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FIBER-OPTIC COMMUNICATIONS

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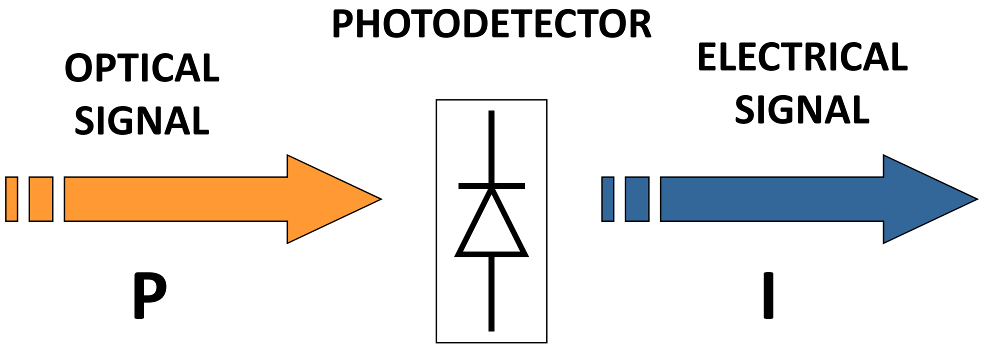
INTRODUCTION TO PHOTODETECTORS



QUADRATIC CONVERSION

$$I \propto P$$

DESIRABLE CHARACTERISTICS

- ✓ HIGH FREQUENCY RANGE
- ✓ HIGH CONVERSION EFFICIENCY
- ✓ LINEAR O/E TRANSFER FUNCTION
- ✓ RAPID RESPONSE TIME (BW)
- ✓ REDUCED NOISE LEVEL
- ✓ STABILITY (TEMPERATURE ...)
- ✓ FIBER SIZE COMPATIBILITY
- ✓ LOW CONSUMPTION
- ✓ REDUCED COST
- ✓ LONG LIFETIME

TYPES OF PHOTODETECTORS

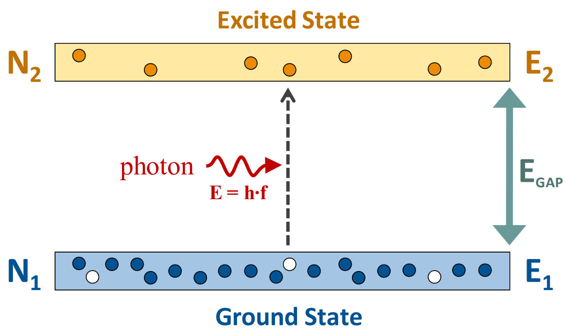
- PHOTOMULTIPLIERS
- PYROELECTRIC DETECTORS
- SEMICONDUCTORS
  - PHOTOCONDUCTORS
  - PHOTOTRANSISTORS
  - PHOTODIODES
    - PN
    - PIN
    - APD

OPTICAL COMMUNICATIONS

PHOTODIODES

Working Principle

STIMULATED ABSORPTION

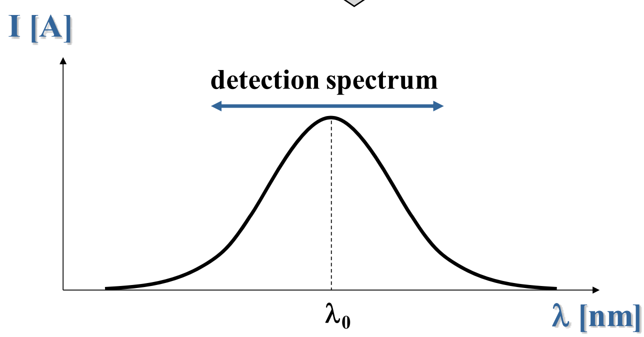


"The incident photon is absorbed by an electron which increments its energy level"

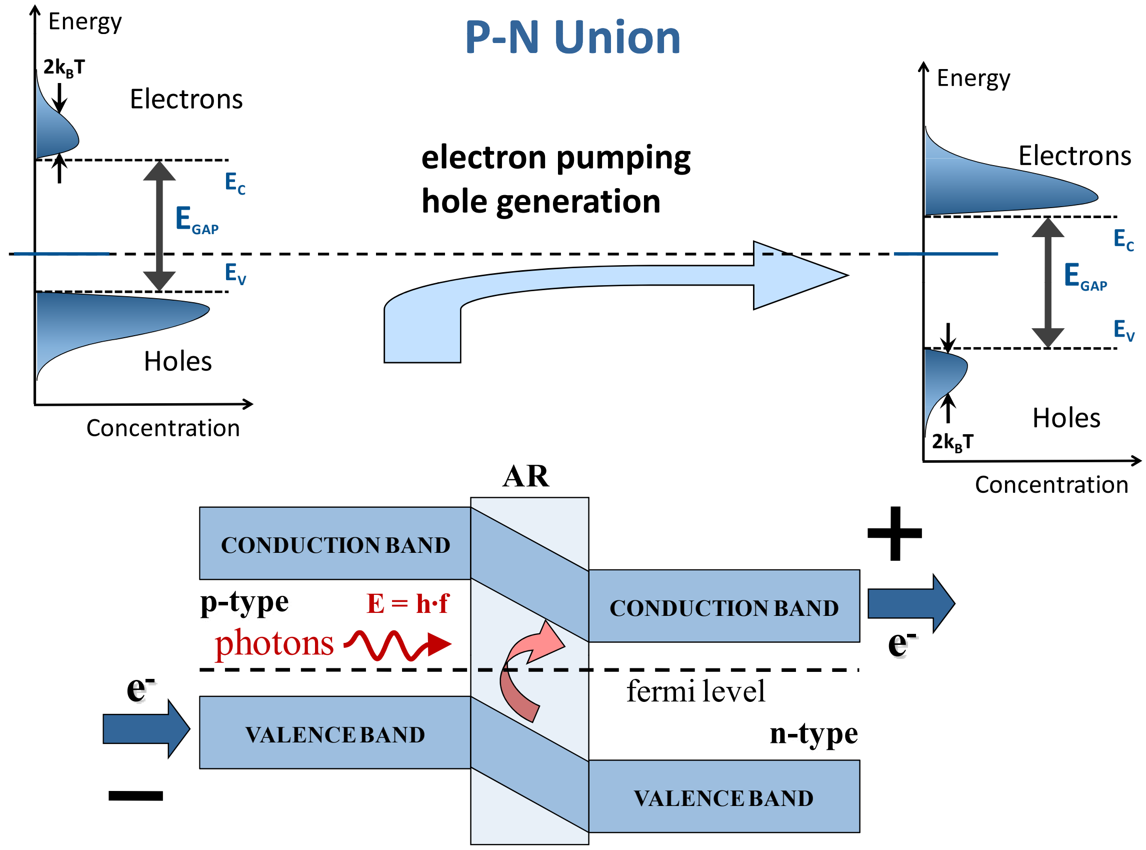
$$hf \geq E_g \rightarrow \lambda \leq \frac{h \cdot c}{E_g} \equiv \lambda_c$$

$$\boxed{\lambda_c = \frac{1.24}{E_g[eV]}[\mu m]}$$

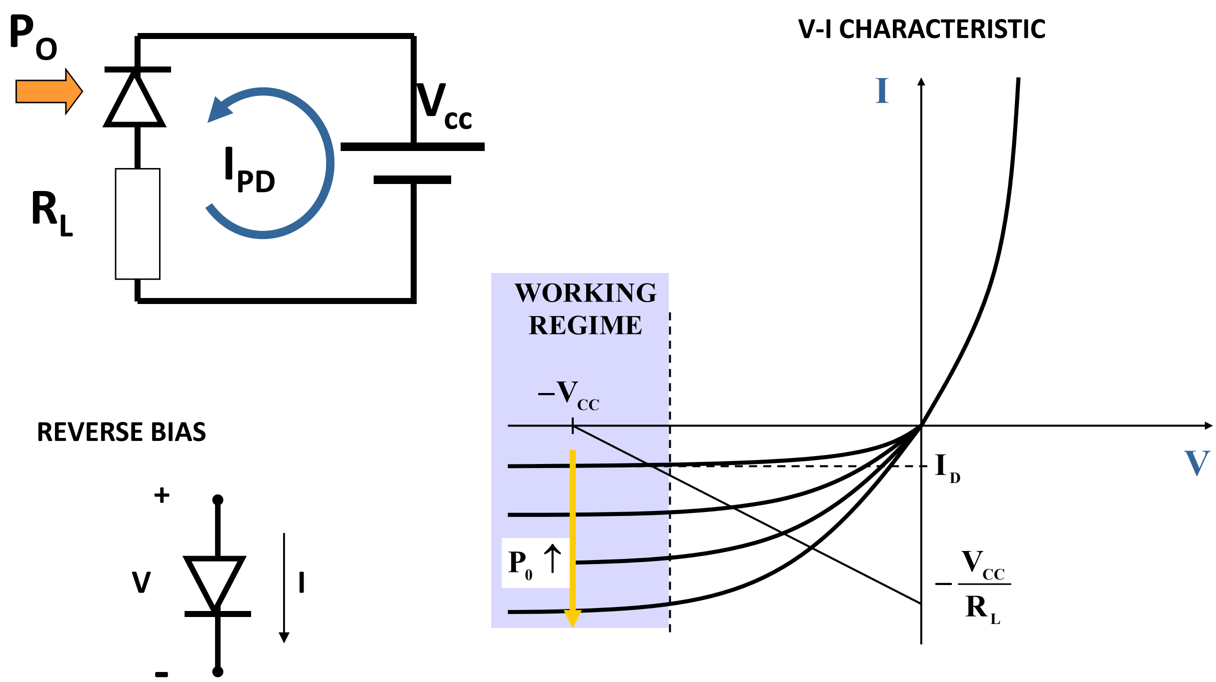
cut-off wavelength



P-N Union



Circuital Model



I_D: dark current

Quantum Efficiency

"Measure of the photon-electron conversion efficiency"

$$\eta \equiv \frac{\left\langle N^o e - h / seg \right\rangle}{\left\langle N^o fot / seg \right\rangle} = \frac{I_P / q}{P_{IN} / hf} \leq 1$$

Depends on:

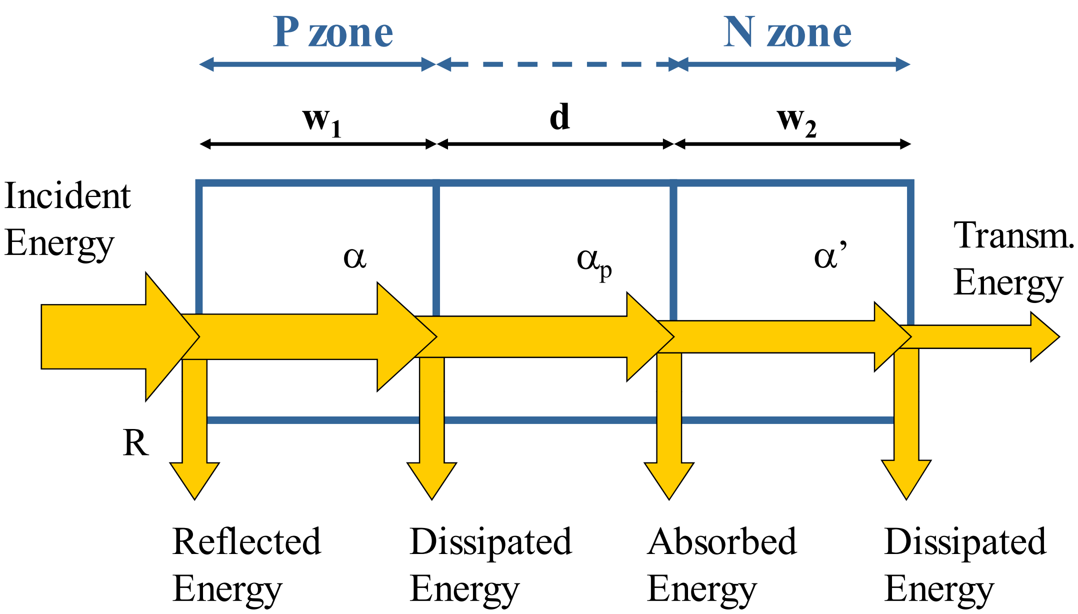
- **materials**
- **structure**

Responsivity

"Average delivered photocurrent over average incident optical power ratio (transfer function)"

$$R \equiv \frac{I_P}{P_{IN}} = \eta \frac{q}{hf} = \eta \frac{q}{h} \frac{\lambda}{c} \quad \text{[A/W]}$$

$$\lambda \uparrow \rightarrow R \uparrow$$



$$\eta \equiv \frac{P_{absorbed}}{P_{incident}} \leq 1$$

α: loss coefficient
α_p: absorption coefficient

$$P_{IN}$$
$$P_{IN}(1-R)$$
$$P_{IN}(1-R)\exp[-\alpha w_1]$$
$$P_{IN}(1-R)\exp[-\alpha w_1](1-\exp[-\alpha_p d])$$

$$\eta \equiv \frac{P_{absorbed}}{P_{incident}} = (1-R)\exp[-\alpha w_1](1-\exp[-\alpha_p d])$$

usually

$$\left.\begin{matrix} R \ll 1 \\ \alpha w_1 \ll 1 \end{matrix}\right\} \rightarrow \boxed{\eta \approx 1-\exp[-\alpha_p d]}$$

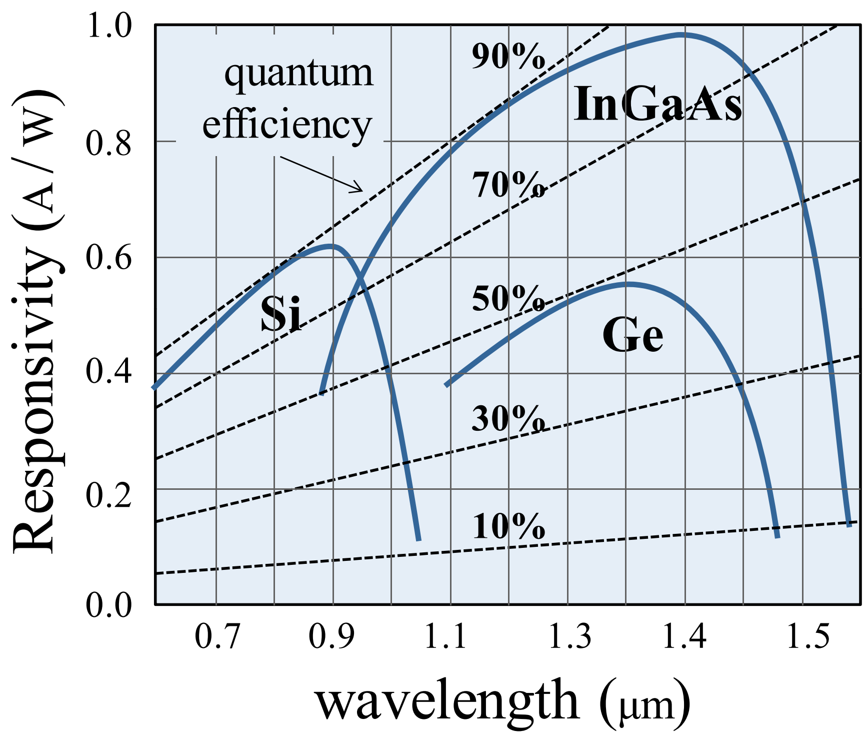
frequency dependence

$$\alpha_p(\lambda) \rightarrow \eta(\lambda)$$

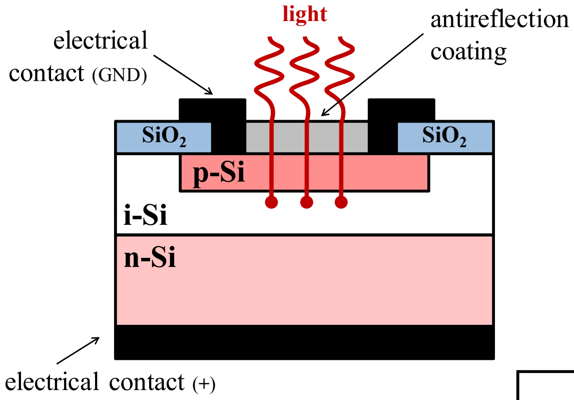
interest → $\alpha_p d \uparrow\uparrow$

typically → $\eta \approx 0.6 - 0.8$

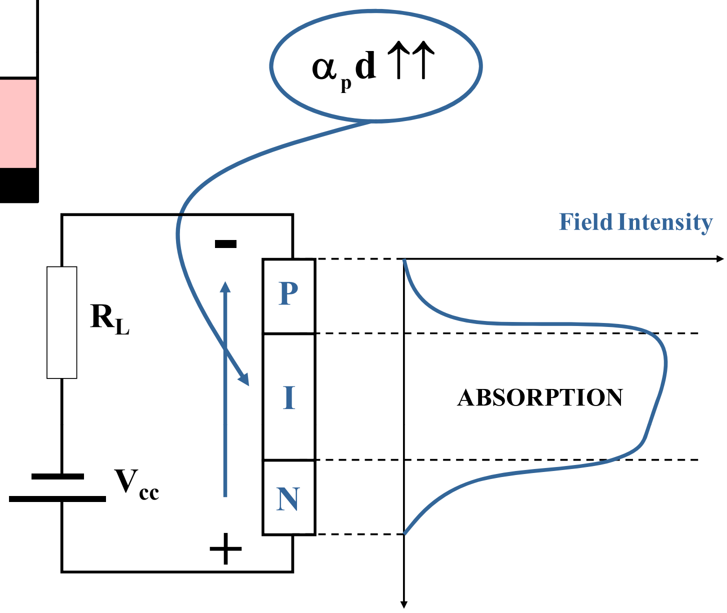
Responsivity for different photodetector materials



