{màxim}}}$

$\boxed{\text{Pla } \vec{H}} \rightarrow$ (Maxim - el que queda)

$(y - x)$



$\phi = \frac{\pi}{2}$

$$\hookrightarrow t(\theta) = (1+0{,}34)^2 \frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta}$$

c) Directividad

$$D = \frac{P_{max}}{\frac{P_r}{4\pi r^2}} = \frac{\frac{60^2 \cancel{I_1^2}}{\cancel{r^2}}(1'34)^2 / 120\pi}{\cancel{I_1^2}\, 81'43 / 4\pi \cancel{r^2}} = 2{,}646$$

$\downarrow$

$10\log \ldots =$

$\boxed{4{,}23 \text{ dB}}$

$$P_{max} = \frac{|E|^2_{max}}{\eta}$$

$$P_r = I_1^2 R_1 + I_2^2 R_2 = I_1^2 (73 + 0{,}34^2 \cdot 73) = \underline{\underline{81'43\, I_1^2}}$$

Recordatori:

27/12/07
Antenas

$\sin^2 a = \frac{1}{2}(1-\cos 2a)$

$\cos^2 a = \frac{1}{2}(1+\cos 2a)$

$\cos^3 a = \frac{1}{4}(3\cos a + \cos 3a)$

$\sin 2a = 2\sin a \cos a$

$\cos 2a = \cos^2 a - \sin^2 a$

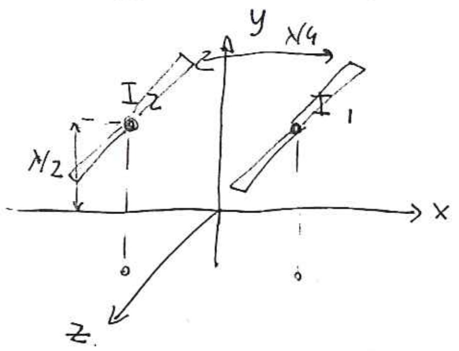
$\sin(a+b) = \sin a \cos b + \cos a \sin b$

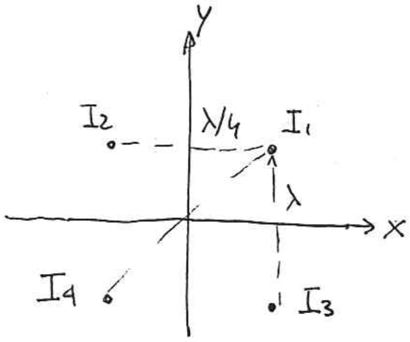
$\cos(a+b) = \cos a \cos b - \sin a \sin b$

$\sin(a-b) = \sin a \cos b - \cos a \sin b$

$\cos(a-b) = \cos a \cos b + \sin a \sin b$

dos (1)





$I_2 = j I_1$

$I_3 = -I_1$

$I_4 = -j I_1$

$Z_{IN_1} = \frac{V_1}{I_1} \longrightarrow V_1 = Z_{11} I_1 + Z_{12} I_2 + Z_{13} I_3 + Z_{14} I_4$

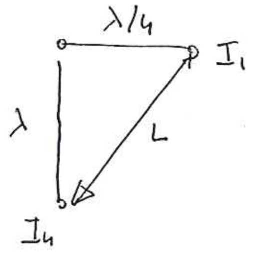
$\Rightarrow Z_{IN_1} = Z_{11} + j Z_{12} - Z_{13} - j Z_{14}$

$\boxed{Z_{11} = 73 + j42}$

Gràfica $\rightarrow$ $Z_{12} = 40 - j30$

$Z_{13} = 6 + j18$

$Z_{14} = Z_{13} = 6 + j18$



$L^2 = \lambda^2 + \left(\frac{\lambda}{4}\right)^2 = \frac{17}{16}\lambda^2$

$L = 1{,}03\lambda \approx \lambda$

$Z_{IN_2} = (Z_{21} I_1 + Z_{22} I_2 + Z_{23} I_3 + Z_{24} I_4)/I_2$

$\Rightarrow Z_{IN_2} = -j Z_{21} + Z_{22} + j Z_{23} - Z_{24}$ //

$Z_{21} = Z_{12}$

$Z_{24} = Z_{13}$

$Z_{23} = Z_{14} = Z_{24}$

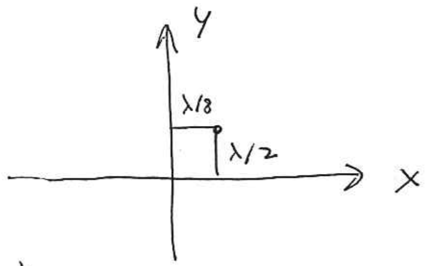
$Z_{22} = Z_{11}$

$Z_{IN_1} = 73 + j42 + j40 + 30 - 6 - j18 - 6j + 18$

$\boxed{Z_{in_1} = 115 + j58}$

$Z_{IN_2} = -j40 - 30 + 73 + j42 + j6 - 18 - 6 - j18$ $\boxed{Z_{in_2} = 19 - j10}$

b) $\vec{E} = E_0 \Big(e^{jk_x\frac{\lambda}{8}} e^{jk_y\frac{\lambda}{2}} +$



$+ j\, e^{jk_y\frac{\lambda}{2}} e^{-jk_x\frac{\lambda}{8}}$

$- e^{jk_x\frac{\lambda}{8}} e^{-jk_y\frac{\lambda}{2}} - j\, e^{-jk_x\frac{\lambda}{8}} e^{-jk_y\frac{\lambda}{2}} \Big)$

$$= E_0 \Big(e^{jk_x\frac{\lambda}{8}} \big(\underbrace{e^{jk_y\frac{\lambda}{2}} - e^{-jk_y\frac{\lambda}{2}}}_{2j\sin k_y\frac{\lambda}{2}}\big) + j\, e^{-jk_x\frac{\lambda}{8}} \big(\underbrace{e^{jk_y\frac{\lambda}{2}} - e^{-jk_y\frac{\lambda}{2}}}_{2j\sin k_y\frac{\lambda}{2}}\big)\Big) =$$

$$= E_0 \Big[e^{jk_x\frac{\lambda}{8}}\, 2j \sin k_y\frac{\lambda}{2} + j\, e^{-jk_x\frac{\lambda}{8}}\, 2j \sin k_y \lambda/8 \Big] =$$

$$= E_0 \cdot 2 \sin k_y\frac{\lambda}{2} \Big[\underset{\downarrow\, e^{j\pi/2}}{j} e^{jk_x\lambda/8} - e^{-jk_x\lambda/8} \Big] =$$

$$= E_0 \cdot 2 \sin(k_y\tfrac{\lambda}{2})\, e^{j\pi/4} \Big[e^{j\pi/4} e^{jk_x\frac{\lambda}{8}} - e^{-j\pi/4} e^{-jk_x\frac{\lambda}{8}} \Big] =$$

$$= E_0\, 2 \sin(k_y\tfrac{\lambda}{2})\, e^{j\pi/4} \cdot 2j \sin\left(k_x\tfrac{\lambda}{8} + \tfrac{\pi}{4}\right) =$$

$$= E_0 \cdot 4j \sin(k_y\tfrac{\lambda}{2})\, e^{j\pi/4} \sin\left(k_x\tfrac{\lambda}{8} + \tfrac{\pi}{4}\right)$$

$$= E_0\, 4j \sin(\pi \sin\theta \sin\phi)\, e^{j\frac{\pi}{4}} \sin\left(\tfrac{\pi}{4}\sin\theta\sin\phi + \tfrac{\pi}{4}\right)$$

$k_y = \underset{\frac{2\pi}{\lambda}}{k} \sin\theta \sin\phi$

$k_x = k \sin\theta \cos\phi$

$k_y\frac{\lambda}{2} = \pi \sin\theta \sin\phi$

$k_x\frac{\lambda}{8} = \frac{\pi}{4} \sin\theta \sin\phi$

on $\left[E_0 = j60 I_1 \frac{e^{-jkr}}{r} \frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta}\, \hat{\theta} \right]$ ← dipol de $\lambda/2$ en z

c) Pla H

E { Intensitat (z), màxim de radiació

H { màxim rad, altre eix } YX

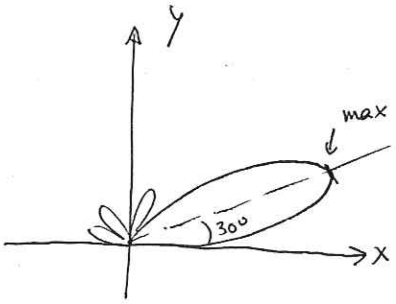
$[E \perp H]$

Pla H → XY → $\theta = \frac{\pi}{2}$

$$E \propto \sin(\pi \sin\phi) \sin\left(\tfrac{\pi}{4}\sin\phi + \tfrac{\pi}{4}\right)$$

$$t_{(\theta,\phi)}\Big|_{\theta=\frac{\pi}{2}} = \frac{|E_{(\theta,\phi)}|^2}{|E_{max}|^2} = \sin^2(\pi\sin\phi)\sin^2\left(\tfrac{\pi}{4}\sin\phi + \tfrac{\pi}{4}\right)$$

ϕ	t
0	0
30°	1 ←
60°	0,142
90°	0
120°	0,024
150°	0,011
180	0



d) $$D = \frac{P_{max}}{\frac{P_r}{4\pi r^2}} = \frac{|E_{max}|^2/\eta}{P_r/4\pi r^2} =$$

$$P_r = R_r I^2 = R_{r_1}|I_1|^2 + R_{r_2}|I_2|^2 = 115 \cdot |I_1|^2 + 19\,|I_2|^2 =$$

$$= 134\,|I_1|^2$$

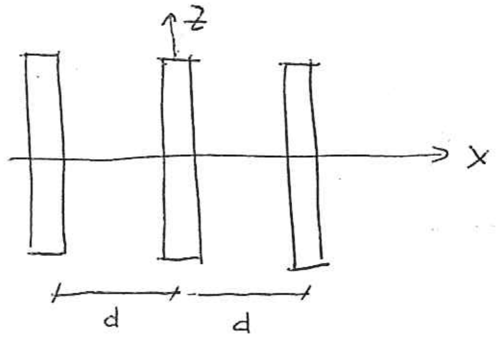
$$|E_{max}| \rightarrow \theta = \pi/2$$
$$\phi = 30°$$

$$|E| = |E_0|\, 4 \cdot \overbrace{\sin(\pi \sin\phi)\ \sin\left(\frac{\pi}{4}\cos\phi + \frac{\pi}{4}\right)}^{1}$$

$$D = \frac{\overset{4^2}{16}\, 60^2\, \cancel{I_1^2} / \cancel{r^2}\cdot 120\cancel{\pi}}{134\, \cancel{I_1^2} / 4\cancel{\pi}\cancel{r^2}} = \frac{16 \cdot 60^2 \cdot 4}{120 \cdot 134} = 14'328$$

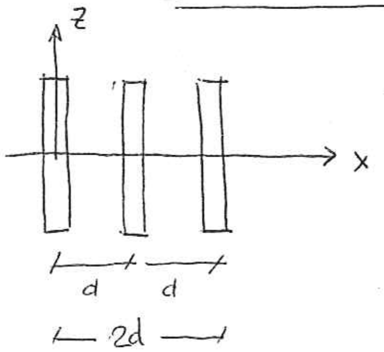
$$\boxed{D = 11'56\ \text{dB}}$$

AGRUPACIONS



$$\vec{E} = E_0\left(e^{-jk_x d} + 1 + e^{jk_x d}\right)$$
$$= E_0\left(1 + 2\cos k_x d\right)$$

$\vec{E} \propto \vec{N}$ $\quad \boxed{\vec{N} = N_0 \cdot \sum_{n=0}^{N-1} I_n e^{jn\tilde{k_x} d}}$ $\quad \begin{cases} k_x \\ k_y \\ k_z \end{cases}$



$$\vec{E} = E_0\left(1 + e^{jk_x d} + e^{j2k_x d}\right) =$$
$$= E_0\left(1 + e^{jk_x d}\left(1 + e^{jk_x d}\right)\right)$$

$$\vec{E} = E_0\, FA(\psi)$$

$I_n \in \mathbb{C}$ $\quad \boxed{I_n = a_n e^{jn\alpha}}$

$$\vec{N} = \vec{N_0} \sum_{n=0}^{N-1} \underbrace{a_n e^{jnk_x d + jn\alpha}}_{e^{jn\boxed{(k_x d + \alpha)}} \rightarrow \psi}$$

$$\vec{N} = N_0 \underbrace{\boxed{\sum_{n=0}^{N-1} a_n e^{jn\psi}}}_{FA(\psi)}$$

on $\boxed{\psi = k_{\substack{x\\y\\z}} d + \alpha}$ $\rightarrow$ fase progresiva

$FA(\psi)$

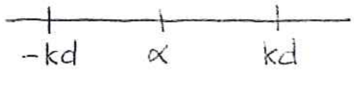
Periòdic cada 2π

$\Rightarrow$ Si el $a_n \in \mathbb{R} > 0$

$\Rightarrow$ màxim $FA(\psi)\Big|_{\psi=0}$

Marge visible del $FA(\psi)$

$[-kd + \alpha,\ kd + \alpha]$



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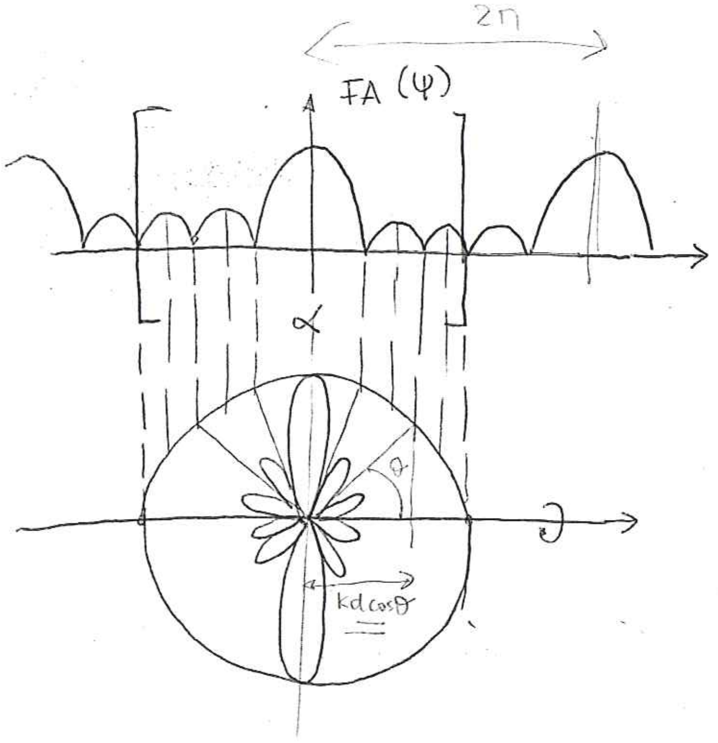
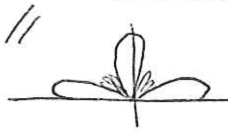
Antenes



→ Si el marge visible és més gran que 2π, apareixen lòbuls de difracció. //



Si augmentem el MV, → no canvia NPLS
→ augmenten els lòbuls de difracció
→ disminueix l'ample de feix

$$P(z) = \sum_{n=0}^{N-1} a_n z^n = a_0 + a_1 z + a_2 z^2 + a_3 z^3 + \dots + a_{N-1} z^{N-1}$$

$$FA(\psi) = P(z)\Big|_{z=e^{j\psi}}$$

$$FA(\psi) = a_0 + a_1 e^{j\psi} + a_2 e^{j2\psi} + \dots$$

$$P(z) = a_{N-1} \prod_{n=1}^{N-1} (z - z_n)$$

→ Modificant α, canviem cap a on van dirigits els lòbuls principals.

$n\frac{2\pi}{N} = \boxed{n\frac{2\pi}{3}}$

- Agrupacions uniformes → distribució dels corrents és uniforme ($a_n = 1$)

$$\left[P(z) = 1 + z + z^2 + \dots + z^{N-1} = \frac{z^N - 1}{z - 1} \right]$$

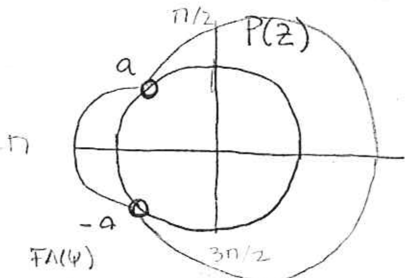
$\boxed{\# \text{ zeros} = N-1}$

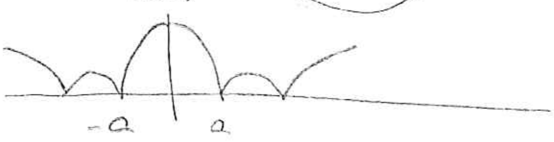
Uniforme de 3 elements

↳ # zeros = 2

$a_n \in \mathbb{R} > 0$

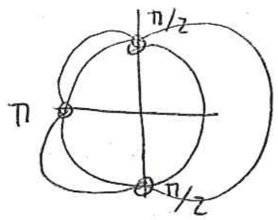
$P(z)\Big|_{\psi=0} = \max$

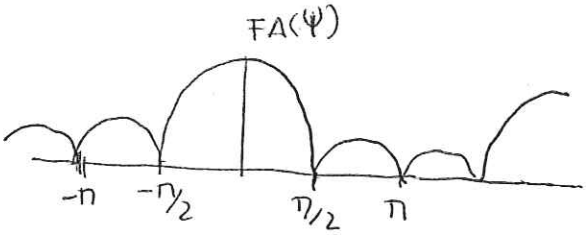




zeros cada $\boxed{n\frac{2\pi}{N}}$

Unif de 4 elem





$\boxed{\text{Uniformes}}$

$$\left[\ |FA(\psi)| = \frac{|\sin(N\Psi/2)|}{|\sin(\Psi/2)|}\ \right]$$

?

