APUNTS PROPIETAT DE:
MÀRIUS SERRA LÓPEZ

PER QUALSEVOL DUBTE O CONSULTA RESPECTE ELS APUNTS O EL QUE FACI FALTA ESCRIURE A:
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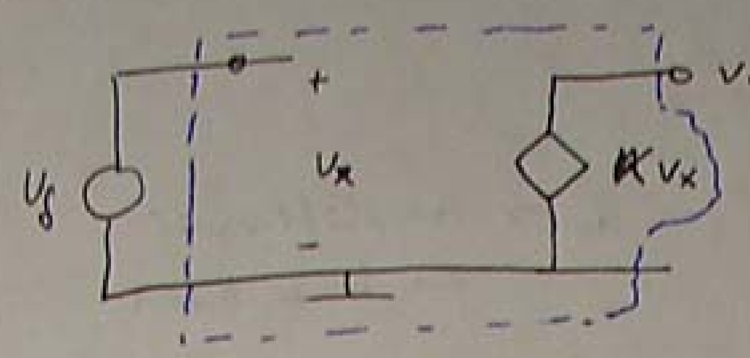
(l'assumpte ha de ser: Apunts ETSETB)

QUALSEVOL ERROR PRESENT S'ATRIBUEIX AL PROFE DE L'ASSIGNATURA QUE ME LA VA IMPARTIR!!

MODEL:



$$K = 1 + \frac{R_2}{R_1}$$

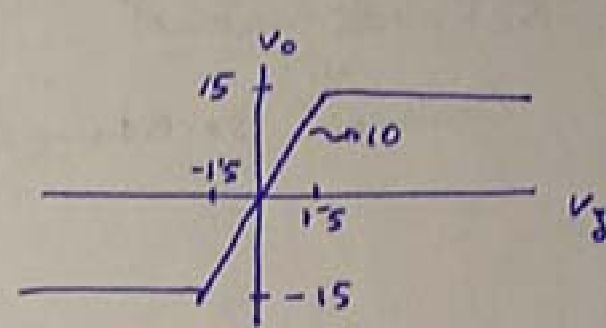
Amplificació en llaç tancat

- K es pot fixar a partir R_1, R_2
- K no depén de A_0
- $K \ll A_0$
- Model vàlid $\Leftrightarrow$ $|v_0| < V_{DC}$

Ex:

$V_{DC} = 15V$

$\left.\begin{array}{l} R_1 = 1K\Omega \\ R_2 = 9K\Omega \end{array}\right\} K = 10$



$-1'5 < v_0 < 1'5$

$\Downarrow$

Treball en zona lineal

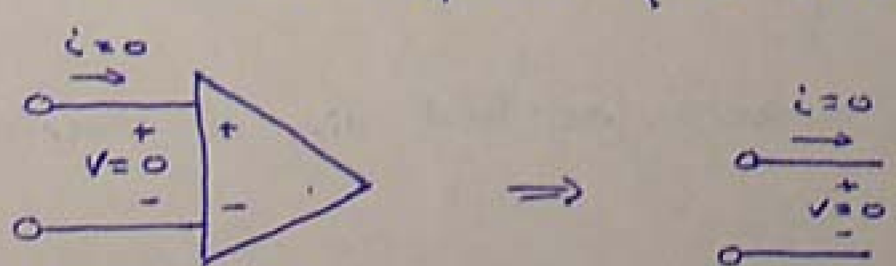
Valors exàctes de v i v_0:

$$v = \frac{1}{\frac{A_0}{K} + 1} v_g = 9'999 \cdot 10^{-5} v_g \Big|_{v_g = 1V} = 100\mu V \simeq 0V \Rightarrow \text{Compleix Hip} < 150\mu V$$

$$v_0 = \frac{A_0}{\frac{A_0}{K} + 1} v_g \simeq 10 v_g$$

Tècnica del curt-circuit virtual:

* Consisteix en suposar que $v=0$.

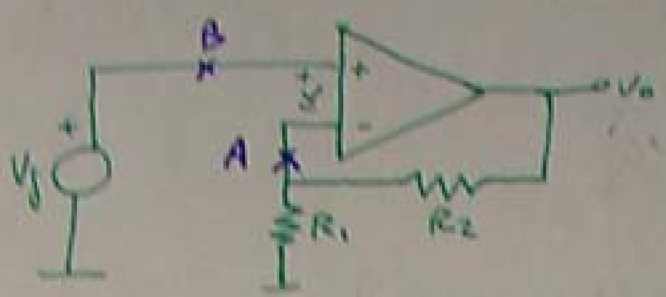


curtcircuit virtual c.c. i c.o.

cc → curtcircuit ($v=0$)

co → circuit obert ($i=0$)

Ex:

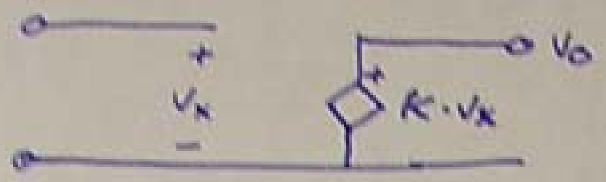


En el Node A hi ha una tensió v_g, que és el mateix que hi ha al Node B.

$CCV \Rightarrow v=0 \Rightarrow v_- = v_g$

$$v_g = \frac{R_1}{R_1+R_2} v_o \Rightarrow \boxed{v_g\left(1+\frac{R_2}{R_1}\right) = v_o}$$

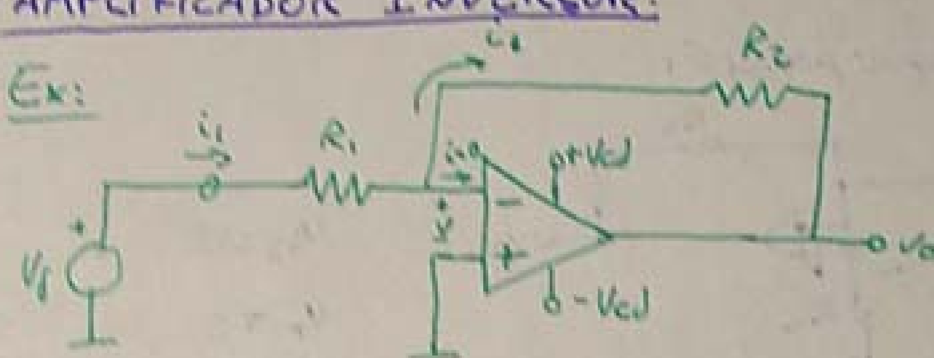
AMPLIFICADOR NO INVERSOR:



$$\boxed{K = 1 + \frac{R_2}{R_1}}$$

AMPLIFICADOR INVERSOR:

Ex:



$\Rightarrow$ Tot el corrent que passa per R_1, s'en va cap a R_2.

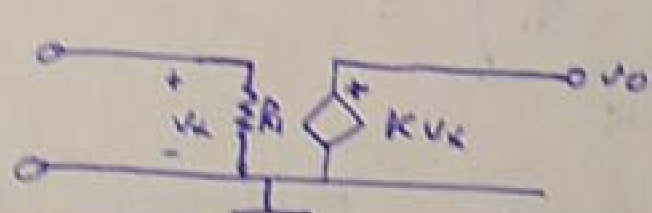
$$R_{eq} = \frac{V}{I_1} = \frac{v_g}{v_g/R_1} = R_1$$

$CCV \Rightarrow v=0 \Rightarrow v_- = 0$

$$KVL \Rightarrow \quad i_1 = \frac{v_g - 0}{R_1} = \frac{0 - v_o}{R_2}$$

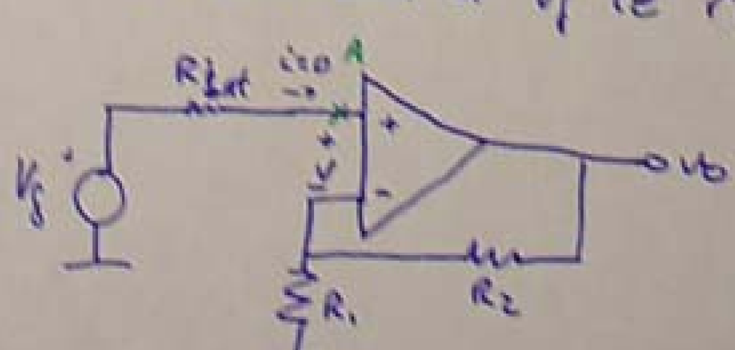
$$\boxed{v_o = -v_g \frac{R_2}{R_1}}$$

MODEL:



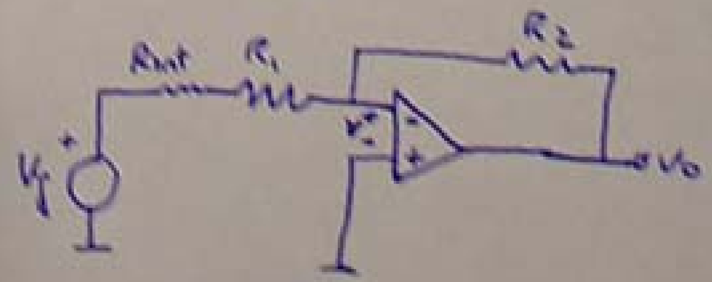
$$\boxed{K = \frac{R_2}{R_1}}$$

Ex: Que succeix si v_g té rint?



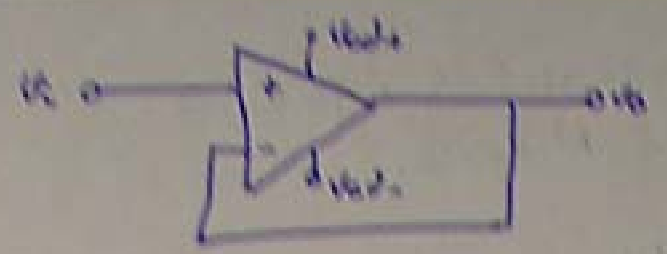
En A hi ha v_g volts. En Rint no cau tensió.

$$v_o = v_g \frac{R_1+R_2}{R_1} = v_g\left(1+\frac{R_2}{R_1}\right) \Rightarrow \text{No té conseqüències.}$$



$$v_o = -\frac{R_2}{R_{int}+R_1} v_g \Rightarrow \text{Provoca una perdua d'amplificació.}$$

SEGUIDOR DE TENSIÓ:

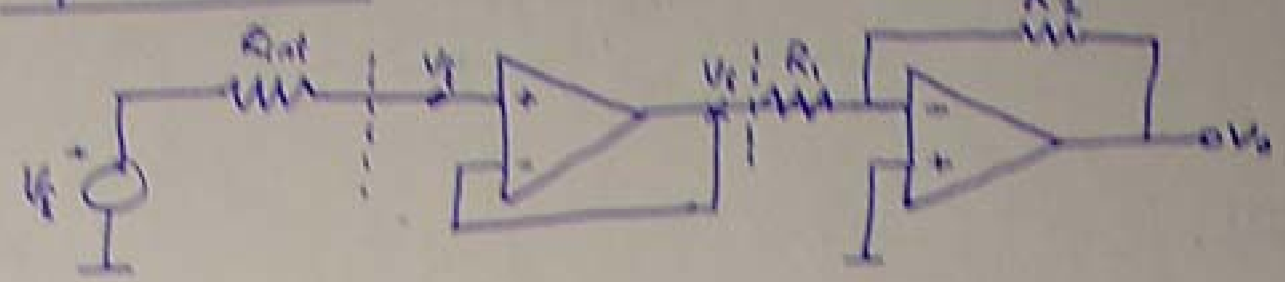


curtcircuit virtual

$CCV \Rightarrow v=0 \Rightarrow \boxed{v_o = v_i}$

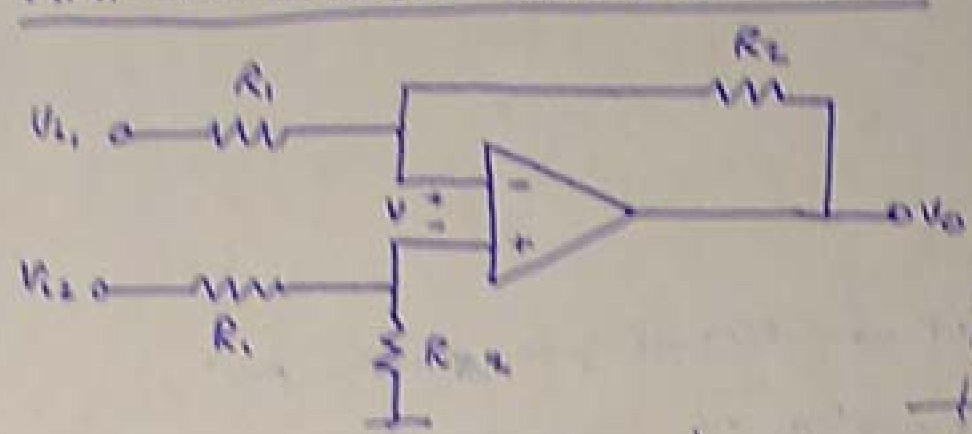
* També s'anomena, etapa separadora. En anglès **buffer**.

Aplicació:



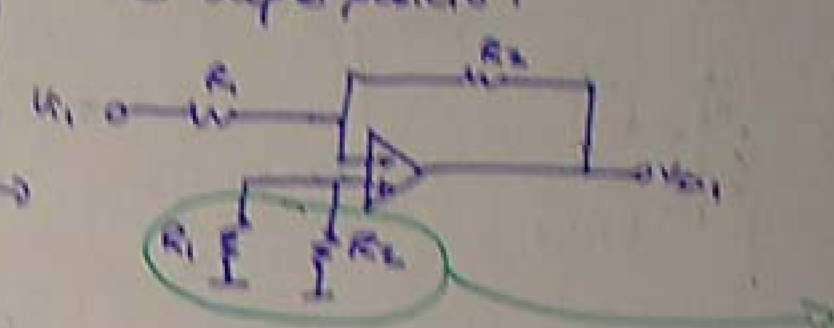
$$v_o = -\frac{R_2}{R_1} v_f$$

AMPLIFICADOR DIFERENCIAL:



CCV

Per superposició:

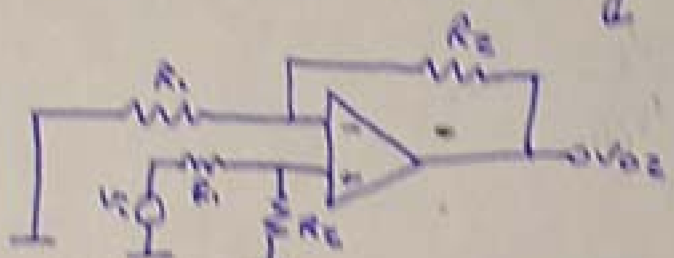


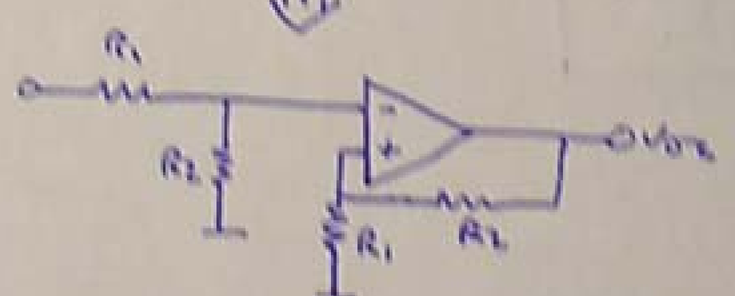
$v_{i2} = 0$

$v_+ = 0$

↳ R_1 i R_2 com si no hi fossin

$$\boxed{v_{o1} = -\frac{R_2}{R_1} v_{i1}}$$





$$v_{o2} = \underbrace{v_{i2}\frac{R_2}{R_1+R_2}}_{v_x}\underbrace{\left(1+\frac{R_2}{R_1}\right)}_{K} =$$

$$v_{o2} = v_{i2}\frac{R_2}{R_1+R_2}\cdot\frac{R_1+R_2}{R_1} = \boxed{v_{i2}\frac{R_2}{R_1}}$$

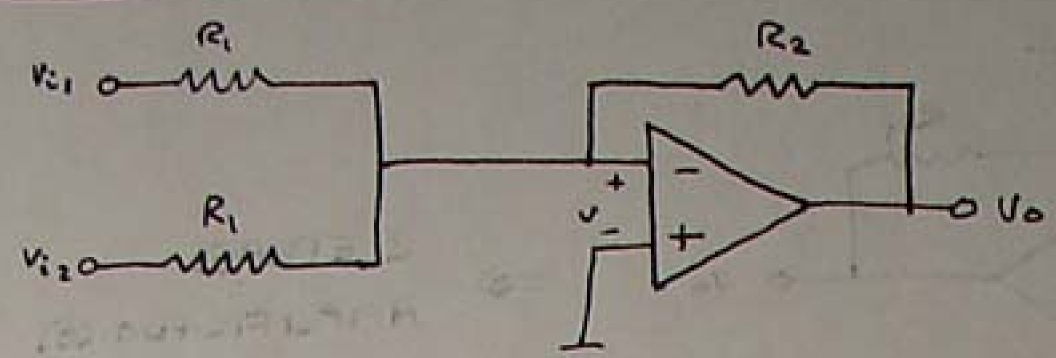
$$v_o = v_{o1} + v_{o2}$$

$$v_o = -\frac{R_2}{R_1}v_{i1} + \frac{R_2}{R_1}v_{i2}$$

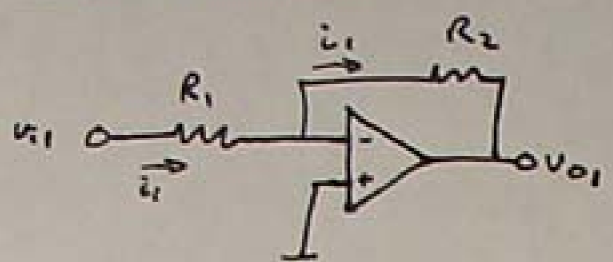
$$\boxed{v_o = \frac{R_2}{R_1}(v_{i2} - v_{i1})}$$

AMPLIFICADOR SUMADOR INVERSOR:

24-10-03



Superposició:

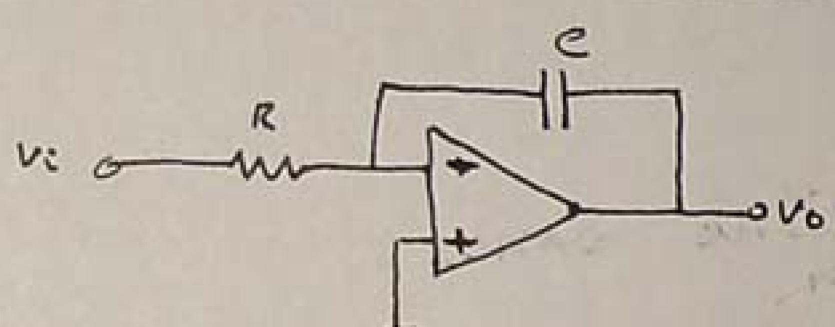


$$i_1 = \frac{V_{i1}}{R_1} = \frac{-V_{o1}}{R_2} \Rightarrow \boxed{V_{o1} = -\frac{R_2}{R_1} V_{i1}}$$