Types of Photodiodes: PIN



intrinsic semiconductor material



trade-off

$$d\uparrow \begin{cases} \eta\uparrow \\ t_r\uparrow \end{cases}$$

$$d \approx 10-20\mu m$$
$$\eta \approx 80-85\%$$

t_r: response time

I –V Characteristic

current = photocurrent + dark current

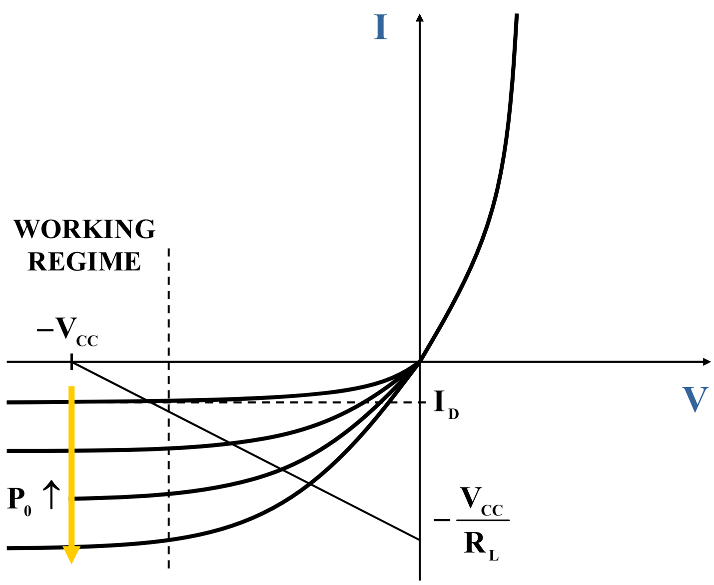
$$i_{PD} = i_{PH} + i_D$$

$$\boxed{\begin{array}{l} I_{PH} \equiv \langle i_{PH} \rangle = R \cdot P_{IN} \\ \sigma_{PH}^2 = 2qB \cdot I_{PH} \end{array}}$$

random variables

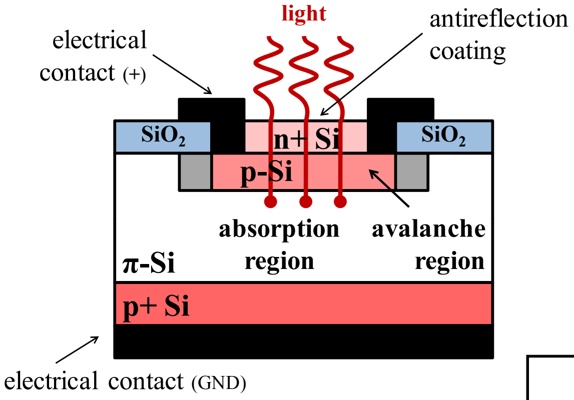
$$\Downarrow$$

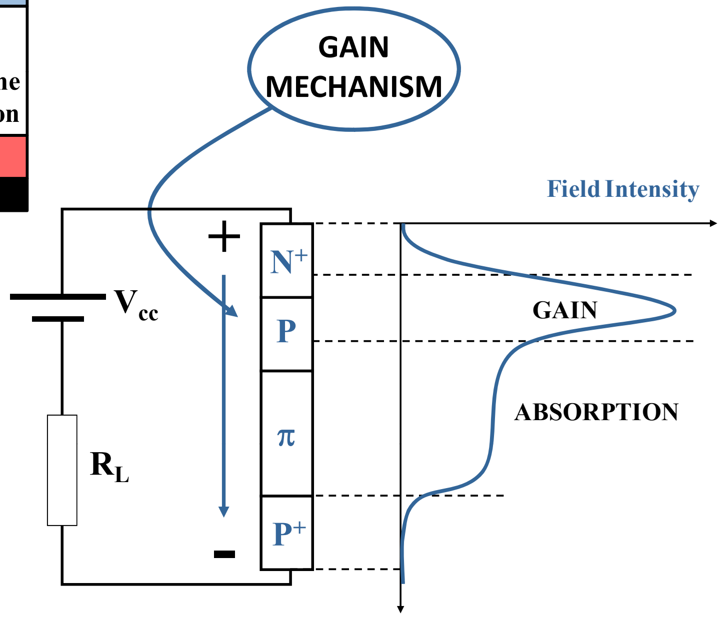
$$SNR \equiv \frac{\langle n \rangle^2}{\sigma_n^2}$$



B: receiver bandwidth

Types of Photodiodes: APD AVALANCHE PHOTODIODE





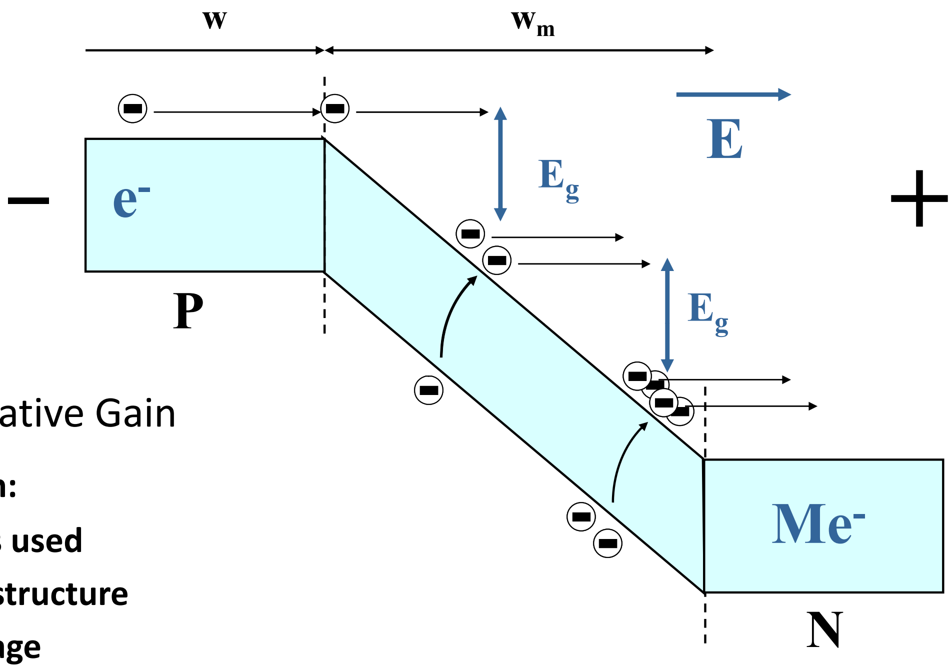
trade-off

$$R_{APD} = M \cdot R_{PIN}$$

$$R_{APD} \approx 20 - 80 \quad A/W$$

$$t_r\big|_{APD} > t_r\big|_{PIN}$$

Avalanche Effect

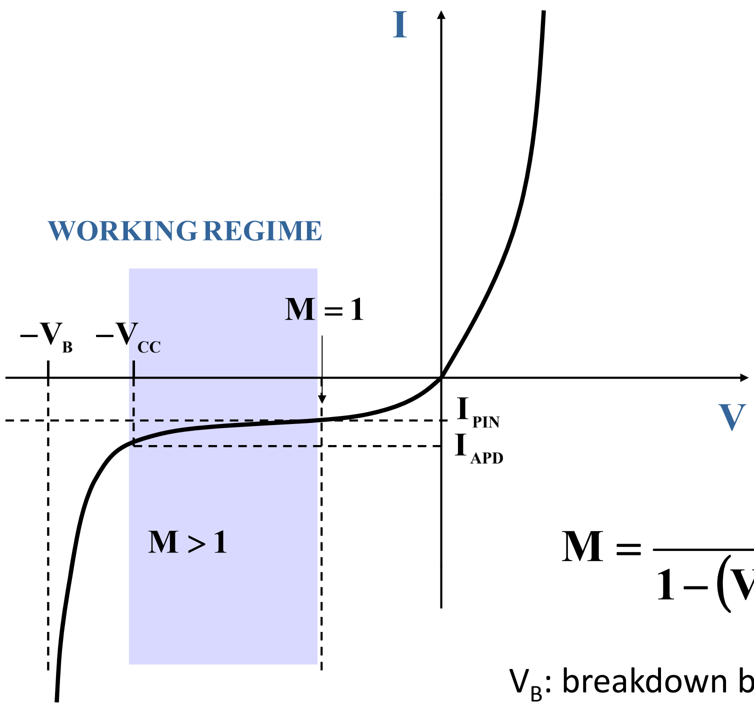


M: Multiplicative Gain

Depends on:

- **materials used**
- **physical structure**
- **bias voltage**
- **temperature**

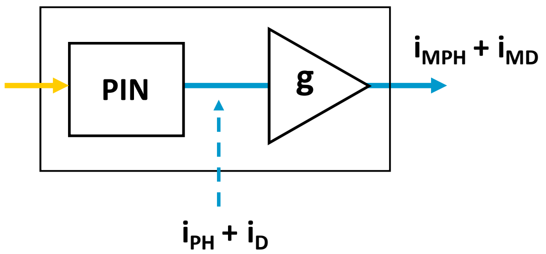
I –V Characteristic



$$M = \frac{1}{1-\left(V/V_B\right)^n}$$

V_B: breakdown bias

Equivalent Statistical Model of an APD



i_{PD}: primary current (v.a.)
i_{APD}: secondary current (v.a.)
g: gain (v.a.)
F: noise factor

$$\langle g \rangle = M \qquad \langle i_D \rangle = I_D$$

$$i_{PD} \begin{cases} \langle i_{PD} \rangle = I_{PD} \\ \sigma^2_{PD} = 2qB \cdot I_{PD} \end{cases} \qquad i_{APD} \begin{cases} \langle i_{APD} \rangle = I_{APD} = M \cdot I_{PD} \\ \sigma^2_{APD} = 2qB \cdot M^2 F(M) \cdot I_{PD} \end{cases}$$

$$I_{APD} = M \cdot \underbrace{\left(I_{PH} + I_D\right)}_{I_{PD}} = M\left(R \cdot P_{IN} + I_D\right)$$

$$\sigma^2_{APD} = M^2 F(M) \cdot \sigma^2_{PD} = M^2 F(M) \cdot 2qB\left(R \cdot P_{IN} + I_D\right)$$

Noise Factor

$$F(M) \approx M^x \qquad 0'2 < x < 1$$

empirically $\longrightarrow$ $F(M) = kM + (1-k)(2 - 1/M) \quad 0 < k < 1$

$$k = 0 \rightarrow F \approx 2 \quad \text{(ideal)}$$
$$k = 1 \rightarrow F = M$$
$$M = 1 \quad \text{(PIN)} \quad \rightarrow F = 1$$

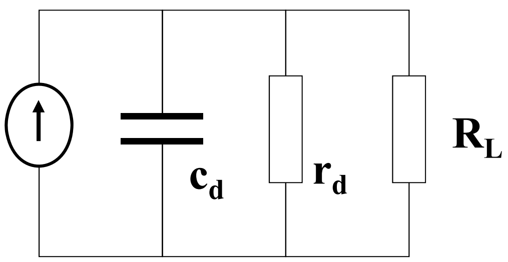
Sensitivity – Bandwidth trade-off

1. AR propagation
2. ⇨ Temporal constant $R_L \cdot c_d$
3. ⇨ Avalanche Effect

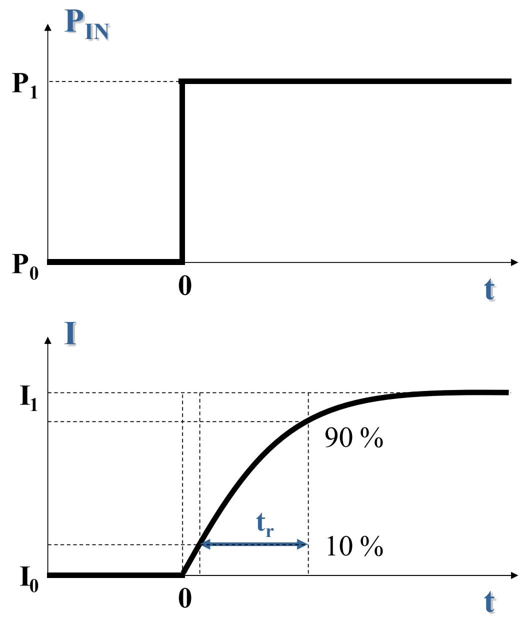
PIN → BW=ct

APD → M·BW=ct

Equivalent Circuit



$r_d >> R_L$



Response Time

$$t_r = \ln\left(\frac{0.9}{0.1}\right) \cdot R_L c_d \approx 2'19 \cdot R_L c_d$$

$$f_{3dB} = \frac{1}{2\pi \cdot R_L c_d}$$

InGaAs – PIN → t_r = 0.01 ns

InGaAs – APD → t_r = 0.1 ns

APD vs PIN

ADVANTAGES

- Better Sensitivity (5-15 dB)
- Reduction of P_{IN} fluctuations

DRAWBACKS

- Lower Bandwidth
- Higher Cost
- Noise Addition
- Higher Consumption
- Temperature Control

PHOTODETECTION NOISE

Definition

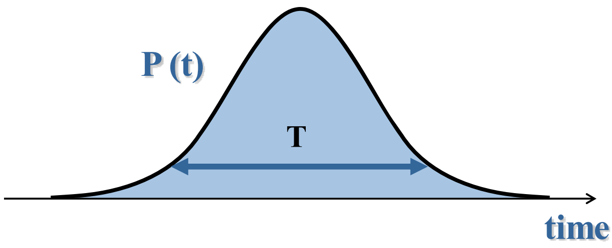
"Perturbation of the transmitted signal which can mask the information contained in it to the point of making the detection impossible"

↓

"Random fluctuation of the electrical current delivered by the photodiode"

LIGHT RANDOM NATURE

Deterministic Concepts



Power → $\mathbf{P(t)}$

Bit Energy → $\mathbf{E_{bit} = \int P(t)dt}$

Photon Energy → $\mathbf{hf}$

Random Concepts

$$\mathrm{N^o\ fot/bit} \equiv \frac{E_{bit}}{hf} = m = \underbrace{\langle m \rangle}_{\text{INFO}} + \underbrace{\left(m - \langle m \rangle\right)}_{\text{FLUCTUATION}}$$

m: random variable

SNR CONCEPT

$$p = m + n = \underbrace{\langle m \rangle}_{\text{INFO}} + \underbrace{\left(m - \langle m \rangle\right)}_{\text{FLUCTUATION}} + \underbrace{n}_{\text{NOISE}}$$

$$\boxed{\mathrm{SNR} \equiv \frac{\langle p \rangle^2}{\sigma_p^2} = \frac{\langle m \rangle^2}{\left\langle \left(m - \langle m \rangle + n\right)^2 \right\rangle} = \frac{\langle m \rangle^2}{\sigma_m^2 + \sigma_n^2} < \infty}$$

Light Randomness

LASER

coherent light →
Poisson statistics

$$\sigma_m^2 = \langle m \rangle$$

$$\mathrm{SNR} = \langle m \rangle$$

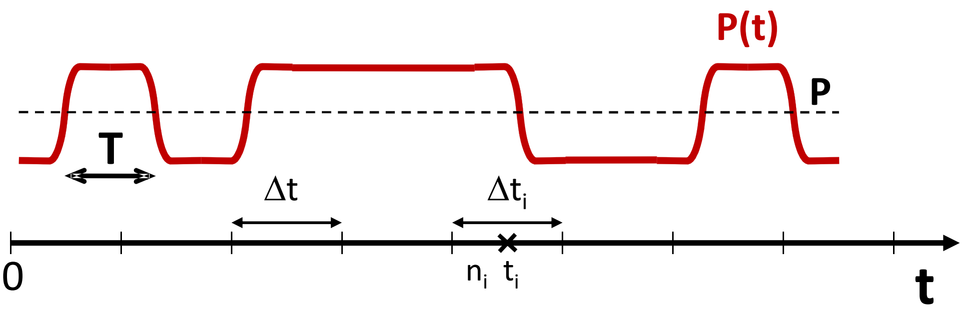
LED

incoherent light →
Bose-Einstein statistics

$$\sigma_m^2 = \langle m \rangle\left(\langle m \rangle + 1\right)$$

$$\mathrm{SNR} = \frac{\langle m \rangle^2}{\langle m \rangle\left(\langle m \rangle + 1\right)} \approx 1$$

Photon arrival statistics → Poisson (coherent light)