$\boxed{NLPS = 13{,}4 \text{ dB}}$

$\vec{E} = E_0\, FA(\psi)$

$\vec{N} = N_0\, FA(\psi)$

$NLPS = N \sin\left(\frac{3\pi}{2N}\right)$

- Agrupacions triangulars

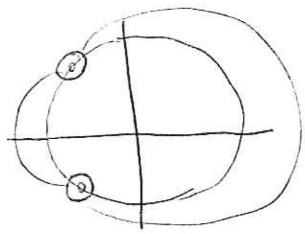
imparell d'antenes

$a_n = \{1, 2, 3, 2, 1\}$

$P(z) = 1 + 2z + 3z^2 + \dots + 3z^{N-3} + 2z^{N-2} + z^{N-1}$

$\#$ zeros: $\left(\frac{N-1}{2}\right) \longrightarrow$ els zeros són <u>dobles</u>

$\Rightarrow$ Triangular de 5 elements



$$\left[\ FA_{triang} = \frac{\left|\sin\left(\frac{N+1}{4}\psi\right)\right|^2}{|\sin \Psi/2|^2}\ \right]$$

<u>NLPS = 26 dB</u>

⇒ BINÒMIQUES

$$a_n = \binom{N-1}{n} = \frac{(N-1)!}{n!\,(N-1-n)!}$$

1

N = 2 unif → 1 1

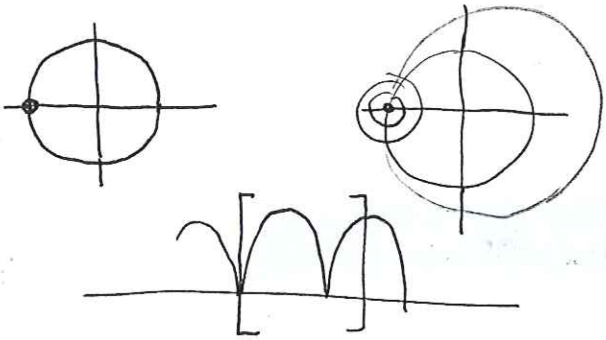
N = 3 triang → 1 2 1

1 3 3 1

1 4 6 4 1

$$P(z) = (z+1)^{N-1}$$

\# zeros sempre és 1
de multiplicitat N−1



$$|FA(\psi)| = |2\cos\psi/2|^{N-1}$$
binòmiques

Propietats

Si $a_n = a_{n1} * a_{n2}$

$$FA(\psi) = FA_1(\psi) \cdot FA_2(\psi)$$

$$P(z) = P_1(z) \cdot P_2(z)$$

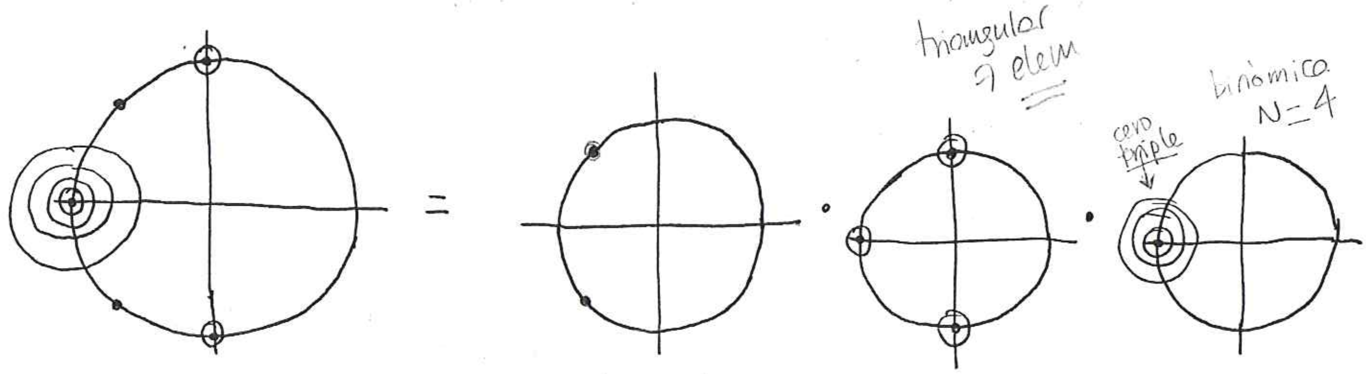
Si $a_n = a_{n1} + a_{n2} \Rightarrow FA_1 + FA_2 = FA$

$$P(z) = P_1(z) + P_2(z)$$

$\frac{N-1}{2} = 3$

triangular 9 elem

binòmica N = 4

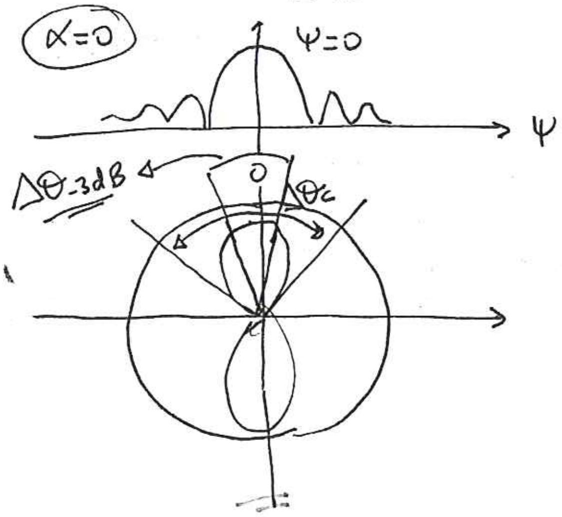


{1, 1, 1} {1, 2, 3, 4, 3, 2, 1} {1,3,3,1}

⟹ UNIFORMES TRANSVERSALS (broadside)

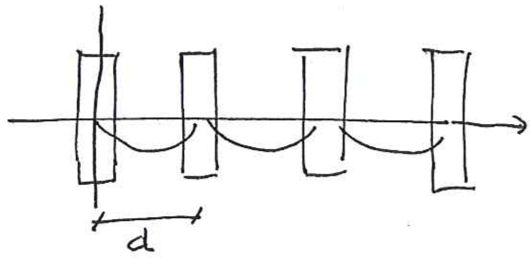
máx está en $\Theta = 90^\circ$

$\alpha = 0$



Si $a_n \in \mathbb{R} > 0$

L = longitud física de l'agrupació
$(N-1)\,d$



$$\boxed{\Delta\Theta_{-3dB} = 0{,}88\,\lambda/L}$$

$$\boxed{\Delta\Theta_c = \frac{2\lambda}{L}}$$

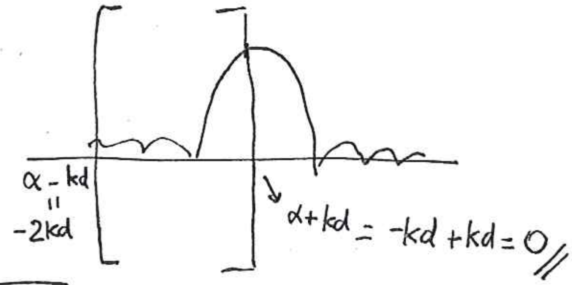
⟹ UNIFORMES LONGITUDINAL (endfire)

max està a $\boxed{\Theta = 0}$

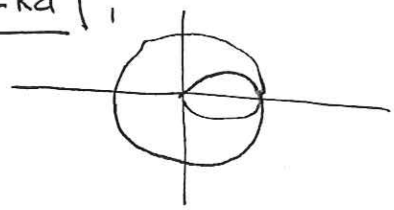
$\alpha = -kd$

$a_n \in \mathbb{R} > 0$

Sabíem que $\Psi = \underbrace{k_z d}_{kd} + \alpha$



el màxim estarà quan $\Psi = 0$ → $\boxed{\alpha = -kd}$!



$$\boxed{\begin{aligned} \Delta\Theta_{-3dB} &\simeq \sqrt{\frac{3{,}5\lambda}{L}} \\ \Delta\Theta_c &= \sqrt{\frac{8\lambda}{Nd}} \end{aligned}}$$

Distancia entre antenas $d = m\frac{\lambda}{2}$ m enter

$$\boxed{D = \frac{\left(\sum_{n=0}^{N-1} a_n\right)^2}{\sum_{n=0}^{N-1} a_n^2}}$$

Directivitat de l'agrupació

Transversal

$d < \lambda \rightarrow \boxed{D_{agrup} = \frac{2d}{\lambda} \frac{(\sum a_n)^2}{\sum a_n^2}}$

Longitudinal

$d < \lambda/2 \rightarrow \boxed{D_{agrup} = \frac{4d}{\lambda} \frac{(\sum a_n)^2}{\sum a_n^2}}$

Si $kd \rightarrow \infty$ $d \rightarrow \infty$

$\Downarrow$

$\boxed{D = D_{\lambda/2}}$

→ Agrupació Hansen-Woodyard

$\alpha = -kd - \pi/N$

$D = 7{,}2\, N\, d/\lambda$

Uniformes: és la que radia més camp en la direcció del principal

Binòmica: és la que radia menys camp en la direcció del principal

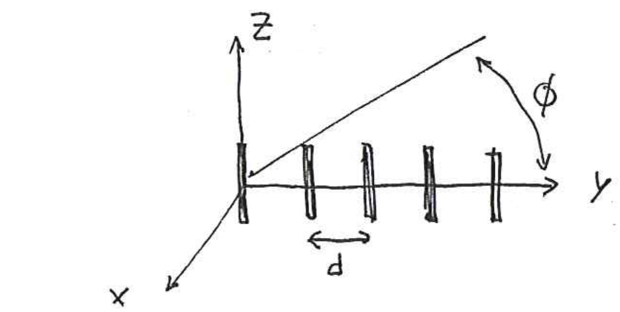
Uniforme: pitjor NLPS

$D_{long} > D_{transv}$

Paramètriques:

- Chebychev → òptim $\Delta\theta$
- Taylor → millor directivitat i NLPS

d06 (2)



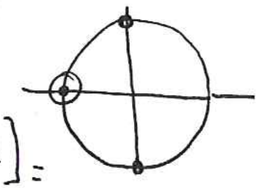
$\psi = 90°, -90°$

zeros simples

$\psi_{1\,\text{zero doble}} = 180°$

Agrup. long → $\boxed{\begin{array}{l}\theta_{max} = 0 \\ \alpha = -kd\end{array}}$

a)



$$P(z) = (z+1)^2 \left(z - e^{j\pi/2}\right)\left(z - e^{-j\frac{\pi}{2}}\right)$$

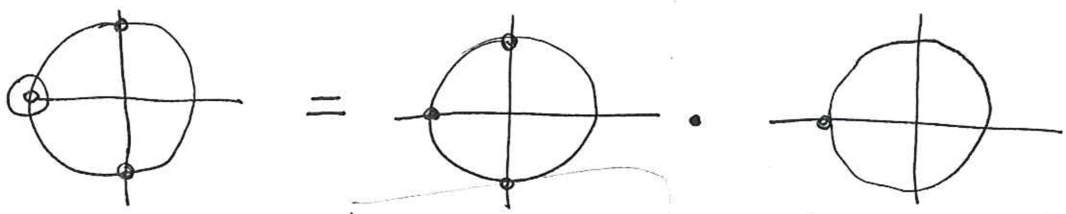
$$= (z+1)^2 \left[z^2 - \underbrace{\left(e^{-j\frac{\pi}{2}} + e^{j\frac{\pi}{2}}\right)}_{0} z + 1 \right] =$$

$$= (z+1)^2 (z^2+1) = (z^2 + 2z + 1)(z^2+1) = z^4 + z^2 + 2z^3 + 2z + 2z^2 + 1 =$$

$$= z^4 + 2z^3 + 2z^2 + 2z + 1$$

$a_n = \{1, 2, 2, 2, 1\} \longrightarrow a_n \in \mathbb{R} > 0$

També es pot fer



$\{1, 1, 1, 1\}$
unif de 4 elements

$\{1, 1\}$
unif de 2 elements

$$\{1, 1\} * \{1, 1, 1, 1\} = a_{n1} * a_{n2} = \boxed{\{1, 2, 2, 2, 1\}}$$

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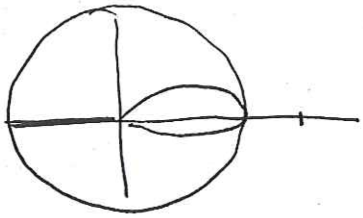
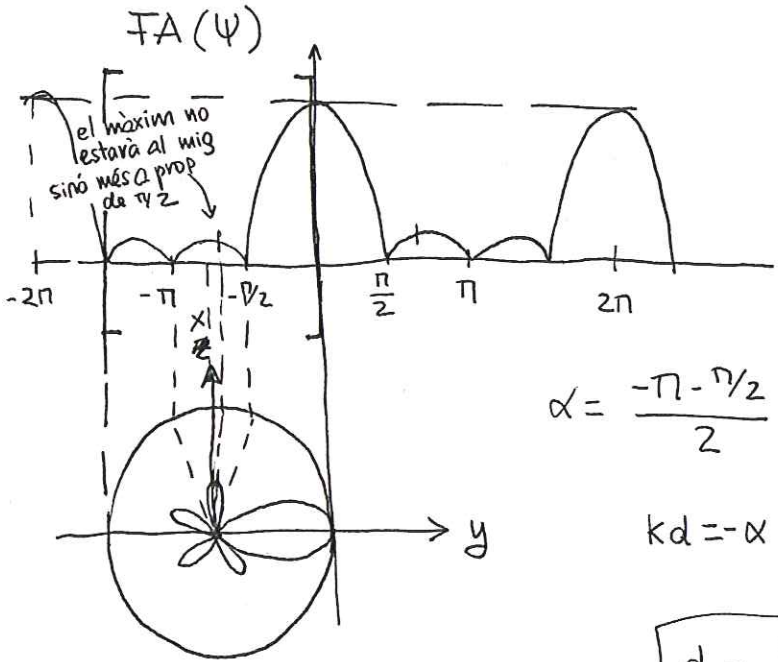
Antenes

b) $\left[\begin{array}{l} \text{long}: \theta_{max} = 0 \\ \alpha = -kd \end{array}\right]$



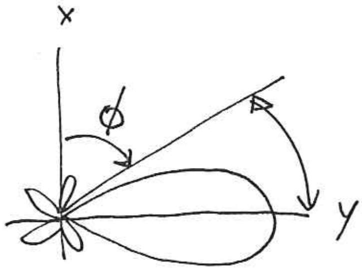


$MV \uparrow \rightarrow D \uparrow$

$$\alpha = \frac{-\pi - \pi/2}{2} = \frac{-3\pi/2}{2} = -\frac{3\pi}{4}$$

$$kd = -\alpha = \frac{3\pi}{4}$$

$$\boxed{d = \frac{3\lambda}{8}}$$



c) $\Psi = k_y d + \alpha$

$$\Psi = kd \sin\theta \sin\phi + \alpha \underset{\theta = \pi/2}{=} kd \sin\phi + \alpha$$

$$\frac{\pi}{2} = \frac{3\pi}{4} \sin\phi - \frac{3\pi}{4} \rightarrow \phi_c = \arcsin(1/3) =$$

$$= 0{,}34 \text{ rad} = 19{,}47^\circ$$

$$\Delta\phi_c = 2(90^\circ - 19'47^\circ) = \boxed{141{,}05^\circ}$$

$$NLPS = 20 \log \frac{|FA(max)|}{|FA(\text{max secundari})|}$$

$$|FA(\Psi)| = \frac{|\sin\Psi|}{|\sin(\Psi/2)|} \frac{|\sin(2\Psi)|}{|\sin(\Psi/2)|}$$

$$\left[FA(0) = \frac{\cancel{\Psi}}{\cancel{\Psi}/2} \frac{2\cancel{\Psi}}{\cancel{\Psi}/2} = 4 \cdot 2 = 8 \right]$$

$$|FA(-3\pi/2)| = 0{,}8284$$

$$NLPS = 20 \log \left|\frac{8}{0{,}8284}\right| \rightarrow$$

$$\rightarrow \boxed{19{,}69 \text{ dB}}$$

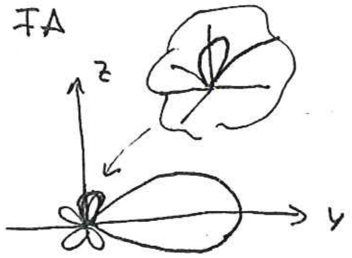
d) Pla E → Z-Y (màxim: Y)
Pla H → X-Y

$$|E| = |E_0| \, |FA(\psi)|$$

Diagrama dipol curt $= \frac{1}{2}$ Diagrama dipol elemental

Dipol curt

Pla E



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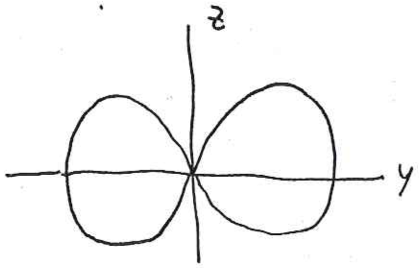