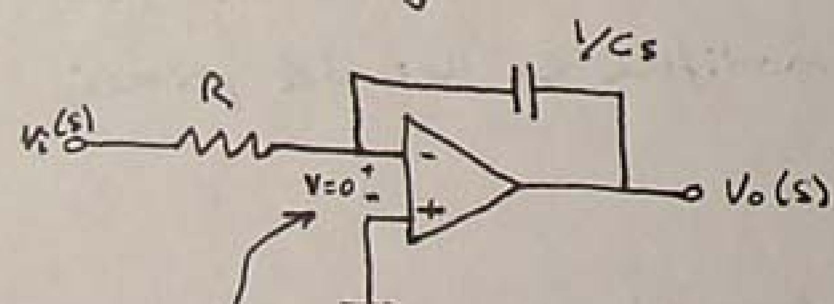
$$\boxed{V_{o2} = -\frac{R_2}{R_1} V_{i2}}$$

$$\boxed{V_o = V_{o1} + V_{o2} = -\frac{R_2}{R_1}\left(V_{i1} + V_{i2}\right)}$$

AMPLIFICADOR INTEGRADOR INVERSOR:



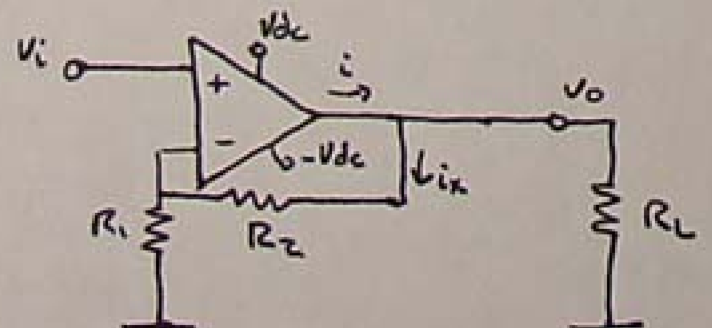
$\Big\} \mathcal{L}$



CCV transformat = CCV

$$V_o(s) = -\frac{\frac{1}{Cs}}{R} V_i(s) = \underbrace{\frac{-1}{RCs}}_{H(s)} V_i(s)$$

EL OPERACIONAL TÉ LÍMITS:

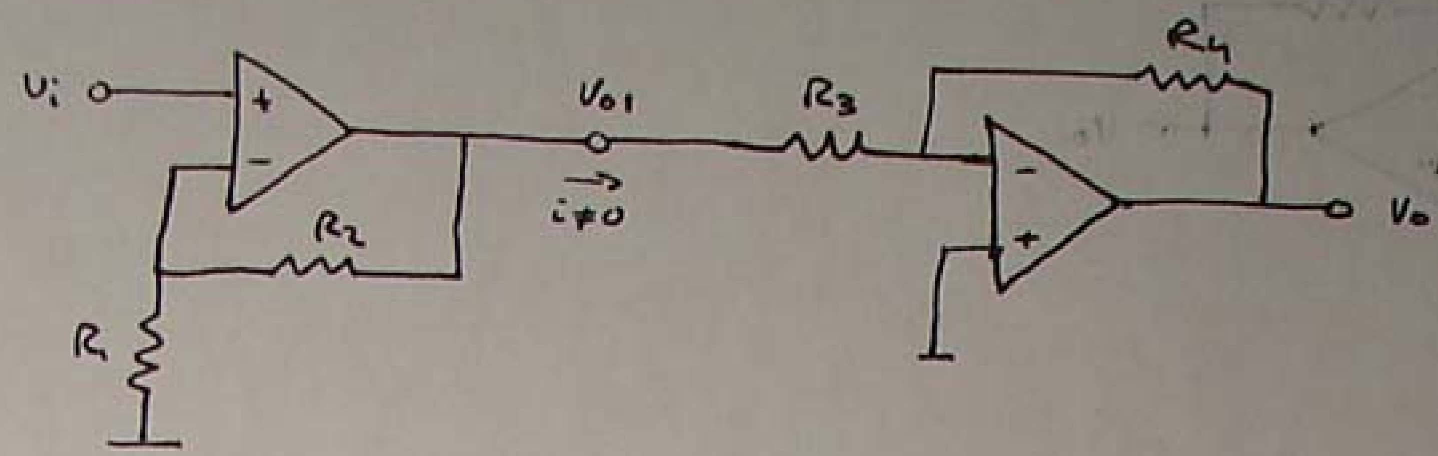


$$V_o = \left(1 + \frac{R_2}{R_1}\right) V_i \Rightarrow \text{No depén de } R_L$$

$i_{max} \simeq 40\,mA$ → Si es supera deixa de complir-se lo anterior o s'esconya.
($\mu A741$)

$$i_x = \frac{-V_o}{R_1 + R_2}$$

CONEXIÓ EN CASCADA D'AO:



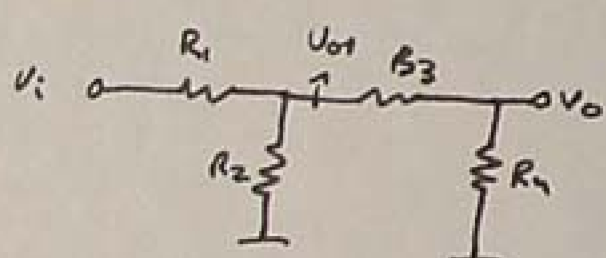
$\Rightarrow$ 2 ETAPES AMPLIFICADORES

$$v_{o1} = \left(1 + \frac{R_2}{R_1}\right) v_i \qquad v_o = -\frac{R_4}{R_3} v_{o1}$$

$$\Downarrow$$

$$v_o = \underbrace{\underbrace{\left(1 + \frac{R_2}{R_1}\right)}_{K_1}\underbrace{\left(\frac{-R_4}{R_3}\right)}_{K_2}}_{K_T} v_i \qquad K_T = K_1 \cdot K_2$$

Ex:



$$v_o = \cancel{\frac{R_2}{R_1+R_2} \cdot \frac{R_4}{R_3+R_4}} \cdot v_i \Rightarrow \text{FALS}$$

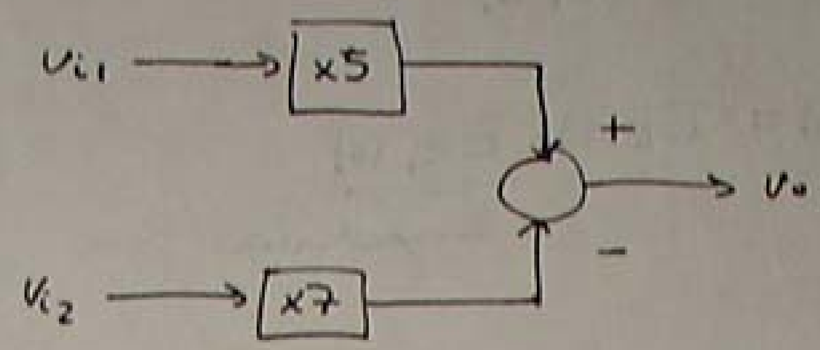
* És fals pq el segon divisor de tensió absorveix intensitat, i això modifica la llei del primer divisor de tensió.

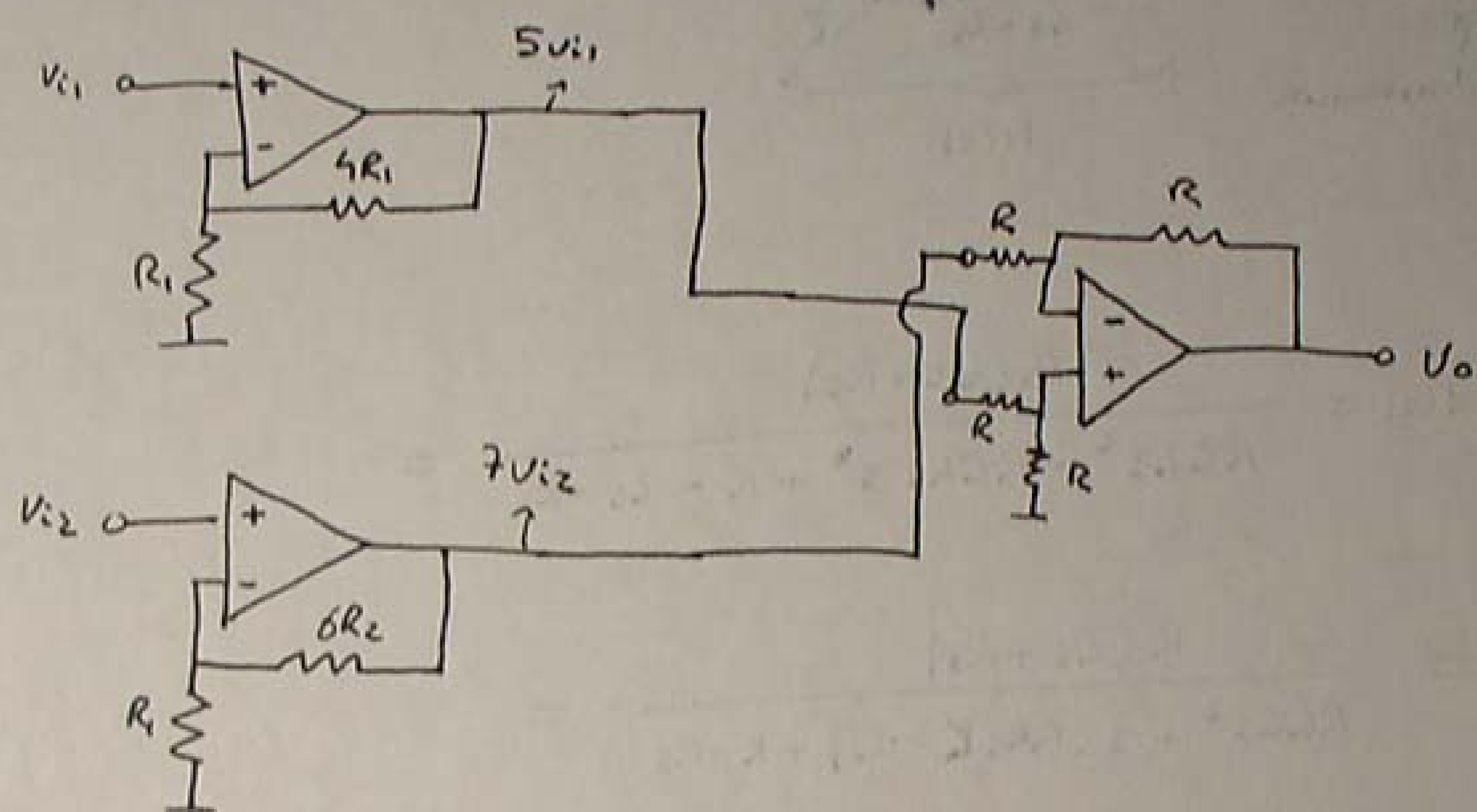
* Això no succeeix en les etapes amplificadores.

* Límit: si intercanviem les dues etapes anteriors, i hi posem un generador amb resistencia interna, hi hauria problemes.

Exercici:

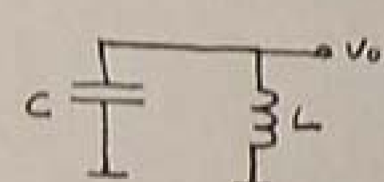
Fer un circuit que produeixi: $V_o = 5V_{i1} - 7V_{i2}$





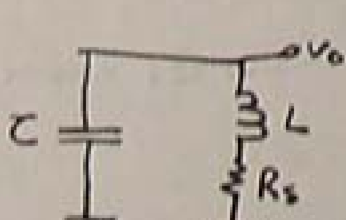
Oscil·lador sinusoidal amb R negativa:

Oscil·lador ideal



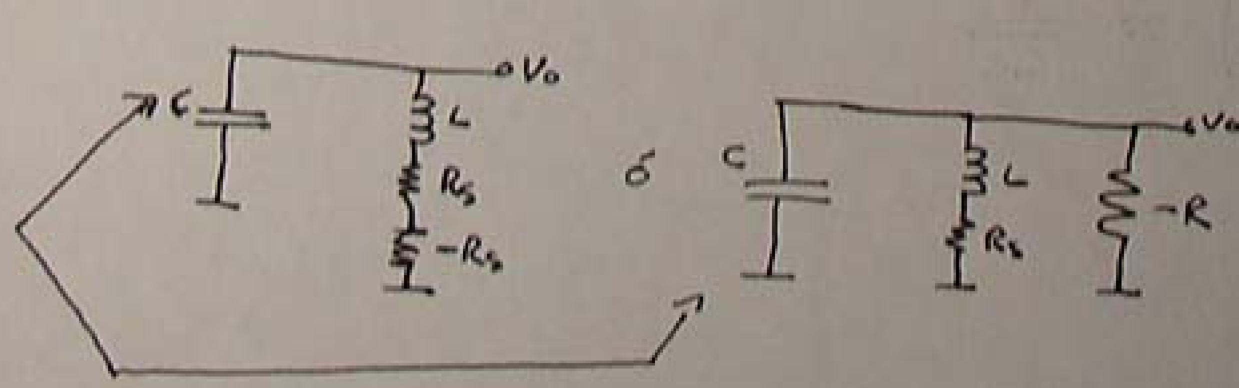
Si CI ≠ 0
Resposta lliure
es manté en el temps

Situació real

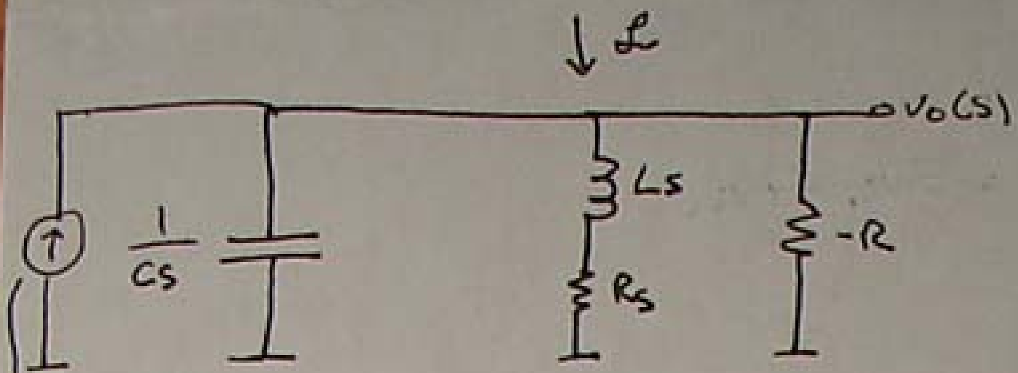


Resposta lliure que s'atenua amb el temps.

Solució



$\downarrow \mathcal{L}$



$C \cdot v_c(0^-) = I_c(s)$

$$H(s) = \frac{V_o(s)}{I_{cz}(s)}$$

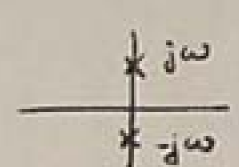
$$V_o(s) = I_c(s) \cdot \underbrace{Z_{ef}(s)}_{\text{Impedancia}}$$

$$V_o(s) = I_c(s) \frac{1}{Y_{ef}(s)} = \underbrace{\frac{1}{Cs + \frac{1}{Ls+R_s} - \frac{1}{R}}}_{H(s)} \cdot I_c(s)$$

$\uparrow$ Admitancia

Oscil·lador

$$H(s) = \frac{N(s)}{s^2 + \omega_0^2}$$



$$H(s) = \frac{R(Ls + R_s)}{RCLs^2 + RCR_s s + R - Ls - R_s} =$$

$$= \frac{R(Ls + R_s)}{RLCs^2 + s(RR_sC - L) + R - R_s} =$$

$$= \frac{\ldots \text{(El que sigui)} \ldots}{s^2 + \left(\frac{R_s}{L} - \frac{1}{RC}\right)s + \frac{1}{LC}\left(1 - \frac{R_s}{R}\right)}$$

Condició d'oscil·lació:

$$\frac{R_s}{L} - \frac{1}{RC} = 0 \qquad \boxed{R = \frac{L}{R_s C}}$$

$$\frac{1}{LC}\left(1 - \frac{R_s}{R}\right) > 0$$

Habitualment $R >> R_s$ → És cumpleix

Freq. d'oscil·lació:

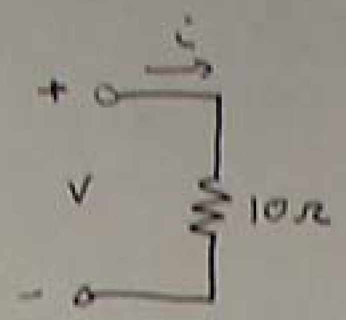
$$\omega_0 = \sqrt{\frac{1}{LC}\left(1 - \frac{R_s}{R}\right)} \overset{R>>R_s}{\approx} \frac{1}{\sqrt{LC}}$$

$\parallel$

$2\pi f$

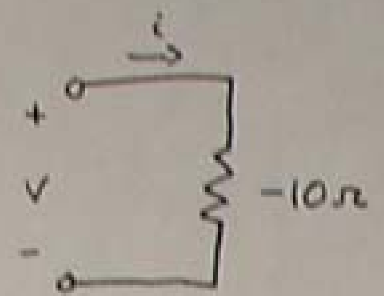
$$\frac{1}{LC} = \frac{R_s}{RLC}$$

Realització d'una resistència negativa



$v = 10i \Rightarrow$ si $v > 0 \Rightarrow i > 0$

$p = v \cdot i = 10i^2 \geq 0$

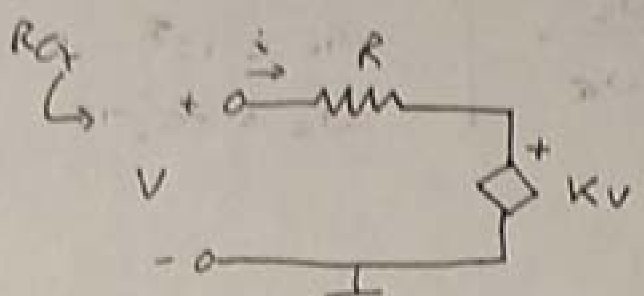


$v = -10i \Rightarrow$ si $v > 0 \Rightarrow i < 0$

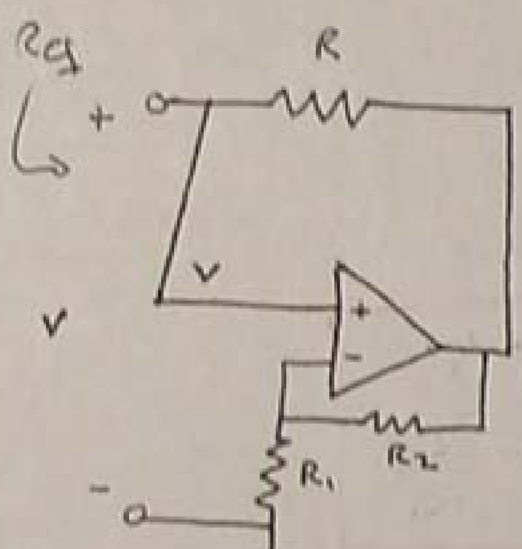
$p = vi = -10i^2 \leq 0$

↳ Lliura energia al circuit

Com realitzar l'últim cas:



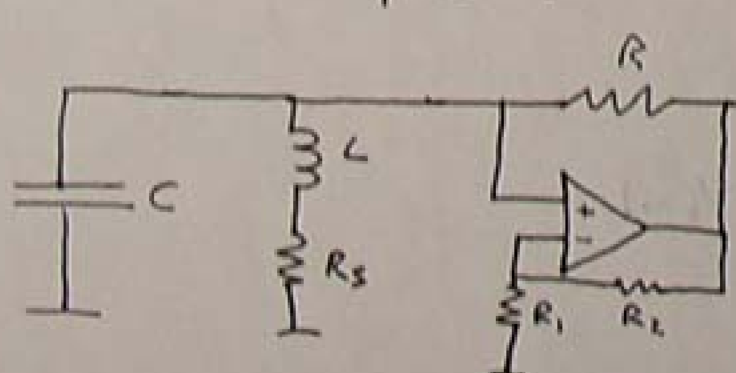
$$R_{eq} = \frac{v}{i} = \frac{v}{\frac{v - Kv}{R}} = \boxed{\frac{R}{1-K} \Rightarrow \text{Resistència Negativa} \Rightarrow K > 1}$$



$$\boxed{K = 1 + \frac{R_2}{R_1}}$$

$$R_{eq} = \frac{R}{-\frac{R_2}{R_1}} = \frac{-R \cdot R_1}{R_2} \overset{R_1 = R_2}{=} -R$$