1. The number of photons arrived in a given temporal interval is independent of the number of photons arrived in any other non-overlapping and disjoint temporal interval
2. The number of received photons in the interval i is:

$$n_i \begin{cases} 1 & p_i = \lambda_n(i \cdot \Delta t)\Delta t \\ 0 & 1 - p_i \end{cases}$$

$$\lambda_n(t) = \frac{P(t)}{hf}$$ mean received photons per unit time

3. The mean number of photon arrived in a given temporal interval follows:

$$p_T(k,t) \equiv \frac{Q_T^k(t)}{k!} e^{-Q_T(t)}$$

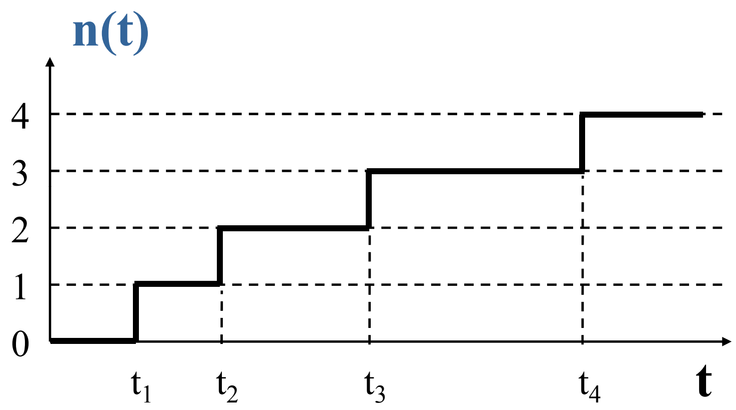
$Q_T(t)$ Mean number of photons received in (t,t+T)

$$Q_T(t) = \int_t^{t+T} \lambda_n(\tau)\,\partial\tau = \underbrace{\int_t^{t+T} \frac{P(\tau)}{hf}\,\partial\tau}_{\text{PHOTONS IN } (t,t+T)}$$

4. The variance of received photons equals its mean value:

$$\sigma_{n,T}^2(t) = \langle n_T \rangle (t)$$

Derivation of mean value and variance of received photons



Photon counter process:
 # Photons in (0 , t)

$$n(t) = \lim_{\Delta t \to 0} \sum_{i=0}^{\infty} n_i U(t - i\Delta t)$$

Photon counter r.v.:
 # Photons in (t , t+T)

$$n_T(t) = \lim_{\Delta t \to 0} \sum_{i=t/\Delta t}^{(t+T)/\Delta t} n_i$$

Mean Value $\langle n_T \rangle(t)$

$$n_i \begin{cases} 1 & p_i = \dfrac{P(i \cdot \Delta t)}{hf}\Delta t \\ 0 & 1 - p_i \end{cases}$$

$$n_T(t) = \lim_{\Delta t \to 0} \sum_{i=t/\Delta t}^{(t+T)/\Delta t} n_i$$

$$E\{n_T(t)\} = E\left\{\lim_{\Delta t \to 0} \sum_{i=t/\Delta t}^{(t+T)/\Delta t} n_i\right\} = \lim_{\Delta t \to 0} \sum_{i=t/\Delta t}^{(t+T)/\Delta t} E\{n_i\} = \lim_{\Delta t \to 0} \sum_{i=t/\Delta t}^{(t+T)/\Delta t} p_i$$

$$E\{n_i\} = 1 \cdot p_i + 0 \cdot (1 - p_i) = p_i = \frac{P(i \cdot \Delta t)}{hf}\Delta t$$

$$= \lim_{\Delta t \to 0} \sum_{i=t/\Delta t}^{(t+T)/\Delta t} \frac{P(i \cdot \Delta t)}{hf}\Delta t = \boxed{\underbrace{\int_t^{t+T} \frac{P(\tau)}{hf}\partial\tau}_{\#\text{photons in } [t,t+T]} = \langle n_T \rangle(t)}$$

Variance $\sigma^2_{n,T}(t)$

$$n_T(t) = \lim_{\Delta t \to 0} \sum_{i=t/\Delta t}^{(t+T)/\Delta t} n_i$$

$$E\left\{n_T^2(t)\right\} = E\left\{\lim_{\Delta t\to 0} \sum_{i=t/\Delta t}^{(t+T)/\Delta t} \sum_{j=t/\Delta t}^{(t+T)/\Delta t} n_i n_j\right\} = \lim_{\Delta t\to 0} \sum_{i=t/\Delta t}^{(t+T)/\Delta t} \sum_{j=t/\Delta t}^{(t+T)/\Delta t} E\left\{n_i n_j\right\} = \lim_{\Delta t\to 0}\left\{\sum_{i=j} p_i + \sum_{i\neq j} p_i p_j\right\}$$

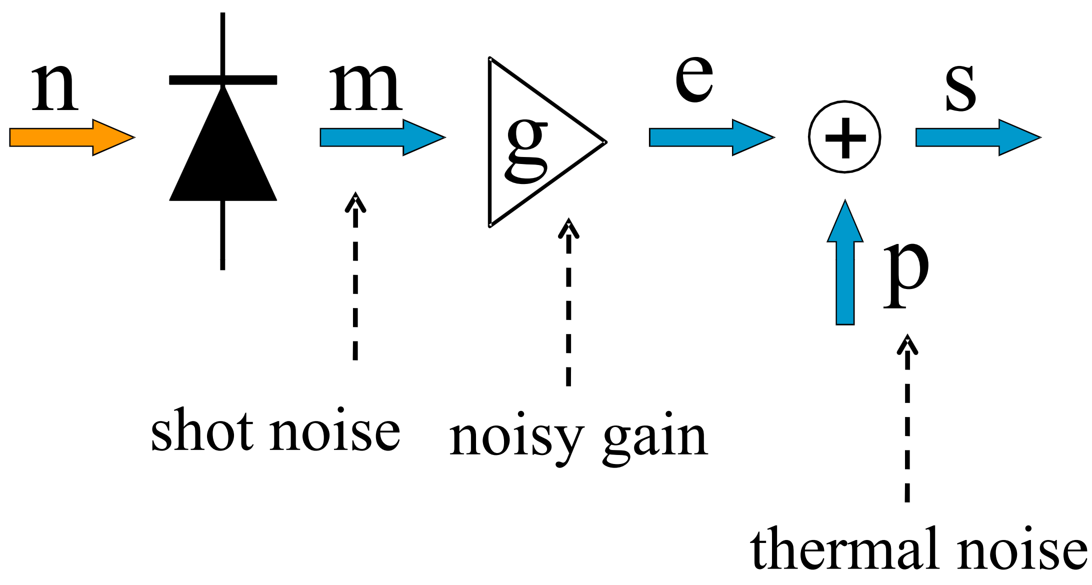
$$E\left\{n_i n_j\right\} = \begin{cases} E\left\{n_i^2\right\} = E\left\{n_i\right\} = p_i = \dfrac{P(i\cdot\Delta t)}{hf}\Delta t & i = j \\ E\left\{n_i\right\}E\left\{n_j\right\} = p_i p_j = \dfrac{P(i\cdot\Delta t)}{hf}\Delta t \dfrac{P(j\cdot\Delta t)}{hf}\Delta t & i \neq j \end{cases}$$

$$= \lim_{\Delta t\to 0}\left\{\sum_{i=t/\Delta t}^{(t+T)/\Delta t} \frac{P(i\cdot\Delta t)}{hf}\Delta t + \sum_{i=t/\Delta t}^{(t+T)/\Delta t} \frac{P(i\cdot\Delta t)}{hf}\Delta t \sum_{j=t/\Delta t}^{(t+T)/\Delta t} \frac{P(j\cdot\Delta t)}{hf}\Delta t - \cancel{\sum_{j=t/\Delta t}^{(t+T)/\Delta t} \frac{P^2(j\cdot\Delta t)}{h^2 f^2}}\Delta t^2\right\}$$

$$= \underbrace{\int_t^{t+T} \frac{P(\tau)}{hf}\partial\tau}_{E\{n_T(t)\}} + \underbrace{\left[\int_t^{t+T} \frac{P(\tau)}{hf}\partial\tau\right]^2}_{E^2\{n_T(t)\}}$$

$$\boxed{\sigma^2_{n,T}(t) = E\left\{n_T^2(t)\right\} - E^2\left\{n_T(t)\right\} = E\left\{n_T(t)\right\} = \langle n_T\rangle(t)}$$

SHOT AND THERMAL NOISE



Shot Noise

"Shot noise refers to random fluctuations in the photocurrent after the photodiode due to light's inner randomness"

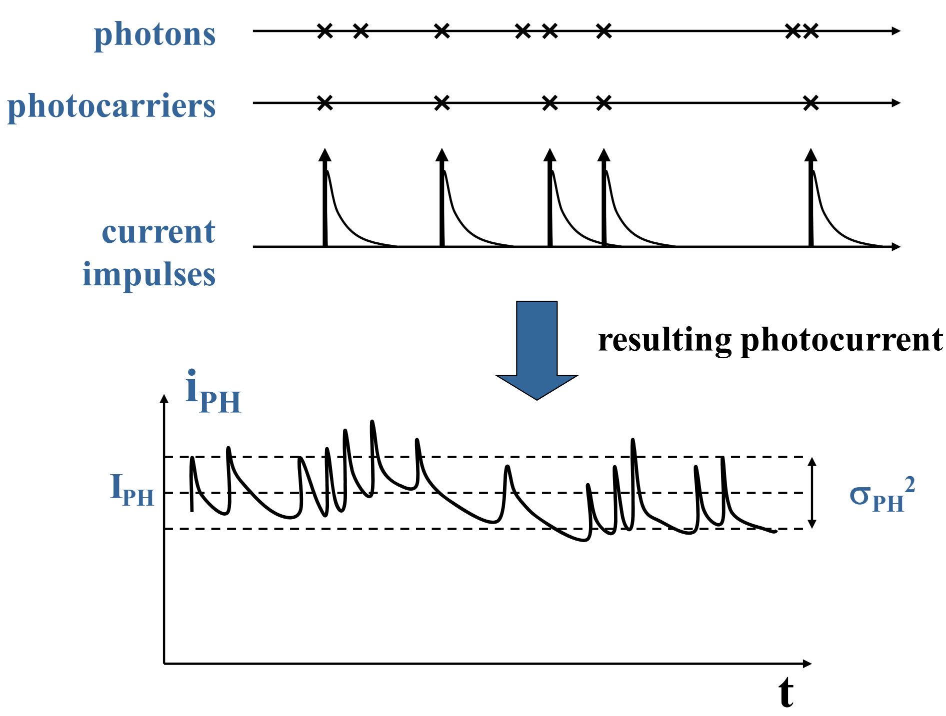
$$\mathbf{i}_{\mathbf{PH}} = \underbrace{\langle \mathbf{i}_{\mathbf{PH}} \rangle}_{\text{SIGNAL}} + \underbrace{\left(\mathbf{i}_{\mathbf{PH}} - \langle \mathbf{i}_{\mathbf{PH}} \rangle\right)}_{\text{SHOT NOISE}}$$

$$\mathbf{s} \equiv \mathbf{i}_{\mathbf{PH}} - \langle \mathbf{i}_{\mathbf{PH}} \rangle$$

$$\langle \mathbf{s} \rangle \equiv \mathbf{S} = \mathbf{0}$$

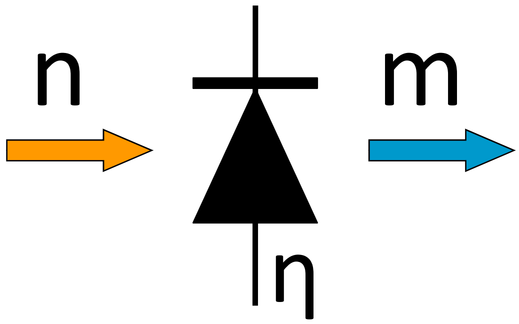
$$\sigma_{\mathbf{S}}^2 = \mathbf{E}\left\{\left(\mathbf{s} - \langle \mathbf{s} \rangle\right)^2\right\} = \mathbf{E}\left\{\mathbf{s}^2\right\} = \mathbf{E}\left\{\left(\mathbf{i}_{\mathbf{PH}} - \langle \mathbf{i}_{\mathbf{PH}} \rangle\right)^2\right\} = \sigma_{\mathbf{PH}}^2$$

Shot Noise



Photocarrier statistics → Poisson (coherent light)

Quantum Efficiency Probability that a photon which arrives at the photodetector generates a photocarrier



$$m_T(t) = \lim_{\Delta t \to 0} \sum_{i=t/\Delta t}^{(t+T)/\Delta t} m_i$$

$$m_i \begin{cases} 1 & p_i = \lambda_m (i \cdot \Delta t)\Delta t \\ 0 & 1 - p_i \end{cases}$$

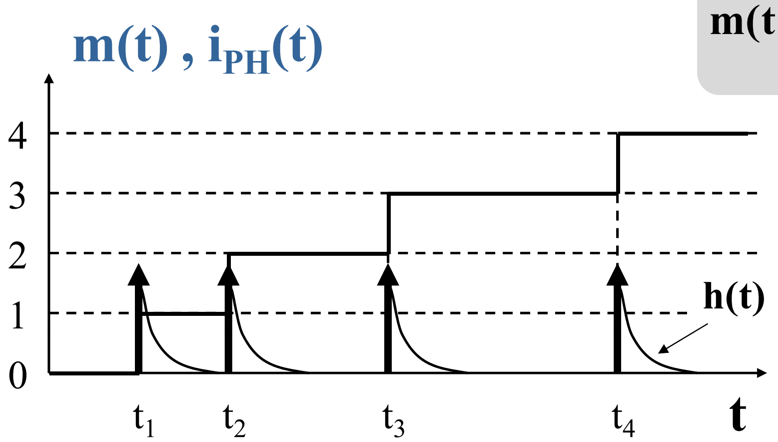
mean value of e^-/h^+ generated per unit time $\quad \lambda_m(t) = \eta \dfrac{P(t)}{hf}$

$$\langle m_T \rangle (t) = \underbrace{\int_t^{t+T} \eta \frac{P(\tau)}{hf} \partial\tau}_{\#\text{photocarriers in } [t,t+T]} = \eta \langle n_T \rangle (t)$$

$$\sigma^2_{m,T}(t) = \langle m_T \rangle (t)$$

Instantaneous Photocurrent $i_{PH}(t)$

$$m(t) = \lim_{\Delta t \to 0} \sum_{i=0}^{\infty} m_i U(t - i\Delta t)$$



$$\int_0^{\infty} h(t)\, \partial t = q$$

$$q \cdot m(t)$$

total cumulated charge

$$i_{PH}(t) = \frac{\partial}{\partial t}\{m(t)\} * h(t) \quad \Rightarrow \quad i_{PH}(t) = \lim_{\Delta t \to 0} \sum_{i=0}^{\infty} m_i\, h(t - i\Delta t)$$

m(t): generated photocarriers counter process
h(t): resulting system's impulsive response

Mean Value $\langle i_{PH}\rangle(t)$

$$i_{PH}(t)=\lim_{\Delta t\to 0}\sum_{i=0}^{\infty} m_i\, h(t-i\Delta t)$$

$$E\{i_{PH}(t)\}=E\left\{\lim_{\Delta t\to 0}\sum_{i=0}^{\infty} m_i\, h(t-i\Delta t)\right\}=\lim_{\Delta t\to 0}\sum_{i=0}^{\infty}E\{m_i\}h(t-i\Delta t)=$$

$$=\lim_{\Delta t\to 0}\sum_{i=0}^{\infty}\eta\frac{P(i\Delta t)}{hf}h(t-i\Delta t)=\frac{\eta}{hf}\int_0^{\infty}P(\tau)h(t-\tau)\partial\tau=\boxed{\frac{\eta}{hf}P(t)*h(t)\equiv\langle i_{PH}\rangle(t)}$$

slow signal approximation

$$h(t)<<P(t)$$

$$\frac{\eta}{hf}\int_0^{\infty}P(\tau)h(t-\tau)\partial\tau\approx\eta\frac{P(t)}{hf}\underbrace{\int_0^{\infty}h(t-\tau)\partial\tau}_{q}=\underbrace{\eta\frac{q}{hf}}_{\Re}P(t)=\boxed{\Re P(t)=\langle i_{PH}\rangle(t)}$$

$$P=ct\longrightarrow I_{PH}=\Re P$$

Constant Power

Variance $\sigma^2_{PH}(t)$

$$i_{PH}(t)=\lim_{\Delta t\to 0}\sum_{i=0}^{\infty} m_i\, h(t-i\Delta t)$$

$$E\{i^2_{PH}(t)\}=E\left\{\lim_{\Delta t\to 0}\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}m_im_j\,h(t-i\Delta t)h(t-j\Delta t)\right\}=\lim_{\Delta t\to 0}\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}E\{m_im_j\}h(t-i\Delta t)h(t-j\Delta t)$$

$$E\{m_im_j\}=\begin{cases}E\{m_i^2\}=\eta\dfrac{P(i\cdot\Delta t)}{hf}\Delta t & i=j\\[2ex] E\{m_i\}E\{m_j\}=\eta\dfrac{P(i\cdot\Delta t)}{hf}\Delta t\;\eta\dfrac{P(j\cdot\Delta t)}{hf}\Delta t & i\neq j\end{cases}$$

$$=\lim_{\Delta t\to 0}\left\{\sum_{i=j}E\{m_i^2\}h^2(t-i\Delta t)+\sum_{i\neq j}E\{m_i\}E\{m_j\}h(t-i\Delta t)h(t-j\Delta t)\right\}=$$

$$=\lim_{\Delta t\to 0}\left\{\sum_{i=0}^{\infty}\eta\frac{P(i\cdot\Delta t)}{hf}h^2(t-i\Delta t)\Delta t+\left[\sum_{i=0}^{\infty}\eta\frac{P(i\cdot\Delta t)}{hf}h(t-i\Delta t)\Delta t\right]^2\right\}=$$

$$=\underbrace{\int_0^{\infty}\eta\frac{P(\tau)}{hf}h^2(t-\tau)\partial\tau}_{\frac{\eta}{hf}P(t)*h^2(t)}+\cancel{\underbrace{\left[\int_0^{\infty}\eta\frac{P(\tau)}{hf}h(t-\tau)\partial\tau\right]^2}_{E^2\{i_{PH}(t)\}}}\Leftarrow\;\sigma^2_{PH}(t)=E\{i^2_{PH}(t)\}-E^2\{i_{PH}(t)\}$$