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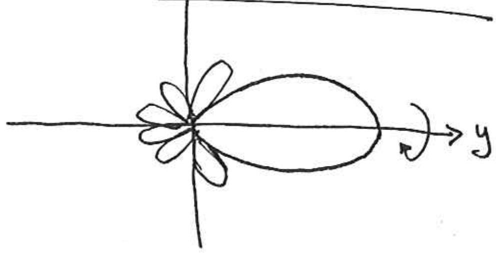
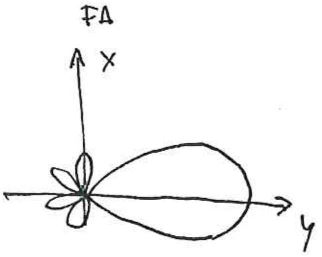
Diagrama del pla E

=



Pla H



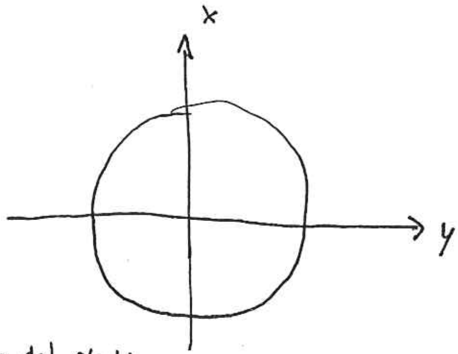


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Diagrama del pla H

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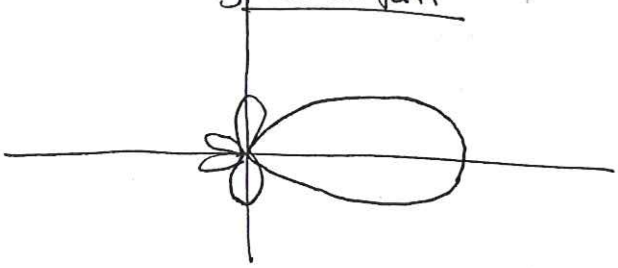

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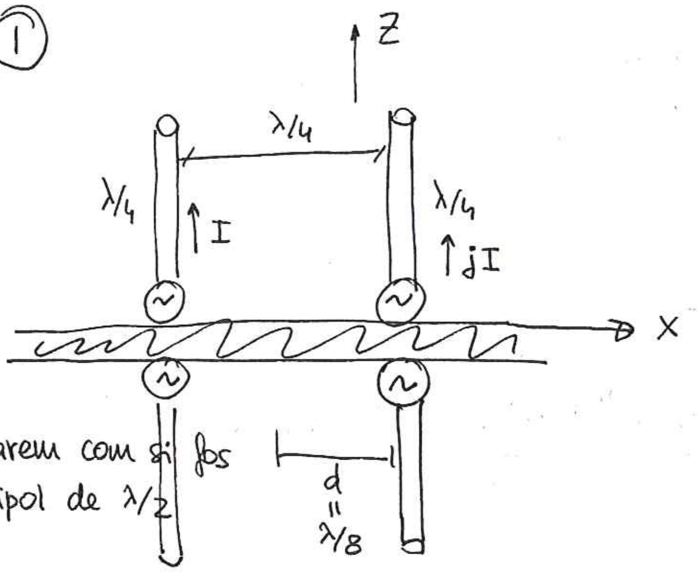
Antenas

$\boxed{\text{Fos}}$ 1



Treballarem com si fos un dipol de λ/2

a) $\vec{E} = E_0 \left(e^{-jk_x d} + j e^{jk_x d} \right) = E_0 \left(\cos k_x d - j \sin k_x d + j \cos k_x d - \sin k_x d \right)$

$= E_0 \left[\cos k_x d \, (1+j) - \sin k_x d \, (1+j) \right] = E_0 (1+j) \left[\cos k_x d - \sin k_x d \right]$

$(1+j) = \sqrt{2}\, e^{j\pi/4}$

$$\vec{E} = j60 \frac{e^{-jkr}}{r} I \frac{\cos(\pi/2 \cos\theta)}{\sin\theta} \sqrt{2}\, e^{j\pi/4} \left(\cos k_x d - \sin k_x d \right)$$

$$\vec{E}_{monopol} = \begin{cases} \vec{E}_{dipol} & z \geq 0 \\ \emptyset & z \geq 0 \end{cases} \qquad z \geq 0 , \ \boxed{\theta \leq \pi/2}$$

Pla H = {XY}
$\theta = \pi/2$

$k_x d = kd \sin\theta \cos\phi \qquad kd = \frac{2\pi}{\lambda} \frac{\lambda}{8} = \frac{\pi}{4}$

$$\vec{E} \Big|_{\theta = \pi/2} = \text{ctes} \left[\cos\left(\pi/4 \cos\phi \right) - \sin\left(\pi/4 \cos\phi \right) \right]$$

$\cancel{\phi = \pi/2}$
$\cancel{\theta = \pi/2}$

$\phi = \pi \rightarrow E = \text{ctes} \left[\cos(-\frac{\pi}{4}) - \sin(-\frac{\pi}{4}) \right] =$

$\Downarrow$ màxim en $-X$

$= \text{ctes} \cdot \sqrt{2}$ //

$$t(\theta,\phi) = \frac{|E(\theta,\phi)|^2}{|E_{max}|^2}$$

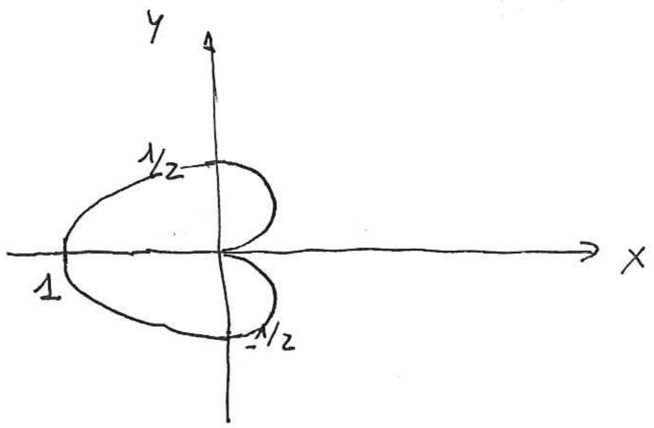
$$t(\theta,\phi) = \frac{|E(\theta,\phi)|^2}{|\sqrt{2}|^2}$$

Pla H (x,y) $\theta = \frac{\pi}{2}$ →

$$t(\tfrac{\pi}{2},\phi) = \frac{\left[\cos(\pi/4 \cos\phi) - \sin(\pi/4 \cos\phi)\right]^2}{2}$$

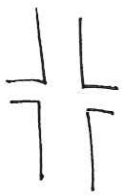
$$= \frac{1}{2}\left[\cos^2(\tfrac{\pi}{4}\cos\phi) + \sin^2(\tfrac{\pi}{4}\cos\phi) - 2\sin(\tfrac{\pi}{4}\cos\phi)\cos(\tfrac{\pi}{4}\cos\phi)\right]$$

$$= \left|\frac{1}{2}\left[1 - \sin(\tfrac{\pi}{2}\cos\phi)\right]\right|$$



c) Zin

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$



$Z_{12} = Z_{21} = 40 - 29j \ \Omega$

$Z_{22} = Z_{11} = \underline{\underline{73}} + j42 \ \Omega$

$I_2 = j I_1$

$$V_1 = Z_{11} I_1 + Z_{12}\, j I_1$$

$$Zin_1 = Z_{11} + jZ_{12} = 73 + j42 + j40 + 29 = 102 + j82$$

$$Zin_2 = -jZ_{21} + Z_{22} = -j40 - 29 + 73 + j42 = 44 + 2j$$

$$Zin_{monopol} = \frac{Zin_{dipol}}{2}$$

$$\boxed{\begin{array}{l} Zin_{m_1} = 51 + j41 \\ Zin_{m_2} = 22 + j \end{array}}$$

d) $P_r = 10\text{ kW}$

$P_r = R_r |I|^2$

$R_r = \mathrm{Re}\{Z_{in_1}\} + \mathrm{Re}\{Z_{in_2}\} = 73$

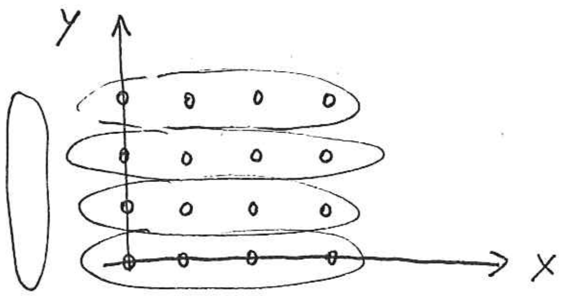
$$\boxed{P_{r\,monopol} = P_{rd}/2}$$

$P_r = ?$

$P_{r\,dipol} = 146\,|I|^2 \rightarrow P_{rm} = 73|I|^2$

$$\sqrt{\frac{10\text{ kW}}{73}} = I \rightarrow \boxed{I = 11'7\text{ A}}$$

AGRUPACIONS
BIDIMENSIONALS



$\Rightarrow$ Planes, rectangulars, equiespaiades
alimentació separable

$$FA(\Psi_x, \Psi_y) = \underline{FA(\Psi_x) \cdot FA(\Psi_y)}$$

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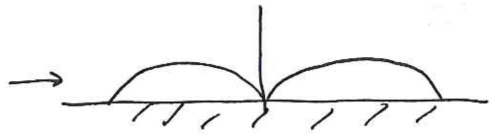
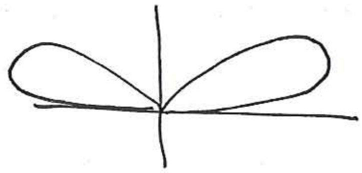
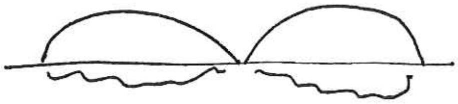
Terra imperfecta

Introdueix un zero a la horitzontal on hi hauria el màxim

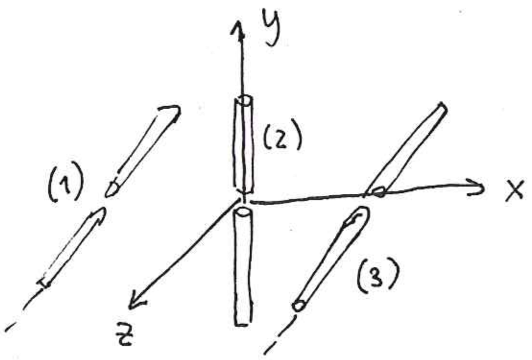




Pla no infinit → hi ha reflexions



$\boxed{J04}$ 1



$I_2 = 2I_0$

$I_1 = -I_0$

$I_3 = I_0$

$Z_{13} = Z_{31} = -(13+j29)\ \Omega$

$Z_{12} = Z_{32} = Z_{23} = Z_{21} \simeq 0$

a)

$$V_1 = Z_{11} I_1 + Z_{12} I_2 + Z_{13} I_3 =$$

$$= -Z_{11} I_0 + I_0 Z_{13} = Z_{11} I_1 - Z_{13} I_1$$

$$\rightarrow Z_{in_1} = \frac{V_1}{I_1} = Z_{11} - Z_{13} = (73+j42) - (-(13+j29)) = \boxed{86+j71}$$

$$V_3 = Z_{31} I_1 + \underbrace{Z_{32}}_{0} I_2 + Z_{33} I_3 = -Z_{31} I_3 + Z_{33} I_3$$

$$\rightarrow Z_{in_3} = \frac{V_3}{I_3} = Z_{33} - Z_{31} = \boxed{86+j71}$$

$\underline{Z_{in_1} = Z_{in_3}}$

$$V_2 = \cancel{Z_{21}}_{0} I_1 + Z_{22} I_2 + \cancel{Z_{23}}_{0} I_3 = Z_{22} I_2$$

$$\rightarrow Z_{in_2} = \frac{V_2}{I_2} = Z_{22} = \boxed{73+j42}$$

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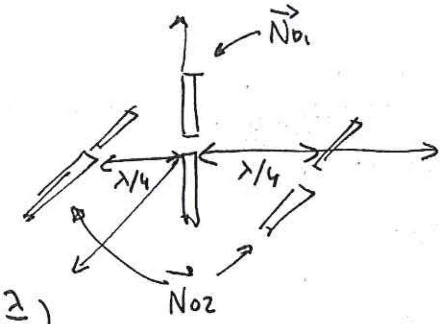
Antenes

b) $\vec{N} = \vec{N}_{01} + \vec{N}_{02} \ldots$



$\downarrow$

$$\vec{N}_{01} + \vec{N}_{02} \underbrace{\left(-e^{-jk_x \lambda/4} + e^{jk_x \frac{\lambda}{4}}\right)}_{2j \sin k_x \lambda/4}$$

$$\vec{N} = \vec{N}_{01} + \vec{N}_{02}\, 2j \sin k_x \lambda/4$$

$$\vec{N}_{02} = 2k I_0 \frac{\cos(k_z H) - \cos(kH)}{k^2 - k_z^2} \qquad (k_z = k\cos\theta)$$

$$= 2\cancel{k} I_0 \frac{\cos(k\cos\theta\, H) - \cancel{\cos(kH)}^{\,0}}{k^{\cancel{2}} \underbrace{(1-\cos^2\theta)}_{\sin^2\theta}} = \qquad kH = \frac{2\pi}{\lambda}\frac{\lambda}{4} = \frac{\pi}{2}$$

$$= 2 I_0 \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{k \sin^2\theta} = \vec{N}_{02}\, \hat{z}$$

Volem trobar $N_{02}\, \hat{\theta}$

$$\begin{pmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \end{pmatrix} = \begin{pmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{pmatrix} \begin{pmatrix} \hat{r} \\ \hat{\theta} \\ \hat{\phi} \end{pmatrix}$$

$$\vec{N}_{02}\, \hat{\theta} \Rightarrow \vec{N}_{02}\, \hat{z} = \vec{N}_{02} \cdot (-\sin\theta\, \hat{\theta}) =$$

$$= -2 I_0 \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{k \sin\theta}$$

$$\vec{N}_{01}\, \hat{y} = 2k I_z \cdot \frac{\cos(k_y H) - \cancel{\cos(kH)}}{k^2 - k_y^2} = 2k \cdot 2 I_0 \frac{\cos\left(\frac{\pi}{2}\sin\theta\sin\phi\right)}{k^2 - k^2\sin^2\theta\sin^2\phi}$$

$$= 4 I_0 \frac{\cos\left(\frac{\pi}{2}\sin\theta\sin\phi\right)}{k(1-\sin^2\theta\sin^2\phi)}$$

$$\vec{N}_{01} \cdot \hat{y} = 4 I_0 \frac{\cos\left(\frac{\pi}{2}\sin\theta\sin\phi\right)}{k(1-\sin^2\theta\sin^2\phi)} \left(\cos\theta\sin\phi\, \hat{\theta} + \cos\phi\, \hat{\phi}\right)$$

$$\vec{N}\hat{\theta} = \frac{4 I_0}{k} \frac{\cos(\pi/2 \sin\theta\sin\phi)}{(1-\sin^2\theta\sin^2\phi)} \cos\theta \sin\phi + 2j \sin\left(k\frac{\lambda}{4}\sin\theta\cos\phi\right) 2\frac{I_0}{k} \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}$$

$$\vec{N}_\phi = \frac{4 I_0}{H} \frac{\cos\left(\frac{\pi}{2} \sin\theta \sin\phi\right)}{(1 - \sin^2\theta \sin^2\phi)} \cos\phi \, \hat{\phi}$$

$$\begin{cases} \vec{E}_\theta = j k \eta \dfrac{e^{-jkr}}{4\pi r} N_\theta \\ \vec{E}_\phi = j k \eta \dfrac{e^{-jkr}}{4\pi r} N_\phi \end{cases}$$

c) Direcció x $\rightarrow \theta = \frac{\pi}{2}, \ \phi = 0$

$$N_\theta = -4j \frac{I_0}{k}$$

$$N_\phi = \frac{4 I_0}{k}$$

$\left(\begin{array}{l} \hat{x} + j\hat{y} \rightarrow \text{esquerres} \\ \hat{x} - j\hat{y} \rightarrow \text{dretes} \end{array} \right)$

dretes ~~esquerres~~

$$\vec{E} \propto \vec{N} = \frac{4 I_0}{k} (\hat{\phi} - j \hat{\theta}) \rightarrow \underline{\text{CIRCULAR}} \text{ !}$$

$$\vec{E} = \frac{k \eta}{4\pi r} \frac{4 I_0}{\cancel{k}} e^{-jkr} \underbrace{(\hat{\theta} + j\hat{\phi})}_{\text{modul} \rightarrow \sqrt{2}} \rightarrow \text{circular esquerres}$$

d) D $= \dfrac{S_{max}}{P_r/4\pi r^2} = \dfrac{|E_{max}|^2/\eta}{P_r/4\pi r^2} = \dfrac{\frac{2 \cdot 120^{\cancel{2}} \cancel{I_0^2}}{\cancel{r^2}} \Big/ \cancel{120\pi}}{464 \cancel{I_0^2} / 4\cancel{\pi} \cancel{r^2}} = ⓧ$

$$|\vec{E}_{max}| = |E \text{ direcció de l'eix x}| = \frac{120\cancel{\pi} I_0}{\cancel{\pi} r} \cdot \sqrt{2} = \frac{\sqrt{2}\, 120 I_0}{r}$$

$$|E_{max}|^2 = \frac{2 \cdot 120^2 I_0^2}{r^2}$$

$$P_r = \underset{86}{R_{r1}} \underset{|I_0|^2}{|I_1|^2} + \underset{73}{R_{r2}} \underset{4|I_0|^2}{|I_2|^2} + \underset{86}{R_{r3}} \underset{|I_0|^2}{|I_3|^2} = |I_0|^2 [86 + 4 \cdot 73 + 86] = \underline{464 I_0^2}$$

$$= ⓧ = \frac{4 \cdot 2 \cdot 120}{464} = \frac{240 \cdot 4}{464} = \frac{960}{464} = 2{,}068 \rightarrow \boxed{D = 3{,}15 \text{ dB}}$$