Circuit complert:



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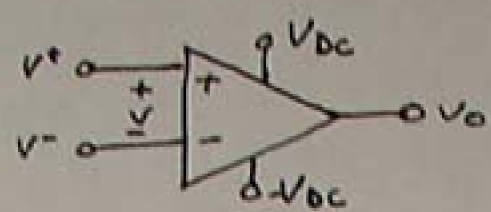
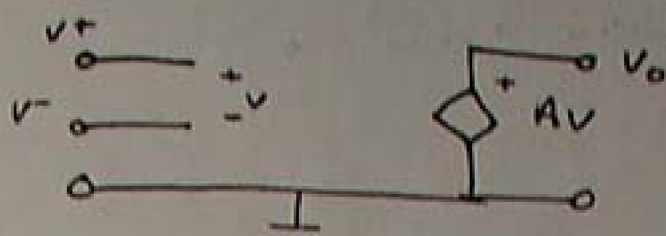
MODEL DINÀMIC DE L'AMPLIFICADOR OPERACIONAL:

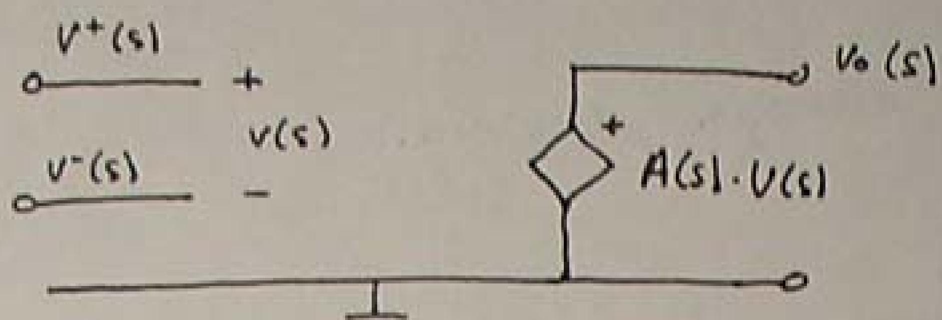
Model simplificat:



$\Longrightarrow$



Model + precís:



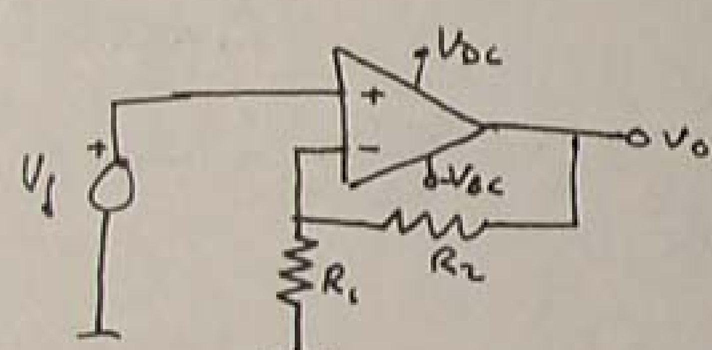
→ Domini transformat

↳ Dif. amb el de dalt. $A(s) \to$ variable

$$A(s) = \frac{A_o \omega_c}{s + \omega_c} \quad \Bigg\} \quad \begin{array}{l} \text{EN } \mu A741 \\ A_o \approx 10^5 \\ \omega_c = 80\,s^{-1} \end{array}$$

$A(s) \equiv$ Funció de xarxa de l'A.O

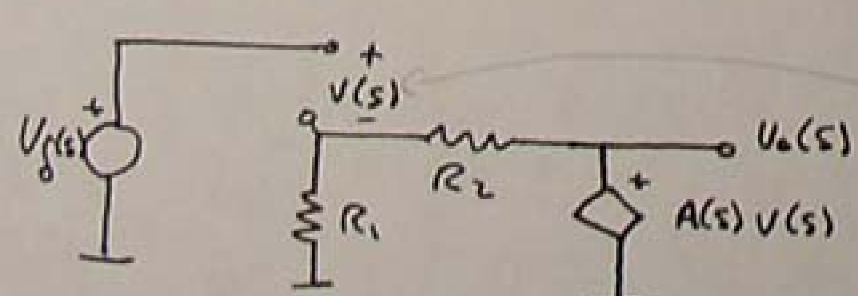
* Com afecta el nou model a l'AO NO INVERSOR:



→ Model simplificat →

$$\boxed{\begin{array}{l} V_o = K \cdot V_g \\ K = 1 + \dfrac{R_2}{R_1} \end{array}}$$

↓ Model dinàmic



$$V_o(s) = H(s) \cdot V_g(s)$$

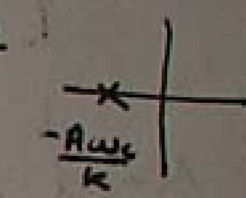
KVL: $V_g(s) = A(s) \cdot V(s) \cdot \underbrace{\frac{R_1}{R_1 + R_2}}_{\frac{1}{K}} + V(s)$

$$V(s) = \frac{1}{\frac{A(s)}{K} + 1} V_g(s)$$

$$V_o(s) = A(s)\,V(s) = \underbrace{\frac{A(s)}{\frac{A(s)}{K}+1}}_{H(s)} V_g(s)$$

$$H(s) = \frac{A(s)}{\frac{A(s)}{K}+1} = \frac{\frac{A_0 \omega_c}{s+\omega_c}}{\frac{A_0\omega_c}{K(s+\omega_c)}+1} = \frac{A_0\omega_c}{\frac{A_0\omega_c}{K}+s+\omega_c} =$$

$$= \frac{A_0\omega_c}{s+\omega_c\left(\frac{A_0}{K}+1\right)} \overset{\frac{A_0}{K}\gg 1}{\approx} \frac{A_0\omega_0}{s+\frac{A_0}{K}\omega_c} \longrightarrow \text{1r ordre estable}$$



Ex:

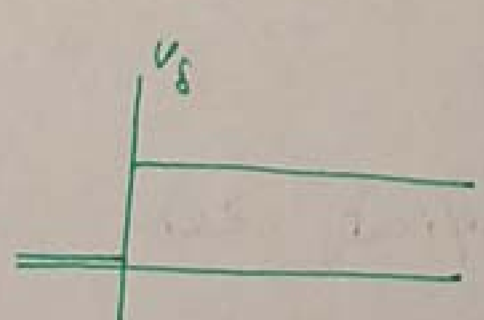
$V_g(t) = u(t)$ $\qquad R_1 = 1K\Omega$, $R_2 = 9K\Omega$ $\Rightarrow K = 10$

$$V_o(s) = H(s)\cdot V_g(s) = \frac{A_0\omega_c}{s+\frac{A_0}{K}\omega_c}\cdot\frac{1}{s} = \frac{K}{s} - \frac{K}{s+\frac{A_0}{K}\omega_c}$$

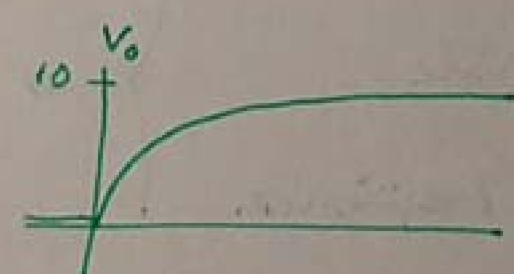
$$V_o(t) = K\cdot u(t) - Ke^{-t/\tau}\cdot u(t) = K\left(1-e^{-t/\tau}\right)u(t) =$$

$$\overset{\text{valors}}{=} 10\left(1-e^{-\frac{t}{1'2\cdot 10^{-6}}}\right)u(t)$$

$$\boxed{\tau = \frac{K}{A_0\omega_c}}$$



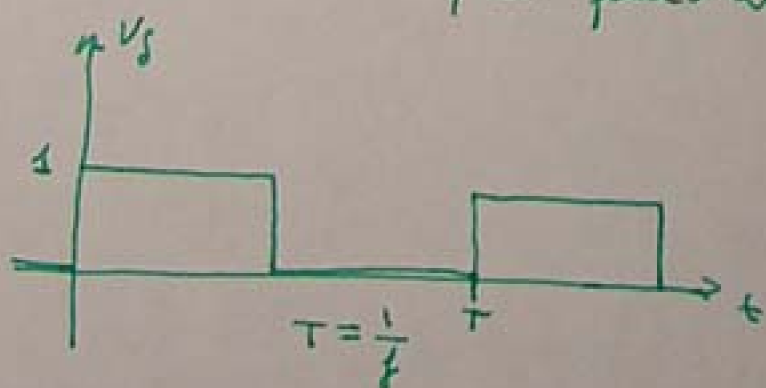
$\Longrightarrow$



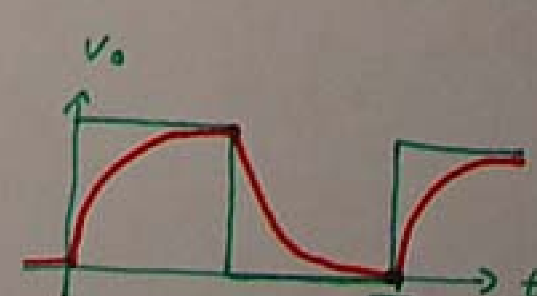
* Tenim un temps de resposta

* En regim permanent l'amplificació es manté

Ex: $V_g(t) \rightarrow$ Senyal quadrat



$\Longrightarrow$

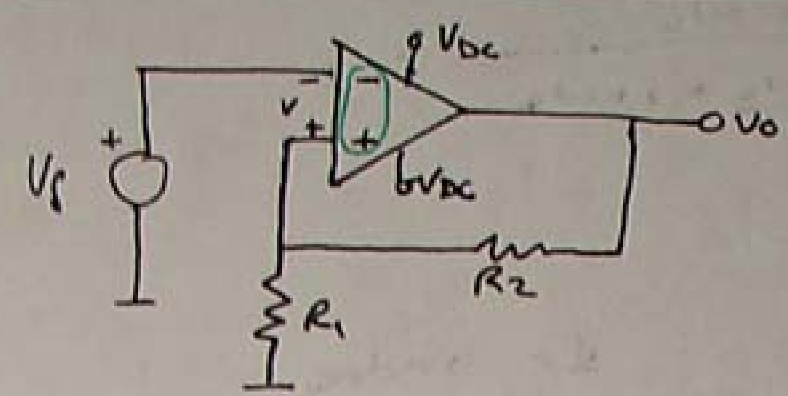


* Perque a V_o hi hagi una senyal quadrada $\Rightarrow 4\tau \ll \frac{T_{min}}{2} \rightarrow 4\tau = \frac{T_{min}}{2}:10$

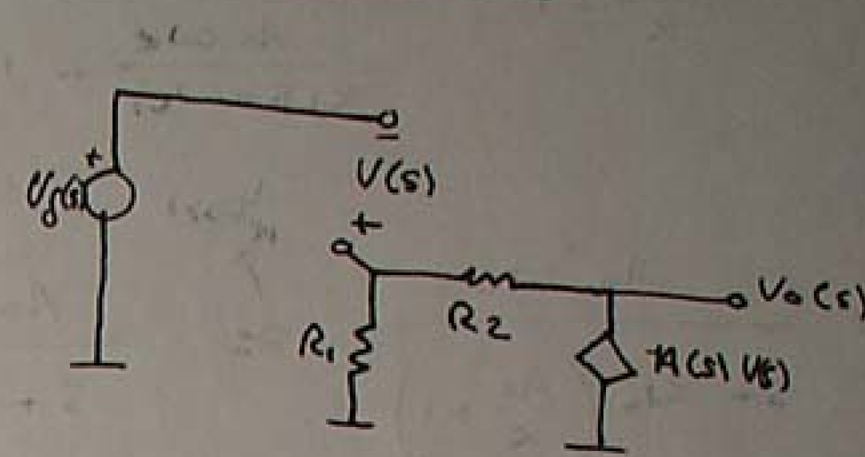
$$4\tau = \frac{1}{20\, f_{max}} \Rightarrow f_{max} = \frac{1}{80\tau} = 10'4\,\text{KHz} \Rightarrow \mu A741$$

$$\tau = 1'2\,\mu s$$

* Com afecta la connexió entrada - sortida?



Model $\xrightarrow{\text{Dinàmic}}$



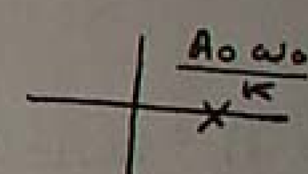
Model simplificat $\Rightarrow$ $\boxed{\begin{array}{l} V_o = K\cdot V_f \\ K = 1 + \frac{R_2}{R_1} \end{array}}$

$$H(s) = \frac{A_o\,\omega_c}{\frac{A_o}{K}\omega_c - s} = -\frac{A_o\,\omega_c}{s - \frac{A_o}{K}\omega_c} \Rightarrow \text{1r Ordre}$$

$\boxed{\text{Inestable}}$

$\Downarrow$

OPERA EN SATURACIÓ



Nota: Si intercanviem les entrades en qualsevol dels AO estudiats, aquests es tornen inestables i no funcionen com a tals.

CONCLUSIONS:

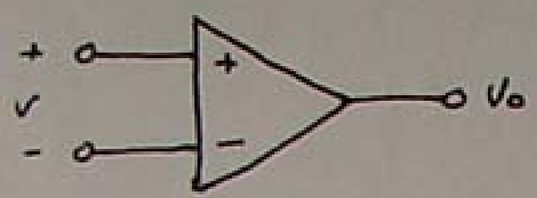
1) $\exists$ retroalimentació
2) Configuració és estable
3) Excitació de baixa freqüència

$\Rightarrow$ Model simplificat vàlid

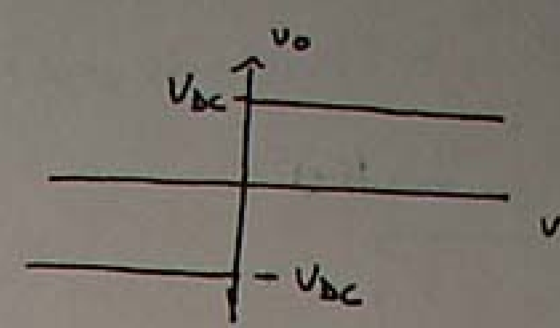
$\Downarrow$

Anàlisi amb CCV

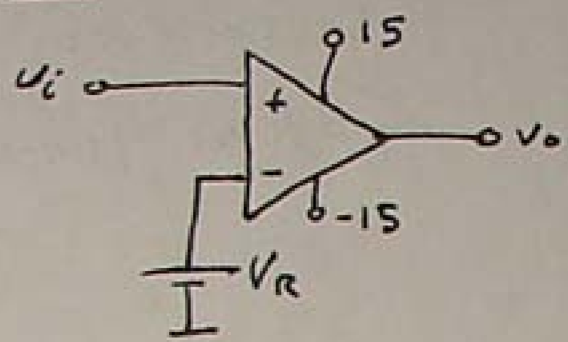
APLICACIÓ DE L'AO OPERANT EN ZONES DE SATURACIÓ:

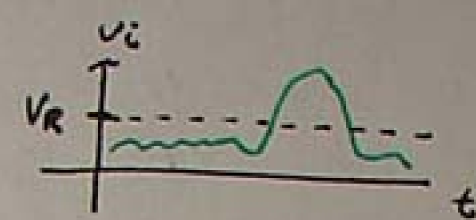


Aspecte Real de $v_o = f(v)$ ⟶

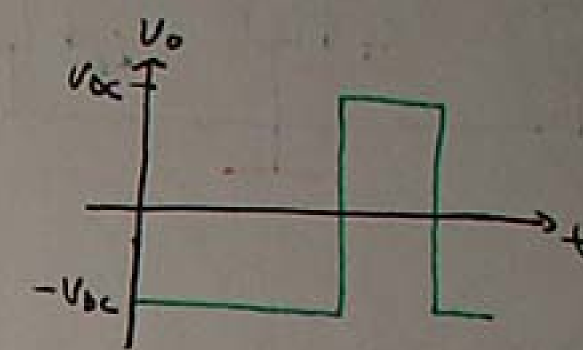


Ex: COMPARADOR

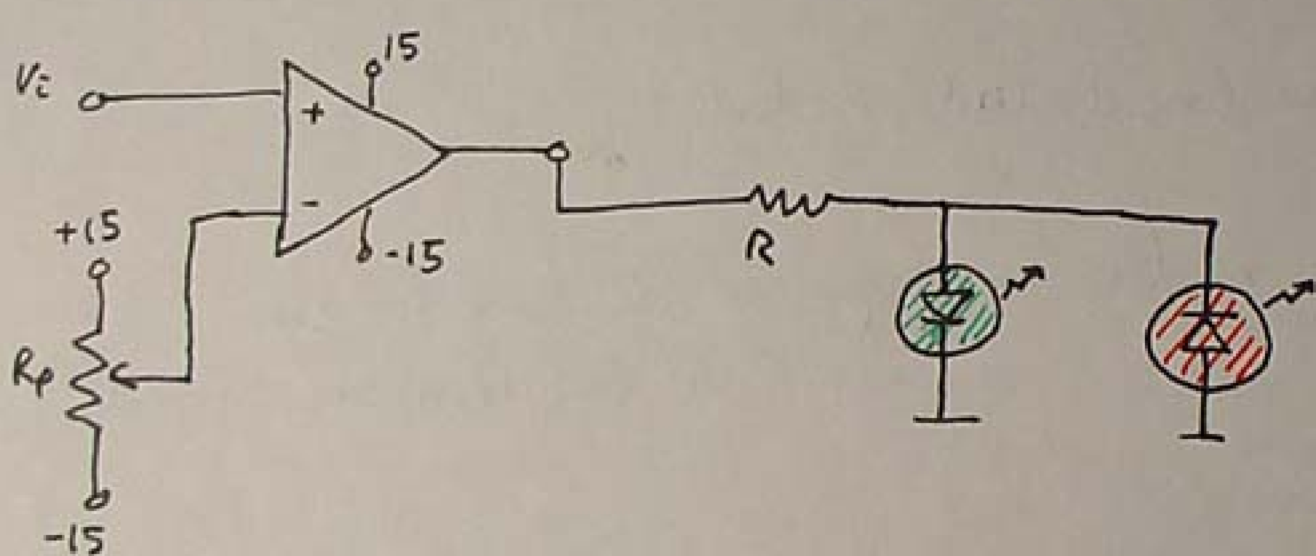




$\Rightarrow$



COMPARADOR AMB LLINDAR AJUSTABLE:



TEMA 5: ANÀLISI SISTEMA DE CIRCUITS DINÀMICS:

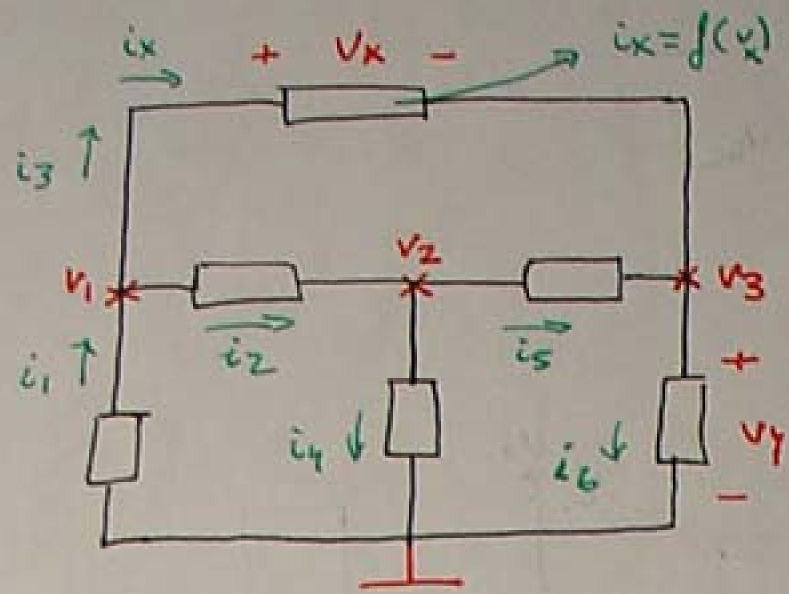
Objectius: Disposar de ~~sistemes~~ mètodes d'anàlisi $\underbrace{\text{generals}}$ i $\underbrace{\text{eficients}}$:

- generals → Aplicables a qualsevol circuit
- eficients → Mínim nombre d'incògnites ⇓ Base generadora

2 mètodes → TENSIONS NODALS
→ CORRENTS DE MALLA

MÈTODE DE LES TENSIONS NODALS:

Base generadora   Tensions nodals



- Comprovació que són base generadora.

a) Tensions:

$v_x = v_1 - v_3$

$v_y = v_3$

b) Corrents:

$i_x = f(v_x) = f(v_1 - v_3)$

Determinació:

N nodes $\rightarrow$ $n = N-1$ tensions nodals

n Equacions linealment independents

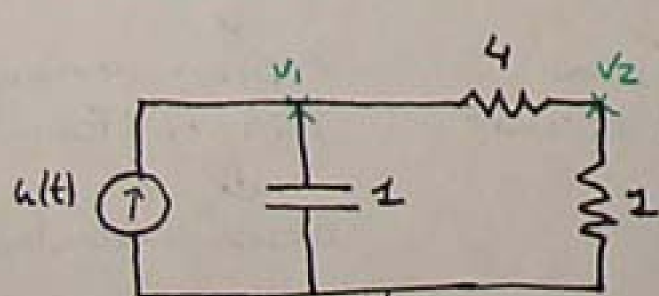
$\downarrow$

KCL en els n nodes $\Rightarrow$ Expressar els corrents en funció de les tensions nodals.