Variance $\sigma^2_{PH}(t)$

$$\boxed{\sigma^2_{PH}(t) = \frac{\eta}{hf} P(t) * h^2(t)}$$

SHOT NOISE VARIANCE

slow signal approximation

$$h^2(t) << P(t)$$

$$\int_0^\infty \eta \frac{P(\tau)}{hf} h^2(t-\tau)\partial\tau \approx \frac{\eta}{hf} P(t) \underbrace{\int_0^\infty h^2(t-\tau)\partial\tau}_{2q^2B} = 2qB \underbrace{\eta \frac{q}{hf}}_{\Re} P(t) = \boxed{2qB\Re P(t) = \sigma^2_{PH}(t)}$$

$$B \equiv \frac{1}{2q^2}\int_0^\infty h^2(t)\partial t = \frac{1}{2}\int_0^\infty \left|\frac{H(f)}{H(0)}\right|^2 \partial f \geq \frac{1}{2T_b}$$

equivalent noise bandwidth

Constant Power

$$P = ct \longrightarrow \sigma^2_{PH} = 2qB\,I_{PH}$$

Dark Current

$$i_D = \langle i_D \rangle + \left(i_D - \langle i_D \rangle\right) \equiv I_D + s_D$$

$$i_{PD} = i_{PH} + i_D = \underbrace{\langle i_{PH} \rangle}_{\text{SIGNAL}} + \underbrace{I_D + s + s_D}_{\text{NOISE}}$$

$$\sigma^2_D = 2qBI_D$$

s: signal's shot noise

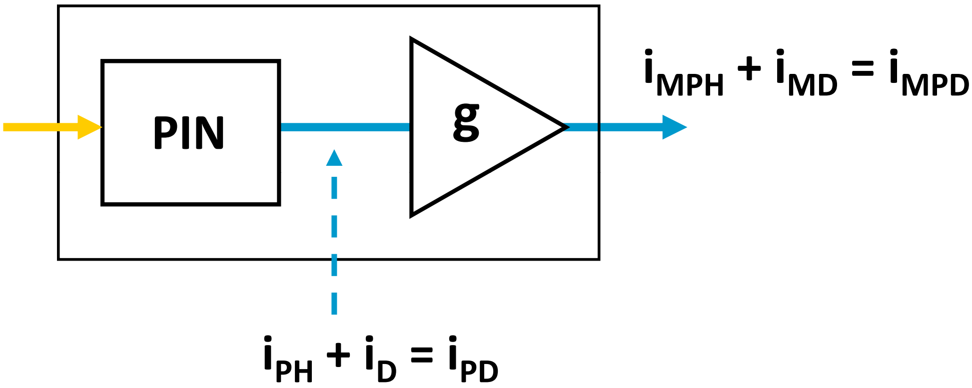
s_D: dark current's shot noise

dark current's shot noise is independent of signal's shot noise

$$\langle i_{PD} \rangle(t) = \langle i_{PH} \rangle(t) + I_D$$

$$\sigma^2_{PD}(t) = \sigma^2_{PH}(t) + 2qBI_D$$

Multiplicative effect on shot noise: APD



delivered carrier counter process

$$e(t) = \lim_{\Delta t \to 0} \sum_{i=0}^{\infty} m_i g_i U(t - i\Delta t)$$

$$E\{g_i\} = M$$

$$E\{g_i^2\} = M^2 F(M)$$

g_i: APD gain factor

$F(M)$: APD noise factor

Mean Value $\langle i_{MPH} \rangle (t)$

$$i_{MPH}(t) = \lim_{\Delta t \to 0} \sum_{i=0}^{\infty} m_i g_i h(t - i\Delta t)$$

$$E\{i_{MPH}(t)\} = E\left\{ \lim_{\Delta t \to 0} \sum_{i=0}^{\infty} m_i g_i h(t - i\Delta t) \right\} = \lim_{\Delta t \to 0} \sum_{i=0}^{\infty} E\{m_i g_i\} h(t - i\Delta t) =$$

$$= \lim_{\Delta t \to 0} \sum_{i=0}^{\infty} E\{m_i\} \underbrace{E\{g_i\}}_{M} h(t - i\Delta t) = M \lim_{\Delta t \to 0} \sum_{i=0}^{\infty} \eta \frac{P(i\Delta t)}{hf} h(t - i\Delta t) =$$

$$= M \frac{\eta}{hf} \int_0^{\infty} P(\tau) h(t - \tau) \partial\tau = M \frac{\eta}{hf} P(t) * h(t) = \boxed{\langle i_{MPH} \rangle (t) = M \langle i_{PH} \rangle (t)}$$

$$\langle i_{PH} \rangle (t) = \frac{\eta}{hf} P(t) * h(t)$$

slow signal approximation

$$h(t) << P(t)$$

$$M \frac{\eta}{hf} \int_0^{\infty} P(\tau) h(t - \tau) \partial\tau \approx MP(t) \underbrace{\frac{\eta}{hf} \underbrace{\int_0^{\infty} h(t - \tau) \partial\tau}_{q}}_{\Re} = \boxed{M \Re P(t) = \langle i_{MPH} \rangle (t)}$$

Variance $\sigma^2_{MPH}(t)$

$$i_{MPH}(t) = \lim_{\Delta t\to 0}\sum_{i=0}^{\infty} m_i g_i\, h(t-i\Delta t)$$

$$E\left\{i^2_{MPH}(t)\right\} = E\left\{\lim_{\Delta t\to 0}\sum_{i=0}^{\infty}\sum_{j=0}^{\infty} m_i m_j g_i g_j\, h(t-i\Delta t)\, h(t-j\Delta t)\right\} =$$

$$= \lim_{\Delta t\to 0}\sum_{i=0}^{\infty}\sum_{j=0}^{\infty} E\{m_i m_j\} E\{g_i g_j\} h(t-i\Delta t) h(t-j\Delta t) =$$

$$E\{m_i m_j\} = \begin{cases} E\{m_i^2\} = \eta \dfrac{P(i\cdot\Delta t)}{hf}\Delta t & i=j \\ E\{m_i\}E\{m_j\} = \eta\dfrac{P(i\cdot\Delta t)}{hf}\Delta t\ \eta\dfrac{P(j\cdot\Delta t)}{hf}\Delta t & i\neq j \end{cases} \qquad E\{g_i g_j\} = \begin{cases} E\{g_i^2\} = M^2F & i=j \\ E\{g_i\}E\{g_j\} = M^2 & i\neq j \end{cases}$$

$$= \lim_{\Delta t\to 0}\left\{ M^2F\sum_{i=0}^{\infty}\eta\frac{P(i\cdot\Delta t)}{hf}\Delta t\, h^2(t-i\Delta t) + M^2\left[\sum_{i=0}^{\infty}\eta\frac{P(i\cdot\Delta t)}{hf}\Delta t\, h(t-i\Delta t)\right]^2\right\} =$$

$$= \underbrace{M^2F\int_0^{\infty}\eta\frac{P(\tau)}{hf}h^2(t-\tau)\partial\tau}_{\frac{\eta}{hf}P(t)*h^2(t)} + \cancel{\underbrace{M^2\left[\int_0^{\infty}\eta\frac{P(\tau)}{hf}h(t-\tau)\partial\tau\right]^2}_{E^2\{i_{MPH}(t)\}}}$$

$$\sigma^2_{PH}(t) = E\left\{i^2_{PH}(t)\right\} - E^2\left\{i_{PH}(t)\right\}$$

Variance $\sigma^2_{MPH}(t)$

$$\sigma^2_{PH}(t) = \frac{\eta}{hf}P(t)*h^2(t)$$

$$\boxed{\sigma^2_{MPH}(t) =} M^2F\frac{\eta}{hf}P(t)*h^2(t) \boxed{= M^2F\cdot\sigma^2_{PH}(t)}$$

SHOT NOISE VARIANCE

slow signal approximation

$$h^2(t) << P(t)$$

$$M^2F\int_0^{\infty}\eta\frac{P(\tau)}{hf}h^2(t-\tau)\partial\tau \approx M^2F\frac{\eta}{hf}P(t)\underbrace{\int_0^{\infty}h^2(t-\tau)\partial\tau}_{2q^2B} = \boxed{M^2F\, 2qB\Re P(t) = \sigma^2_{MPH}(t)}$$

$$B \equiv \frac{1}{2q^2}\int_0^{\infty}h^2(t)\partial t = \frac{1}{2}\int_0^{\infty}\left|\frac{H(f)}{H(0)}\right|^2\partial f \geq \frac{1}{2T_b}$$

equivalent noise bandwidth

Constant Power

$$P = ct \longrightarrow \sigma^2_{MPH} = M^2F\ 2qB\, I_{PH}$$

Dark Current

$$i_D = \underbrace{i_D\big|_M}_{\text{multip.}} + \underbrace{i_D\big|_{NM}}_{\text{no-multip.}}$$

$$i_{MD} = \langle i_{MD} \rangle + \left(i_{MD} - \langle i_{MD} \rangle\right) \equiv I_{MD} + s_{MD}$$

$$i_{MPD} = i_{MPH} + i_{MD} = \underbrace{\langle i_{MPH} \rangle}_{\text{SIGNAL}} + \underbrace{I_{MD} + s_M + s_{MD}}_{\text{NOISE}}$$

s_M: signal's shot noise

s_{MD}: dark current's shot noise

$$I_{MD} = M \cdot I_D\big|_M + I_D\big|_{NM} \approx M \cdot I_D$$

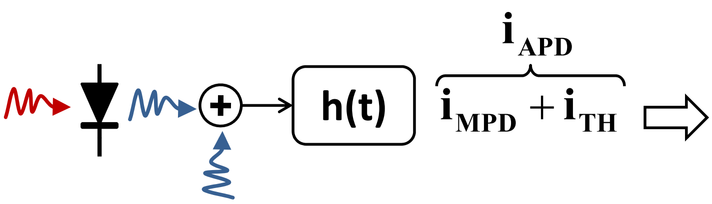
$$\sigma_{MD}^2 = 2q\left[M^2F(M) \cdot I_D\big|_M + I_D\big|_{NM}\right]B \approx 2qM^2F(M)I_D B$$

$$\langle i_{MPD} \rangle(t) \approx M\left(\langle i_{MPH} \rangle(t) + I_D\right)$$

$$\sigma_{MPD}^2(t) \approx M^2F(M)\left(\sigma_{PH}^2(t) + 2qBI_D\right)$$

Thermal Noise

"Thermal noise refers to the random fluctuations in the photocurrent delivered by the photodiode due to the chaotic movement of electrons in any electronic circuitry"



Thermal Noise (white Gaussian)

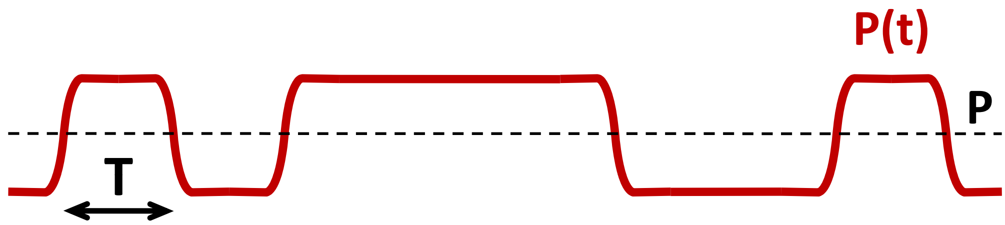
$$\begin{cases} \langle i_{TH} \rangle = 0 \\ \sigma_{TH}^2 = \langle i_{TH}^2 \rangle = 4\dfrac{KT}{R_L}B \end{cases}$$

B: equivalent noise bandwidth

Shot Noise vs. Thermal Noise

- Both depend on receiver's bandwidth
- Both show a uniform spectrum in the whole band
- Thermal noise is independent of i_{ph} while shot noise is proportional to it

MODULATED SIGNAL



$$P(t) = p(t) * \sum_{k=0}^{\infty} a_k \partial(t - kT_b)$$

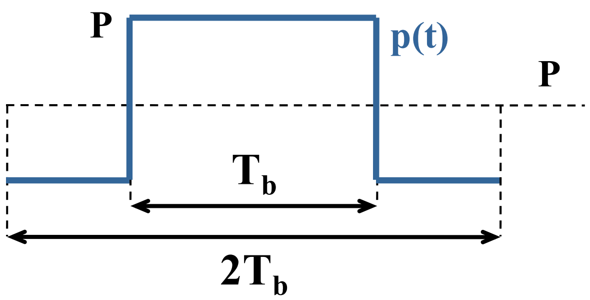
PAM

$$\langle i_{PH} \rangle (t) = \frac{\eta}{hf} P(t) * h(t) = \frac{\eta}{hf} \boxed{p(t) * h(t)} * \sum_{k=0}^{\infty} a_k \partial(t - kT_b)$$

$$\sigma_{PH}^2(t) = \frac{\eta}{hf} P(t) * h^2(t) = \frac{\eta}{hf} \boxed{p(t) * h^2(t)} * \sum_{k=0}^{\infty} a_k \partial(t - kT_b)$$

IDEAL PULSES AND FILTERS

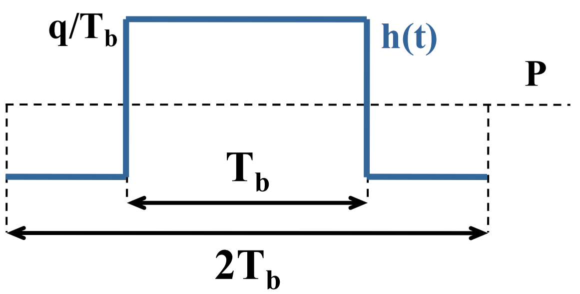
ideal square pulses



$$\langle n_{T_b} \rangle = \frac{1}{hf} \int_0^{\infty} p(\tau)\, \partial\tau = \frac{P}{hf} T_b$$

$$\langle i_{PH} \rangle (t) = \frac{\eta}{hf} p(t) * h(t) * \sum_{k=0}^{\infty} \partial(t - kT_b)$$

$$\sigma_{PH}^2(t) = \frac{\eta}{hf} p(t) * h^2(t) * \sum_{k=0}^{\infty} \partial(t - kT_b)$$



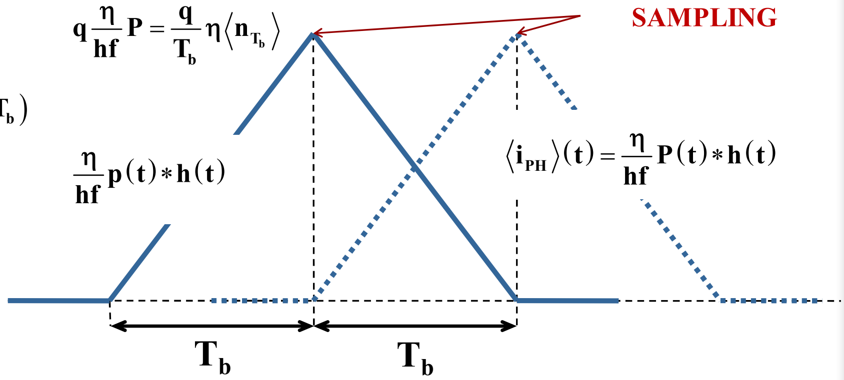
$$\int_0^{\infty} h(t)\, \partial\tau = q$$

$$B \equiv \frac{1}{2q^2} \underbrace{\int_0^{\infty} h^2(t) \partial t}_{q^2/T_b} = \frac{1}{2T_b}$$

IDEAL PULSES AND FILTERS — ideal square pulses

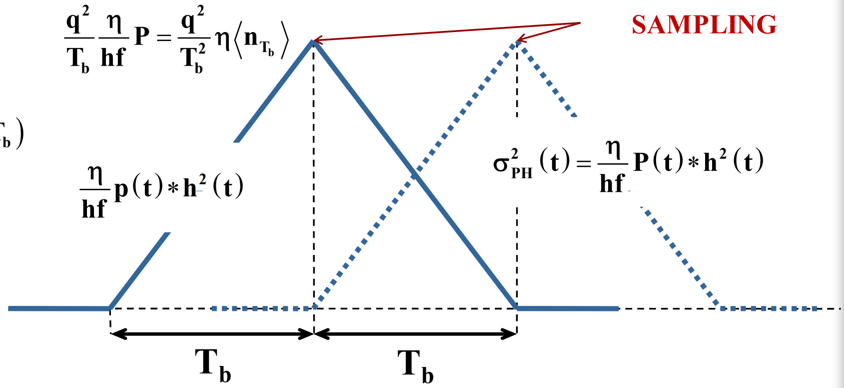
MEAN

$$\langle i_{PH}\rangle(t)=\frac{\eta}{hf}p(t)*h(t)*\sum_{k=0}^{\infty}\partial(t-kT_b)$$