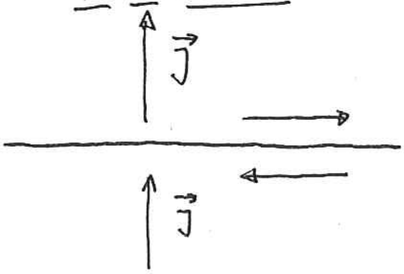
ANTENES d'OBERTURA

Taula de dualitat

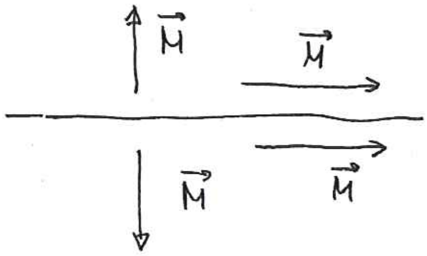
Font electrica	$\vec{E}$	$\vec{H}$	$\vec{J}$	ρ	ε	μ	η	k
Font magnètica	$\vec{H}$	$-\vec{E}$	$\vec{M}$	τ	μ	ε	$1/\eta$	k

→ pla cond. electric

Font electrica



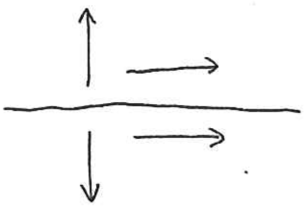
Font magnètica



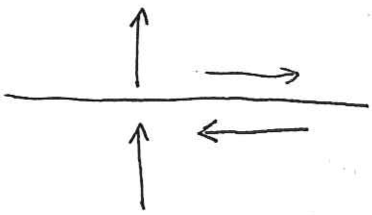
$\hat{n} \times \vec{E} = 0$

→ pla cond magnètic

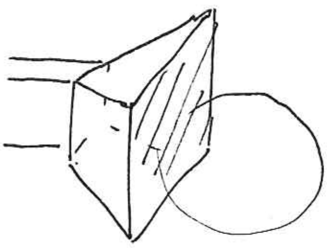
Font elèctrica

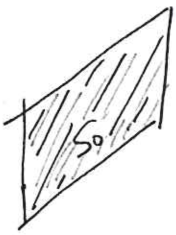


Font magnètica



$\hat{n} \times \vec{H} = 0$





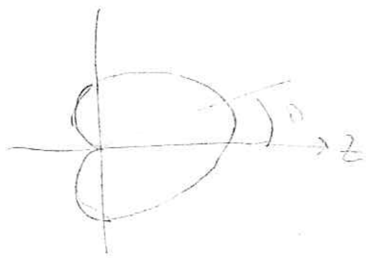
∘ Obertura elemental (dimensions $<< \lambda$)

$$\Downarrow$$

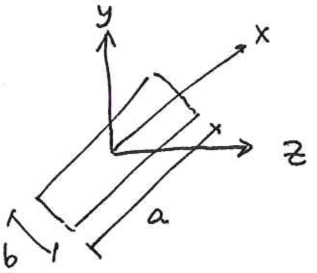
$$\underline{\underline{Z_0 \simeq \eta}}$$

$\boxed{D = 3}$

$$t(\theta, \phi) = \frac{1 + \cos\theta}{2}$$

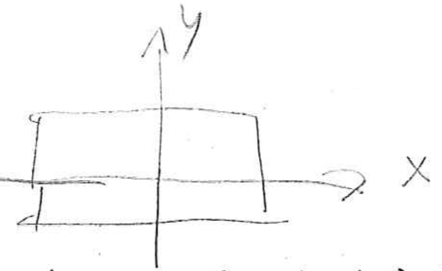


○ Obertura Rectangular



Normalment

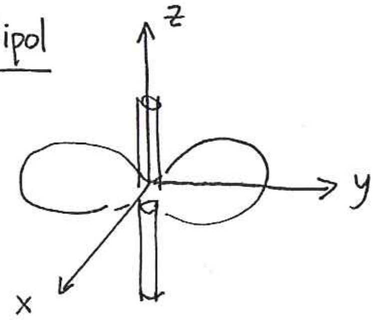
$a, b >> \lambda$



$$\vec{E} = E_x(x', y')\,\hat{x} = E_0\, f(x')\, g(y')\,\hat{x}$$

[Pla $\vec{E}$ = XZ Pla $\vec{H}$ = ZY]

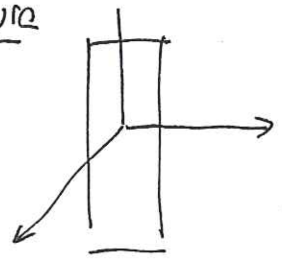
Dipol



[Pla E = { ZX, ZY }] $\rightarrow \underline{E_\theta}$ $\rightarrow \underline{H_\phi}$

Pla H = { XY }

Ranura



$$\vec{H} = \vec{E_\theta}$$

$$\vec{E} = -\vec{H_\phi}$$

El pla H de la ranura será el pla E del dipol

El pla E de la ranura sera el -H del dipol

$$E_\theta = j\,\frac{e^{-jkr}}{2\lambda r}\,(1+\cos\theta)\,\cos\phi \cdot TF_{2D}\{E_x\}$$

$$E_\phi = j\,\frac{e^{-jkr}}{2\lambda r}\,(1+\cos\theta)\,\sin\phi \cdot TF_{2D}\{E_x\}$$

$$TF_{2D} = \int_{-a/2}^{a/2}\int_{-b/2}^{b/2} \underbrace{E_x(x', y')\; e^{-j(k_x x' + k_y y')}}_{\text{separable}}\, dx'dy'$$

$$TF_{2D} = E_0\, \underbrace{F(k_x, a)}_{\text{taula}}\, \underbrace{G(k_y, b)}_{\text{taula}}$$

Pla E = {X, Z} $\rightarrow \phi = 0, \pi$

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$$\underline{\text{Pla } \vec{E} = \{x, z\}} \rightarrow \phi = 0, \pi$$

$$k_x = k \sin\theta \cos\phi = k \sin\theta$$

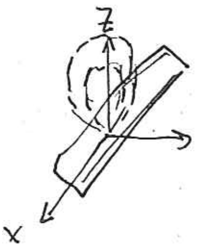
$$k_y = k \sin\theta \sin\phi = 0$$

$$TF_{2D} = E_0 \, F(k\sin\theta, a) \cdot G(0, b)$$

$$TF_{2D} = E \boxed{F(k_x, a)} \boxed{G(k_y, b)}$$

la transformada d'il·luminació en $\hat{x}$ en la direcció $\vec{E}$

transf. de il·luminació en $\hat{y}$ en la direcció $\vec{H}$



$$\underline{\text{Pla } H = \{zy\}} \rightarrow \phi = \pi/2$$

$$k_x = k \sin\theta \cos\phi = 0$$

$$k_y = k \sin\theta \sin\phi = k \sin\theta$$

distrib del camp ← $f(s)$	$F(k_x, a)$ / $G(k_y, b)$ ← canviant a per b	η_{ilx}	$\Delta\theta_{3dB}$	NLPS
1 	$\dfrac{a \sin(k_x a/2)}{k_x a/2}$	1	$50° \lambda/a$	13dB
cosinus 	$\dfrac{1/2 \, \pi a \cos(k_x a/2)}{[(\pi/2)^2 - (k_x a/2)^2]}$	0,81	$68° \lambda/a$	23dB
triang 	$\dfrac{a}{2}\left(\dfrac{\sin(k_x a/4)}{k_x a/4}\right)^2$	0'75	$73° \lambda/a$	26dB

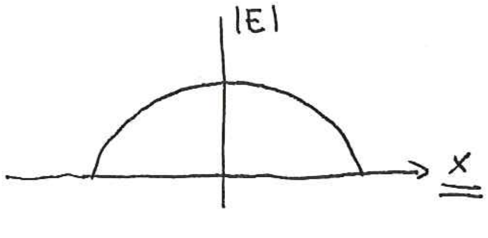
$$D = \frac{4\pi}{\lambda^2} a \cdot b \cdot \eta_{il}$$

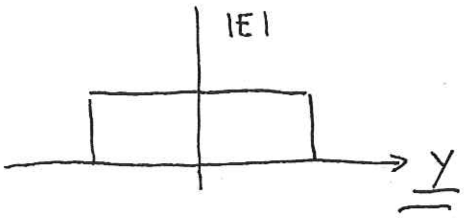
$$\left[D = \frac{4\pi}{\lambda^2} \text{ Area geomètrica} \cdot \underbrace{\eta_{ilx}}_{0,81} \, \underbrace{\eta_{ily}}_{1} \right]$$

Modo TE10

$$\vec{E} = E_0 \cos \frac{\pi}{a} x \ \hat{y}$$

Com més plana la forma del camp, més directiva és la obertura.





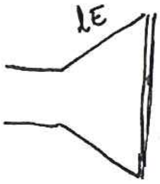
$$D = \frac{S_{max}}{P_r/4\pi r^2} = \frac{4\pi}{\lambda^2}\left[\frac{\left|\iint E_x(x',y')\,dx'dy'\right|^2 + \left|\iint E_y(x',y')\,dx'dy'\right|^2}{\iint \left(|E_x(x',y')|^2 + |E_y(x',y')|^2\right) dx'dy'}\right]$$

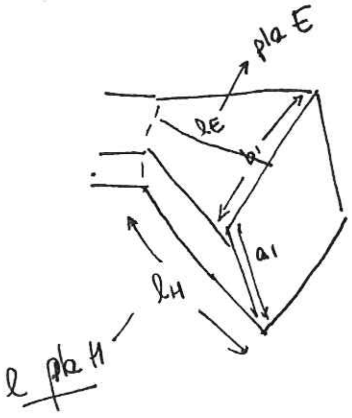
$$\eta_{il} = \frac{1}{A_{geomètrica}}\left[\quad\right]$$

Ranura ressonant → utilitzaríem el dipol ressonant ?

$$b_1 = \sqrt{2\lambda l_E}$$

$$a_1 = \sqrt{3\lambda l_H}$$



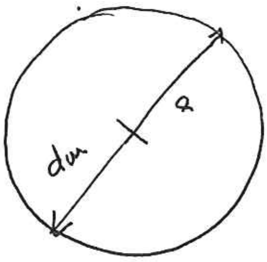


$$t(\beta) = E^2\left(\frac{1+\cos\beta}{2}\right)^2$$

no és el camp

on E → valor del lateral de la gràfica.

Obertures circulars



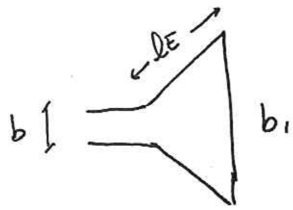
$$\Delta\theta_{-3dB} = \frac{\lambda}{d_m}$$
↳ diàmetre

$$A_{ef} = \pi a^2$$

$$D = \frac{4\pi}{\lambda^2} \underline{\underline{\pi a^2}} \eta_{il}$$

$$NLPS = 17{,}6\ dB$$

BOCINES



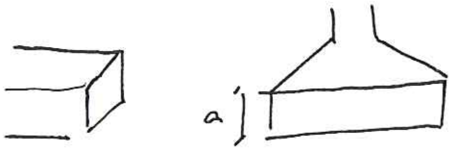
⇒ Bocines Pla E

$$\Delta\theta_{3dB_E} = 56^\circ \lambda/b_1$$

$$\Delta\theta_{3dB_H} = 67^\circ \lambda/a$$

$$NLPS_E = 10\ dB$$

$$NLPS_H \sim 23\ dB$$



BOCINA ÒPTIMA del pla E $\rightarrow \left[S = \frac{b_1^2}{8\lambda l_E} = 1/4 \right] \rightarrow \boxed{b_1 = \sqrt{2\lambda l_E}}$

$\eta_{il} = 0'64$; $D = \frac{8ab_1}{\lambda^2}$ ← Per totes

⇒ Bocines Pla H

$$\Delta\theta_{3dB_E} = 50^\circ \lambda/b$$

$$\Delta\theta_{3dB_H} = 78^\circ \lambda/a_1$$

$$NLPS_E \sim 13\ dB$$

$$NLPS_H \sim 17\ dB$$

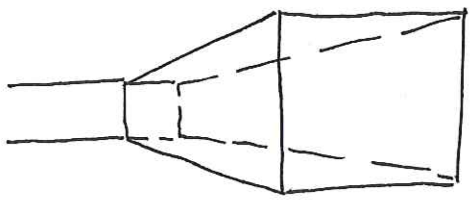
Òptima del pla H $\longrightarrow \left[t = \dfrac{a_1^2}{8\, l_H \lambda} = 3/8 \right]$

$\eta_{il} = 0'62$

$\hookrightarrow a_1 = \sqrt{3 \lambda\, l_H}$

$D = 7'9\ a_1 b / \lambda^2$

Bocina Piramidal



$\Delta\theta_{3dB}^{E} = 56^\circ \lambda / b_1$

$\Delta\theta_{-3dB}^{H} = 78^\circ \lambda / a_1$

$$\left[D = D_E \frac{\lambda}{a_1} D_H \frac{\lambda}{b_1} \frac{\pi}{32} \right]$$

$\eta_{il} = 0'51 \leftarrow$ òpt

òptima $\left[D = 6'4\ a_1 b_1 / \lambda^2 \right]$

òptim E $\longrightarrow S = \dfrac{b_1^2}{8 \lambda l_E} = 1/4$

òptim H $\longrightarrow t = \dfrac{a_1^2}{8 \lambda l_H} = 3/8$

Bocines còniques

$t = \dfrac{d_m^2}{8 \lambda l_c}$ on d_m = diàmetre

òptim $\rightarrow t = 3/8$

$d_m = \sqrt{3\, l_c \lambda}$

$\eta_{il} = 0'52$

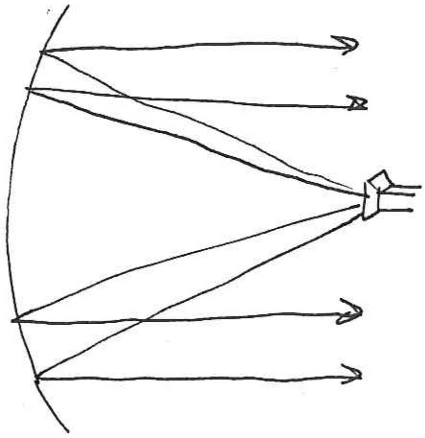
NLPS $\simeq$ 13 dB

$$D = \eta_{il} \left(\pi d_m / \lambda \right)^2$$

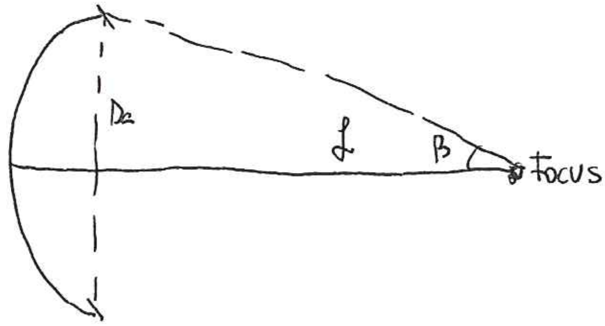
$\Delta\theta_{3dB\,E} = 60^\circ \lambda / d_m$ $\qquad$ $\Delta\theta_{3dB\,H} = 70^\circ \lambda / d_m$

Reflectors

- Reflector parabòlic



Paràmetres:



Da = diàmetre
f = distància focal

$s = \frac{b_i^2}{8\lambda \ell_E}$

En coordenades:

Cartesianes: $y^2 = 4f(f-z)$

Polars: $r = f/\cos^2(\beta/2)$

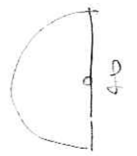
Paramétricas: $y = 2f\,tg(\beta/2)$

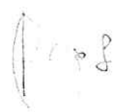
$z = f(1 - tg^2(\beta/2))$

Decaimiento als bordes: $Z = 10 \log t(\beta) + 40 \log(\cos \beta/2)$

$t(\beta) = E^2\left(\frac{1+\cos\beta}{2}\right)^2$

$$\left[\frac{f}{D_a} = \frac{1}{4\,tg(\beta/2)}\right]$$

$\frac{f}{D_a} = 0{,}25 \rightarrow$ 

$\frac{f}{D_a} > 0{,}25 \rightarrow$ 

$\frac{f}{D_a} < 0{,}25$ 

$$\eta_{il} = \frac{\eta_t}{\eta_s}$$

$$\eta_t = \cot g^2(\beta/2) \left| \int_0^{\beta} \sqrt{D_f} \cdot tg(\beta/2)\, d\theta \right|^2$$

$$\eta_s = \frac{1}{4\pi} \int_0^{2\pi} \int_0^{\beta} D_f(\beta) \sin\theta \, d\theta \, d\phi$$

$$\left[P_{rad} \cdot \underline{D_f(\beta)} = (1+\cos\theta)^2 \, t^2(\beta) \right]$$

$$\boxed{D_f(\beta) = D_f \, t(\theta, \phi)}$$

D_f: Directivitat bocina

$t(\theta,\phi)$: diagrama de radiació normalitzat

$$\frac{|E(\theta,\phi)|^2}{|E_{max}|^2} = \frac{|E_\theta^2 + E_\phi|^2}{|E_{max}|^2}$$

$$D = \frac{4\pi}{\lambda^2} A_{geo} \cdot \eta_{il} \cdot \eta_x \, \eta_s$$

$$\left[\eta_t = \eta_{il} \cdot \eta_x \, \eta_s \right]$$

(η_x: suposem que és 1)