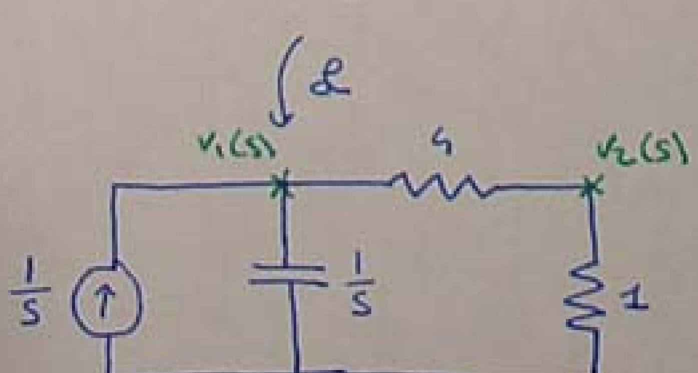
KCL

$$\left.\begin{array}{ll} ① & i_1 - i_2 - i_3 = 0 \\ ② & i_2 - i_4 - i_5 = 0 \\ ③ & i_5 + i_3 - i_6 = 0 \end{array}\right\} \Rightarrow \text{L Independents}$$

Exemple:



Determinar les tensions nodals v_1 i v_2



KCL

① $\dfrac{V_1(s)}{1/s} + \dfrac{V_1(s) - V_2(s)}{4} = \dfrac{1}{s}$

② $\dfrac{V_1(s) - V_2(s)}{4} = \dfrac{V_2(s)}{1}$

$$\begin{bmatrix} s+\frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{4} & 1+\frac{1}{4} \end{bmatrix} \begin{bmatrix} V_1(s) \\ V_2(s) \end{bmatrix} = \begin{bmatrix} \frac{1}{s} \\ 0 \end{bmatrix}$$

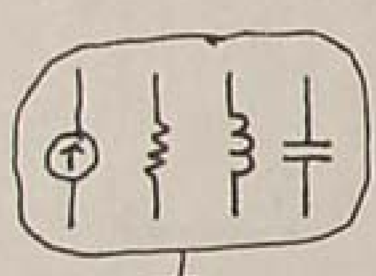
Kramer:

$$V_1(s) = \frac{\Delta_1}{\Delta} = \frac{\begin{bmatrix} \frac{1}{s} & -\frac{1}{4} \\ 0 & 1+\frac{1}{4} \end{bmatrix}}{\begin{vmatrix} s+\frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{5}{4} \end{vmatrix}} = \frac{\frac{1}{s}+\frac{1}{4s}}{\frac{5}{4}s+\frac{5}{16}-\frac{1}{16}} = \frac{\frac{5}{4}\cdot\frac{1}{s}}{\frac{5}{4}s+\frac{1}{4}} = \frac{\frac{5}{4}}{s\left(\frac{5}{4}s+\frac{1}{4}\right)} = \frac{1}{s\left(s+\frac{1}{5}\right)}$$

$$V_2(s) = \frac{\Delta_2}{\Delta} = \frac{\begin{bmatrix} s+\frac{1}{4} & \frac{1}{s} \\ -\frac{1}{4} & 0 \end{bmatrix}}{\frac{5}{4}s-\frac{1}{4}} = \frac{\frac{1}{4}\cdot\frac{1}{s}}{\frac{5}{4}s+\frac{1}{4}} = \frac{\frac{1}{4}}{s\left(\frac{5}{4}s+\frac{1}{4}\right)} = \frac{1}{s(5s+1)} = \frac{\frac{1}{5}}{s\left(s+\frac{1}{5}\right)}$$

$$V_1(s),\ V_2(s) \xrightarrow{\mathcal{L}^{-1}} V_1(t),\ V_2(t)$$

* Cas general:



$n = N-1$ tensions nodals

$KCL \rightarrow \quad Y(s)\cdot V(s) = I(s)$

↑ Admitància

Matriu d'admitàncies:

$$Y(s) = \begin{bmatrix} Y_{11}(s) & \cdots & Y_{1n}(s) \\ \vdots & & \vdots \\ Y_{n1}(s) & \cdots & Y_{nn}(s) \end{bmatrix}$$

$Y_{KK}(s) = \sum$ admitàncies connectades al node K.

$Y_{K\ell}(s) = -\sum$ admitàncies connectades als nodes K i ℓ. ($K \neq \ell$)

↳ "Entremig d'ells"

Vector de tensions:

$$V(s) = \begin{bmatrix} V_1(s) \\ \vdots \\ V_n(s) \end{bmatrix}$$

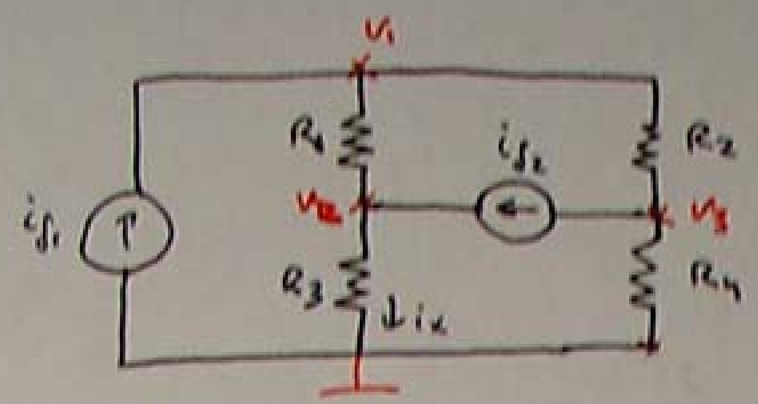
V_{Ks} = Tensió al node K

Vector de corrents:

$$I(s) = \begin{bmatrix} I_1(s) \\ \vdots \\ I_n(s) \end{bmatrix}$$

$I_K(s) = \sum$ de corrents entrants al node K procedents de generadors independents.

Exemple 2:



Determinar i_x:

$R_1 = R_2 = R_3 = R_4 = 1K\Omega$

$i_{g_2} = 2mA \quad i_{g_1} = 1mA$

$G_k = \frac{1}{R_k}$

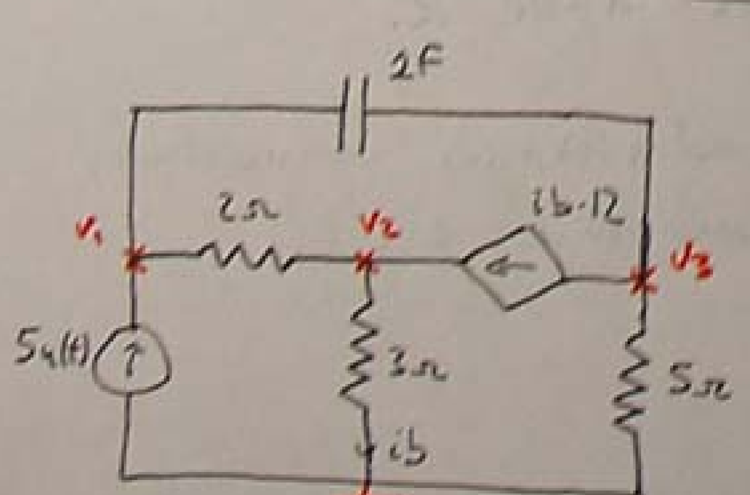
$$\begin{bmatrix} G_1+G_2 & -G_1 & -G_2 \\ -G_1 & G_1+G_3 & 0 \\ -G_2 & 0 & G_2+G_4 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} i_{g_1} \\ i_{g_2} \\ -i_{g_2} \end{bmatrix}$$

$$\cancel{10^{-3}} \cdot \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & 0 \\ -1 & 0 & 2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \cancel{10^{-3}} \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}$$

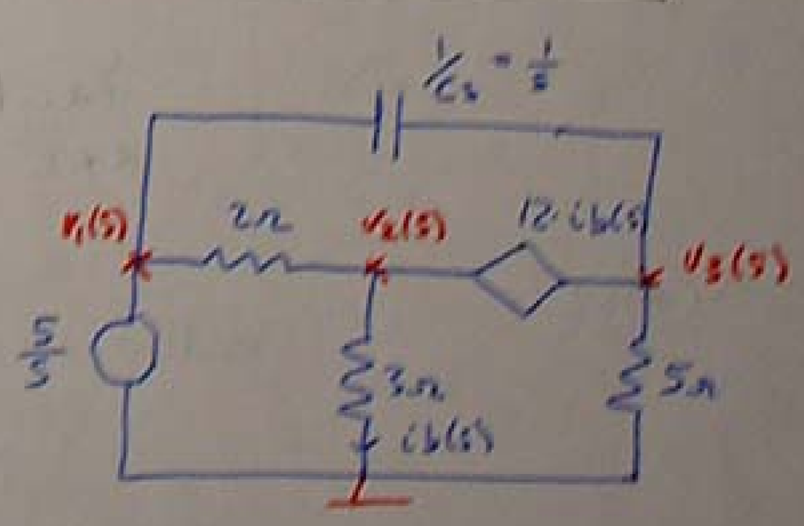
$$v_2 = \frac{\Delta_2}{\Delta} = \frac{\begin{vmatrix} 2 & 1 & -1 \\ -1 & 2 & 0 \\ -1 & -2 & 2 \end{vmatrix}}{\begin{vmatrix} 2 & -1 & -1 \\ -1 & 2 & 0 \\ -1 & 0 & 2 \end{vmatrix}} = \frac{6}{4} = \frac{3}{2} \qquad i_x = \frac{V_2}{R_3} = \frac{3/2}{1K} = 1'500\ mA$$

31-10-03

- Mètode nodal amb fonts de corrent controlades:



$\xrightarrow{\mathcal{L}}$

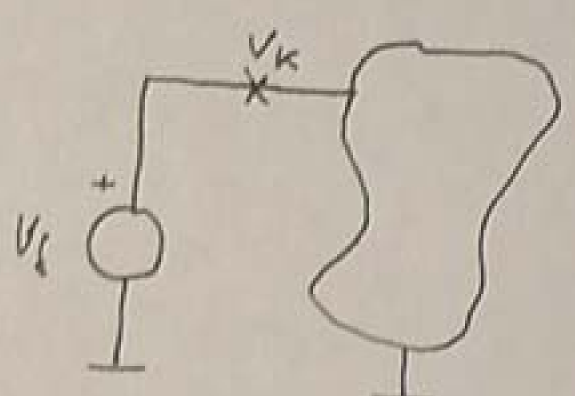


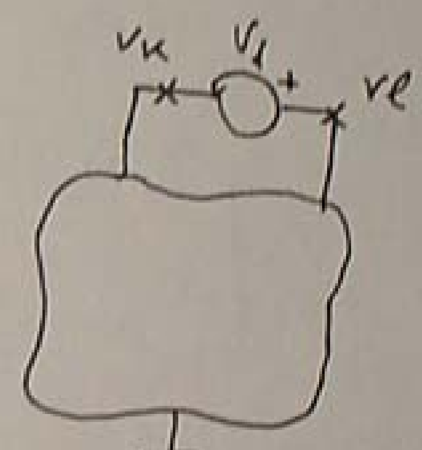
$$\begin{bmatrix} s+\frac{1}{2} & \frac{-1}{2} & -s \\ \frac{-1}{2} & \frac{1}{2}+\frac{1}{3} & 0 \\ -s & 0 & s+\frac{1}{5} \end{bmatrix} \begin{bmatrix} V_1(s) \\ V_2(s) \\ V_3(s) \end{bmatrix} = \begin{bmatrix} \frac{5}{s} \\ 12\, i_b(s) \\ -12\, i_b(s) \end{bmatrix} \overset{i_b = \frac{V_2}{3}}{=} \begin{bmatrix} \frac{5}{s} \\ 4V_2(s) \\ -4\cdot V_2(s) \end{bmatrix}$$

$$\Downarrow$$

$$\begin{bmatrix} s+\frac{1}{2} & \frac{-1}{5} & -s \\ \frac{-1}{2} & \frac{1}{2}+\frac{1}{3}-4 & 0 \\ -s & 4 & s+\frac{1}{5} \end{bmatrix} \begin{bmatrix} V_1(s) \\ V_2(s) \\ V_3(s) \end{bmatrix} = \begin{bmatrix} 5/s \\ 0 \\ 0 \end{bmatrix}$$

- Mètode nodal amb fonts de tensió:





$V_k = V_f$

$\Downarrow$

1 incògnita menys

1 KCL menys

$V_l = V_k + V_f$

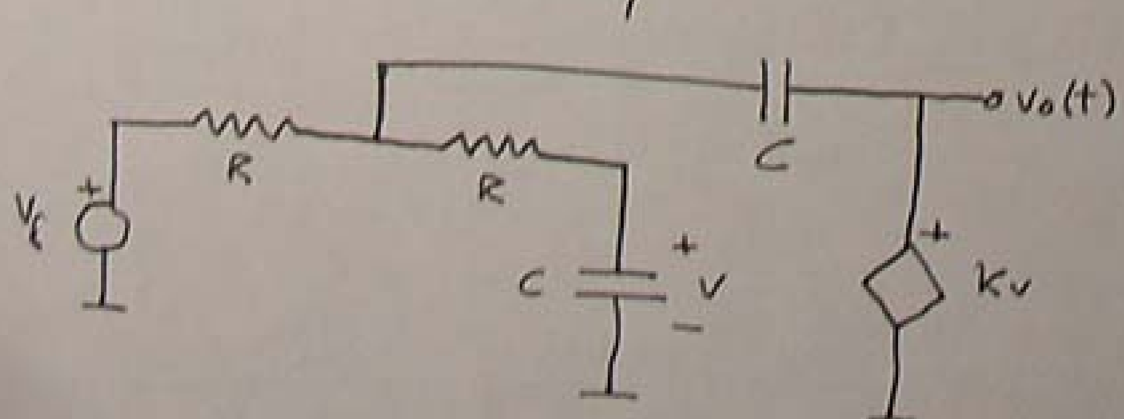
$\Downarrow$

1 incògnita menys

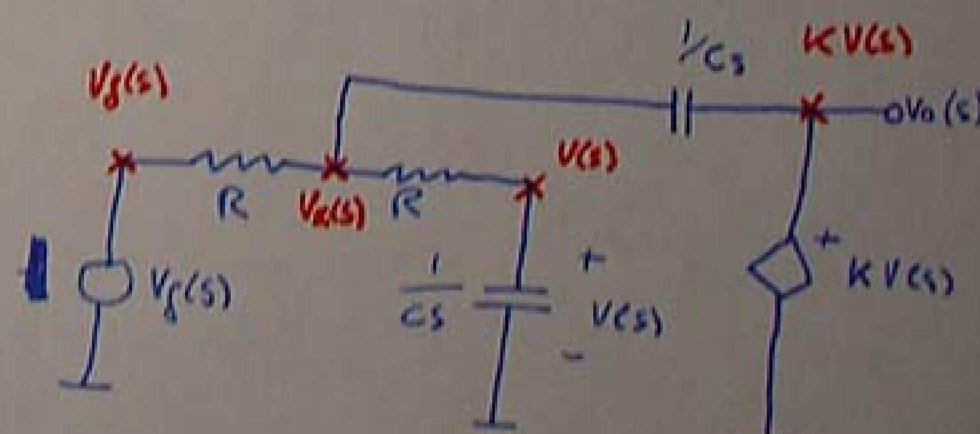
1 KCL menys

Ex:

Circuit de Saller-Key



$\overset{\mathcal{L}}{\Longrightarrow}$



* 4 tensions nodals → 2 incògnites ($V_x(s)$, $V(s)$)

KCL

$$V_x(s) \qquad \frac{V_g(s) - V_x(s)}{R} = \frac{+V_x(s) - V(s)}{R} + \frac{V_x(s) - KV(s)}{1/Cs}$$

$$V(s) \qquad \frac{V_x(s) - V(s)}{R} = \frac{V(s)}{1/Cs}$$

Forma alternativa KCL de forma sistemàtica

KCL

$$V_x(s) \qquad (2G + Cs)\,V_x(s) - G V_g(s) - G\,V(s) - Cs\,K\,V(s) = 0$$

$$V(s) \qquad -G\,V_x(s) + (G + Cs)\,V(s) = 0$$

SISTEMA:

$$\begin{bmatrix} 2G + Cs & -G - KCs \\ -G & G + Cs \end{bmatrix} \begin{bmatrix} V_x(s) \\ V(s) \end{bmatrix} = \begin{bmatrix} G \cdot V_g(s) \\ 0 \end{bmatrix}$$