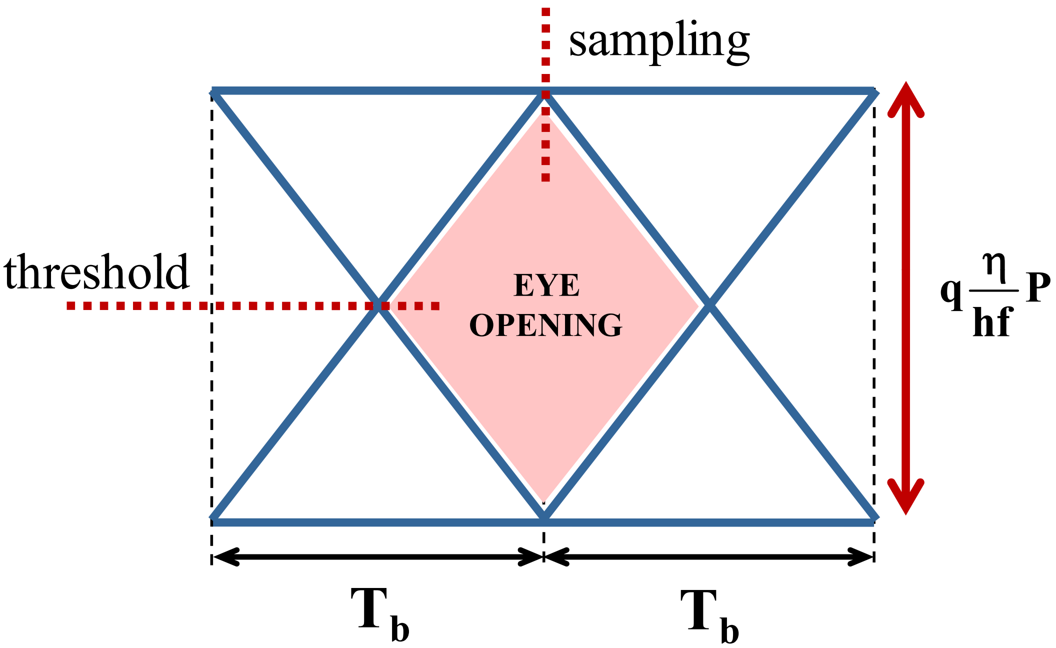


VARIANCE

$$\sigma^2_{PH}(t)=\frac{\eta}{hf}p(t)*h^2(t)*\sum_{k=0}^{\infty}\partial(t-kT_b)$$



EYE DIAGRAM



Particles – Currents Relationship

$$\langle s\rangle = \langle e\rangle + \langle p\rangle = M\eta\langle n\rangle + \langle p\rangle$$

$$\langle n\rangle \equiv \langle n_{T_b}\rangle$$

Thermal Noise

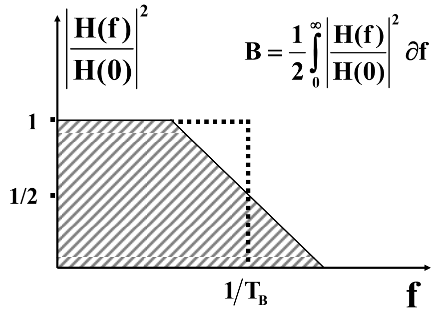
$$\langle p\rangle = \frac{I_{TH}}{q}T_b$$

$$\begin{cases} I_{TH} = 0 \\ \sigma_{TH}^2 = \dfrac{4KTB}{R_L} \end{cases} \qquad \begin{cases} \langle p\rangle = 0 \\ \sigma_p^2 = \left(\dfrac{T_b}{q}\right)^2 \dfrac{4KTB}{R_L} \end{cases}$$

Shot Noise

$$\langle n\rangle = \frac{P\,T_b}{hf}$$

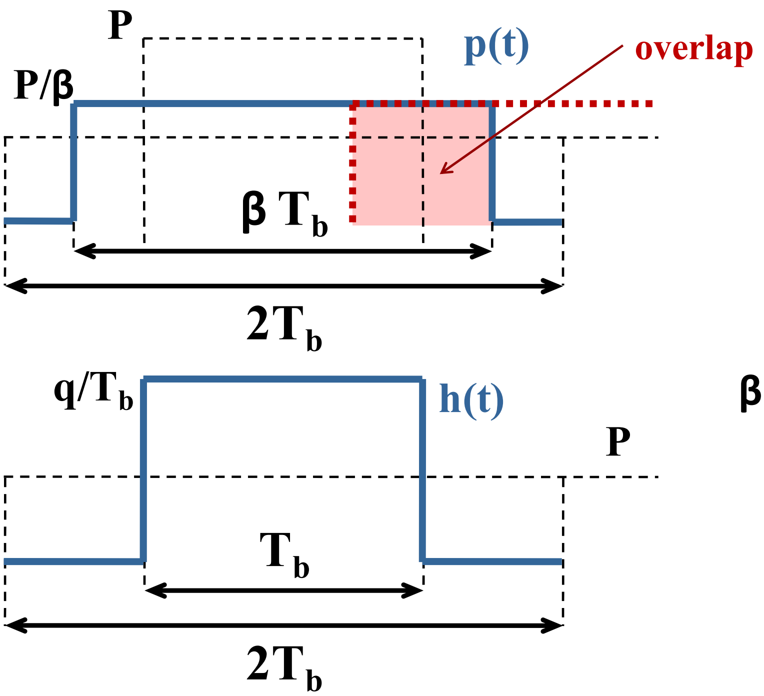
$$\begin{cases} \langle e\rangle \equiv M\eta\langle n\rangle \\ \sigma_e^2 \equiv M^2F\eta\langle n\rangle \end{cases}$$



$$B = \frac{1}{2}\int_0^\infty \left|\frac{H(f)}{H(0)}\right|^2 \partial f = \frac{1}{2T_b}$$

$$\begin{cases} I_{MPH} = M\Re P = M\eta\dfrac{q}{hf}P = \dfrac{q}{T_b}M\eta\langle n\rangle \\ \sigma_{MPH}^2 = 2qBM^2F(M)\Re P = \dfrac{q}{T_b}M^2F(M)\eta\dfrac{q}{hf}P = \left(\dfrac{q}{T_b}\right)^2 M^2F(M)\eta\langle n\rangle \end{cases}$$

SIMPLIFIED ISI MODEL



Bit Energy

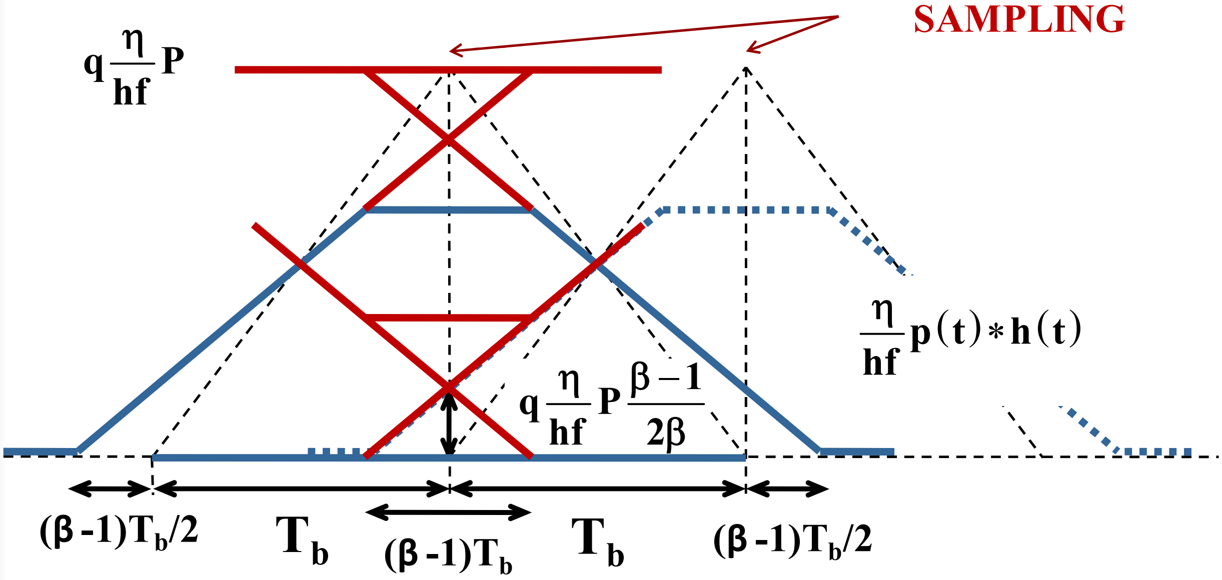
$$E_b = P\cdot T_b$$

β : broadening factor

SIMPLIFIED ISI MODEL

MEAN

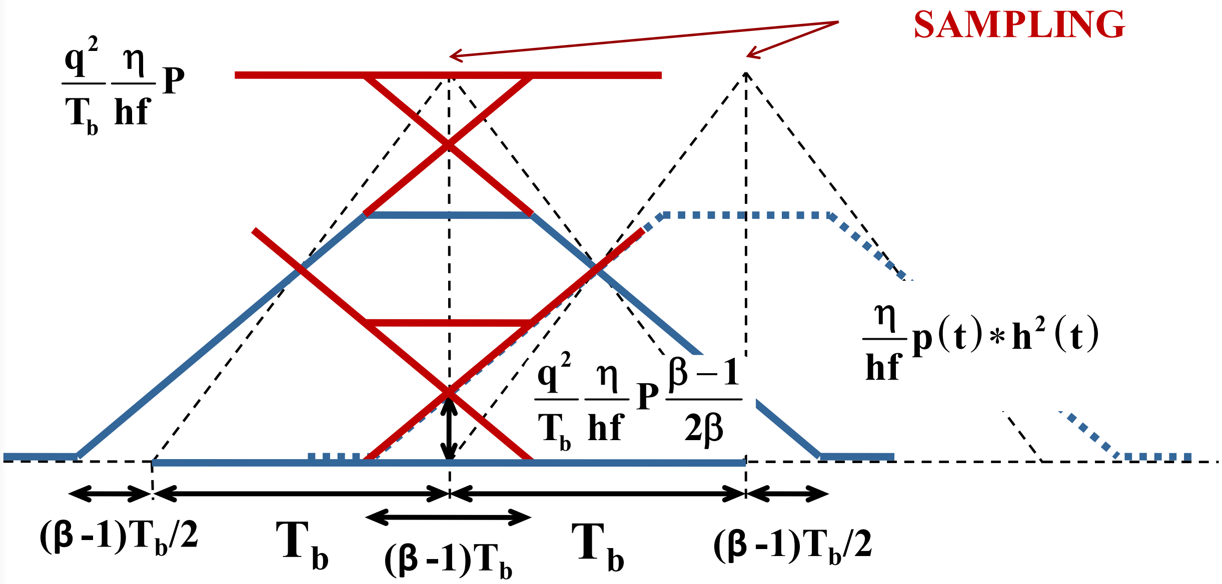
$$\langle i_{PH}\rangle(t) = \frac{\eta}{hf}P(t)*h(t)$$



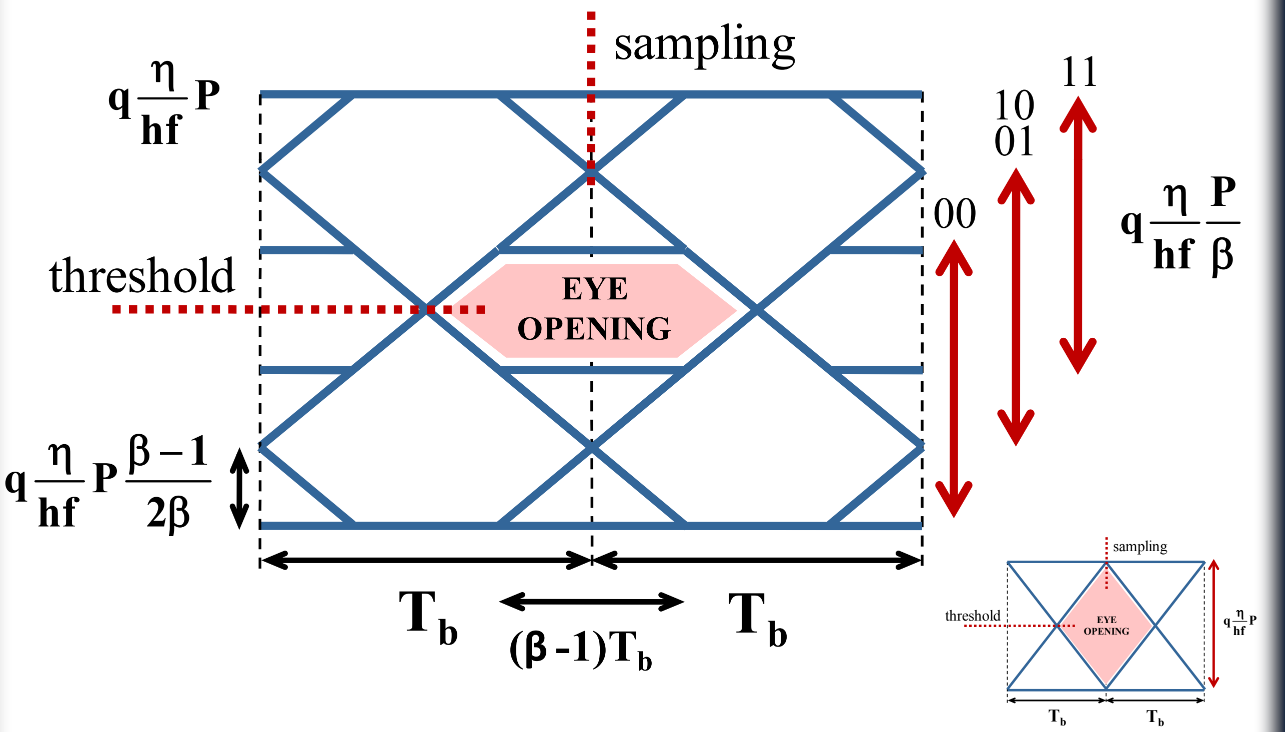
SIMPLIFIED ISI MODEL

VARIANCE

$$\sigma^2_{PH}(t) = \frac{\eta}{hf}P(t)*h^2(t)$$



ISI EYE DIAGRAM



SIGNAL to NOISE RATIO (SNR)

Definition

coherent light

Optical Domain ⇨ $$SNR_O \equiv \frac{\langle n\rangle^2}{\sigma_n^2} = \langle n\rangle = \frac{P}{hf}T_b$$

photon statistics

square pulses

$$\langle i\rangle = \frac{q}{T_b}\langle n\rangle$$

$$\sigma_i^2 = \left(\frac{q}{T_b}\right)^2 \sigma_n^2$$

⇨ $$SNR_E = \frac{\langle i\rangle^2}{\sigma_i^2} = \frac{\langle n\rangle^2}{\sigma_n^2} = SNR_O$$

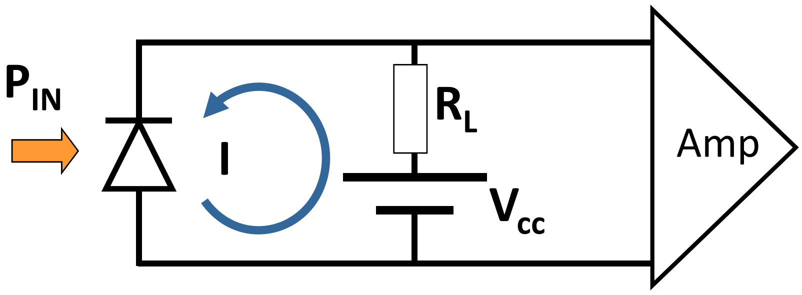
Electrical Domain ⇨ $$SNR_E \equiv \frac{\langle i\rangle^2}{\sigma^2}$$

current statistics

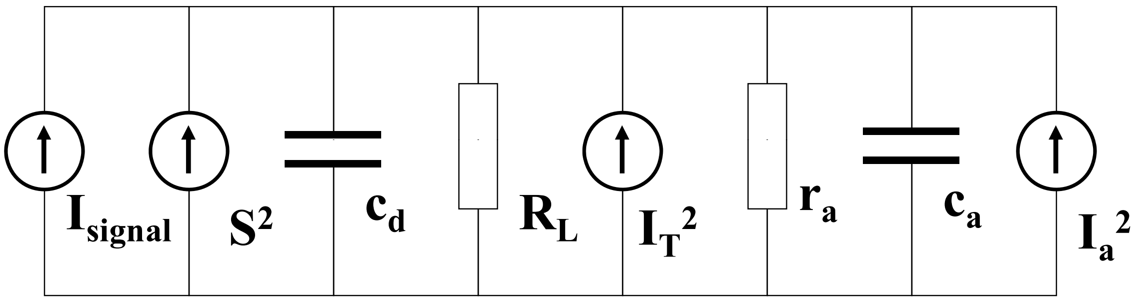
n: number of photons

T_b: bit period

Direct Detection



Equivalent Circuit



PIN – general expression

$$\mathrm{SNR}(t) \equiv \frac{\langle i \rangle_{ph}^2(t)}{\sigma(t)^2} = \frac{\left(\frac{\eta}{hf}\int_0^\infty P(\tau)h(t-\tau)\,\partial\tau\right)^2}{\frac{\eta}{hf}\int_0^\infty P(\tau)h^2(t-\tau)\,\partial\tau + 2qBI_D + 4\frac{KT}{R_L}BF_A}$$

high bandwidth photodetector

F_A: RF amplifier noise factor

$$\frac{\eta}{hf}\int_0^\infty P(\tau)h(t-\tau)\,\partial\tau \approx \Re P(t) \qquad \frac{\eta}{hf}\int_0^\infty P(\tau)h^2(t-\tau)\,\partial\tau \approx 2qB\Re P(t)$$

$$\boxed{\mathrm{SNR}(t) \approx \frac{\Re^2 P^2(t)}{2qB\left(\Re P(t) + I_D\right) + 4\frac{KT}{R_L}BF_A}}$$

constant power

$$\Re P(t) = I_{PH}$$

PIN – constant power

$$SNR = \frac{I_{PH}^2}{\sigma_{PH}^2 + \sigma_D^2 + \sigma_{TH}^2 + \sigma_A^2} = \frac{I_{PH}^2}{2qBI_{PH} + 2qBI_D + 4\frac{KT}{R_L}B + I_A^2}$$

$$= \frac{I_{PH}^2}{2qB\left(I_{PH} + I_D\right) + 4\frac{KT}{R_L}BF_A} = \frac{\left(\eta\frac{q}{hf}P\right)^2}{2qB\left(\eta\frac{q}{hf}P + I_D\right) + 4\frac{KT}{R_L}BF_A}$$

$$I_{PH} = R \cdot P = \eta\frac{q}{hf}P$$

$$\boxed{SNR_{PIN} = \frac{(\Re P)^2}{2qB\left(\Re P + I_D\right) + 4\frac{KT}{R_L}BF_A}}$$