Normalment $\underline{\eta_t \geq 0'7}$

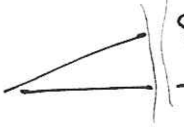
$$\boxed{D = \frac{4\pi}{\lambda^2} \pi \left(\frac{D_a}{2}\right)^2 \eta_t}$$

$\frac{f}{D_a} \uparrow\uparrow \longrightarrow BW \downarrow\downarrow$

$\frac{f}{D_a} \downarrow\downarrow \longrightarrow$ $\downarrow$ pèrdues
$\downarrow$ soroll extern

G'OS ③

a) Bocina Piramidal Òptima



$S = 1/4$
$t = 3/8$

$f = 10\,GHz$

$\Delta\theta_{-3dB} = 45°$ - als plans principals: pla E, pla H

$$\lambda = \frac{3 \cdot 10^8}{10G} = 0{,}03\,m$$

$$\Delta\theta_{-3dB}^{E} = \frac{56°\lambda}{b_1} = 45° \longrightarrow \underline{b_1 = 3{,}73\,cm}$$

$$\Delta\theta_{-3dB}^{H} = \frac{78°\lambda}{a_1} = 45° \longrightarrow \underline{a_1 = 5{,}2\,cm}$$

$b_1 = \sqrt{2\lambda l_E} \longrightarrow \boxed{l_E = 2'32 \text{ cm}}$

$a_1 = \sqrt{3\lambda l_H} \longrightarrow \boxed{l_H = 3 \text{ cm}}$

b) $\frac{f}{D} = 0,5 \rightarrow$ decaimiento en bordes

$$\tau(\beta) = 40 \log(\cos(\beta/2)) + 10 \log t(\beta)$$

$$\frac{f}{D_a} = \frac{1}{4 \, tg(\beta/2)} = 0'5$$

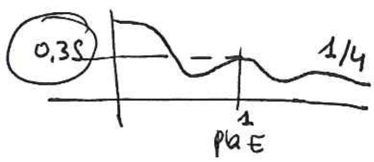
$$tg(\beta/2) = 0,5 \longrightarrow \left[\begin{array}{l} \beta/2 = 26'56^\circ \\ \beta = 53,13^\circ \end{array}\right]$$

no és el camp

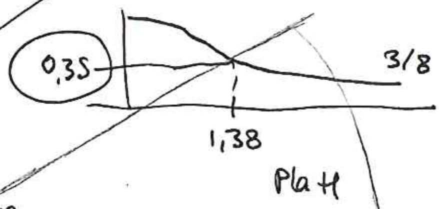
$$t(\beta) = E^2 \left(\frac{1+\cos\beta}{2}\right)^2$$

$$10 \log t(\beta) = \boxed{20 \log E} + \overbrace{20 \log\left(\frac{1+\cos\beta}{2}\right)}^{-1,938 \text{ dB}}$$

pla $\vec{E} \rightarrow \frac{b_1}{\lambda} \sin(\beta) \sim 1$ $\quad S = 1/4$



pla $\vec{H} \rightarrow \frac{a_1}{\lambda} \sin(\beta) \simeq 1,38$



$\tau_E = \tau_H$

Dona el mateix valor a les 2 gràfiques. Hi ha les mateixes pèrdues a les 2 bandes. Si no fos així, tindríem $\tau_E(\beta)$
$\tau_H(\beta)$

$$20 \log E = 20 \log 0,35 = -9,11 \text{ dB}$$

$$\tau = -11'05 \text{ dB} + \underbrace{40 \log(\cos 26,56^\circ)}_{-1,9} \simeq \underline{-13 \text{ dB}}$$

$$\eta_t = (tg^{-1}\beta/2)^2 \left| \int_0^\beta \sqrt{D_f(\beta)} \; tg(\beta/2) d\beta \right|^2$$

c) $\Delta\theta = 2^\circ \rightarrow \pi/90$

$$D = \frac{4\pi}{\lambda}\pi\left(\frac{D_m}{2}\right)^2 \eta_t$$

$$D = \frac{4\pi}{\Omega_{eq}} = \frac{4\pi}{\Delta\theta_{-3dB}\,\Delta\phi_{-3dB}} = \frac{4\pi}{\Delta^2\theta_{-3dB}}$$

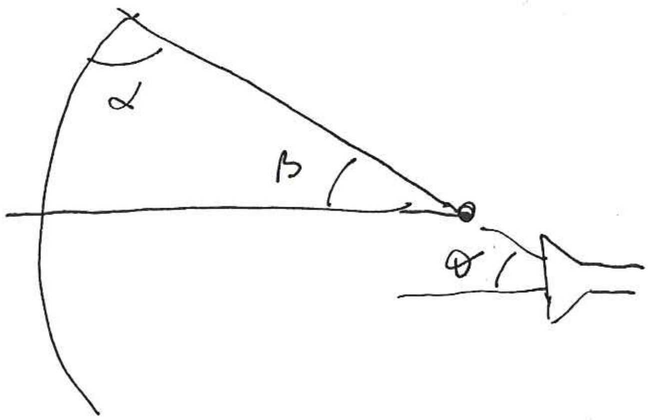
$$D = \frac{4\pi}{\pi/90} = \frac{90^2 \cdot 4}{\pi} = \underline{10.313'24}$$

$$D = \frac{4\pi}{\lambda}\pi\left(\frac{D_a}{2}\right)^2 \eta_t = 10.313'24$$

suposem que el reflector està ben disseuyat $\rightarrow$ $\underline{\eta_t \simeq 0'7}$

$$D_a^2 = \frac{10.313{,}24 \cdot \lambda}{\pi^2 \cdot 0{,}7}$$

$$\boxed{D_a = \sqrt{\frac{10.313{,}24 \cdot \lambda}{\pi^2 \cdot 0{,}7}} = 1{,}158\ m}$$



dos (2)

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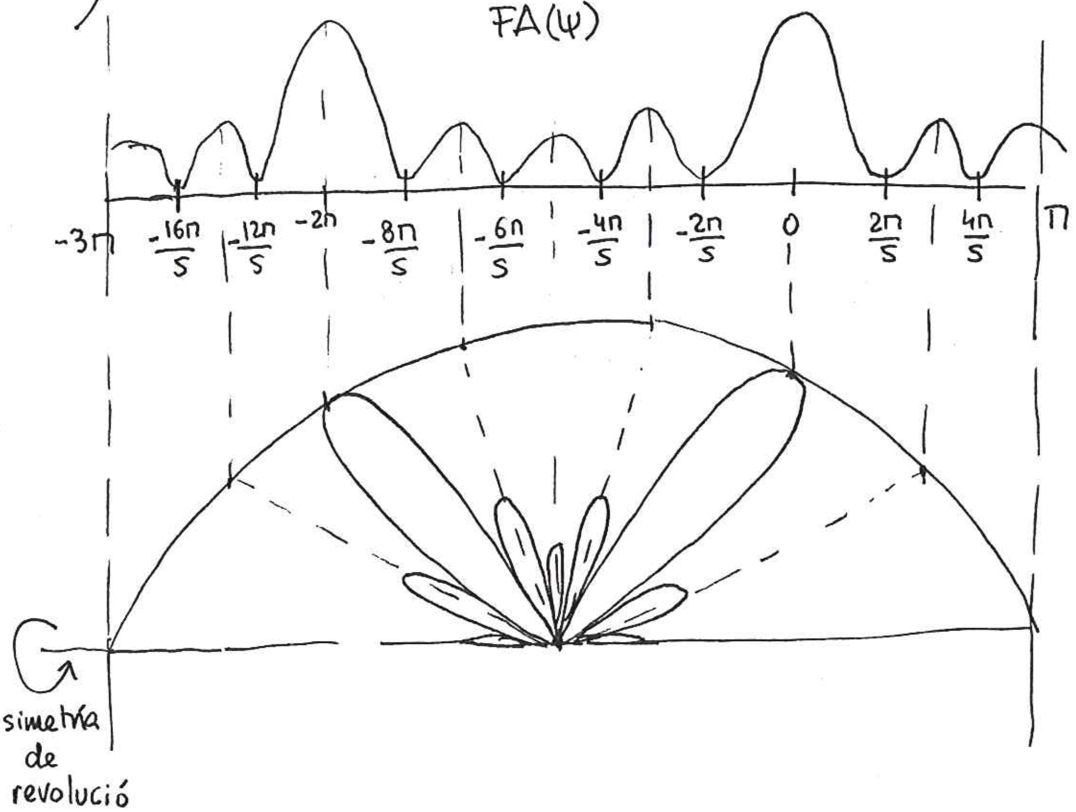
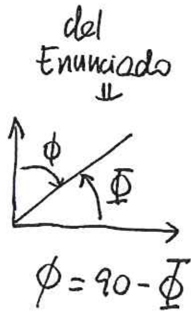
b) $MV = [\alpha - kd, \alpha + kd] = [-\pi - \frac{2\pi}{\lambda}\lambda, -\pi + 2\pi] =$

$$= \underline{[-3\pi, \pi]}$$

c) # zeros = cada $2\pi/5$ (dobles)





$\Psi = \alpha + kd\sin\phi$

$$\frac{2\pi}{5} = -\pi + 2\pi\sin\phi_{c1} \rightarrow \phi_{c1} = 44{,}42°$$

$$-\frac{2\pi}{5} = -\pi + 2\pi\sin\phi_{c2} \rightarrow \phi_{c2} = 17'45°$$

$$\boxed{\Delta\phi_c = \phi_{c1} - \phi_{c2} = 26{,}96°}$$

$$NLPS = 20\log\left|\frac{FA(0)^{\leftarrow max}}{FA(\Psi)_{\leftarrow secundari}}\right|$$

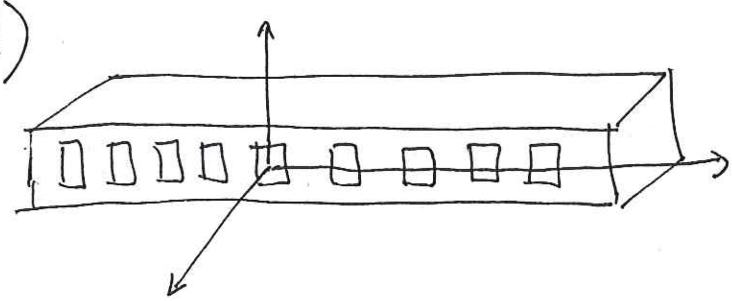
$$\boxed{24{,}277 \text{ dB}}$$

$$FA(0) = \left(\frac{\frac{5}{2}\Psi}{\Psi/2}\right)^2 = 5^2 = 25$$

$$FA\left(\frac{3\pi}{5}\right) = \left|\frac{\sin\left(\frac{5}{2}\frac{3\pi}{5}\right)}{\sin\left(\frac{3\pi}{10}\right)}\right|^2 = 1{,}5278$$

20

d)



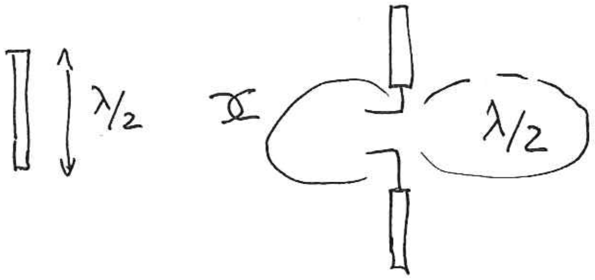
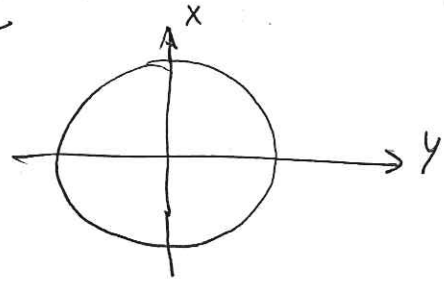


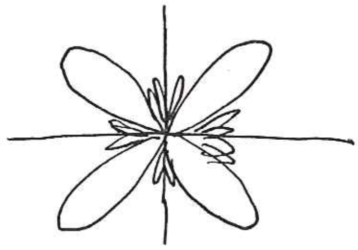
Diagrama de radiació de la ranura

$\Rightarrow$ Element bàsic : dipol

Pla H (x-y)

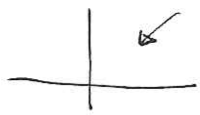


Ho multipliquem pel de l'agrupació



$\times$

$\downarrow$



igual que el de l'agrupació ?

3

tenim $\left\{ \begin{array}{l} \dfrac{f}{D_a} = 1{,}5 \\ f = 2{,}5\ \text{GHz} \end{array} \right.$ $\rightarrow \lambda = \dfrac{3 \cdot 10^8}{2{,}5 \cdot 10^9} = 0{,}12$

volem $\left\{ \begin{array}{l} D = 48\,\text{dB} \\ NLPS = 32\ \text{dB} \end{array} \right.$

a) $D = \dfrac{4\pi}{\lambda^2} \pi \left(\dfrac{D_a}{2}\right)^2 \eta_t =$ (η_t: 0,7)

$= \dfrac{4\pi}{0{,}12^2} \pi \left(\dfrac{12}{2}\right)^2 \cdot 0{,}7 =$

$= 69.182{,}918$

$\boxed{D = 48{,}40\ \text{dB}}$

Sí

b) Bocina piramidal óptima

Volem NLPS = 32

Z_H necessari

$Z = 40 \log(\cos(\beta/2)) + 10 \log t(\beta)$

$\dfrac{f}{D_a} = \dfrac{1}{4\,tg(\beta/2)} = 1'5 \longrightarrow \boxed{\beta = 18{,}92^\circ}$

A les gràfiques, per NLPS 32 dB surt $Z = 14{,}5$ dB

b) $Z = \overbrace{40 \log\left(\cos(\beta/2)\right)}^{-0,23798} + 10 \log\left(t(\beta)\right) = \underline{-14,5\ \mathrm{dB}}$

$10 \log\left(t(\beta)\right) = -14,262\ \mathrm{dB}$

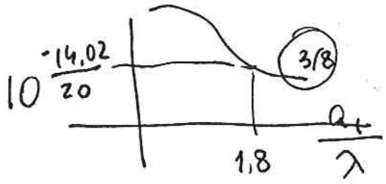
$20 \log E + 20 \log \left[\frac{1+\cos\beta}{2}\right] = -14,26\ \mathrm{dB}$

$E \Rightarrow -14,02\ \mathrm{dB} \xrightarrow[\text{lineal}]{} 10^{-14,02/20}$

$\Downarrow$

GRÀFICA $\boxed{t = 3/8}$

óptima



$\frac{a_1}{\lambda} \sin\beta = 1'8$

$\Downarrow$

$\boxed{a_1 = 66,55\ \mathrm{cm}}$

$t = \frac{a_1^2}{8\lambda l_H} = 3/8 \longrightarrow l_H = \frac{a_1^2}{3\lambda} \boxed{= 1,23\ \mathrm{m}}$

30/12/07

Maxwell

$\nabla \times H = J + j\omega\varepsilon E$

$\nabla \times E = -M - j\omega\mu H$

$\nabla \cdot D = \rho$

$\nabla \cdot B = Z$

$\nabla \cdot J + j\omega\rho = 0$

$\nabla \cdot M + j\omega Z = 0$

Relaciones

$\left[H_\theta = -E_\phi / \eta\right]$

$\left[H_\phi = E_\theta / \eta\right]$

$$D = \frac{4\pi}{\iint_{4\pi} t(\theta,\phi)\, d\Omega} = \frac{4\pi}{\int_0^{2\pi}\int_0^{\pi} t(\theta,\phi) \sin\theta\, d\theta\, d\phi}$$