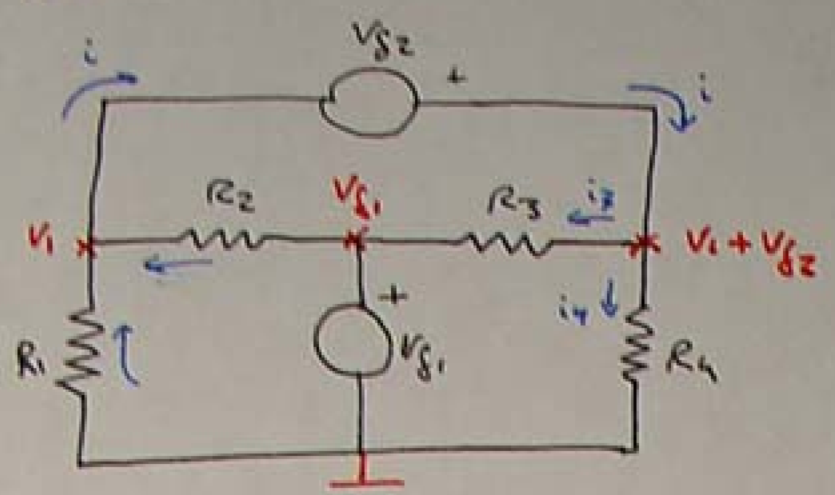
$$V(s) = \frac{\begin{vmatrix} 2G + Cs & G\,V_g(s) \\ -G & 0 \end{vmatrix}}{\begin{vmatrix} 2G + Cs & -G - KCs \\ -G & G + Cs \end{vmatrix}} = \frac{G^2 \cdot V_g(s)}{2G^2 + 2GCs + GCs + C^2s^2 - (G^2 + GKCs)} =$$

$$= \frac{G^2 \cdot V_g(s)}{C^2s^2 + CGs(3-K) + G^2} \overset{G = \frac{1}{R}}{=} \frac{\frac{1}{R^2} \cdot V_g(s)}{C^2s^2 + \frac{C}{R}s(3-K) + \frac{1}{R^2}} =$$

$$= \frac{\frac{1}{(RC)^2} \cdot V_g(s)}{s^2 + \frac{(3-K)}{RC}s + \frac{1}{C^2R^2}}$$

$$H(s) = \frac{V_o(s)}{V_g(s)} = \frac{K\,V(s)}{V_g(s)} = \frac{\frac{K}{(RC)^2}}{s^2 + \frac{3-K}{RC}s + \frac{1}{(RC)^2}}$$

Ex2:



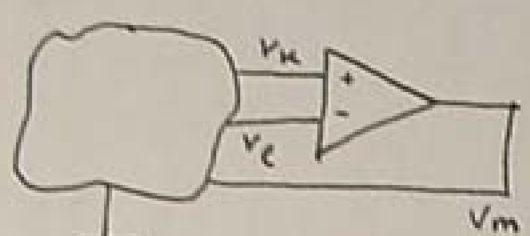
$\Rightarrow$ Per anar bé se sol aplicar KCL en els nodes de tensió desconeguts.

$$i = i_3 + i_4$$

V_1
$$\frac{0 - V_1}{R_1} + \frac{V_{g1} - V_1}{R_2} = \frac{V_1 + V_{g2} - V_{g1}}{R_3} + \frac{V_1 + V_{g2} - 0}{R_4} \Bigg\} \underline{\text{Aillar } V_1}$$

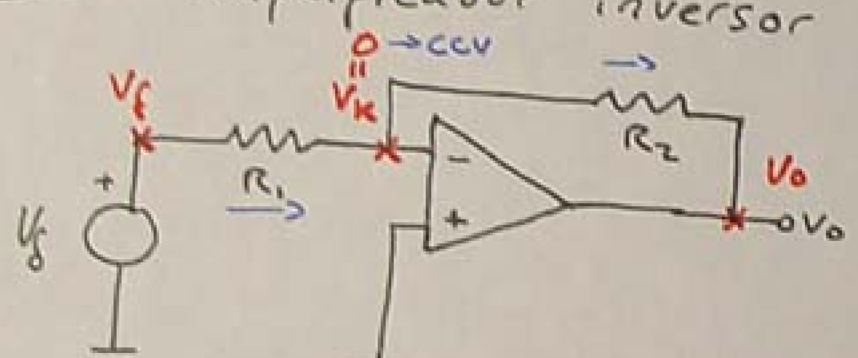
- Mètode nodal i A.O.:

(Zona lineal)



$\xRightarrow{\text{CCV}}$ $V_K = V_l \Rightarrow$ Només 2 incògnites

Ex: Amplificador inversor

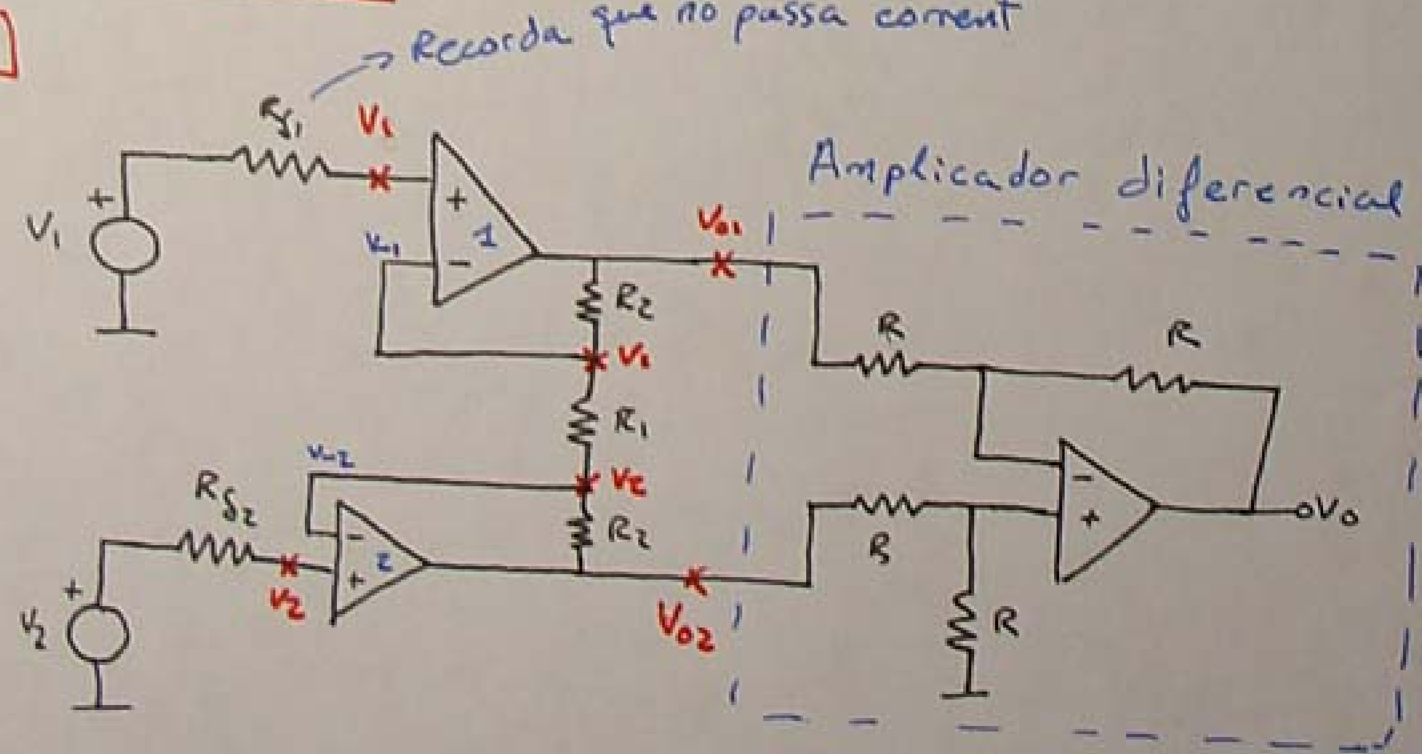


V_K
$$\frac{V_g - V_K}{R_1} = \frac{V_K - V_0}{R_2}$$

$$\frac{V_g}{R_1} = \frac{-V_0}{R_2} \Rightarrow \boxed{V_0 = -V_g \frac{R_2}{R_1}}$$

PROBLEMA T4

P11



$\boxed{V_0 = V_{02} - V_{01}}$

Incògnites V_{01} i V_{02}

KCL

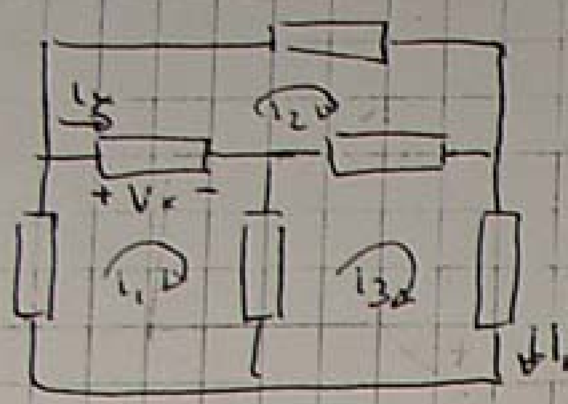
$V_{-1})$
$$\underbrace{\frac{V_{01} - V_1}{R_2} = \frac{V_1 - V_2}{R_1}}_{V_{01}} \quad \Bigg| \quad \underbrace{\frac{V_1 - V_2}{R_1} = \frac{V_2 - V_{02}}{R_2}}_{V_{02}}$$

$$V_{01} = V_1\left(\frac{R_2}{R_1} + R_1\right) - V_2\frac{R_2}{R_1}$$

$$V_{02} = V_1\left(\frac{-R_2}{R_1}\right) + V_2\left(\frac{R_2}{R_1} + R_1\right)$$

Metode dels corrents de malla.

Base generadora $\Rightarrow$ corrents de malla



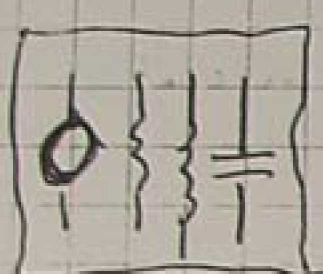
$i_x = I_1 - I_2$

$I_1 = I_3$

$V_x = f(I_x) = f(I_1 - I_2)$

Mètode aplicable només a circuits plans

Cas General



m mallas $\Rightarrow$ m corrents de mallas $\Rightarrow$

m ecuacions aplicant KVL

Expresar les tensions en funció dels corrents.

$Z(s) \cdot I(s) = V(s)$

Matriu de Impedancies

$$Z(s) = \begin{bmatrix} Z(s)_{11} & \cdots & Z(s)_{1m} \\ \vdots & & \vdots \\ Z(s)_{m1} & \cdots & Z(s)_{mm} \end{bmatrix}$$

$Z_{kk}(s)$ = Suma de impedancies que trobem al recorre la malla nº k

$Z_{kl}(s) = -\Sigma$ impedancies compartides entre les malles k y l
$k \neq l$

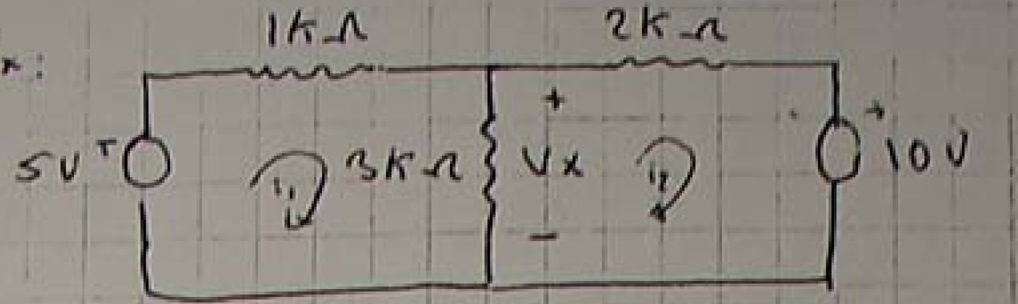
Vector de corrents $I(s) = \begin{bmatrix} I(s)_1 \\ \vdots \\ I(s)_n \end{bmatrix}$ ...

I_k = corrent de malla k

Vector de tensions $V(s)$ $\begin{bmatrix} V(s)_1 \\ \vdots \\ V(s)_m \end{bmatrix}$

$V_k = \Sigma$ tensions amb polaritat creixent provinents de generadors de tensió independents

Ex:



Calcula V_x pel mètode de malles

$$\begin{array}{cc} 1+3 & \\ \left[\begin{matrix} 4 & -3 \\ -3 & 5 \end{matrix}\right] \\ k\Omega & 2+3 \end{array} \begin{array}{c} \\ \left[\begin{matrix} i_1 \\ i_2 \end{matrix}\right] \\ mA \end{array} = \begin{array}{c} \\ \left[\begin{matrix} 5 \\ -10 \end{matrix}\right] \\ V \end{array}$$

$$i_1 = \frac{\begin{vmatrix} 5 & -3 \\ -10 & 5 \end{vmatrix}}{11} = \frac{-5}{11} = -0'45\ mA$$

$$i_2 = \frac{\begin{vmatrix} 4 & 5 \\ -3 & -10 \end{vmatrix}}{11} = \frac{-25}{11} = -2'27\ mA$$

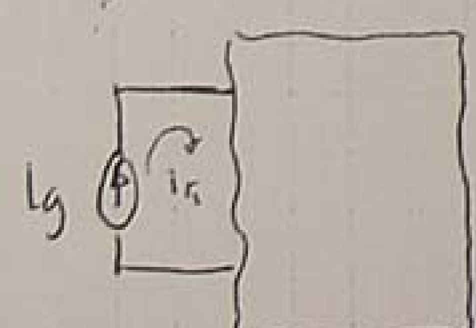
$$V_x = 3(i_1 - i_2) = 3(-0'45 + 2'27) = \boxed{5'45V}$$

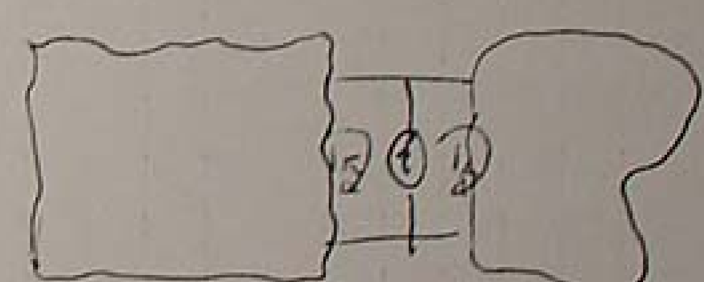
- Mètode de malles amb fonts de tensió controlades.

  Idem mètode de nodes

  Construir sist. equacions directament $\Longrightarrow$ Reagrupar variables (corrents de malles)

- Mètode de malles amb fonts de corrents

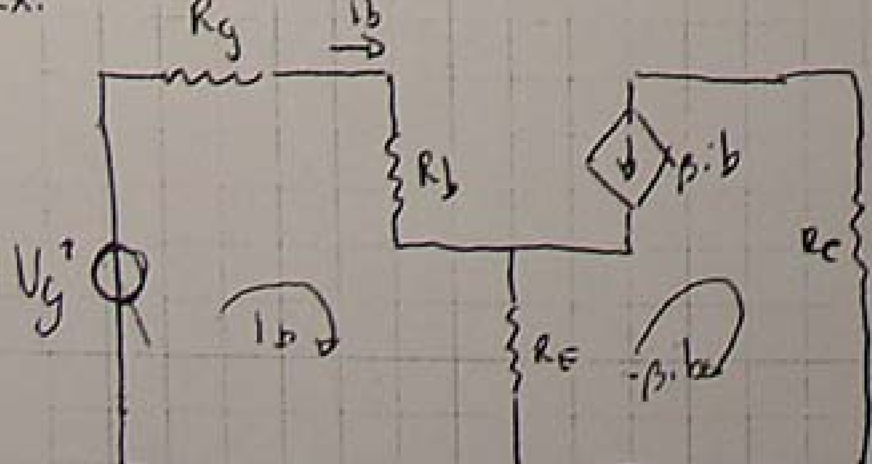




$I_e = -I_k + I_g$

$I_k = I_g \Rightarrow 1$ incognita meny $\Rightarrow 1$ KVL meny

Ex:



+ 2 malles

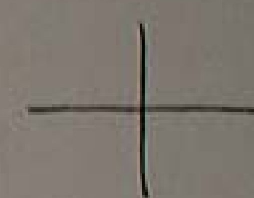
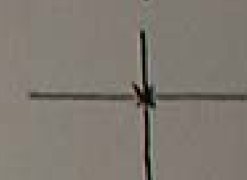
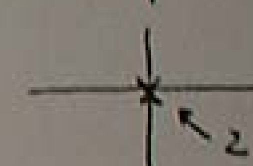
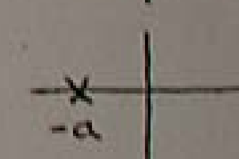
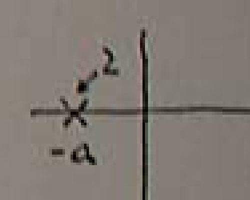
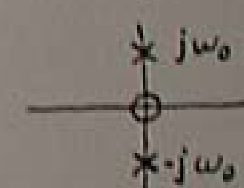
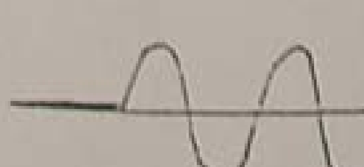
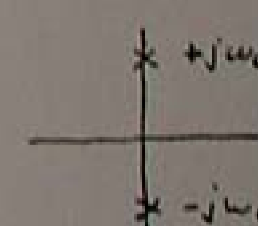
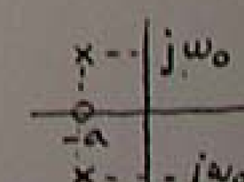
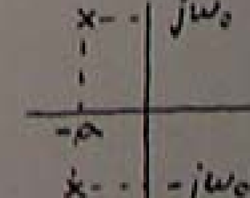
$V_0 = ?$ 1 incognita → 1 equació

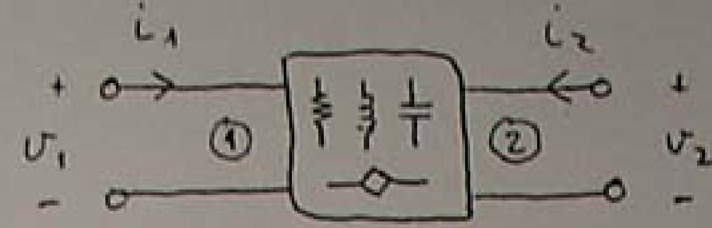
KVL $R_g \cdot i_b + R_b \cdot i_b + R_E(\beta+1) i_b = V_g \Rightarrow$

$\Rightarrow [R_g + R_b + R_E(\beta+1)] \cdot i_b = V_g$

$$i_b = \frac{V_g}{R_g + R_b + R_E(\beta+1)} \Longrightarrow \boxed{V_0 = i_b \cdot R_c}$$

TAULA DE TRANSFORMADES BÀSIQUES

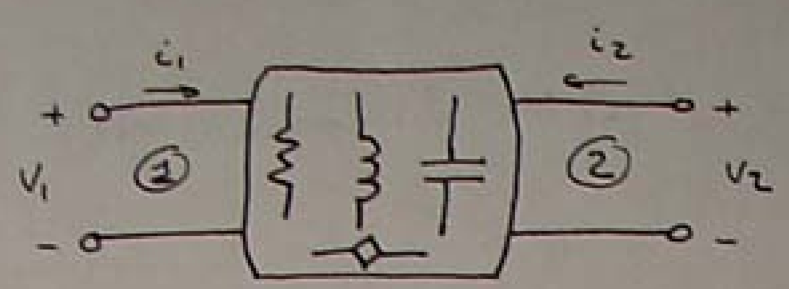
		$v(t)$	$V(s)$	
Delta de Dirac		$\delta(t)$	1	
esglaó unitari		$u(t)$	$\frac{1}{s}$	
Rampa		$t\,u(t)$	$\frac{1}{s^2}$	
exponencial		$e^{-at}u(t)$	$\frac{1}{s+a}$	
		$t\,e^{-at}u(t)$	$\frac{1}{(s+a)^2}$	
		$\cos\omega_0 t\;u(t)$	$\frac{s}{s^2+\omega_0^2}$	
		$\sin\omega_0 t\;u(t)$	$\frac{\omega_0}{s^2+\omega_0^2}$	
		$e^{-at}\cos\omega_0 t\;u(t)$	$\frac{s+a}{(s+a)^2+\omega_0^2}$	
		$e^{-at}\sin\omega_0 t\;u(t)$	$\frac{\omega_0}{(s+a)^2+\omega_0^2}$	



→ 6 maneres de caracteritzar un biport:

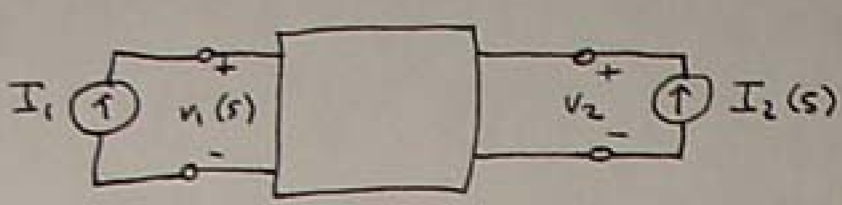
Paràmetres	Forma matricial	
IMPEDÀNCIA	$\begin{bmatrix} V_1(s) \\ V_2(s) \end{bmatrix} = \underline{\underline{Z}}(s) \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix}$	
ADMITÀNCIA	$\begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} = \underline{\underline{Y}}(s) \begin{bmatrix} V_1(s) \\ V_2(s) \end{bmatrix}$	$\underline{\underline{Y}}(s) = \underline{\underline{Z}}^{-1}(s)$
HÍBRIDS - 1	$\begin{bmatrix} V_1(s) \\ I_2(s) \end{bmatrix} = \underline{\underline{H}}(s) \begin{bmatrix} I_1(s) \\ V_2(s) \end{bmatrix}$	
HÍBRIDS - 2	$\begin{bmatrix} I_1(s) \\ V_2(s) \end{bmatrix} = \underline{\underline{H}}'(s) \begin{bmatrix} V_1(s) \\ I_2(s) \end{bmatrix}$	$\underline{\underline{H}}'(s) = \underline{\underline{H}}^{-1}(s)$
TRANSMISSIÓ - 1	$\begin{bmatrix} V_1(s) \\ I_1(s) \end{bmatrix} = \underline{\underline{T}}(s) \begin{bmatrix} V_2(s) \\ -I_2(s) \end{bmatrix}$	
TRANSMISSIÓ - 2	$\begin{bmatrix} V_2(s) \\ -I_2(s) \end{bmatrix} = \underline{\underline{T}}'(s) \begin{bmatrix} V_1(s) \\ I_1(s) \end{bmatrix}$	$\underline{\underline{T}}'(s) = \underline{\underline{T}}^{-1}(s)$

BIPORTS:



4 terminals, 2 ports.