F_A: RF amplifier noise factor

Particular Cases

- negligible dark current
- dominant shot noise

$$\Rightarrow \quad SNR \approx \eta\frac{P}{2B \cdot hf}$$

$$SNR_{PIN} = \frac{\left(\eta\frac{q}{hf}P\right)^2}{2qB\left(\eta\frac{q}{hf}P + \cancel{I_D}\right) + \cancel{4\frac{KT}{R_L}BF_A}}$$

$$\eta = 1 \quad \rightarrow \quad \boxed{SNR_{PIN} \approx \frac{P}{2B \cdot hf} \equiv SNR_{LQ}}$$

quantum limit

"Even though an ideal situation is considered the SNR is not infinite. This effect is known as quantum limit and is due to light's inner randomness" coherent light

$$SNR_{IN} = SNR_O = \langle n \rangle = \frac{P}{hf} T_b \qquad T_b = \frac{1}{2B}$$

$$SNR_{OUT} = SNR_{LQ} = \frac{P}{2B \cdot hf} = \frac{P}{hf} T_b = SNR_{IN}$$

signal quality is maintained

- dominant thermal noise

$$SNR_{PIN} = \frac{(\Re P)^2}{\cancel{2qB(\Re P + I_D)} + 4\frac{KT}{R_L}BF_A}$$

$$SNR_{PIN} \approx \frac{(\Re P)^2}{4\frac{KT}{R_L}BF_A} << SNR_{LQ}$$

APD – constant power

$$SNR = \frac{M^2 I_{PH}^2}{\sigma_{MPH}^2 + \sigma_{MD}^2 + \sigma_{TH}^2 + \sigma_A^2}$$

$$= \frac{M^2 I_{PH}^2}{2qB\left[M^2 F(M)\left(I_{PH} + I_D|_M\right) + I_D|_{NM}\right] + 4\frac{KT}{R_L}BF_A}$$

$$\approx \frac{M^2 I_{PH}^2}{2q\left[M^2 F(M)\left(I_{PH} + I_D\right)\right]B + 4\frac{KT}{R_L}BF_A} = \frac{I_{PH}^2}{2q\left[F(M)\left(I_{PH} + I_D\right)\right]B + \frac{1}{M^2}4\frac{KT}{R_L}BF_A}$$

$$I_{PH} = \underbrace{\eta \frac{q}{hf}}_{\Re} P$$

$$\boxed{SNR_{APD} = \frac{(\Re P)^2}{2qB\left(\Re P + I_D\right)F(M) + \frac{1}{M^2}4\frac{KT}{R_L}BF_A}}$$

usually

F(M) = M^x: APD noise factor

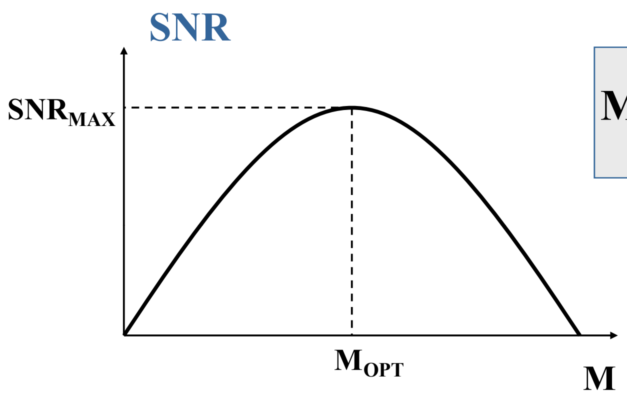
$SNR_{APD}(M)$

$$SNR_{APD} = \frac{\left(\eta \frac{q}{hf} P\right)^2}{2qB\left(\eta \frac{q}{hf} P + I_D\right) M^x + \frac{1}{M^2} 4\frac{KT}{R_L} BF_A} \Leftarrow \quad F(M) = M^x$$

$M \uparrow\uparrow \rightarrow$ shot noise dominant

$M \downarrow\downarrow \rightarrow$ thermal noise dominant

Optimum M



$$\boxed{M_{OPT}^{x+2} = \frac{4KT \cdot F_A}{x \cdot q \cdot R_L \left(I_{PH} + I_D\right)}}$$

Particular Cases

- negligible dark current
- dominant shot noise

$$\Rightarrow \quad SNR \approx \eta \frac{P}{2B \cdot hf \cdot F(M)}$$

$$SNR_{APD} = \frac{(\Re P)^2}{2qB(\Re P + \cancel{I_D})F(M) + \cancel{\frac{1}{M^2} 4\frac{KT}{R_L} BF_A}}$$

$$\eta = 1 \rightarrow \boxed{SNR_{APD} \approx \frac{P}{2B \cdot hf \cdot F(M)} \equiv \frac{SNR_{LQ}}{F(M)}}$$

coherent light

$$T_b = \frac{1}{2B}$$

$$SNR_{IN} = SNR_O = \langle n \rangle = \frac{P}{hf} T_b$$

$$SNR_{OUT} = \frac{SNR_{LQ}}{F(M)} = \frac{P}{hf \cdot F(M)} T_b = \frac{SNR_{IN}}{F(M)}$$

signal quality degrades

SNR improvement

$$I_D = 0$$

$$SNR_{APD} = \frac{(\Re P)^2}{2qB\,\Re P\,F(M) + \frac{1}{M^2}\sigma_T^2} \qquad SNR_{PIN} = \frac{(\Re P)^2}{2qB\,\Re P + \sigma_T^2}$$

$$\frac{(\Re P)^2}{2qB\,\Re P\,F(M) + \frac{1}{M^2}\sigma_T^2} > \frac{(\Re P)^2}{2qB\,\Re P + \sigma_T^2} \qquad SNR_{APD} > SNR_{PIN}$$

$$2qB\,\Re P + \sigma_T^2 > 2qB\,\Re P\,F(M) + \frac{1}{M^2}\sigma_T^2$$

$$\boxed{F(M) < 1 + \frac{\sigma_T^2}{2qB\,\Re P}\left(1 - \frac{1}{M^2}\right) \approx 1 + \frac{\sigma_T^2}{2qB\,\Re P} = 1 + \frac{\sigma_T^2}{\sigma_{PH}^2} \approx \frac{\sigma_T^2}{\sigma_{PH}^2}}$$

$$M >> 1 \qquad \sigma_T^2 >> \sigma_{PH}^2$$

Types of Pre-amplifiers

- High impedance amplifier → noise reduction

$$\left.\begin{array}{l} \text{soroll} \propto \dfrac{1}{R_L // R_A} \approx \dfrac{1}{R_L} \\ BW \propto \dfrac{1}{R_L} \end{array}\right\}$$

trade-off

reduced dynamic range

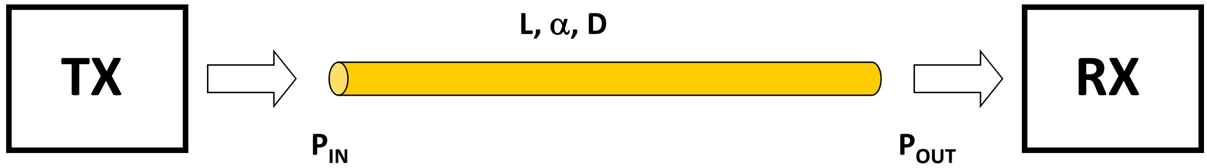
- Transimpedance amplifier → high BW & DR

In this case neither the bandwidth nor the noise level are reduced. What is improved is the dynamic range.

High speed systems require this type of amplifiers.

ERROR PROBABILITY AND SENSITIVITY

DIRECT DETECTION



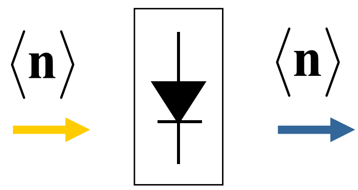
$$\frac{P_{IN}T_b}{hf} = \langle p \rangle \qquad \boxed{\frac{P_{OUT}}{P_{IN}} = 10^{-\frac{\alpha(dB/Km)L(Km)}{10}} = \frac{\langle n \rangle}{\langle p \rangle}} \qquad \langle n \rangle = \frac{P_{OUT}T_b}{hf}$$

<p>: mean photons per bit “1” @ TX

<n>: mean photons per bit “1” @ RX

Ideal Receiver (quantum limit)

- Digital Intensity Modulation NRZ
- Equiprobable Messages
- No Thermal Noise
- No Dark Current
- 100% Quantum Efficiency
- Monochromatic Light → Poisson



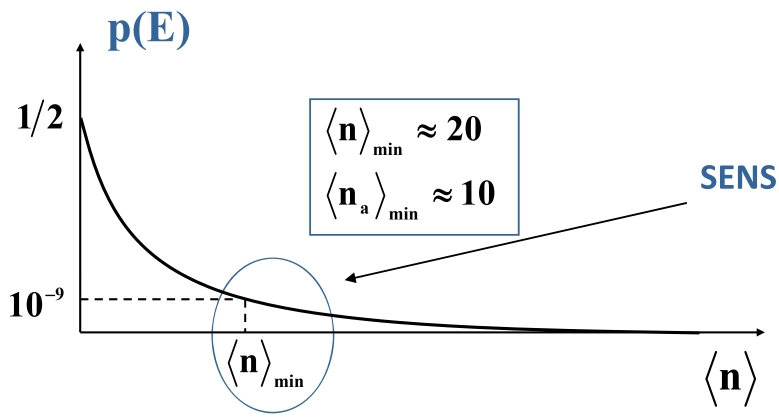
$$p(n) = \frac{\langle n \rangle^n}{n!} e^{-\langle n \rangle}$$

decision criterion

$$n \geq 0 \rightarrow \text{"1"}$$
$$n = 0 \rightarrow \text{"0"}$$

$$\boxed{p(E) = p(E/0)p(0) + p(E/1)p(1) = \frac{1}{2}e^{-\langle n \rangle} = \frac{1}{2}e^{-2\langle n_a \rangle}}$$

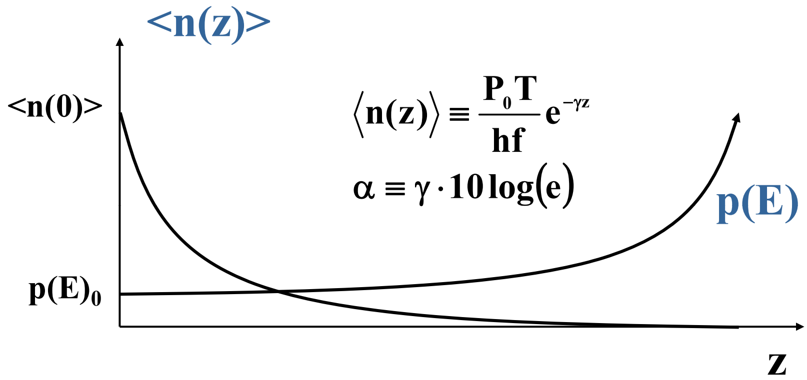
$\langle n_a \rangle$: mean photons per bit @ RX



$$\langle n \rangle_{min} \approx 20$$
$$\langle n_a \rangle_{min} \approx 10$$

SENSITIVITY

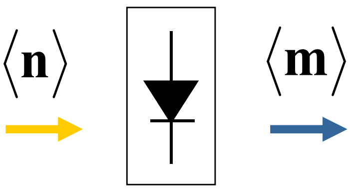
$$\frac{\text{photons}}{\text{bit}}$$



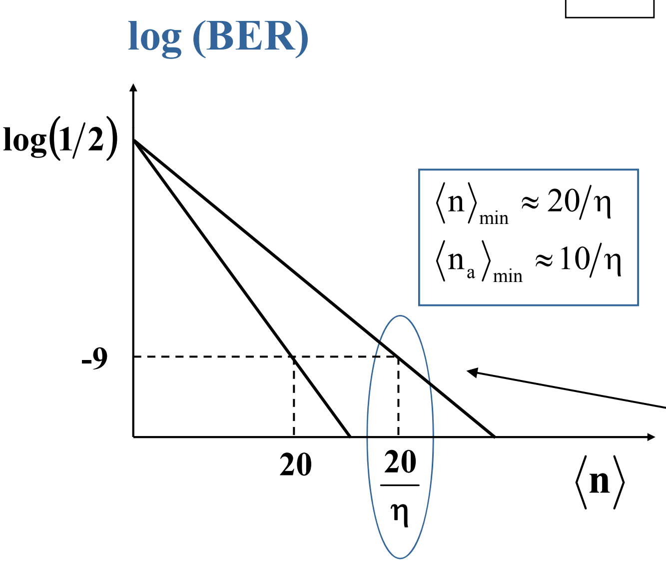
$$\langle n(z) \rangle \equiv \frac{P_0 T}{hf} e^{-\gamma z}$$
$$\alpha \equiv \gamma \cdot 10 \log(e)$$

Non-Ideal Quantum Efficiency

$\langle n \rangle$



$$\langle m \rangle = \eta \langle n \rangle$$



$$\langle n \rangle_{min} \approx 20/\eta$$
$$\langle n_a \rangle_{min} \approx 10/\eta$$

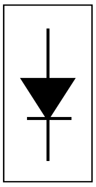
$$p(E) = \frac{1}{2} e^{-\langle m \rangle} = \frac{1}{2} e^{-2\langle m_a \rangle}$$
$$p(E) = \frac{1}{2} e^{-\eta\langle n \rangle} = \frac{1}{2} e^{-2\eta\langle n_a \rangle}$$

SENSITIVITY

$$\frac{\text{photons}}{\text{bit}}$$

Thermal Noise

$$\langle n \rangle \rightarrow$$



$$\langle s \rangle = \langle m \rangle + \langle p \rangle = \eta \langle n \rangle + \langle p \rangle$$

thermal noise

$$p = \frac{i_{TH}}{q} T_b \rightarrow \quad i_{TH} \begin{cases} \langle i_{TH} \rangle = 0 & \rightarrow \quad \langle p \rangle = 0 \\ \sigma_{TH}^2 = \dfrac{4KTB}{R_L} & \rightarrow \quad \sigma_p^2 = \left(\dfrac{T_b}{q}\right)^2 \dfrac{4KTB}{R_L} \end{cases}$$

$$s = m + p \quad \begin{cases} \langle s \rangle = \langle m \rangle \\ \sigma_s^2 = \sigma_m^2 + \sigma_p^2 \end{cases} \equiv \text{Poiss} + \text{Gauss} \approx \text{Gauss}$$

independent processes

$$\boxed{f_s(s) \cong \frac{1}{\sqrt{2\pi}\sigma_s} \exp\left[-\frac{1}{2}\left(\frac{s - \langle s \rangle}{\sigma_s}\right)^2\right]}$$

$$\mu \equiv \langle m \rangle = \eta \langle n \rangle \qquad \xrightarrow{\text{APD}} \qquad \mu \equiv M\eta\langle n \rangle$$

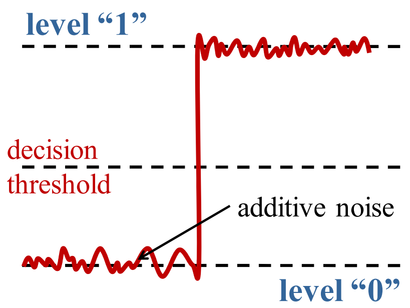
$$\sigma^2 \equiv \sigma_m^2 + \sigma_p^2 = \eta\langle n \rangle + \sigma_p^2 \qquad \qquad \sigma^2 \equiv M^2F\eta\langle n \rangle + \sigma_p^2$$