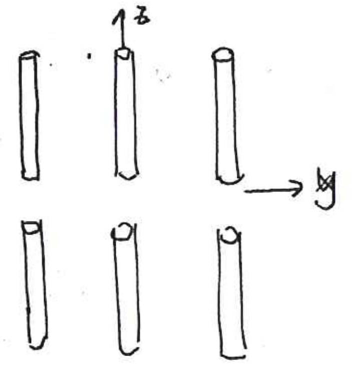
G07

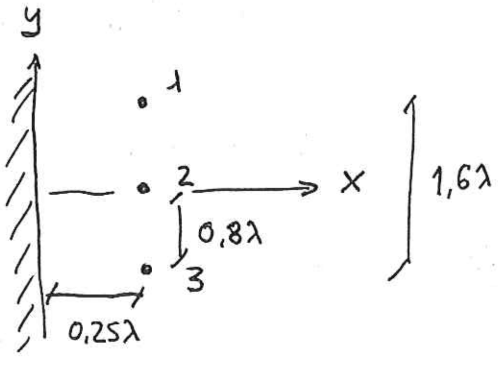
① 3 dipolos de media onda

$I_a = \{ I_0, 2I_0, I_0 \}$

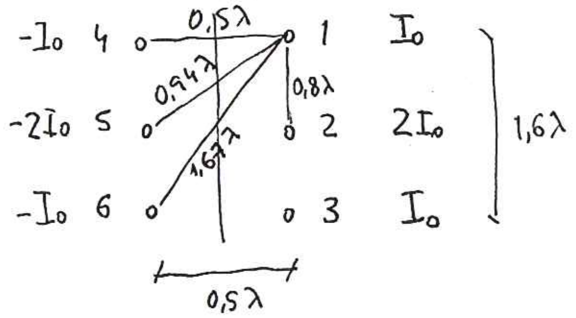
$d = 0{,}8\lambda$ entre si

$d_2 = 0{,}25\lambda$ de un pla conductor





a) Z_{in} de cada dipolo



$$Z_{in_1} = \frac{V_1}{I_1 = I_0} = \frac{Z_{11} I_0 + Z_{12}\, 2 I_0 + Z_{13} I_0 - Z_{14} I_0 - 2 Z_{15} I_0 - Z_{16} I_0}{I_0} =$$

$$= Z_{11} + 2Z_{12} + Z_{13} - Z_{14} - 2Z_{15} - Z_{16} = \boxed{54 + j54}$$

$Z_{11} = 73 + j42$

$Z_{12} = -19 + j12$ ← Taula

$Z_{13} = -8 - j8$

$Z_{14} = -12 - j30$

$Z_{15} = -2 + j18$

$Z_{16} = -11 - j2$

veiem que és simètric

$$Z_{in_3} = \frac{V_3}{I_3} = Z_{in_1}$$

$$Zin_2 = \frac{V_2}{I_2} = Z_{22} + \underbrace{\frac{Z_{21}}{2} + \frac{Z_{23}}{2}}_{Z_{21}} - Z_{25} - \underbrace{\frac{Z_{24}}{2} - \frac{Z_{26}}{2}}_{-Z_{24}} =$$

$$= \underset{Z_{12}}{\overset{\shortparallel}{Z_{22}}} + Z_{21} - \underset{Z_{14}}{\overset{\shortparallel}{Z_{25}}} - \underset{Z_{15}}{\overset{\shortparallel}{Z_{24}}} = \boxed{68 + j66}$$

$$\boxed{\begin{array}{l} Zin_1 = Zin_3 = 54 + j54 \\ Zin_2 = 68 + j66 \end{array}}$$

b) Expresión de los campos (analítica)

$$\vec{E}_{d\lambda/2} = \hat{\theta}\, j60\, I_0 \frac{e^{jkr}}{r} \frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta} = \vec{E}_0$$

$$\vec{E} = \vec{E}_0 \Big[e^{jk_x 0'25\lambda} (2 + e^{jk_y 0'8\lambda} + e^{jk_y 0'8\lambda}) -$$

$$- e^{-jk_x 0'25\lambda} (2 + \underbrace{e^{jk_y 0,8\lambda} + e^{-jk_y 0'8\lambda}}_{2\cos(k_y 0,8\lambda)}) \Big] =$$

$$= \vec{E}_0 \left[e^{jk_x 0,25\lambda} \cdot 2(1 + \cos k_y 0'8\lambda) - 2(1 + \cos k_y 0,8\lambda) e^{-jk_x 0'25\lambda} \right]$$

$$= \vec{E}_0\, 2(1 + \cos k_y 0,8\lambda) \left[e^{jk_x 0'25\lambda} - e^{-jk_x 0,25\lambda} \right]$$

$$= 4j\vec{E}_0 (1 + \cos k_y 0,8\lambda) \cdot \sin(k_x 0'25\lambda)$$

$$\left\{ \begin{array}{l} k_x = k\sin\theta\cos\phi \\ k_y = k\sin\theta\cos\phi \end{array} \right.$$

$$= 4j\vec{E}_0 (1 + \cos(1{,}6\pi \sin\theta \sin\phi)) (\sin(\pi/2 \sin\theta\cos\phi))$$

$k \cdot 0{,}25\lambda = \pi/2$

$k \cdot 0{,}8\lambda = 1{,}6\pi$

$E_\phi = 0$

c) Representar corte plano H

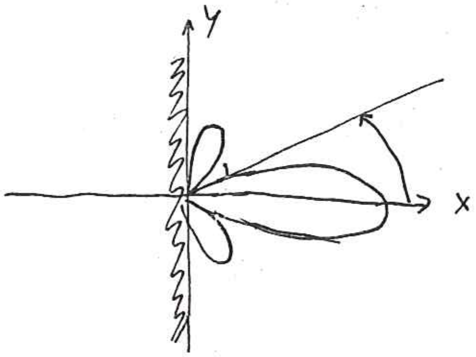
Pla H $\rightarrow \{XY\} \rightarrow \theta = \pi/2$

$$\vec{E}\Big|_{\theta=\frac{\pi}{2}} = j60 I_0 \frac{e^{jkr}}{r} \cdot 4j\left(1+\cos(1{,}6\pi \sin\phi)\right) \cdot \underbrace{\sin(\pi/2 \cos\phi)}_{\text{para que dé } 1 \;\downarrow\; \phi = 0}$$

$$E_{max} = j60 I_0 \frac{e^{jkr}}{r} \cdot 4j \underbrace{(1+1)}_{2} =$$

$$= j60 \cdot 8 \cdot I_0 \frac{e^{jkr}}{r} j =$$

$$t(\theta,\phi)\Big|_{\pi/2} = \left[\frac{1+\cos(1{,}6\pi\sin\phi)\ \sin(\pi/2\cos\phi)}{2}\right]^2$$



$$t(\theta,\phi) = 0 \Big|_{\substack{\phi = 38{,}7^\circ \\ \theta = \pi/2}}$$

d) Directivitat

$$D = \frac{\wp_{max}}{P_r/4\pi r^2} = \frac{|E_{max}|^2/\eta}{P_r/4\pi r^2}$$

$$|E_{max}|^2 = \frac{60^2 \cdot 8^2 I_0^2}{r^2}$$

$$P_r = R_{r1}|I_1|^2 + R_{r2}|I_2|^2 + R_{r3}|I_3|^2 = 54 I_0^2 + 68 \cdot 4I_0^2 + 54 I_0^2 =$$

$$= I_0^2 \cdot (2 \cdot 54 + 4 \cdot 68)$$

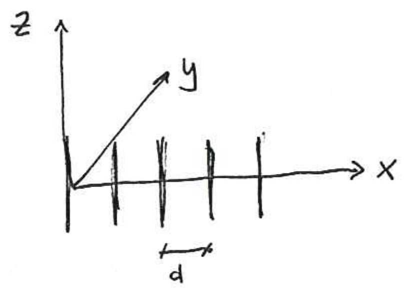
$$D = \frac{60^2 I_0^2 \cdot 8^2 \cdot 4\cancel{\pi}\cancel{r^2}}{\cancel{r^2}\ 120\cancel{\pi}} = 20{,}2 \rightarrow \boxed{13\,dB}$$

Exercici 2

Agrupació a l'eix $\hat{x}$

Dipols curts orientats en $\hat{z}$

$a_n = \{1, 3, 4, 3, 1\} \rightarrow a_n \text{ unif} * a_n \text{ triang}$



$\rightarrow \{1, 1, 1\} * \{1, 2, 1\}$

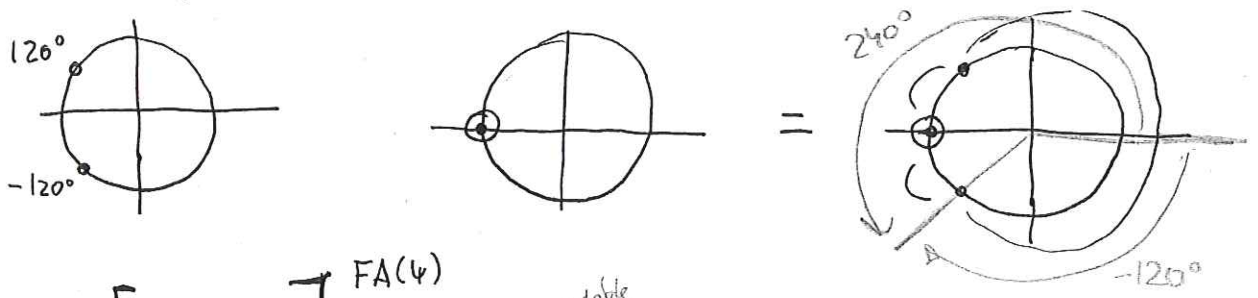
a) Calculem d, α

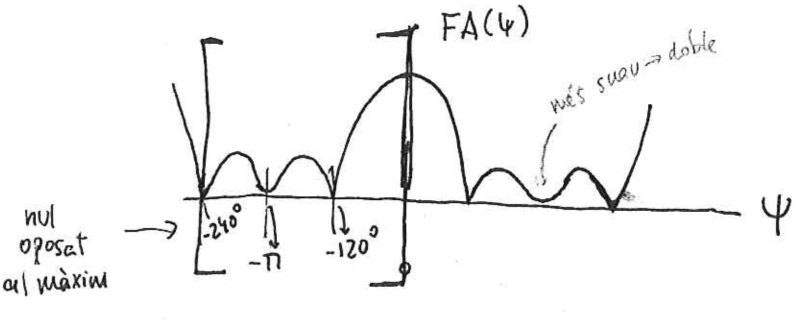
Radiació longitudinal → només tenen 1 lòbul principal

Max. ~~diferent~~ directivitat → màxim marge visible

Nul oposat al màxim

P(z) uniforme — P(z) triangular — P(z) agrupació





$MV = [\alpha - kd, \alpha + kd]$

$\alpha + kd = 0$

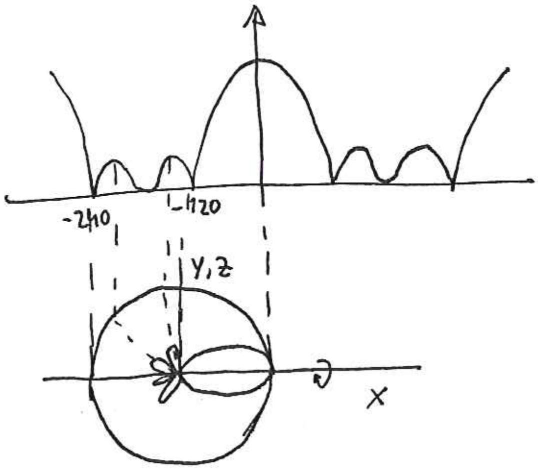
$\boxed{\alpha = -kd}$

$[\alpha = -120° = -2\pi/3]$

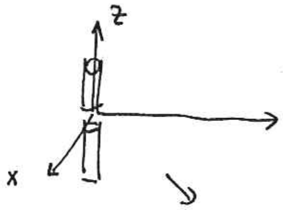
$\alpha = -kd \rightarrow kd = 120° = \frac{2\pi}{3} \rightarrow \left[d = \frac{\lambda}{2\pi}\frac{2\pi}{3} = \frac{\lambda}{3}\right]$

$MV = \left[-\frac{4\pi}{3}, 0\right]$

b) Dibuixar $FA(\psi) \rightarrow FA(\theta)$



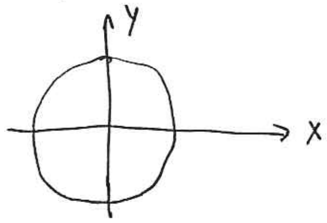
$\psi = k_x d + \alpha$ on $k_x = \sin\theta \cos\phi$

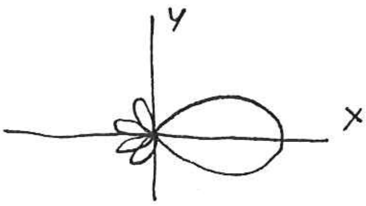


Pla H $\rightarrow (x, y)$

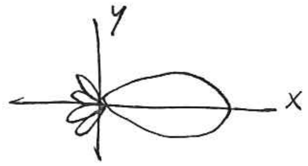
el seu pla H era

Dipol curt





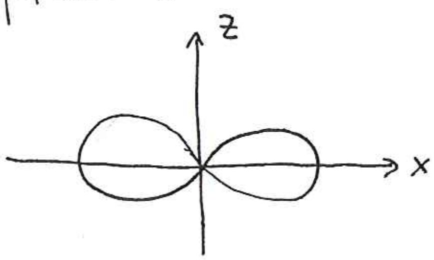
$\Longrightarrow$



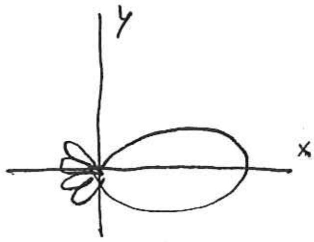
Pla E $\rightarrow \{z, x\}$

Dipol curt E

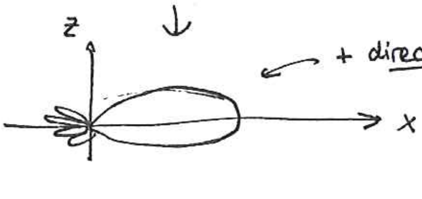
FA^E ↙ igual que el del pla H



×



↓



+ directiu

c) Directivitat (aprox. lineal)

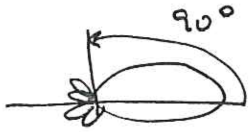
$\Delta\theta_c$

$$d = \lambda/3 < \lambda$$

$$D \simeq \frac{4d}{\lambda} \frac{(\sum a_n)^2}{\sum a_n^2} =$$

$$= \frac{4\cdot\frac{\cancel{\lambda}}{3}}{\cancel{\lambda}} \cdot \frac{(1+3+4+3+1)^2}{1^2+3^2+4^2+3^2+1} = \frac{12^2}{1+1+2\cdot 9+16} = \frac{12^2}{36} =$$

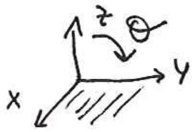
$$\Delta\theta_c = 180^\circ$$



$$= \underline{\underline{5{,}3}}$$

El zero el tenim a

$$-120^\circ = -\frac{2\pi}{3} = \frac{2\pi}{\lambda}\frac{\lambda}{3}\cos\phi_c - \frac{2\pi}{3} \rightarrow \cos\phi_c = 0$$



$\hookrightarrow \theta = \frac{\pi}{2}$

$$\boxed{\phi_c = \pi/2}$$

~~$\Delta\phi_c$~~

$$\Delta\theta_c = \Delta\phi_c = 180^\circ$$

Ex. 3

Reflector parabólico

$D_a = 90$ cm

$f/D = 0{,}5$

$f = 10$ GHz

a) Dissenyar la <u>bocina piramidal óptima</u> perquè la caiguda bordes del reflector sigui de 15 dB

$\rightarrow$ [$\eta_{il} = \boxed{}$ óptimes !]

valors exactes

$$\underline{\underline{\tau = -15\text{ dB}}}$$

$$\frac{f}{D} = \frac{1}{4\,tg\,\beta/2} = 0{,}5 \rightarrow \boxed{\beta = 53^\circ}$$

$$f = 10\text{ GHz} \rightarrow \lambda = \frac{c}{f} = \frac{3\cdot 10^8}{10\,10^9} = \underline{0{,}03\text{ m}}$$

$\underline{\tau = -15\,dB}$

$$\tau = \underbrace{40\log(\cos\overset{53°}{\beta}/2)}_{-1{,}93\,dB} + 10\log t(\beta)$$

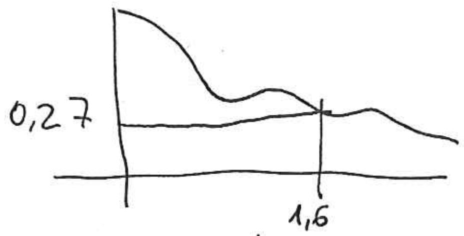
$$10\log(t(\beta)) = -15 + 1{,}93$$

$$10\log(t(\beta)) = 10\log E^2 + \overbrace{20\log\left(\frac{1+\cos\beta}{2}\right)}^{-1{,}93\,dB}$$

$$-15\,dB = -3{,}86\,dB + 20\log E \rightarrow 20\log E = -11{,}14$$

Pla E $\rightarrow S = \frac{1}{4}$

$$E = 10^{\frac{-11{,}14}{20}} = 0'27$$



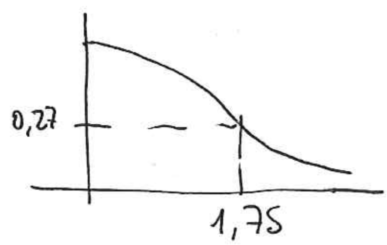
$$\frac{b_1}{\lambda}\sin\beta = 1{,}6$$

$$\boxed{b_1 = 2\lambda = 0{,}06\ m}$$

$$S = \frac{b_1^2}{8\lambda l_E} = 1/4 \rightarrow b_1^2 = 2\,l_E\,\lambda$$

$$l_E = \frac{b_1^2}{2\lambda} = \frac{\cancel{4}2\lambda^2}{\cancel{2}\lambda} = \boxed{2\lambda}$$

Pla H $\rightarrow t = 3/8$



$$1{,}75 = \frac{a_1}{\lambda}\sin\beta \rightarrow \boxed{a_1 = 2{,}2\lambda}$$

$$t = \frac{a_1^2}{8\lambda l_H} = 3/8 \longrightarrow l_H$$

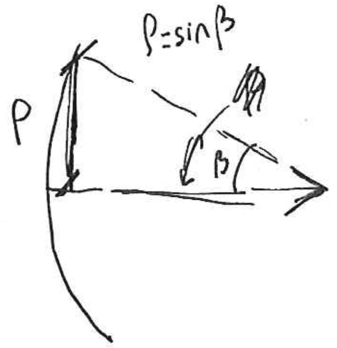
b) Directividad de la bocina

$$D = 6{,}4\,\frac{a_1 \cdot b_1}{\lambda^2} = \frac{6{,}4 \cdot 2\cancel{\lambda} \cdot 2{,}2\cancel{\lambda}}{\cancel{\lambda^2}} = \underline{28{,}6}$$

$$\Longrightarrow \boxed{14{,}49\ \text{dB}}$$

c) Distribució de camps es pot aproximar $f^{\underline{o}}$ triangular sobre pedestal