$\downarrow \mathcal{L}$



Superposició:

$$V_1(s) = Z_{11}(s) \cdot I_1(s) + Z_{12}(s) I_2(s)$$

$$V_2(s) = Z_{21}(s) \cdot I_2(s) + Z_{22}(s) \cdot I_2(s)$$

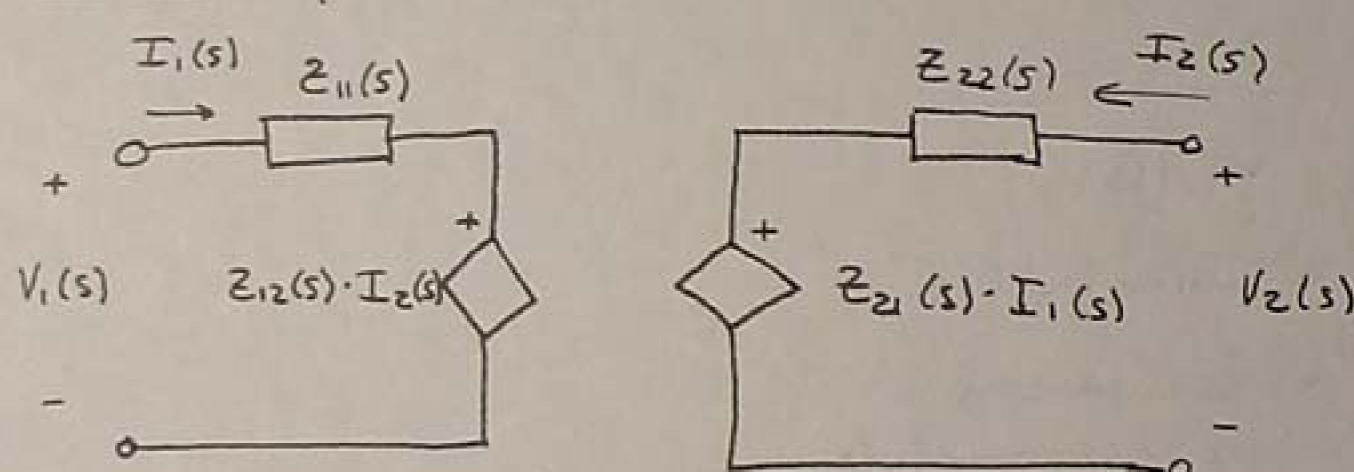
$$\begin{bmatrix} V_1(s) \\ V_2(s) \end{bmatrix} = \begin{bmatrix} Z_{11}(s) & Z_{12}(s) \\ Z_{21}(s) & Z_{22}(s) \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix}$$

$$V(s) = \underline{\underline{Z}}(s) \cdot I(s)$$

$\downarrow$

Matriu de pràmetres Z (impedancia)

Model equivalent:

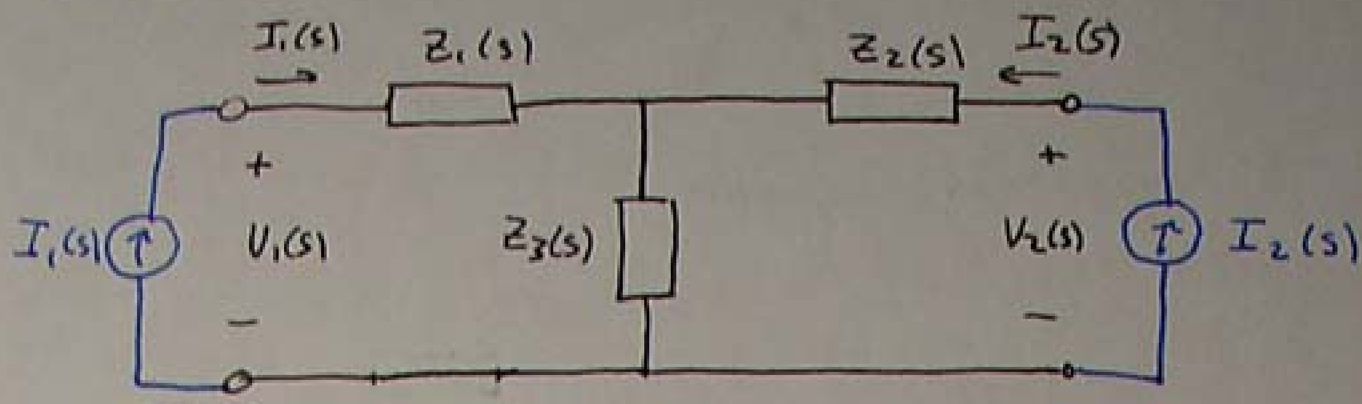


Interpretació física dels parametres Z:

$$Z_{11}(s) = \left. \frac{V_1(s)}{I_1(s)} \right|_{I_2(s)=0} \qquad Z_{22}(s) = \left. \frac{V_2(s)}{I_2(s)} \right|_{I_1(s)=0}$$

$$Z_{12}(s) = \left. \frac{V_1(s)}{I_2(s)} \right|_{I_1(s)=0} \qquad Z_{21}(s) = \left. \frac{V_2(s)}{I_1(s)} \right|_{I_2(s)=0}$$

Ex:



Matriu d'impedàncies).

$$V_1(s) = I_1(s)\cdot Z_1(s) + Z_3(s)\big(I_1(s) + I_2(s)\big)$$

$$V_2(s) = I_2(s)\cdot Z_2(s) + Z_3(s)\big(I_1(s) + I_2(s)\big)$$

$$\begin{bmatrix} V_1(s) \\ V_2(s) \end{bmatrix} = \begin{bmatrix} Z_1(s) + Z_3(s) & Z_3(s) \\ Z_3(s) & Z_2(s) + Z_3(s) \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix}$$

EXÀMEN DEL 5-11-03 :

$\boxed{1}$

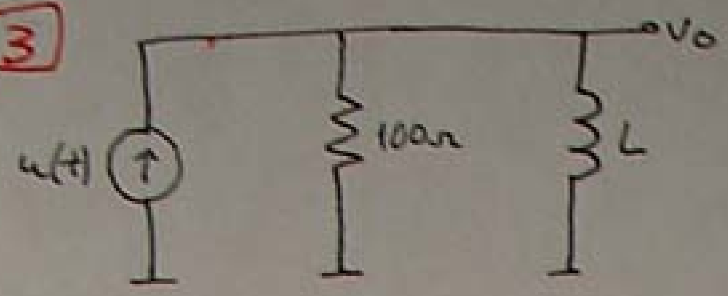
Resistius	Dinàmics
* $H(s) = $ Ktant	* $H(s)$
* Estables	* Inestables
* Resposta. combinació lineal de l'excitació	* En general no es compleix
* Sistema d'eq. algebraics	* Sistema d'equacions integrodiferencials.
* Resp. lliure	* Resp. Lliure + Forçada

$\boxed{2}$

$$\mathcal{L}\{10\cdot\cos 4t\cdot u(t)\} = \frac{10s}{s^2+16}$$

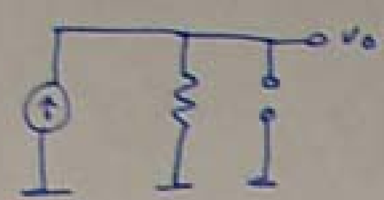
$$N(s) = K(s^2+16)$$

3



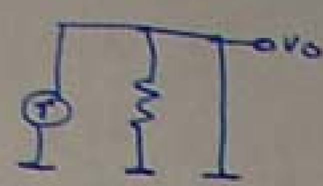
$L = ?$ $\quad v_o(L) \geq 50$ a 10ms

$t=0$



$v_o = 100V$

$t=\infty$

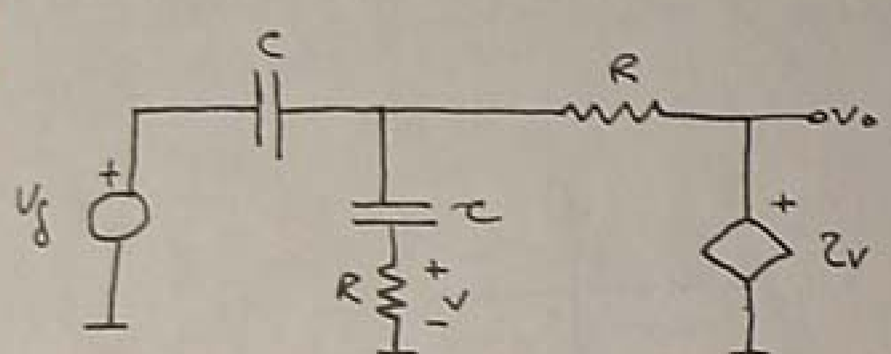


$v_o = 0V$

$$v_o(t) = 100\, e^{-\frac{Rt}{L}} \cdot u(t) = 100 \cdot e^{-\frac{100t}{L}} \cdot u(t)$$

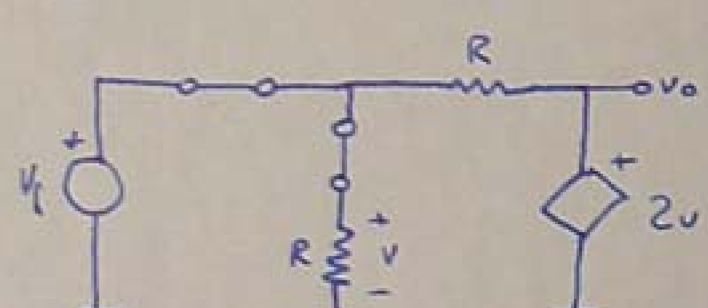
$$\boxed{L = 1'44\, H}$$

4



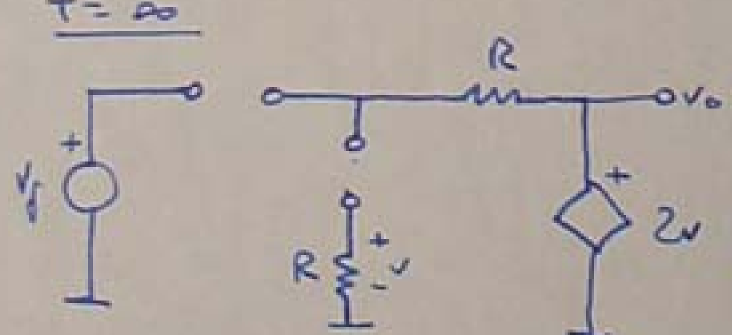
$v_g = u(t)$

$t=0$



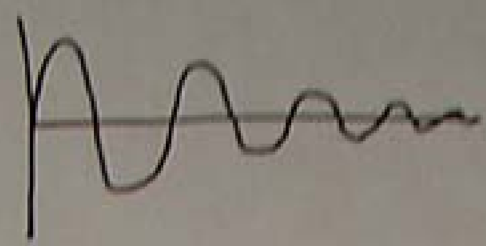
$$\boxed{v_o = 2v_g = 2}$$

$t=\infty$



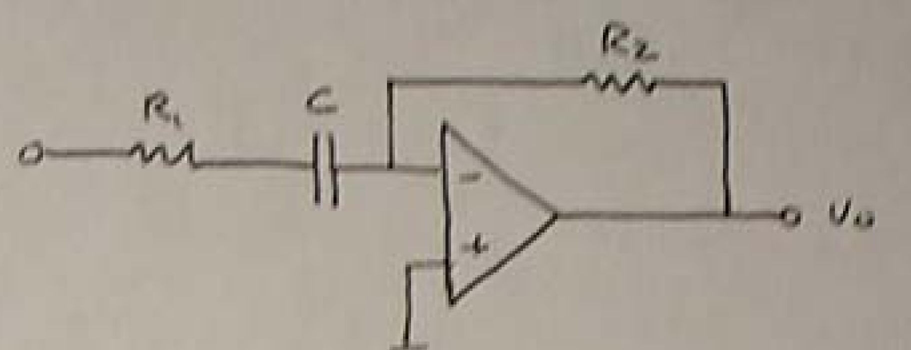
$$\boxed{v_o = 0V}$$

5



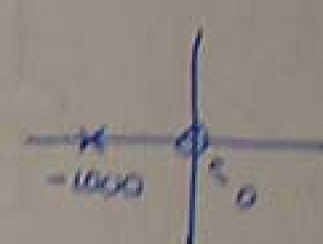
- 2^n ordre, pols complexes, subesmorteït
- Estable
- $H(0) = 0 \Rightarrow s = N(s)$
- $H(\infty) = 0 \Rightarrow D(s)$ comença amb s^2

6



Amplificador inversor:

$$H(s) = -\frac{R_2}{R_1 + \frac{1}{Cs}} = -\frac{\frac{R_2}{R_1}s}{s + \frac{1}{R_1 C}}$$



TEMA 6:

RÈGIM PERMANENT SINUSOÏDAL:

$$v_i(t) = A\cos\omega t \longrightarrow \boxed{\begin{matrix} H(s) \\ \text{Estable} \end{matrix}} \longrightarrow v_o(t) = ?$$

RPS
- Sistemes d'energia
- Sistemes de comunicació

$$v_o(s) = H(s)\cdot v_i(s) = H(s)\cdot\frac{As}{s^2+\omega^2}$$

$$v_o(t) = \mathcal{L}^{-1}\{H(s)\cdot v_i(s)\} = \mathcal{L}^{-1}\Big\{\underbrace{\frac{K_1}{s-p_1}+\dots+\frac{K_n}{s-p_n}}_{\substack{\text{Lliure} \\ \shortparallel \\ 0 \to RPS}} + \underbrace{\frac{K}{s-j\omega}+\frac{K^*}{s+j\omega}}_{\text{Forçada}}\Big\}$$

* La part lliure s'esvaeix, pK estem a RPS ($t=\infty$)

$$K = \Big[H(s)\cdot V_i(s)\cdot(s-j\omega)\Big]_{s=j\omega} = \Big[H(s)\cdot\frac{As}{s^2+\omega^2}(s-j\omega)\Big]_{s=j\omega} =$$

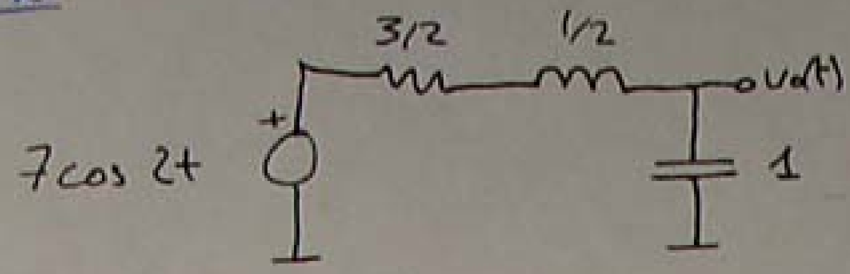
$$= \Big[\frac{H(s)\cdot As}{s+j\omega}\Big]_{s=j\omega} = \frac{H(j\omega)\cdot Aj\omega}{2j\omega} = \frac{A}{2}H(j\omega) = \frac{A}{2}|H(j\omega)|\cdot e^{j\arg(H(j\omega))}$$

$$v_o(t) = \mathcal{L}^{-1}\Big\{\frac{K}{s-j\omega}+\frac{K^*}{s+j\omega}\Big\} = 2\cdot\frac{A}{2}\cdot|H(j\omega)|\,e^{0t}\cdot\cos(\omega t + \arg(H(j\omega)))$$

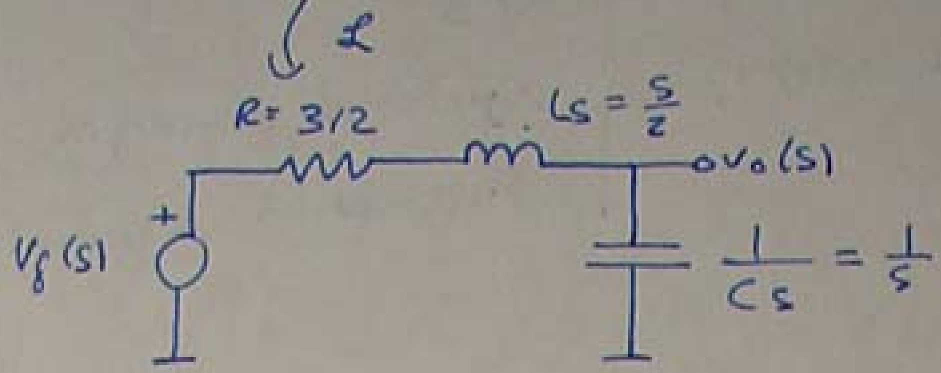
$$\boxed{v_o(t) = A\cdot|H(j\omega)|\cdot\cos(\omega t + \arg H(j\omega))}$$

- Mateixa freq. que $v_i(t)$
- Sinusoidal
- Canvia amplitud i fase

Ex:



$\downarrow \mathcal{L}$



1) $$H(s) = \frac{V_o(s)}{V_g(s)} = \frac{\frac{1}{s}}{\frac{3}{2}+\frac{s}{2}+\frac{1}{s}} = \frac{2}{s^2+3s+2}$$