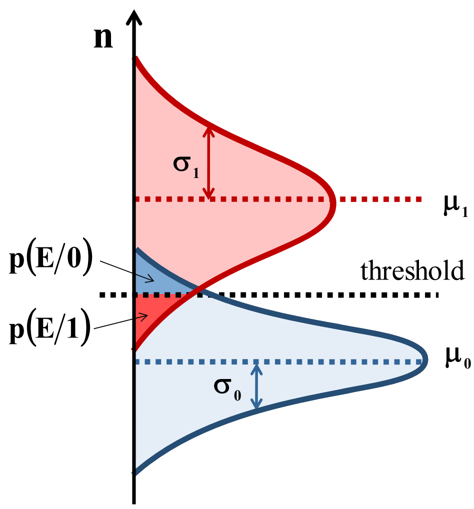
No Poisson

$$\text{"0"} \begin{cases} \mu_0 = 0 \\ \sigma_0^2 = \sigma_p^2 \end{cases} \rightarrow \text{Gauss}$$

$$\sigma_0^2 < \sigma_1^2$$

$$\text{"1"} \begin{cases} \mu_1 = M\eta\langle n \rangle \\ \sigma_1^2 = M^2F\eta\langle n \rangle + \sigma_p^2 \end{cases} \rightarrow \text{Gauss}$$





Bit Error Ratio (BER)

$$f_0(s)=\frac{1}{\sqrt{2\pi}\sigma_0}\exp\left[-\frac{1}{2}\left(\frac{s-\cancel{\mu_0}}{\sigma_0}\right)^2\right]=\frac{1}{\sqrt{2\pi}\sigma_0}\exp\left[-\frac{1}{2}\left(\frac{s}{\sigma_0}\right)^2\right]$$

$$f_1(s)=\frac{1}{\sqrt{2\pi}\sigma_1}\exp\left[-\frac{1}{2}\left(\frac{s-\mu_1}{\sigma_1}\right)^2\right]$$

$$\begin{cases} p(E/0)\equiv\int_{\ell}^{\infty}f_0(s)\partial s \\ p(E/1)\equiv\int_{-\infty}^{\ell}f_1(s)\partial s\end{cases}$$

$$p(E)=p(E/0)p(0)+p(E/1)p(1)=\frac{1}{2}p(E/0)+\frac{1}{2}p(E/1)$$

$$\frac{\partial p(E)}{\partial \ell}=0\ \rightarrow\ \boxed{\ell_{OPT}=\frac{\sigma_1\mu_0+\sigma_0\mu_1}{\sigma_1+\sigma_0}}\ \xrightarrow{\mu_0=0}\ \ell_{OPT}=\frac{\mu_1}{1+\sigma_1/\sigma_0}<\frac{\mu_1}{2}$$

Dominant shot $\longrightarrow$ $\ell_{OPT}\approx\mu_0+\mu_1\dfrac{\sigma_0}{\sigma_1}$

Dominant thermal $\longrightarrow$ $\ell_{OPT}\approx(\mu_0+\mu_1)/2$

Function erf(x)

$$\mathrm{erf}(x)\equiv\frac{2}{\sqrt{\pi}}\int_0^x\exp\left[-t^2\right]\partial t$$

$$\mathrm{erf}(x)+\mathrm{erfc}(x)=1$$

$$\mathrm{erfc}(x)\equiv\frac{2}{\sqrt{\pi}}\int_x^{\infty}\exp\left[-t^2\right]\partial t$$

$$t\equiv\frac{s-\mu_0}{\sqrt{2}\sigma_0}\qquad \partial s=\sqrt{2}\sigma_0\partial t$$

$$p(E/0)=\frac{1}{\sqrt{2\pi}\sigma_0}\int_{\ell}^{\infty}\exp\left[-\frac{1}{2}\left(\frac{s-\mu_0}{\sigma_0}\right)^2\right]\partial s=\frac{1}{\sqrt{\pi}}\int_{\ell_T}^{\infty}\exp\left[-t^2\right]\partial t=\frac{1}{2}\mathrm{erfc}(\ell_T)$$

$$\ell_T=\frac{\ell_{OPT}-\mu_0}{\sqrt{2}\sigma_0}=\frac{\dfrac{\sigma_1\mu_0+\sigma_0\mu_1}{\sigma_1+\sigma_0}-\mu_0}{\sqrt{2}\sigma_0}=\frac{\sigma_1\mu_0+\sigma_0\mu_1-\mu_0\sigma_1-\mu_0\sigma_0}{(\sigma_1+\sigma_0)\sqrt{2}\sigma_0}$$

$$=\frac{\sigma_0\mu_1-\mu_0\sigma_0}{(\sigma_1+\sigma_0)\sqrt{2}\sigma_0}=\frac{1}{\sqrt{2}}\boxed{\frac{\mu_1-\mu_0}{\sigma_1+\sigma_0}}\ Q$$

$$p(E/1)=p(E/0)\ \Rightarrow\ p(E)=p(E/0)=\frac{1}{2}\mathrm{erfc}(\ell_T)=\frac{1}{2}\mathrm{erfc}\left(\frac{Q}{\sqrt{2}}\right)$$

Quality parameter Q

$$Q \equiv \frac{\mu_1 - \mu_0}{\sigma_1 + \sigma_0}$$

$$BER \equiv \frac{1}{2}\operatorname{erfc}\left(\frac{Q}{\sqrt{2}}\right)$$

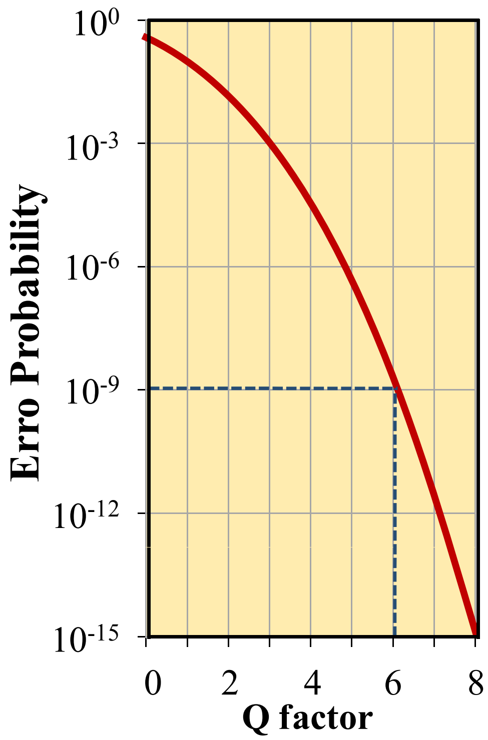
square pulses

$$\langle i \rangle = \frac{q}{T_b}\langle n \rangle$$

$$\sigma_i^2 = \left(\frac{q}{T_b}\right)^2 \sigma_n^2$$

$$\Downarrow$$

$$Q \equiv \frac{\langle n \rangle_1 - \langle n \rangle_0}{\sigma_n^1 + \sigma_n^0} = \frac{I_1 - I_0}{\sigma_i^1 + \sigma_i^0}$$



$$Q = 0 \rightarrow BER = \frac{1}{2}$$

$$Q = \infty \rightarrow BER = 0$$

$$Q = 6 \rightarrow BER \approx 10^{-9}$$

Particular Cases: PIN

$$I_D = 0$$

$$\left.\begin{aligned} \mu_1 - \mu_0 &= \eta\langle n \rangle \\ \sigma_1^2 &= \eta\langle n \rangle + \sigma_p^2 \\ \sigma_0^2 &= \sigma_p^2 \end{aligned}\right\} \rightarrow Q \equiv \frac{\mu_1 - \mu_0}{\sigma_1 + \sigma_0} = \frac{\eta\langle n \rangle}{\sqrt{\eta\langle n \rangle + \sigma_p^2} + \sigma_p}$$

$$BER \le 10^{-9} \rightarrow Q \ge 6 \rightarrow \langle n \rangle \ge \frac{12}{\eta}(3 + \sigma_p) \rightarrow \boxed{\langle n_a \rangle \ge \frac{6}{\eta}(3 + \sigma_p)} \quad \frac{\text{photons}}{\text{bit}}$$

No thermal noise

quantum limit

$$\left.\begin{aligned} \mu_1 - \mu_0 &= \eta\langle n \rangle \\ \sigma_1^2 &= \eta\langle n \rangle \\ \sigma_0^2 &= 0 \end{aligned}\right\} \rightarrow Q \equiv \frac{\mu_1 - \mu_0}{\sigma_1 + \sigma_0} = \sqrt{\eta\langle n \rangle}$$

$$BER \le 10^{-9} \rightarrow \langle n \rangle \ge 36/\eta \rightarrow \boxed{\langle n_a \rangle \ge 18/\eta} \quad \frac{\text{photons}}{\text{bit}}$$

wrong model !!

APD

$I_D = 0$

$$\left.\begin{aligned}\mu_1 - \mu_0 &= M\eta\langle n\rangle \\ \sigma_1^2 &= M^2F\eta\langle n\rangle + \sigma_p^2 \\ \sigma_0^2 &= \sigma_p^2\end{aligned}\right\} \rightarrow Q \equiv \frac{\mu_1 - \mu_0}{\sigma_1 + \sigma_0} = \frac{M\eta\langle n\rangle}{\sqrt{M^2F\eta\langle n\rangle + \sigma_p^2} + \sigma_p}$$

$$\mathrm{BER} \le 10^{-9} \rightarrow \begin{array}{l}\langle n\rangle \ge \dfrac{12}{\eta}\left(3F + \sigma_p/M\right) \\ \langle n_a\rangle \ge \dfrac{6}{\eta}\left(3F + \sigma_p/M\right)\end{array} \xrightarrow{M\uparrow\uparrow} \boxed{\langle n_a\rangle \ge 18\frac{F}{\eta}} \quad \frac{\text{photons}}{\text{bit}}$$

No thermal noise

$$\left.\begin{aligned}\mu_1 - \mu_0 &= M\eta\langle n\rangle \\ \sigma_1^2 &= M^2F\eta\langle n\rangle \\ \sigma_0^2 &= 0\end{aligned}\right\} \rightarrow Q \equiv \frac{\mu_1 - \mu_0}{\sigma_1 + \sigma_0} = \sqrt{\frac{\eta\langle n\rangle}{F}}$$

$$\mathrm{BER} \le 10^{-9} \rightarrow \langle n\rangle \ge 36\ F/\eta \rightarrow \boxed{\langle n_a\rangle \ge 18\ F/\eta} \quad \frac{\text{photons}}{\text{bit}}$$

Receiver Sensitivity

$\langle s\rangle \equiv M\eta\langle n\rangle$ $\qquad I = M\Re P$ $\qquad$ Gaussian statistics

$\sigma_s^2 \equiv M^2F\eta\langle n\rangle + \sigma_p^2$ $\qquad \sigma_I^2 = 2qBM^2F(M)\Re P + \sigma_T^2$

$$Q_{APD} \equiv \frac{I_1 - I_0}{\sigma_1 + \sigma_0} = \frac{M\Re P_1}{\sqrt{2qBM^2F(M)\Re P_1 + \sigma_T^2} + \sigma_T} \qquad \bar{P} \equiv \frac{1}{2}\left(P_1 + P_0\right)$$

$F(M) = M^x$

$$\frac{M\Re P_1}{\sqrt{2qBM^2F(M)\Re P_1 + \sigma_T^2} + \sigma_T} \ge Q \longrightarrow \boxed{\bar{P} \ge Q^2qB\frac{F(M)}{\Re} + Q\frac{\sigma_T}{M\Re} \equiv S_{APD}}$$

$$\langle n_a\rangle \ge \frac{Q}{2\eta}\left(QF + 2\sigma_p/M\right)$$

$$S_{PIN} = Q^2q\frac{B}{\Re} + Q\frac{\sigma_T}{\Re} \approx Q\frac{\sigma_T}{\Re} \quad \text{Thermal Dominant}$$

$$\frac{\partial S}{\partial M} = Q^2xqB\frac{M^{x-1}}{\Re} - Q\frac{\sigma_T}{M^2\Re} = 0 \longrightarrow \boxed{M_{OPT}^{x+1} = \frac{\sigma_T}{QxqB}} \longrightarrow S_{OPT}$$

Sensitivity Improvement

$$S_{APD} = Q^2 qB \frac{F(M)}{\Re} + Q\frac{\sigma_T}{M\Re} \qquad S_{PIN} = Q^2 qB \frac{1}{\Re} + Q\frac{\sigma_T}{\Re}$$

$$S_{APD} < S_{PIN}$$

$$Q^2 qB \frac{F(M)}{\Re} + Q\frac{\sigma_T}{M\Re} < Q^2 qB \frac{1}{\Re} + Q\frac{\sigma_T}{\Re}$$

$$QqBF(M) + \frac{\sigma_T}{M} < QqB + \sigma_T$$

$$\boxed{F(M) < 1 + \frac{\sigma_T}{QqB}\left(1 - \frac{1}{M}\right) \approx 1 + \frac{\sigma_T}{QqB}}$$

SNR – BER relationship

$$Q \equiv \frac{\mu_1 - \mu_0}{\sigma_1 + \sigma_0} = \frac{\text{signal}}{\sigma_1 + \sigma_0} \rightarrow Q^2 = \frac{\text{signal}^2}{(\sigma_1 + \sigma_0)^2}$$

$$BER(Q) \equiv \frac{1}{2}\text{erfc}\left(\frac{Q}{\sqrt{2}}\right)$$

$$\sigma_1 >> \sigma_0 \rightarrow Q^2 = \frac{(\mu_1 - \mu_0)^2}{\sigma_1^2} = SNR \rightarrow \boxed{BER = \frac{1}{2}\text{erfc}\left(\sqrt{\frac{SNR}{2}}\right)}$$

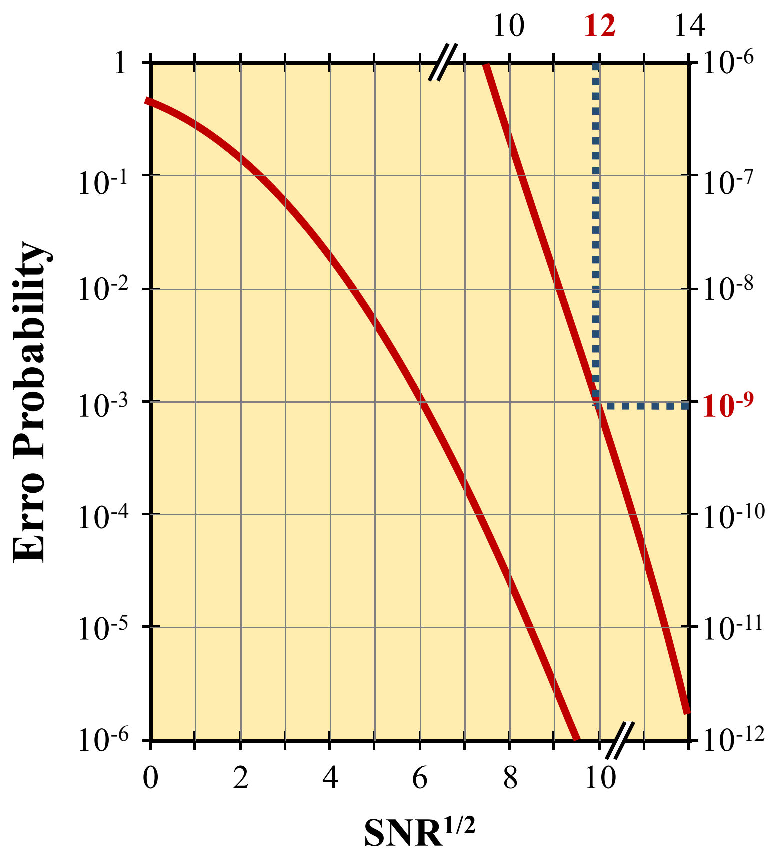
SHOT

$$BER \geq 10^{-9} \rightarrow Q \geq 6 \rightarrow SNR \geq 36 \quad (15.56 \text{ dB})$$

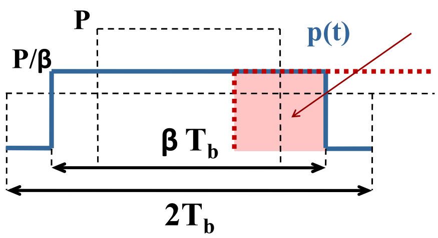
$$\sigma_1 \approx \sigma_0 \rightarrow Q^2 = \frac{(\mu_1 - \mu_0)^2}{4\sigma_1^2} = \frac{SNR}{4} \rightarrow \boxed{BER = \frac{1}{2}\text{erfc}\left(\sqrt{\frac{SNR}{8}}\right)}$$

THERMAL

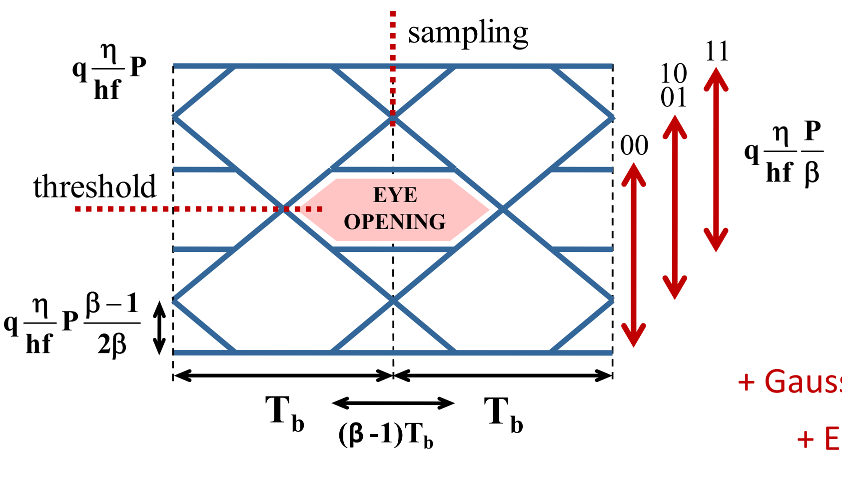
$$BER \geq 10^{-9} \rightarrow Q \geq 6 \rightarrow SNR \geq 144 \quad (21.58 \text{ dB})$$