Calcular η_{il} $\Downarrow$



$$E_a = \left(1 - 0{,}8\,\frac{\rho}{D_a/2}\right)$$

lineal

$$-15\,\text{dB} \rightarrow \boxed{E_a = 0'2}$$

$$\eta_{il} = \frac{\left|\int_0^{2\pi}\int_0^{D_a/2} E_a\,\rho\, d\rho\, d\theta\right|^2}{\pi\left(\frac{D_a}{2}\right)^2 \int_0^{2\pi}\int_0^{D_a/2} E_a^2\,\rho\, d\rho\, d\theta} = 0{,}85 \,//$$

d) $\eta_s \Rightarrow$ eficiencia de desbordament

$\eta = 0{,}7$

$$D = \frac{4\pi}{\lambda^2}\,\pi\left(\frac{D_a}{2}\right)^2 \eta_t \qquad D = 5285 = 37'2\ \text{dB}$$

on $\eta_t = \eta_{il} \cdot \eta_s$

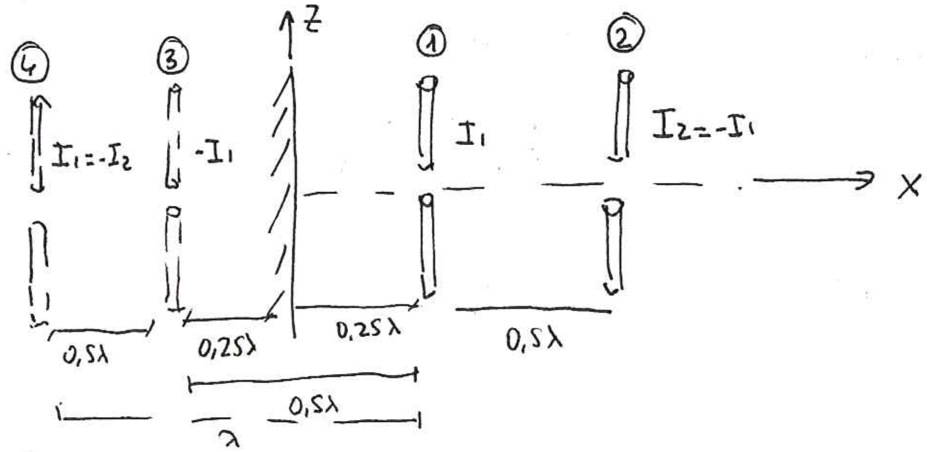
$\eta_{il} \to 0{,}85$, $\eta_s \to 0{,}7$

$\boxed{J07}$

1. 2 dipols $\frac{\lambda}{2}$

$I_2 = -I_1$

$d = \frac{\lambda}{2}$ entre sí



a) Z_{in_1}

Z_{in_2}

$Z_{21} I_1 + Z_{22} I_2 + Z_{23} I_3$

$$V_1 = Z_{11} I_1 + Z_{12} I_2 + Z_{13} I_3 + Z_{14} I_4 =$$

$$= Z_{11} I_1 - Z_{12} I_1 - Z_{13} I_1 + Z_{14} I_1$$

$$Z_{in_1} = \frac{V_1}{I_1} = Z_{11} - Z_{12} - Z_{13} + Z_{14}$$

$$= \cancel{58}\ 101 + j120$$

$Z_{11} = 73 + j42$

$Z_{12} = -12 - j30$

$Z_{13} = -12 - j30$

$Z_{14} = 4 + j18$

$$\frac{V_2}{I_2} = Z_{22} - Z_{21} + Z_{23} - Z_{24} =$$

$$= 91 + j102$$

b) Camps

$$\vec{E} = E_0 \left(e^{jk_x \lambda/4} - e^{jk_x 3\lambda/4} - e^{-jk_x \lambda/4} + e^{-jk_x 3\lambda/4} \right)$$

30/12/07
Antenas

b) $\vec{E}_0 = \vec{E}_{d\lambda/2} = \hat{\theta} \cdot 60 I_1 j \dfrac{e^{-jkr}}{r} \cdot \dfrac{\cos(\pi/2 \cos\theta)}{\sin\theta}$

$\vec{E} = \vec{E}_0 \left(\sin k_x \lambda/4 - \sin k_x 3\lambda/4 \right) 2j$

$k_x = k \sin\theta \cos\phi \quad \rightarrow k\, \lambda/4 = \dfrac{2\pi}{\lambda} \dfrac{\lambda}{4} = \pi/2$

$\qquad\qquad\qquad\qquad \searrow k\, 3\lambda/4 = 3\pi/2$

$\vec{E} = \hat{\theta}\, 60 I j \dfrac{e^{-jkr}}{r} \dfrac{\cos(\pi/2 \cos\theta)}{\sin\theta} 2j \left[\sin(\pi/2 \sin\theta \cos\phi) - \sin(3\pi/2 \sin\theta \cos\phi) \right]$

$E\phi = 0$

$E\phi = 0$

$E\theta \Rightarrow \boxed{H\phi = E\theta/\eta}$

$H\theta = E\phi/\eta = \phi$

c) Màxim, plans

Màxim

$\hookrightarrow \theta = \pi/2$

$E \propto \left[\sin(\pi/2 \cos\phi) - \sin\left(\frac{3\pi}{2} \cos\phi\right) \right] \longrightarrow$ màxim $= 2$

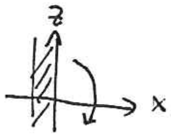
$\cos\phi = 1$

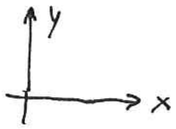
$\downarrow$

$\boxed{\phi = 0}$

$\downarrow$

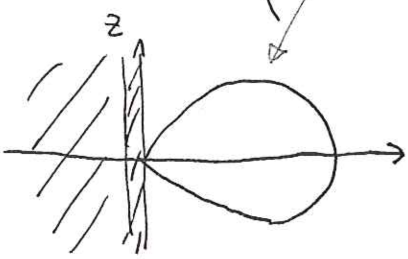
màxim en X

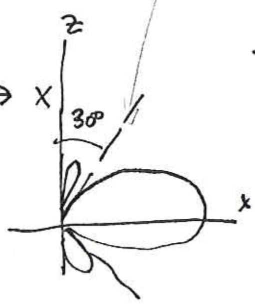
Pla E $= \{X, Z\}$  $0 \leq \theta \leq \pi$

Pla H $= \{X Y\}$  $-\frac{\pi}{2} \leq \phi \leq \pi/2$

d) Pla E $\Rightarrow \{X, Z\} \Rightarrow \phi = 0$

$$t(\theta, \phi) = \left(\dfrac{\dfrac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta} \cdot \left[\sin(\frac{\pi}{2} \sin\theta) - \sin(\frac{3\pi}{2} \sin\theta) \right]}{2} \right)^2$$





$\pi/2 \sin\theta + 2\pi = 3\pi/2 \sin\theta$

$\boxed{\theta = 0}$

$\theta = 30 \rightarrow$ también 0

35

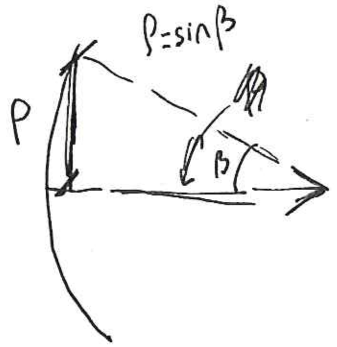
b) Directividad de la bocina

$$D = 6{,}4\,\frac{a_1 \cdot b_1}{\lambda^2} = \frac{6{,}4 \cdot 2\cancel{\lambda} \cdot 2{,}2\cancel{\lambda}}{\cancel{\lambda^2}} = \underline{28{,}6}$$

$$\Longrightarrow \boxed{14{,}49\ \mathrm{dB}}$$

c) Distribució de camps es pot aproximar f° triangular sobre pedestal

Calcular η_{il}



$$\Downarrow$$

$$\underset{\text{lineal}}{E_a} = \left(1 - \frac{0{,}8\,\rho}{D_a/2}\right)$$

$$-15\,\mathrm{dB} \rightarrow \boxed{E_a = 0'2}$$

$$\eta_{il} = \frac{\left|\int_0^{2\pi}\int_0^{D_a/2} E_a\,\rho\,d\rho\,d\theta\right|^2}{\pi\left(\frac{D_a}{2}\right)^2 \int_0^{2\pi}\int_0^{D_a/2} E_a^2\,\rho\,d\rho\,d\theta} = 0{,}85\,//$$

d) $\eta_s \Rightarrow$ eficiencia de desbordament

$\eta = 0{,}7$

$$D = \frac{4\pi}{\lambda^2}\,\pi\left(\frac{D_a}{2}\right)^2 \eta_t \quad D = 5285 = 37'2\ \mathrm{dB}$$

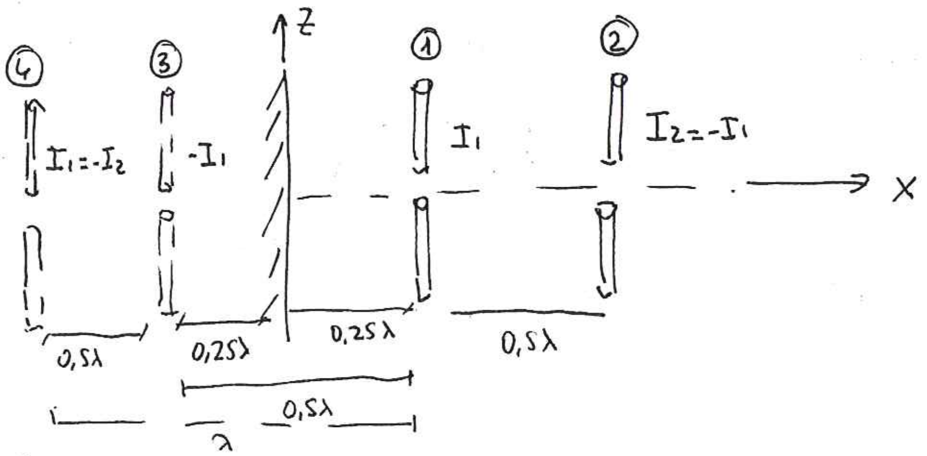
on $\eta_t = \underbrace{\eta_{il}}_{0{,}85} \cdot \underbrace{\eta_s}_{0{,}7}$

J07

1. 2 dipols $\frac{\lambda}{2}$

$I_2 = -I_1$

$d = \frac{\lambda}{2}$ entre sí



a) Z_{in_1}
Z_{in_2}

$Z_{21}I_1 + Z_{22}I_2 + Z_{23}I_3$

$$V_1 = Z_{11}I_1 + Z_{12}I_2 + Z_{13}I_3 + Z_{14}I_4 =$$
$$= Z_{11}I_1 - Z_{12}I_1 - Z_{13}I_1 + Z_{14}I_1$$

$$Z_{in_1} = \frac{V_1}{I_1} = Z_{11} - Z_{12} - Z_{13} + Z_{14}$$
$$= \cancel{58}\ 101 + j120$$

$Z_{11} = 73 + j42$
$Z_{12} = -12 - j30$
$Z_{13} = -12 - j30$
$Z_{14} = 4 + j18$

$$\frac{V_2}{I_2} = Z_{22} - Z_{21} + Z_{23} - Z_{24} =$$
$$= 91 + j102$$

b) Camps

$$\vec{E} = E_0\left(e^{jk_x \lambda/4} - e^{jk_x 3\lambda/4} - e^{-jk_x \lambda/4} + e^{-jk_x 3\lambda/4}\right)$$

$\boxed{J07}$ (2)

30/12/07
Antenas

Red longitudinal

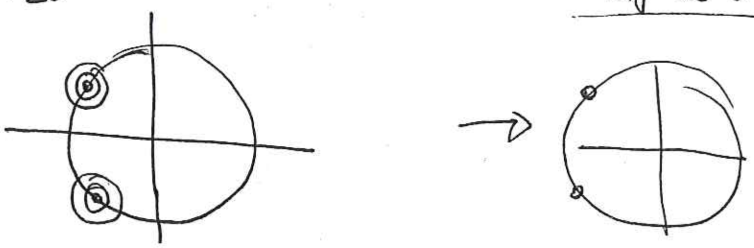
Red $z>0$

$\theta_{max} = 0$

lòbul trasero $\theta = 180°$

$\psi_c = \pm 120°$

Unif de 3 elements

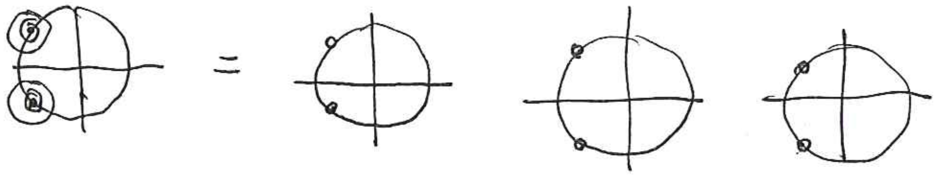


a) $d = \lambda/4$

$\alpha = -90°$

zeros cada $\frac{2\pi}{3} = \pm 120°$

$P(z)$

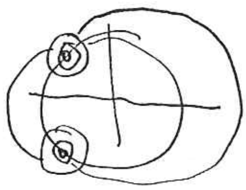


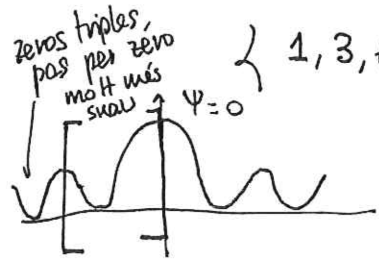
$$a_n = a_{n1} * a_{n2} * a_{n3}$$

$$\underbrace{\{1,1,1\} * \{1,1,1\}}_{\{1,2,3,2,1\}} \quad \{1,1,1\}$$

$$\{1,3,6,7,6,3,1\}$$

b) $FA(\psi)$, $FA(\theta)$

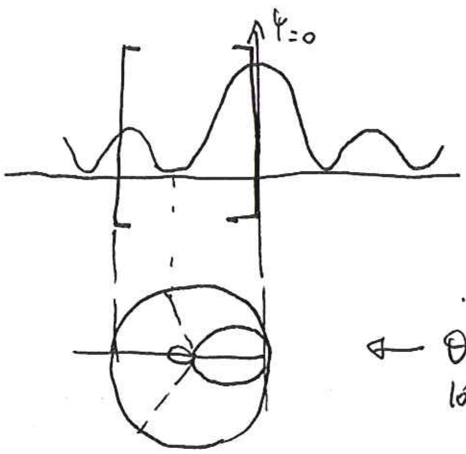




$\psi_{max} = 0 \rightarrow a_n \in \mathbb{R} > 0$

MV: $[-kd+\alpha,\ \alpha+kd]$ $\qquad kd = \frac{2\pi}{\lambda}\lambda/4 = \frac{\pi}{2}$

MV: $[-\pi, 0]$ $\qquad \alpha = -\pi/2$



$\leftarrow \theta_{max}=0$

lóbulo trasero $\theta=180°$

c) $$FA(\psi) = \left| \frac{\sin(N \cdot \psi/2)}{\sin(\psi/2} \right|^3$$

$$= \left| \frac{\sin(3\psi/2)}{\sin \psi/2} \right|^3$$

$$FA(0) = \left| \frac{3\psi/2}{\psi/2} \right|^3 = 3^3 = \underline{\underline{27}}$$

$$FA(-\pi) = |1|^3 = \underline{\underline{1}}$$

$$\Rightarrow NLPS = 20 \log \left| \frac{\overset{27}{\overset{\shortparallel}{FA(0)}}}{FA(-\pi)} \right| = \boxed{28{,}6 \text{ dB}}$$

d) Directividad

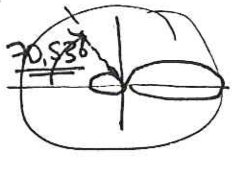
$$D = \frac{(\sum a_n)^2}{\sum a_n^2} = \frac{729}{141} = 5'17 \rightarrow \underline{\underline{7{,}13 \text{ dB}}}$$

e) $\Delta\theta_c =$

$$\underset{-120}{\psi} = \underbrace{kd}_{kz} \cos\theta + \alpha \qquad -120 = 90 \cdot \cos\theta - 90$$

$$\Downarrow$$

$$\underline{\theta_c = 70'53°}$$



$$\theta_{c_1} = 180 - 70{,}53 = 109{,}57°$$

$$\Delta\theta_c \simeq 219°$$

30/12/07
Antenas

(2)

f) $\Delta\theta_{-3dB} \rightarrow FA(0) = 27$

$FA(\Psi_{-3dB}) = \dfrac{27}{\sqrt{2}}$

$$\left(\frac{\sin 3\Psi_{-3dB}/2}{\sin \Psi_{-3dB}/2}\right) = \frac{27}{\sqrt{2}} \rightarrow (1 + 2\cos\Psi_{-3dB})^3 = \frac{27}{\sqrt{2}}$$

$$P(z) = (1+z+z^2)^3 = (1 + e^{j\Psi} + e^{2j\Psi})^3$$

$$FA(\Psi) = P(z)\Big|_{z=e^{j\Psi}}$$

$$\Psi_{-3dB} = -33{,}24^\circ$$

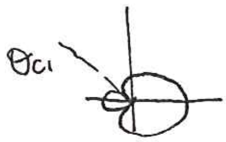
$$-33{,}24^\circ = 90\cos\theta_{-3dB} - 90$$

$$\theta_{-3dB} = 51^\circ$$

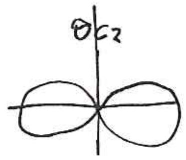
$\times 2$

$$\Delta\theta_{-3dB} = 102^\circ$$

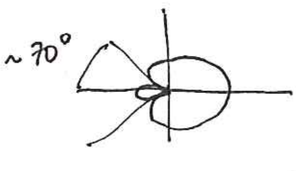
g) Pla $\vec{E}$ agrupació



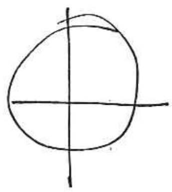
Pla $\vec{E}$ dipol $\lambda/2$



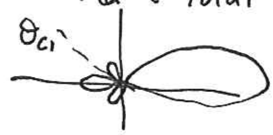
Pla $\vec{H}$ agrup



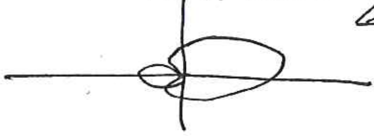
Pla $\vec{H}$ dipol



Pla E total



Pla H total



igual

(3)

$D_a = 50\,cm$

distancia focal $\rightarrow f = 21{,}6\ cm$

$f = 30\,GHz$

PART BOCINA $\rightarrow$ Cònica òptima $(s = 3/8)$

$ZH = -20\,dB$

a) $f/D_a = \dfrac{1}{4\,tg\,\beta/2} \longrightarrow \beta = 60^\circ$

d_m?