2) $|H(j\omega)|$, $\arg(H(j\omega))$

$\omega = 2$

$$H(2j) = \frac{2}{(2j)^2+3(2j)+2} = \frac{2}{-4+6j+2} = \frac{2}{-2+6j} = \frac{1}{-1+3j} =$$

$$= \frac{1}{\sqrt{10}}\, e^{j\,\mathrm{arctg}(-3)}$$

$$v_o(t) = \frac{7}{\sqrt{10}} \cos\left(2t - \mathrm{arctg}(-3)\right)$$

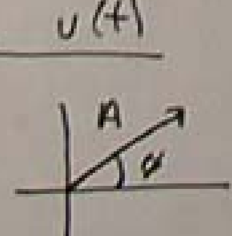
REPRESENTACIÓ FASORIAL:

$v(t) = A\cos(\omega t + \phi)$

$$v(t) = A\cos(\omega t+\phi) = \mathrm{Re}\left[A e^{j(\omega t+\phi)}\right] = \mathrm{Re}\Big[\underbrace{Ae^{j\phi}}_{\vec{V}}\cdot e^{j\omega t}\Big] = \mathrm{Re}\left[\vec{V}e^{j\omega t}\right]$$

Fasor associat a $v(t)$

$\bar{V} \triangleq A e^{j\phi}$



1) Representació "compacta"

2) Suma de senyals sinusoïdals

$$v_s(t) = v_1(t) + v_2(t) = A_1 \cos(\omega t + \phi_1) + A_2 \cos(\omega t + \phi_2) = \circledast$$

$$\circledast = \mathrm{Re}\Big[\underbrace{A_1 e^{j\phi_1}}_{\overline{V}_1} \cdot e^{j\omega t}\Big] + \mathrm{Re}\Big[\underbrace{A_2 e^{j\phi_2}}_{\overline{V}_2} \cdot e^{j\omega t}\Big] = \mathrm{Re}\Big[\underbrace{(\overline{V}_1 + \overline{V}_2)}_{\overline{V}_s} e^{j\omega t}\Big]$$

$$\boxed{\overline{V}_s = \overline{V}_1 + \overline{V}_2} \qquad \boxed{v_s(t) = |\overline{V}_s| \cdot \cos(\omega t + \arg \overline{V}_s)}$$

3)

$$v_i(t) = A\cos(\omega t + \phi) \longrightarrow \boxed{\begin{matrix} H(s) \\ \text{ESTABLE} \end{matrix}} \longrightarrow v_o(t) = A|H(j\omega)| \cdot \cos(\omega t + \phi + \mathrm{Arg}(H(j\omega)))$$

$$\overline{V}_i = A e^{j\phi}$$

$$\overline{V}_o = e^{j\phi} A |H(j\omega)| \cdot e^{j \arg H(j\omega)}$$

$$\boxed{\overline{V}_o = \overline{V}_i \cdot H(j\omega)}$$

$V_o(s) = H(s) \cdot V_g(s)$
SIMILITUT

CIRCUIT TRANSFORMAT FASORIAL:

CONNEXIONS:

KVL

$$\sum_{\circlearrowleft} v_k(t) = 0 \overset{RPS}{\Longrightarrow} \sum_{\circlearrowleft} A_k \cdot \cos(\omega t + \phi_k) = \mathrm{Re}\Big[\big(\sum_{\circlearrowleft} A_k e^{j\phi_k}\big) e^{j\omega t}\Big] =$$

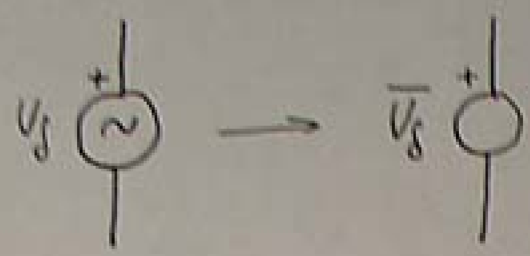
$$= \mathrm{Re}\Big[\big(\sum_{\circlearrowleft} \overline{V}_k\big) e^{j\omega t}\Big] = 0 \Rightarrow \boxed{\sum_{\circlearrowleft} \overline{V}_k = 0}$$

Fasors compleixen Kirtchoff

KCL

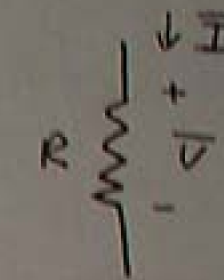
$$\boxed{\sum_{\circlearrowleft} \overline{I}_k = 0}$$

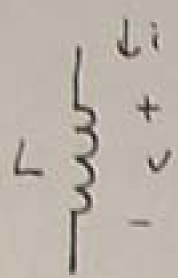
ELEMENTS:



El mateix per:
- Fonts de corrent
- Fonts controlades

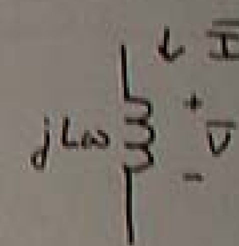
$$v = iR \xrightarrow{\mathcal{L}} V(s) = \underbrace{R}_{H(s)} \cdot I(s) \xrightarrow{RPS} \boxed{\overline{V} = R \cdot \overline{I}}$$





$$v = L\frac{di}{dt} \xrightarrow{\mathcal{L}} V(s) = \underbrace{Ls}_{H(s)} \cdot I(s) \xrightarrow{RPS} \boxed{\overline{V} = jL\omega \cdot \overline{I}}$$

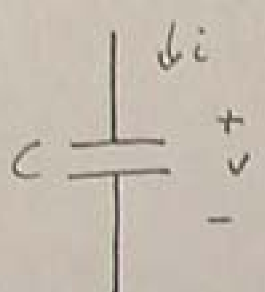
$$\boxed{Z = \frac{\overline{V}}{\overline{I}} = jL\omega}$$



Ex:

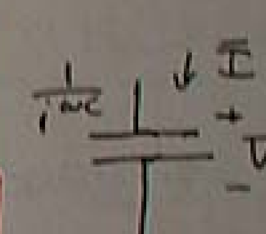
$$\overline{I} = Ae^{j\phi} \Rightarrow \overline{V} = j\omega L \cdot A \cdot e^{j\phi} = AL\omega \cdot e^{j\frac{\pi}{2}} \cdot e^{j\phi} = AL\omega \cdot e^{j(\phi + \frac{\pi}{2})}$$

* La impedància creix amb ω
  - $\omega = 0 \rightarrow Z = 0$ — curt circuit
  - $\omega = \infty \rightarrow Z = \infty$ — circuit obert
* Adelanta la tensió $\frac{\pi}{2}$ respecte del corrent



$$v = \frac{1}{C}\int i(t)\,dt \xrightarrow{\mathcal{L}} V(s) = \frac{I(s)}{Cs} \xrightarrow{RPS} \boxed{\overline{V} = \frac{1}{jC\omega} \cdot \overline{I}}$$

$$\boxed{Z = \frac{\overline{V}}{\overline{I}} = \frac{1}{j\omega C}}$$



Ex:

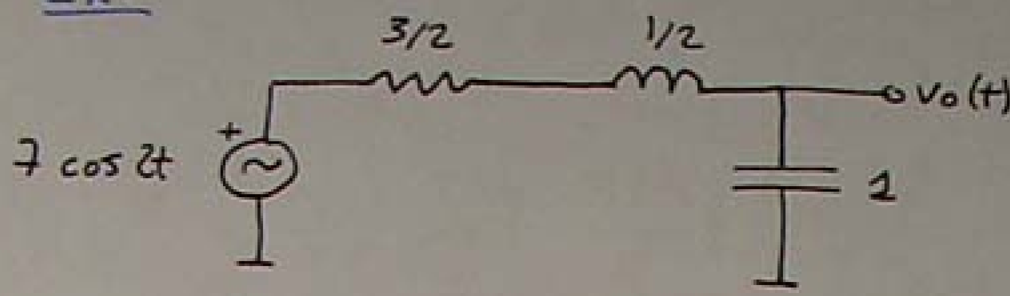
$$\overline{I} = Ae^{j\phi} \Rightarrow \overline{V} = \frac{1}{j\omega C}Ae^{j\phi} = \frac{A}{\omega C}e^{-j\frac{\pi}{2}} \cdot e^{j\phi} = \frac{A}{\omega C}e^{j(\phi - \frac{\pi}{2})}$$

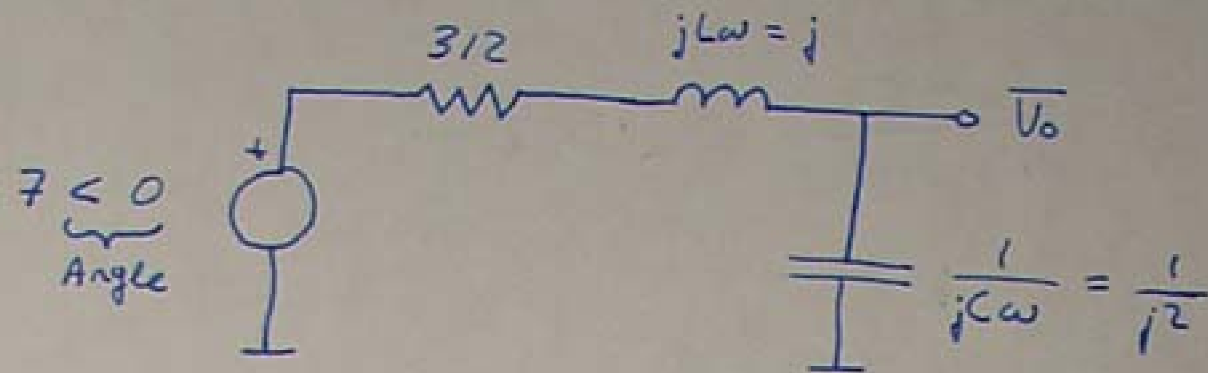
* La impedancia disminueix amb ω
  - $\omega = 0 \rightarrow Z = \infty$ — circuit obert
  - $\omega = \infty \rightarrow Z = 0$ — curt circuit
* Retarda la tensió $\frac{\pi}{2}$ respecte del corrent

Ex: $C = 10\,pF$

Z	f
$16\,M\Omega$	1kHz
$16\,\Omega$	1 GHz

Ex:





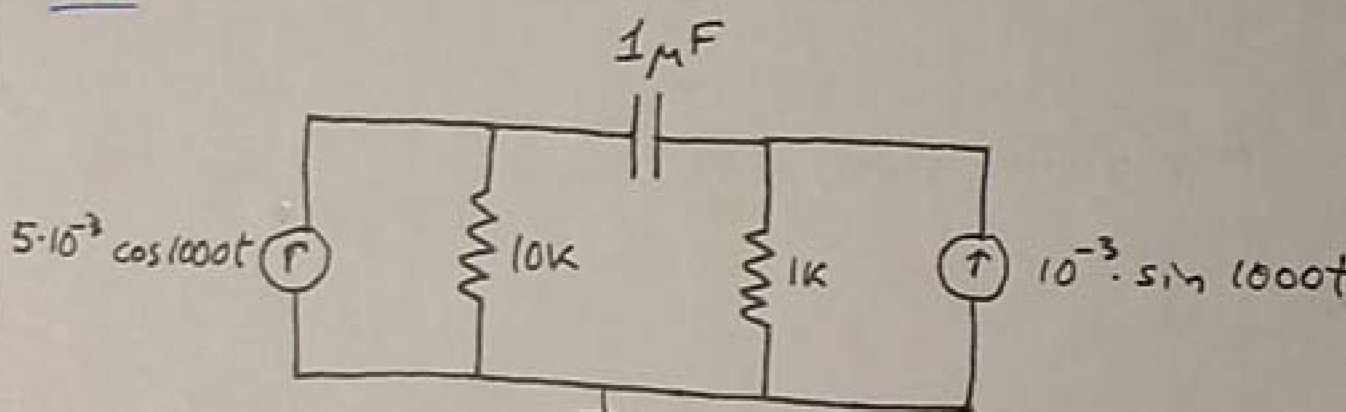
$$\overline{V_0} = 7\,\frac{\frac{1}{j2}}{\frac{1}{j2}+\frac{3}{2}+j} =$$

$$= \frac{7}{1+3j-2} = \frac{7}{-1+3j} = \frac{7<0}{\sqrt{10}<1'9}$$

$$= \frac{7}{\sqrt{10}} < -1'9$$ radians

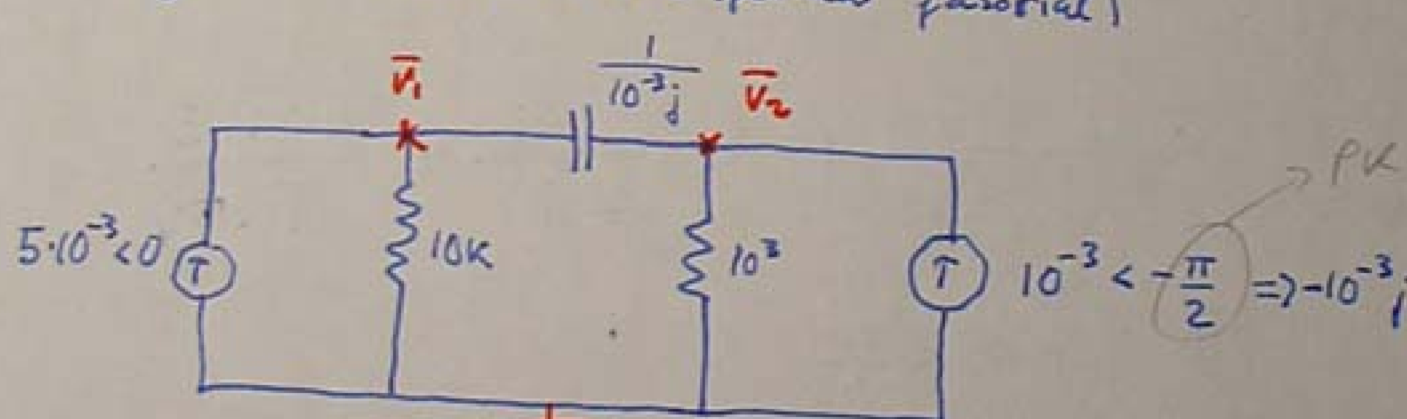
$$\boxed{v(t) = \frac{7}{\sqrt{10}}\cos(2t - 1'9)}$$

Ex:



Determinen les tensions nodals:

CTF (Circuit transformat fasorial)



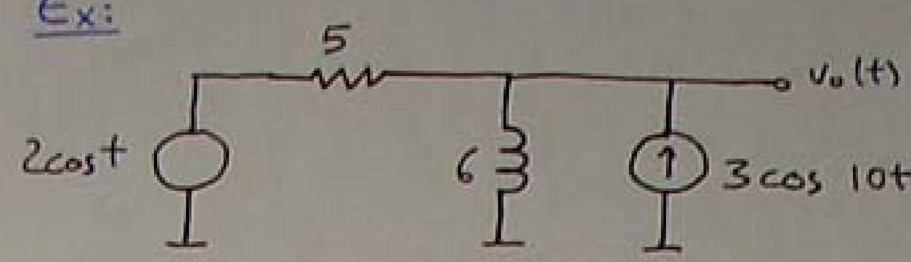
PK l'excitació és un sin, en comptes de cos

$$\begin{bmatrix} 10^{-4}+10^{-3}j & -10^{-3}j \\ -10^{-3}j & 10^{-3}+10^{-3}j \end{bmatrix} \begin{bmatrix} \overline{V_1} \\ \overline{V_2} \end{bmatrix} = \begin{bmatrix} 5\cdot10^{-3} \\ -10^{-3}j \end{bmatrix}$$

$$\boxed{\overline{V_1} = 7'07 < -0'79}$$

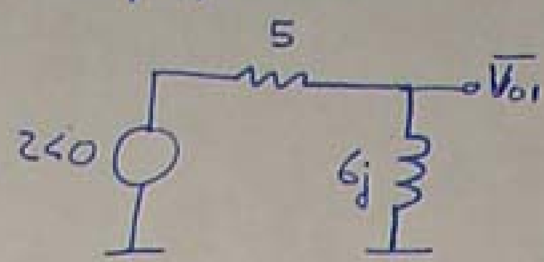
$$\boxed{\overline{V_2} = 4'53 < -0'11}$$

Ex:

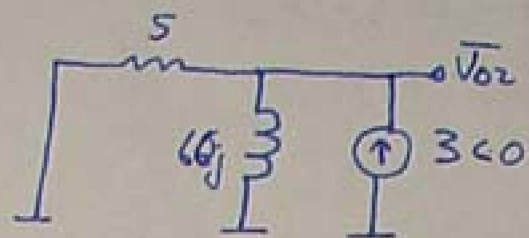


↓ CTF

Superposició:



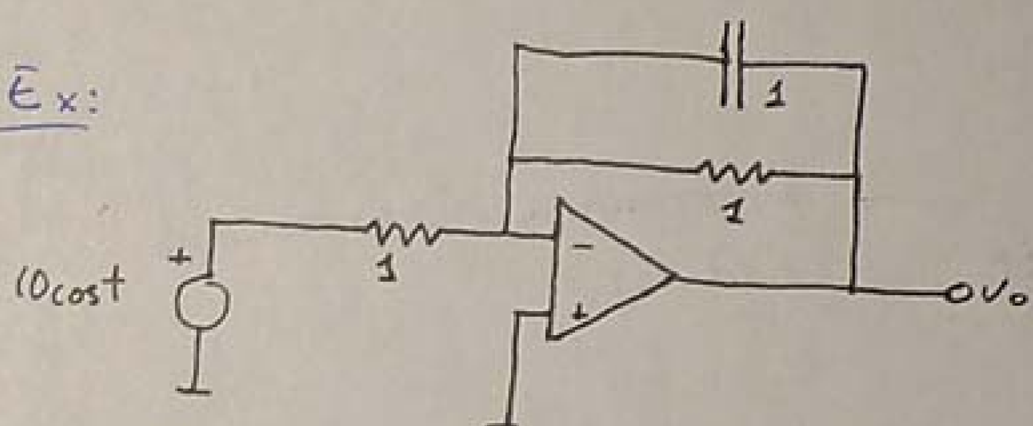
$$\overline{V_{o1}} = \frac{6j}{5+6j}\cdot 2 = \frac{12\angle\frac{\pi}{2}}{7'81\angle 0'88} = 1'54\angle 0'69$$



$$\overline{V_{o2}} = 3\,\frac{60j\cdot 5}{5+60j} = \frac{900\angle\frac{\pi}{2}}{60'2\angle 1'49} = 14'9\angle 0'08$$

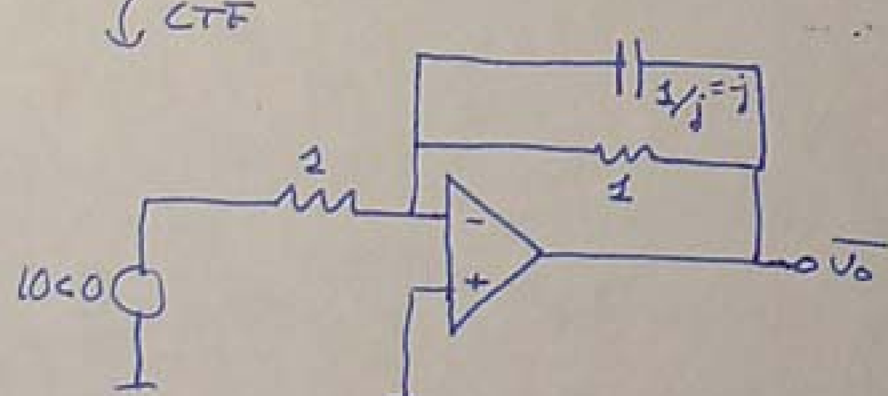
$$\boxed{v_o(t) = 1'54\cdot\cos(t+0'69) + 14'9\cdot\cos(10t+0'08)}$$