BER using simplified ISI model



$$BER = \frac{1}{4}\sum_{i=0}^{1}\sum_{i=0}^{1}\frac{1}{2}\mathrm{erfc}\left(\frac{Q_{ij}}{\sqrt{2}}\right)$$



+ Gaussian Assumption

+ Equiprobable bits

THERMAL NOISE DOMINANT

signal independent

$$Q_{00}=\frac{q\dfrac{\eta}{hf}\dfrac{P}{\beta}}{2\sigma_{th}}=\frac{q\dfrac{\eta}{hf}P}{2\sigma_{th}}\frac{1}{\beta}$$

$$Q_{01}=Q_{10}=\frac{q\dfrac{\eta}{hf}\dfrac{P}{\beta}+q\cancel{\dfrac{\eta}{hf}P\dfrac{\beta-1}{2\beta}}-q\cancel{\dfrac{\eta}{hf}P\dfrac{\beta-1}{2\beta}}}{2\sigma_{th}}=Q_{00}$$

$$Q_{11}=\frac{q\dfrac{\eta}{hf}\dfrac{P}{\beta}+q\cancel{\dfrac{\eta}{hf}P\dfrac{\beta-1}{\beta}}-q\cancel{\dfrac{\eta}{hf}P\dfrac{\beta-1}{\beta}}}{2\sigma_{th}}=Q_{00}$$

ISI penalty

$$BER=\frac{1}{4}\sum_{i=0}^{1}\sum_{i=0}^{1}\frac{1}{2}\mathrm{erfc}\left(\frac{Q_{ij}}{\sqrt{2}}\right)=\frac{1}{2}\mathrm{erfc}\left(\frac{Q_{00}}{\sqrt{2}}\right)=\frac{1}{2}\mathrm{erfc}\left(\frac{q\dfrac{\eta}{hf}P}{2\sqrt{2}\sigma_{th}}\frac{1}{\beta}\right)$$

SHOT NOISE DOMINANT

signal dependent

$$Q_{00}=\frac{q\dfrac{\eta}{hf}\dfrac{P}{\beta}}{\sqrt{\dfrac{q^2}{T_b}\dfrac{\eta}{hf}\dfrac{P}{\beta}}}=\sqrt{T_b\frac{\eta}{hf}P}\sqrt{\frac{1}{\beta}}$$

$$Q_{01}=Q_{10}=\frac{q\dfrac{\eta}{hf}\dfrac{P}{\beta}+q\cancel{\dfrac{\eta}{hf}P\dfrac{\beta-1}{2\beta}}-q\cancel{\dfrac{\eta}{hf}P\dfrac{\beta-1}{2\beta}}}{\sqrt{\dfrac{q^2}{T_b}\dfrac{\eta}{hf}\dfrac{P}{\beta}+\dfrac{q^2}{T_b}\dfrac{\eta}{hf}P\dfrac{\beta-1}{2\beta}}+\sqrt{\dfrac{q^2}{T_b}\dfrac{\eta}{hf}P\dfrac{\beta-1}{2\beta}}}=\sqrt{T_b\frac{\eta}{hf}P}\sqrt{\frac{1}{\beta}}\frac{\sqrt{2}}{\sqrt{\beta+1}+\sqrt{\beta-1}}$$

ISI penalty

noise penalty

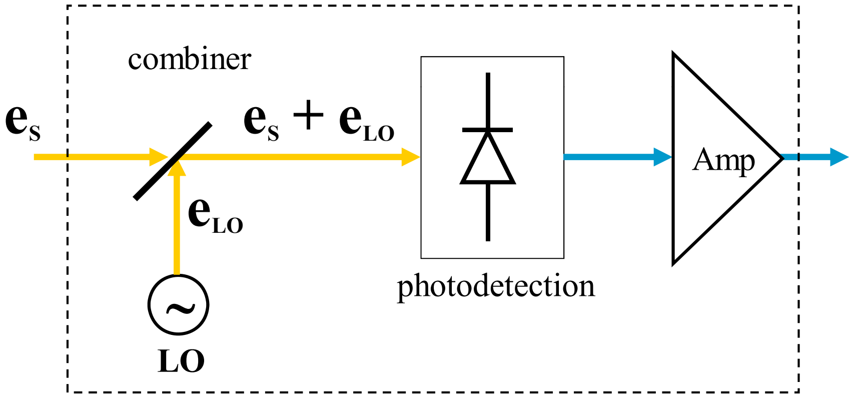
$$Q_{11}=\frac{q\dfrac{\eta}{hf}\dfrac{P}{\beta}+q\cancel{\dfrac{\eta}{hf}P\dfrac{\beta-1}{\beta}}-q\cancel{\dfrac{\eta}{hf}P\dfrac{\beta-1}{\beta}}}{\sqrt{\dfrac{q^2}{T_b}\dfrac{\eta}{hf}\dfrac{P}{\beta}+\dfrac{q^2}{T_b}\dfrac{\eta}{hf}P\dfrac{\beta-1}{\beta}}+\sqrt{\dfrac{q^2}{T_b}\dfrac{\eta}{hf}P\dfrac{\beta-1}{\beta}}}=\sqrt{T_b\frac{\eta}{hf}P}\sqrt{\frac{1}{\beta}}\frac{1}{\sqrt{\beta+1}+\sqrt{\beta-1}}$$

worst case

$$BER=\frac{1}{4}\sum_{i=0}^{1}\sum_{i=0}^{1}\frac{1}{2}\mathrm{erfc}\left(\frac{Q_{ij}}{\sqrt{2}}\right)=\frac{1}{8}\mathrm{erfc}\left(\frac{Q_{00}}{\sqrt{2}}\right)+\frac{1}{4}\mathrm{erfc}\left(\frac{Q_{01}}{\sqrt{2}}\right)\frac{1}{8}\mathrm{erfc}\left(\frac{Q_{11}}{\sqrt{2}}\right)\geq\frac{1}{2}\mathrm{erfc}\left(\frac{Q_{11}}{\sqrt{2}}\right)$$

COHERENT DETECTION

Concept



$$\left.\begin{aligned} e_S &\equiv E_S \cos\left(\omega_S t + \theta(t)\right) \\ e_{LO} &\equiv E_{LO}\cos\left(\omega_{LO} t\right)\end{aligned}\right\} \rightarrow \quad e_{IN} \equiv e_S + e_{LO}$$

$\omega_S\ \omega_{LO}$: optical frequencies $\qquad$ $\theta(t)$: phase difference

$$e_S = E_S \cos\left(\omega_S t + \theta(t)\right) \qquad e_{LO} = E_{LO}\cos\left(\omega_{LO} t\right)$$

$$I_{PH} = R \cdot P_{IN} = R \cdot \left\langle e_{IN}^2 \right\rangle = R \cdot \left\langle \left(e_S + e_{LO}\right)^2 \right\rangle = R \cdot \left(e_S^2 + 2 e_S e_{LO} + e_{LO}^2\right)$$

$$I_{PH} = R \cdot \left\{ \frac{E_S^2}{2}\left[1 + \cos\left(2\cancel{\omega_S} t + 2\theta(t)\right)\right] + 2E_S E_{LO}\cos\left(\omega_S t + \theta(t)\right)\cos\left(\omega_{LO} t\right) + \frac{E_{LO}^2}{2}\left[1 + \cos\left(2\cancel{\omega_{LO}} t\right)\right]\right\}$$

$$2\cos\left(\omega_S t + \theta(t)\right)\cos\left(\omega_{LO} t\right) = \cos\left[\left(\omega_S \cancel{+} \omega_{LO}\right)t + \theta(t)\right] + \cos\left[\left(\underbrace{\omega_S - \omega_{LO}}_{\omega_{FI}}\right)t + \theta(t)\right]$$

$$\boxed{I_{PH} = R \cdot \left\{ \frac{E_S^2}{2} + \frac{E_{LO}^2}{2} + E_S E_{LO}\cos\left[\omega_{FI} t + \theta(t)\right]\right\}} \Leftarrow$$

low-pass filtering

ω_{FI}: intermediate frequency

HETERODYNE DETECTION

$$\omega_{FI} = 0 \quad \rightarrow \qquad I_{PH} = R \cdot \left\{ \frac{E_S^2}{2} + \frac{E_{LO}^2}{2} + E_S E_{LO} \cos[\theta(t)] \right\}$$

HOMODYNE DETECTION

$$\boxed{P_{IN} = P_S + P_{LO} + 2\sqrt{P_S P_{LO}} \cos[\omega_{FI} t + \theta(t)]}$$ optical power

$$P_S = 0 \rightarrow P_{IN} = P_{LO}$$

amplification effect

$$P_{LO} >> P_S$$

$$I_{PH} = I_{DC} + I_{FI}$$

$$I_{DC} = \eta \frac{q}{hf}(P_S + P_{LO}) \approx \eta \frac{q}{hf} P_{LO}$$ DC current

$$I_{FI} = 2\eta \frac{q}{hf}\sqrt{P_S P_{LO}} \cos[\omega_{FI} t + \theta(t)]$$ information

Signal to Noise Ratio (SNR)

$$I_{DC} = \eta \frac{q}{hf}(P_S + P_{LO})$$

$$P_S << P_{LO}$$

$$I_{FI} = 2\eta \frac{q}{hf}\sqrt{P_S P_{LO}} \cos(\omega_{FI} t + \theta(t))$$

$$\sigma_{OL}^2 = \langle i_{OL}^2 \rangle = 2qB \cdot I_{OL} = 2qB \cdot R \cdot P_{OL}$$ LO shot noise

effective value

$$SNR_{HET} \equiv \frac{2\left(\eta \frac{q}{hf}\right)^2 P_S P_{LO}}{2qB\left(\cancel{\eta \frac{q}{hf} P_S} + \eta \frac{q}{hf} P_{LO} + \cancel{2\eta \frac{q}{hf}\sqrt{P_S P_{LO}}} + \cancel{I_D}\right) + \cancel{\frac{4KTB}{R_L}} F_A}$$

$$SNR_{HET} \approx \eta \frac{P_S}{B \cdot hf} \qquad SNR_{HOM} \approx \eta \frac{2P_S}{B \cdot hf} = 2 \cdot SNR_{HET}$$

Amplitude Modulation (ASK)

$$e_M \equiv e_S\left(1+m\cdot f(t)\right)=E_S\left(1+m\cdot f(t)\right)\cos\left(\omega_S t\right)$$

$$I_{PH}=R\cdot\left\{\frac{E_S^2}{2}\left(1+m\cdot f(t)\right)^2+\frac{E_{LO}^2}{2}+E_S E_{LO}\left(1+m\cdot f(t)\right)\cos\left(\omega_{FI} t\right)\right\}$$

$$I_{DC}\approx\eta\frac{q}{hf}P_{LO}$$

$$E_S \ll E_{LO}$$

$$\boxed{I_{FI}=2\eta\frac{q}{hf}\sqrt{P_S P_{LO}}\left(1+m\cdot f(t)\right)\cos\left(\omega_{FI} t\right)}$$

Intensity Modulation (IM)

$$P_M \equiv P_S\left(1+m\cdot f(t)\right)$$

$$e_M = E_S\left(1+m\cdot f(t)\right)^{1/2}\cos\left(\omega_S t\right)$$

$$\boxed{I_{FI}\propto\left(1+m\cdot f(t)\right)^{1/2}\approx 1+\frac{m\cdot f(t)}{2}}$$

$$\left|m\cdot f(t)\right| \ll 1$$

Phase Modulation (PSK)

$$e_M \equiv E_S\cos\left(\omega_S t+m\cdot\theta(t)\right)$$

$$I_{PH}=R\cdot\left\{\frac{E_S^2}{2}+\frac{E_{LO}^2}{2}+E_S E_{LO}\cos\left(\omega_{FI} t+m\cdot\theta(t)\right)\right\}$$

$$I_{DC}\approx\eta\frac{q}{hf}P_{LO}$$

$$E_S \ll E_{LO}$$

$$\boxed{I_{FI}=2\eta\frac{q}{hf}\sqrt{P_S P_{LO}}\cos\left(\omega_{FI} t+m\cdot\theta(t)\right)}$$

Frequency Modulation (FSK)

$$e_M \equiv E_S\cos\left(\left(\omega_S+m\cdot\Delta\omega\right)t\right)$$

$$I_{DC}\approx\eta\frac{q}{hf}P_{LO}$$

$$E_S \ll E_{LO}$$

$$\boxed{I_{FI}=2\eta\frac{q}{hf}\sqrt{P_S P_{LO}}\cos\left(\left(\omega_{FI}+m\cdot\Delta\omega\right)t\right)}$$

ERROR PROBABILITY

$$\left.\begin{array}{l} e_S \equiv E_S A(t)\cos\left(\omega_S t + \theta(t)\right) \\ e_{LO} \equiv E_{LO}\cos\left(\omega_{LO} t\right) \end{array}\right\} \rightarrow \quad e_{IN} \equiv e_S + e_{LO}$$

$$P_{LO} >> P_S$$

$$I_{PH} = I_{DC} + I_{FI} \rightarrow \begin{cases} I_{DC} = \eta \dfrac{q}{hf}\left(P_S + P_{LO}\right) \approx \eta \dfrac{q}{hf} P_{LO} \\ I_{FI} = 2\eta \dfrac{q}{hf}\sqrt{P_S P_{LO}}\, A(t)\cos\left[\omega_{FI} t + \theta(t)\right] \end{cases}$$

$$\sigma_S^2 \approx \sigma_{OL}^2 = \sigma_1^2 = \sigma_0^2 = 2qB \cdot I_{LO} = 2qB \cdot \eta \frac{q}{hf} \cdot P_{LO}$$

dominant noise

Heterodyne ASK

$$I_D = 0$$

$$\mu_0 = 0$$

$$\mu_1 = \sqrt{2}\eta \frac{q}{hf}\sqrt{P_S P_{LO}}$$

$$\sigma_1 = \sigma_0 = \sqrt{2qB\eta \frac{q}{hf} P_{LO}}$$

$$B = \frac{R_B}{2} = \frac{1}{2T_B}$$

$$\rightarrow \quad \boxed{Q \equiv \frac{\mu_1 - \mu_0}{\sigma_1 + \sigma_0} = \sqrt{\frac{\eta P_S T_B}{2hf}} = \sqrt{\frac{\eta\langle n\rangle}{2}}}$$

$$Q = 6 \rightarrow \begin{array}{l} \langle n\rangle = 72/\eta \\ \langle n_a\rangle = 36/\eta \end{array} \quad \frac{\text{photons}}{\text{bit}}$$

Homodyne ASK

$$\mu_0 = 0$$

$$\mu_1 = 2\eta \frac{q}{hf}\sqrt{P_S P_{LO}}$$

$$\sigma_1 = \sigma_0 = \sqrt{2qB\eta \frac{q}{hf} P_{LO}}$$

$$\rightarrow \quad \boxed{Q \equiv \frac{\mu_1 - \mu_0}{\sigma_1 + \sigma_0} = \sqrt{\frac{\eta P_S T_B}{hf}} = \sqrt{\eta\langle n\rangle}}$$

$$Q = 6 \rightarrow \begin{array}{l} \langle n\rangle = 36/\eta \\ \langle n_a\rangle = 18/\eta \end{array} \quad \frac{\text{photons}}{\text{bit}}$$

Heterodyne PSK

$$I_D = 0$$

$$\mu_0 = -\sqrt{2}\eta\frac{q}{hf}\sqrt{P_S P_{LO}}$$

$$\mu_1 = \sqrt{2}\eta\frac{q}{hf}\sqrt{P_S P_{LO}} \quad \rightarrow \quad \boxed{Q \equiv \frac{\mu_1 - \mu_0}{\sigma_1 + \sigma_0} = \sqrt{2\frac{\eta P_S T_B}{hf}} = \sqrt{2\eta\langle n\rangle}}$$

$$\sigma_1 = \sigma_0 = \sqrt{2qB\eta\frac{q}{hf}P_{LO}}$$

$$Q = 6 \quad \rightarrow \quad \langle n\rangle = \langle n_a\rangle = 18/\eta \quad \frac{\text{photons}}{\text{bit}}$$

Homodyne PSK

$$\mu_0 = -2\eta\frac{q}{hf}\sqrt{P_S P_{LO}}$$

$$\mu_1 = 2\eta\frac{q}{hf}\sqrt{P_S P_{LO}} \quad \rightarrow \quad \boxed{Q \equiv \frac{\mu_1 - \mu_0}{\sigma_1 + \sigma_0} = \sqrt{4\frac{\eta P_S T_B}{hf}} = \sqrt{4\eta\langle n\rangle}}$$

$$\sigma_1 = \sigma_0 = \sqrt{2qB\eta\frac{q}{hf}P_{LO}}$$

$$Q = 6 \quad \rightarrow \quad \langle n\rangle = \langle n_a\rangle = 9/\eta \quad \frac{\text{photons}}{\text{bit}}$$

Coherent Detection vs. Direct Detection

ADVANTAGES

- Allows you to detect frequency and phase information
- Improves receiver sensitivity
- Demultiplexing filter can be much more selective

DRAWBACKS

- Much higher complexity
- Much lower stability (temperature)
- LO phase noise is a limiting factor (phase tracking)
- Polarization tracking requirement