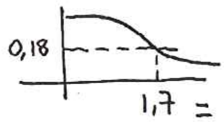
d_m?

$$\tau = -20\,dB = \underbrace{40 \log(\cos\beta/2)}_{-2'5\,dB} + 10\log(t(\beta))$$

$$10\log(t(\beta)) = \underbrace{20\log\left(\frac{1+\cos\beta}{2}\right)^2}_{-2{,}5\,dB} + 20\log E$$ ← lateral de la gràfica

$$E = -15\,dB = E = 10^{-15/20} = 0{,}178 \sim 0'18$$

Gràfica $\rightarrow$ $\frac{a_1}{\lambda}\sin\beta \sim 1{,}7 \rightarrow \boxed{d_m = 2\lambda}$ (with $a_1 = d_m$)



$1{,}7 = \frac{a_1}{\lambda}\sin\theta$

Pla H

b) Camps a l'obertura de la bocina $\propto e^{-jkr}$

$$\boxed{\Delta\measuredangle_{max} = \frac{2\pi}{\lambda}\frac{(d_m/2)^2}{2\ell_c} = 2\pi t}$$

máximo error de fase en la boca de la bocina

$= 2\pi \cdot 3/8 = 3\pi/4$

(si fos rectangular $\rightarrow$ $\Delta\measuredangle_{E\,max}$, $\Delta\measuredangle_{H\,max}$)

c) D $\rightarrow$ $t = 3/8$ òptim

$$D = \eta_{il}\,(\pi\, d_m/\lambda)^2 = 20'53 \rightarrow \underline{13\,dB's}$$ (with $d_m = 2\lambda$)

$\uparrow$ cònica, òptima $\underline{0{,}52}$

d) $$\Delta\theta_{3dB}^{H} = \frac{70^\circ\lambda}{d_m} = 35^\circ$$ ($d_m = 2\lambda$)

PART ~~RECEPTOR~~ REFLECTOR $\rightarrow$

$$D(\theta) \simeq 20\cos^{10}\theta \,, \quad 0 \le \theta \le \beta \quad \rightarrow \eta_s$$

Diagrama de Rad Botzina

$$\eta_s = \frac{\int_0^{2\pi}\int_0^{\beta} D(\theta)\sin\theta\, d\theta\, d\phi}{\int_0^{2\pi}\int_0^{\pi} D(\theta)\sin\theta\, d\theta\, d\phi} = \frac{\int_0^{2\pi}\int_0^{\beta} 20\cos^{10}\theta\sin\theta\, d\theta\, d\phi}{\int_0^{2\pi}\int_0^{\pi} 20\cos^{10}\theta\sin\theta\, d\theta\, d\phi} =$$

$$= \frac{10}{11}\left(1-\cos^{11}\overset{\beta}{60}\right)$$

$$\boxed{\eta_s = \frac{10}{11}\left(1-\cos^{11}60\right) \simeq \frac{10}{11} = 0{,}91}$$

f) $\tau = -20\,dB \longrightarrow$ lineal $0'1$

$$E_a = 1 - 0{,}9\,\rho^2/(D_a/2)^2 \qquad [D_a/2 = a]$$

$$\eta_{il} = \frac{1}{\pi a^2}\,\frac{\left|\int_0^{2\pi}\int_0^{a}\left[1-0{,}9(\rho/a)^2\right]\rho\, d\rho\, d\phi\right|^2}{\int_0^{2\pi}\int_0^{a}\left[1-0{,}9(\rho/a)^2\right]^2\rho\, d\rho\, d\phi} =$$

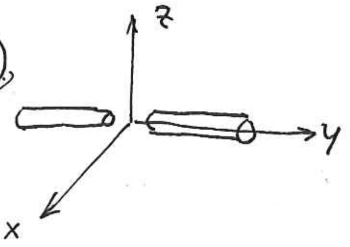
$$= \frac{2\pi}{\pi a^2}\,\frac{\left|a^2\int_0^{1}\left[1-0{,}9x^2\right]x\, dx\right|^2}{a^2\int_0^1\left[1-0{,}9x^2\right]^2 x\, dx} = \dots = \underline{\underline{0'83}}$$

g) Directivitat

$$D = \underset{\text{frec}}{\frac{4\pi}{\lambda^2}}\,\pi\left(\frac{D_a}{2}\right)^2 \cdot \underset{0{,}91}{\eta_s}\,\underset{0{,}83}{\eta_{il}} = 18'6 \rightarrow \boxed{D = 12{,}7\ dB}$$

J07 Test (Perm O)

(1)



$\vec{E} \propto \vec{A} \propto \vec{N}$

$N_y \rightarrow \hat{r}$

$\rightarrow \left[\vec{E} = j\omega \hat{r} \times (\hat{r} \times A\hat{y}) \right]$ (a)

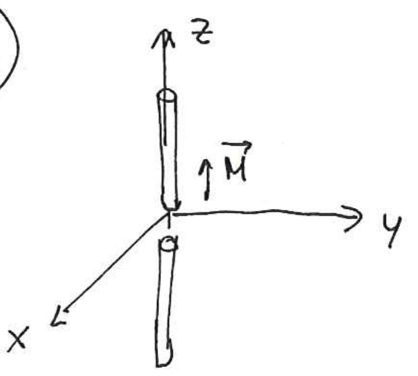
(2) 100 espiras nucli ferrita

Si el nucli era de ferrita

$\left[R_{resp} = N^2 R_{r\,1\,espira} \cdot |\mu_f^2|^{*} \right]$

$\hookrightarrow 1000 = \mu_f$

(3)

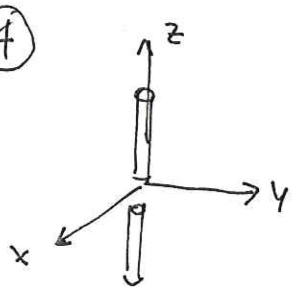


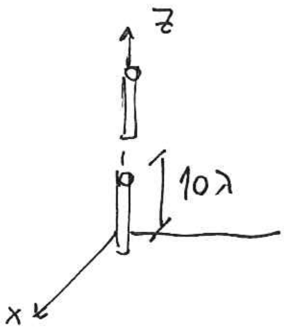
$E \{ ZX, ZY \}$, $H \{ XY \}$ } elèctric

Dipol magnetic $E \Leftrightarrow H$

$\hookrightarrow E = H_{electric} = \{XY\}$ (a)

(4)





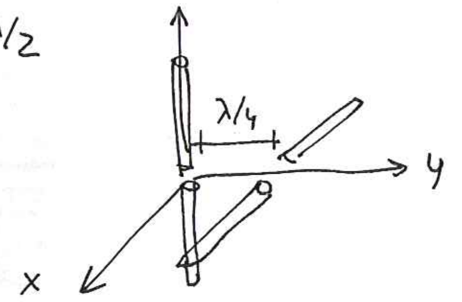
$E = E_0 e^{j \underline{k_z \cdot 10\lambda}} =$ ~~$= E_0 e^{jk}$~~

canviem la fase

(c)

30/12/07
Antenes

(5) $\lambda/2$

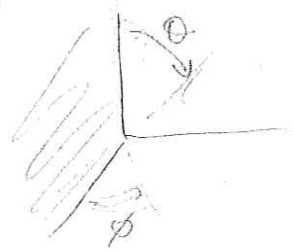


d) X → ~~és un pla~~ $\theta = 0°$ → $\boxed{\text{eix } z}$

↓

$(\hat{z} + \hat{x})$

↳ lineal !



$k_y = k \sin\theta \sin\theta$

$$E = E_{0_{\lambda/2}} \left[\hat{z} + e^{j \lambda/4 \, k_y} \hat{x} \right] =$$

$$= E_{0\lambda/2} \left[\hat{z} + e^{j \pi/2 \sin\theta \sin\phi} \hat{x} \right]$$

a) $\sin\phi = 0 \rightarrow \hat{z} + \hat{x}$

↳ lineal !

b) $\hat{z} + e^{j \pi/2 \cdot \frac{1}{\sqrt{2}}} \hat{x}$

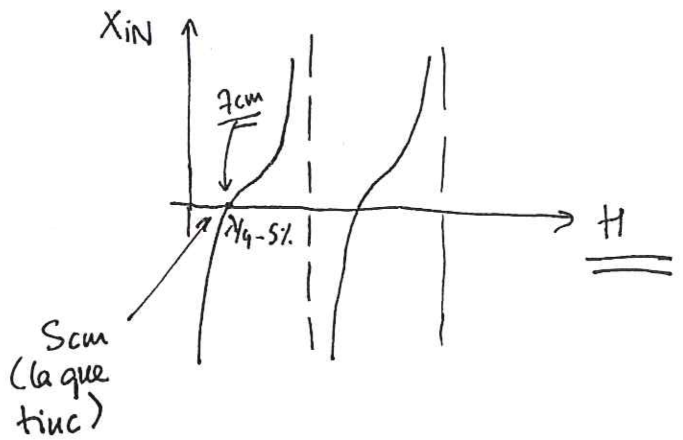
$[a\hat{z} + bj\hat{x}]$

c) $\hat{z} + e^{j\frac{\pi}{2}} \hat{x}$

$\boxed{\hat{z} + j\hat{x}}$ ← ✓

(6) $f = 1\,\text{GHz} \longrightarrow \lambda = 0{,}3\,\text{m} \rightarrow \frac{\lambda}{4} = \frac{0{,}3}{4} = 0{,}07\,\text{m}$ → 7cm

Model de línia de transmissió:



Cond $X_{in} < 0$

Bobines $X_{in} > 0$

↳ $X_{in} < 0$ — Volem carregar-nos-la → hem d'afegir alguna cosa amb $X_{in} > 0$!

Tenim 5, volem 7, l'afegim a 2cm de la base !

(b)

7) Dipolo doblado $\lambda/2$ $\longrightarrow$ Dipolo Simple $\lambda/2$

(2 dipolos en $\lambda/2$ muy prox)

a) $l_{ef\,dd} = 2\, l_{ef\,d}$ ✓

b) diagrama igual ✓

c) $Z_{in\,dd} = 4 Z_{ind}$ ✓

d) $l_{ef}^2 = 4 R_r A_{ef} / \eta$

$$A_{ef\,d} = \frac{l_{ef}^2}{4R_r}\eta \longrightarrow A_{ef\,dd} = \frac{(\cancel{2}\, l_{ef})^2}{\cancel{4}(\cancel{4}R_r)}\eta$$

$\boxed{A_{ef\,dd} = A_{ef\,d}}$ X ← la d está mal ¿?

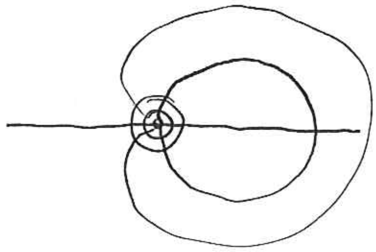
8) Binómica de 3 elementos

$d = \frac{3\lambda}{8}$ $\quad \alpha = -135° = $

$$\begin{matrix} & 1 & \\ 1 & & 1 \\ 1 & 2 & 1 \end{matrix}$$

unif

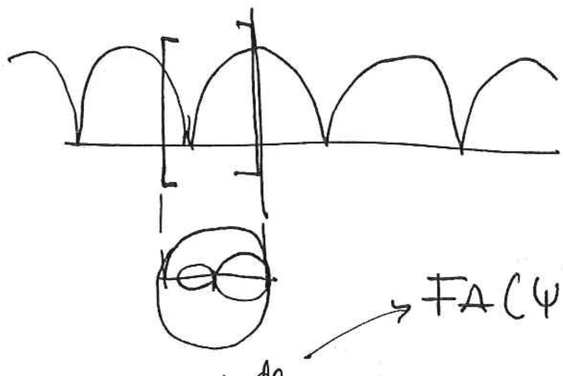
← se considera una triangular



$MV = [-kd+\alpha,\ kd+\alpha]$

$kd = \frac{2\pi}{\cancel{\lambda}}\frac{3\cancel{\lambda}}{8} = \frac{3\pi}{4} = 135°$

$MV = [-270°, 0]$



fórmula de la triangular

$$FA(\psi) = \left|\frac{\sin 2\psi/2}{\sin \psi/2}\right|^2$$

$FA(0) = 4$

$FA(\frac{3\pi}{2} = -\frac{\pi}{2}) = \left|\frac{1}{\frac{1}{\sqrt{2}}}\right|^2 = 2$

$20\log(\frac{4}{2}) = \boxed{6\,dB}$

(b)

9 N elementos

$d = \frac{\lambda}{2} \rightarrow d = \lambda$

$$\left[D = \left(\text{distancia} = m\frac{\lambda}{2} \right) = \frac{\left(\sum a_n\right)^2}{\sum a_n^2} \right]$$

(Si canviem la distància,
els a_n no varien; la direct tampoc) d

$\boxed{\begin{matrix} d < \lambda \\ d < \lambda/2 \end{matrix}}$ ← aquí és quan té numerets davant.

10 $TE_{10} \Rightarrow \left(\vec{E} = E_0 \cos\frac{\pi x}{a}\hat{y},\ \eta_{il} = 0'81 \right)$

$\downarrow$

$TE_{11} \Rightarrow \left(\vec{E} = E_0 \cos\frac{\pi x}{a} \cos\frac{\pi y}{a} \right)$

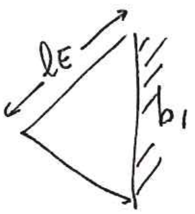
$D = [\text{ctes}] \cdot \eta_{il_x}\, \eta_{il_y}$

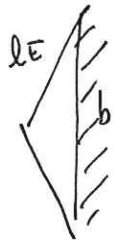
$TE_{10} \rightarrow 0{,}81\ ,\ 1$

$TE_{11} \rightarrow \text{ctes} \cdot 0{,}81\ \boxed{0{,}81}$ ← varía b

11 B. plano E óptima

b_1 / l_E



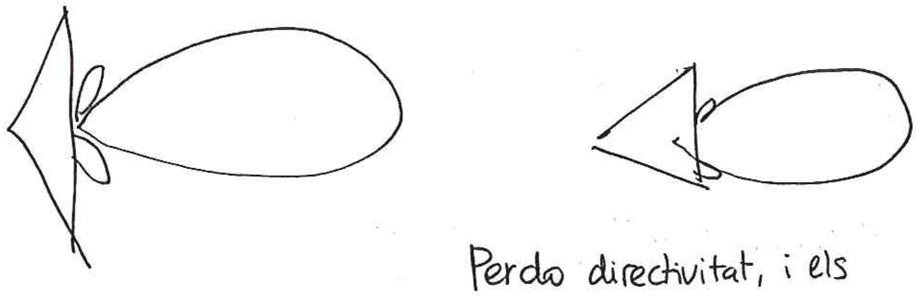


b) D ↑ → mentida, si obrim més la bocina empitjora
A part teniem ja l'òptima, no pot millorar

c) η_{il} ↑? → no, la millor és per l'òptima

d) Tampoc

a)



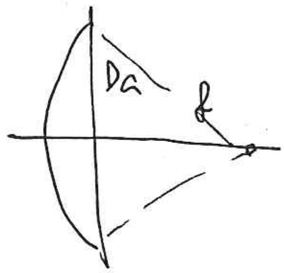
Perdo directivitat, i els lòbuls secundaris guanyen importancia

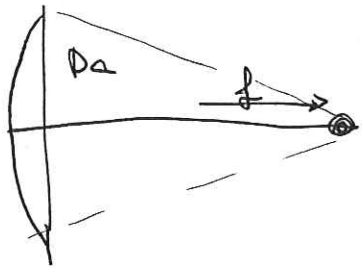
NLPS $\downarrow\downarrow$

(12) $t(\theta) = \cos^4(\theta/2)$

$f/D_a = 0'25$ $\longrightarrow$ correcta ?

$f/D_a = 0'3$





a) η_s no augmenta ?

b) $\eta_x \to 1$ (sempre és això, no canvia)

d) η_T no augmenta ?

↳ $\eta_T = \eta_s + \eta_{il}$

$\eta_s \downarrow\downarrow$

c) $\eta_{il} \uparrow\uparrow$ $\eta_s \downarrow\downarrow$