Ex:



Amplif. inversor

↓ CTF



$$\overline{V_o} = -\frac{-j}{1-j}\,10 = \frac{10j}{1-j} = \frac{10\angle\frac{\pi}{2}}{\sqrt{2}\angle -\pi/4}$$

$$\overline{V_o} = 7'07\angle\frac{3\pi}{4}$$

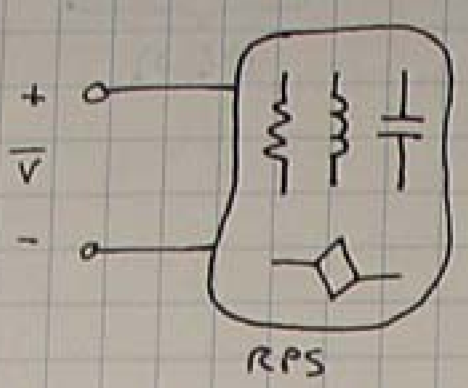
$$v_o(t) = 7'07\cdot\cos\left(t+\frac{3\pi}{4}\right)$$

Titulació

Assignatura

Cognoms: SERRA LÓPEZ   Nom: MARIUS

IMMITÀNCIES FASORIALS:



IMPEDÀNCIA:

$$Z(j\omega) = \frac{\overline{V}}{\overline{I}} = R(\omega) + jX(\omega) \quad (\Omega)$$

Resistència   Reactància

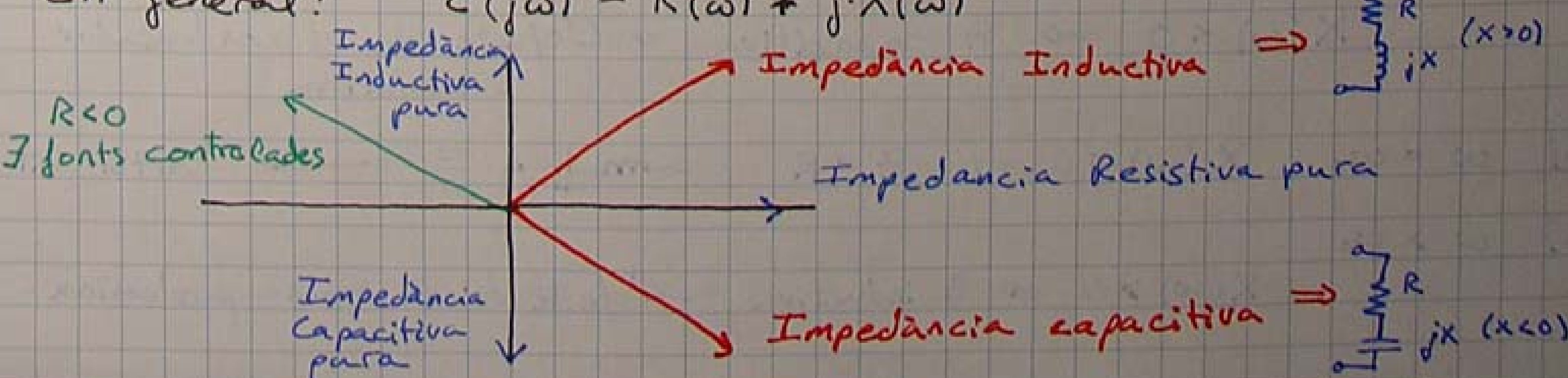
ADMITÀNCIA:

$$Y(j\omega) = \frac{\overline{I}}{\overline{V}} = \frac{1}{Z(\omega)} = G(\omega) + j\,B(\omega) \quad (\mho)$$

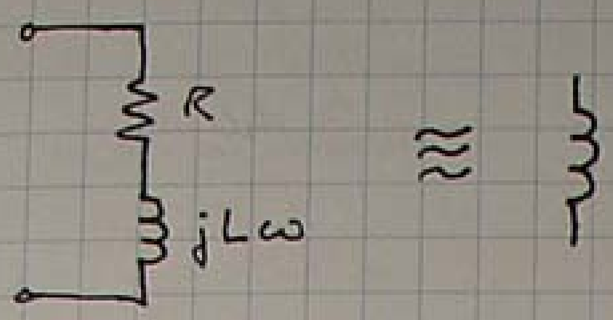
Conductància   Susceptància

Element	$Z(\omega j)$	$Y(\omega j)$
Resistència	R	$\frac{1}{R}$
Bobina	$jL\omega$	$\frac{-j}{L\omega}$
Condensador	$-\frac{j}{C\omega}$	$jC\omega$

En general: $Z(j\omega) = R(\omega) + j\cdot X(\omega)$



Ex: Model d'una bobina



$$\boxed{Z(j\omega) = R + jL\omega}$$

$$\boxed{Y(j\omega) = \frac{1}{Z(j\omega)} = \frac{1}{R + jL\omega} = \frac{R - j\omega L}{R^2 + (\omega L)^2} = \underbrace{\frac{R}{R^2 + (\omega L)^2}}_{G(\omega)} - j\underbrace{\frac{\omega L}{R^2 + (\omega L)^2}}_{B(\omega)}}$$

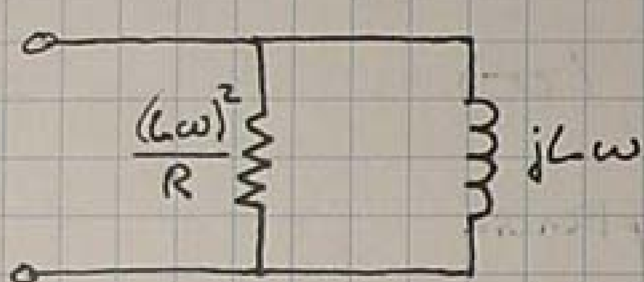
Factor de qualitat:

$$\boxed{Q_b \triangleq \frac{L\omega}{R}} \Rightarrow \text{Si } \boxed{Q_b >> 1} \rightarrow (L\omega >> R)$$

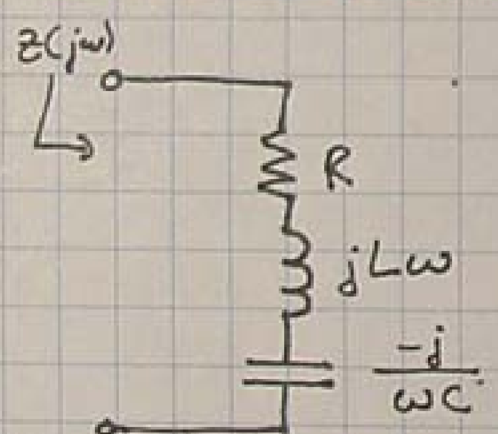
$$\boxed{Y(j\omega) = \underbrace{\frac{R}{(L\omega)^2}}_{G_1} - j\frac{1}{L\omega}}$$

$$G_1 = \frac{R}{(L\omega)^2} \quad R_1 = \frac{(L\omega)^2}{R} = Q_b^2 \cdot R$$

$$G_1 = \frac{1}{R_1}$$



RESSONÀNCIA:



$$Z(j\omega) = R + j\underbrace{\left(L\omega - \frac{1}{C\omega}\right)}_{X(\omega)}$$

Freq. de resonància fa que: $X(\omega_r) = 0$

$$L\omega_r = \frac{1}{C\omega_r} \qquad \boxed{\omega_r = \frac{1}{\sqrt{LC}}} \rightarrow \text{freqüència de ressonància}$$

CASOS:

1) $\omega < \omega_0 = \omega_r$

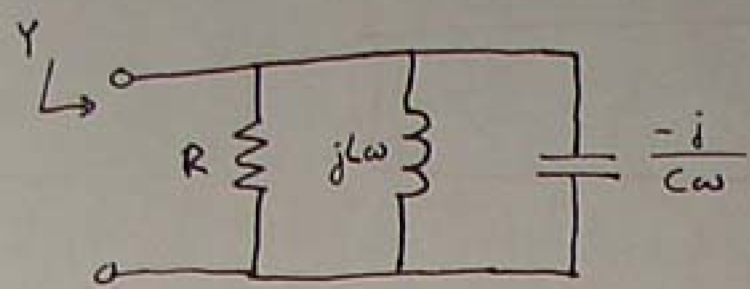
$X(\omega) < 0 \Rightarrow Z$ capacitiva $\rightarrow$ L'efecte de el condensador predomina

2) $\omega = \omega_0$ $X(\omega) = 0 \Rightarrow Z = R$  curtcircuit

3) $\omega > \omega_0$

$X(\omega) > 0 \Rightarrow Z$ inductiva $\rightarrow$ L'efecte de la bobina predomina

Ex: Circuit ressonant paral·lel



$$Y(j\omega) = \frac{1}{R} + \frac{1}{jL\omega} + jC\omega$$

$$Y(j\omega) = \frac{1}{R} + j\underbrace{\left(C\omega - \frac{1}{L\omega}\right)}_{B(\omega)}$$

Freq. de ressonància fa que: $B(\omega_r) = 0$

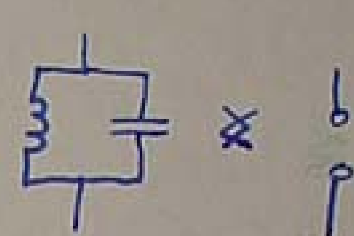
CASOS:

1) $\omega < \omega_0 = \omega_r$

$B(\omega) < 0 \Rightarrow Y, Z$ inductiva

2) $\omega = \omega_r$

$B(\omega) = 0 \Rightarrow Y = \frac{1}{R} \quad Z = R$

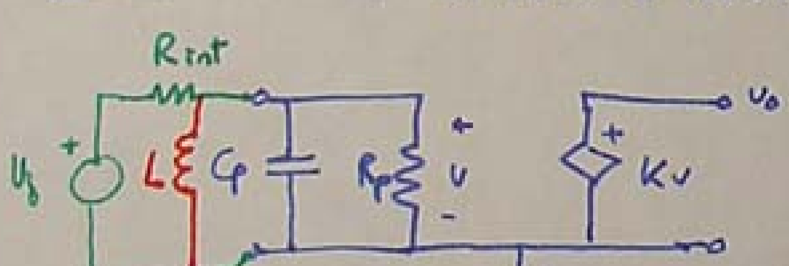


circuit obert

3) $\omega > \omega_r$

$B(\omega) > 0 \Rightarrow Y, Z$ capacitiva

Ex: AMPLIFICADOR D'ALTA FREQÜÈNCIA



$\omega >>> 0$

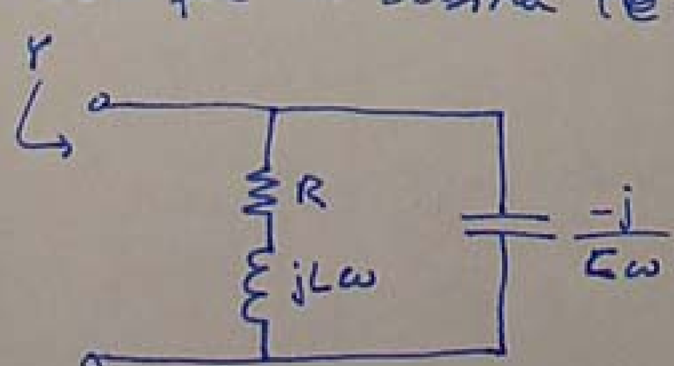
$\omega \approx \frac{1}{jC_p\omega} \to 0 \;/\; v \to 0 \;/\; v_0 \to 0$

SOLUCIÓ:

- Si posem una bobina en paral·lel i fem que LC estigui en resonància per així neutralitzar els dos elements.

Ex:

Com que la bobina té resistència:



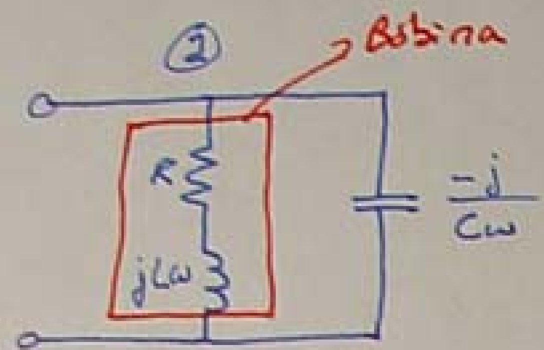
$$Y(j\omega) = \frac{1}{R + jL\omega} + jC\omega = \frac{R - jL\omega}{R^2 + (L\omega)^2} + jC\omega$$

$$= \frac{R}{R^2 + (L\omega)^2} + j\left(C\omega - \frac{L\omega}{R^2 + (L\omega)^2}\right)$$

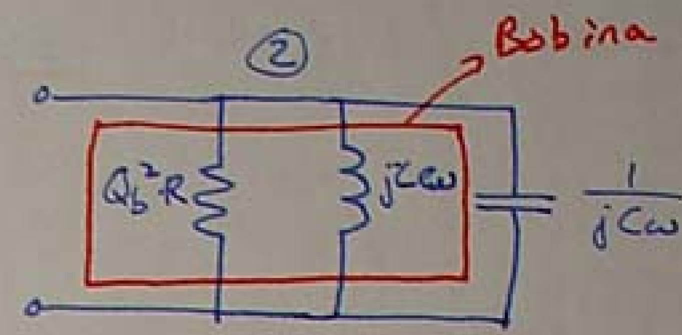
$B(\omega_r) = 0 \implies \boxed{\omega_r = \sqrt{\frac{1}{LC} + \left(\frac{R}{L}\right)^2}}$ $\quad \boxed{Y(\omega_r) = \frac{RC}{L}}$ $\quad \boxed{Z(\omega_r) = \frac{L}{RC}}$

Per a $\boxed{Q_b = \frac{L\omega}{R} >> 1}$ MODEL D'UNA BOBINA

$R \cdot Q_b^2 = \frac{(L\omega)^2}{R}$



$\approx$



A més si $\omega_0 = \omega_r$

Problemes: P2, P3, P6

$\omega_r = \frac{1}{\sqrt{LC}} \implies Z(j\omega_r) = Q_b^2 \cdot R$ $\quad Q_b^2 \cdot R$ $\quad$ Vàlid per 2

EXEMPLE D'AMPLIFICACIÓ: AMPLIFICADOR D'ÀUDIO

- Senyal d'àudio, senyal capaç de ser captat per l'oïda. Ex: veu, música, tro...
- Amplificador d'àudio: present a (walkman, diskman, reproductor MP3, Hi-Fi...)

TRANSDUCTOR
- Capçal del walkman
- Convertidor $\boxed{A/D}$

(x → tensió)

TRANSDUCTOR
- Altaveu

(tensió → pressió)

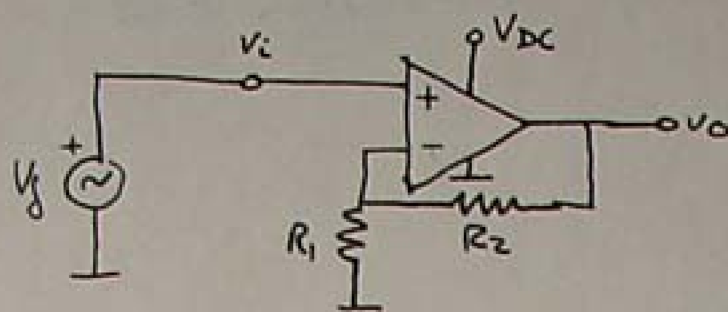
RESTRICCIÓ DE DISSENY:

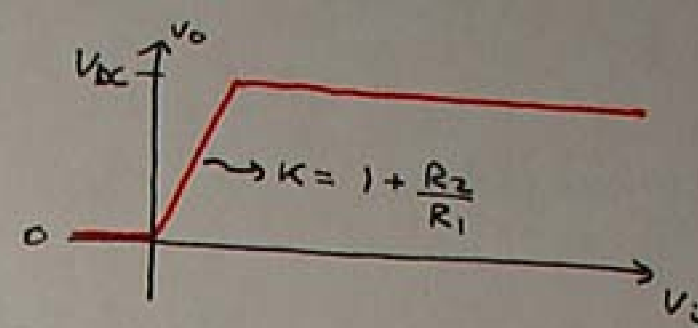
- Ha de ser capaç d'amplificar senyals sinusoïdals de freq. entre 15-20Hz i 15-20KHz (freq. audibles).

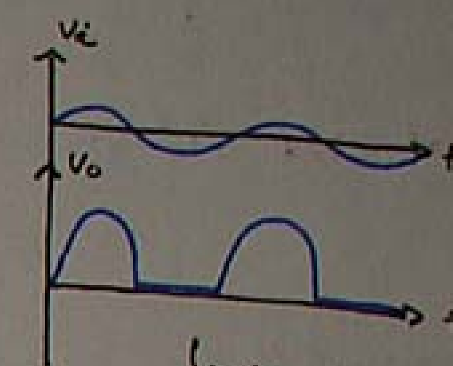
– Restricció addicional (no imprescindible)

Alimentació unipolar ⇒ Una única bateria o Font de tensió.

PRIMER MODEL:



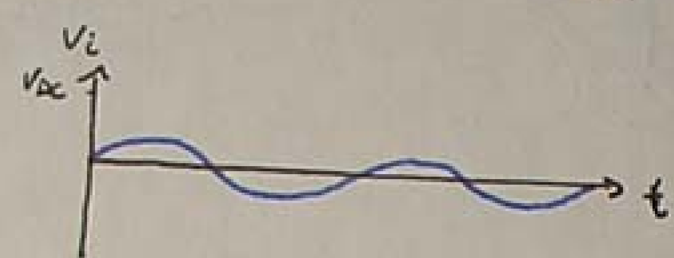


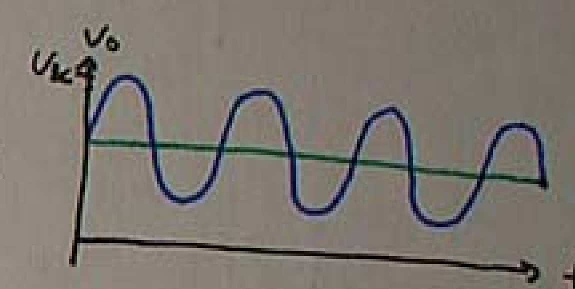


↳ Distorsió

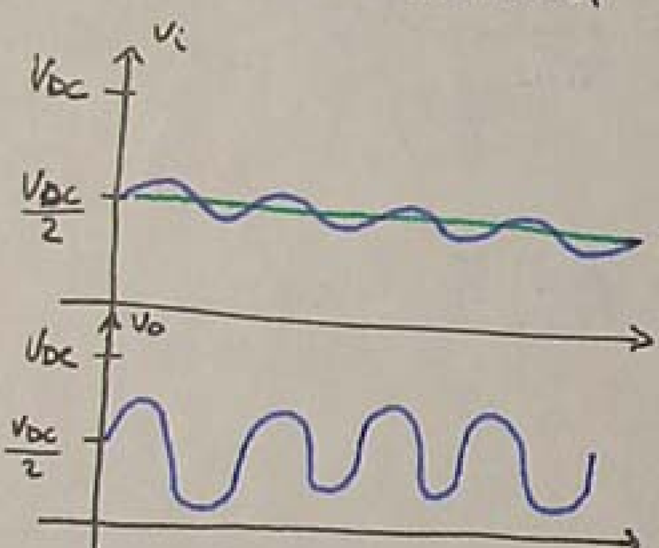
* Distorsió: El senyal de sortida no és = al de entrada.

* Solució: ① Sumar a v_i una senyal constant que fes $v_i > 0 \ \forall t$





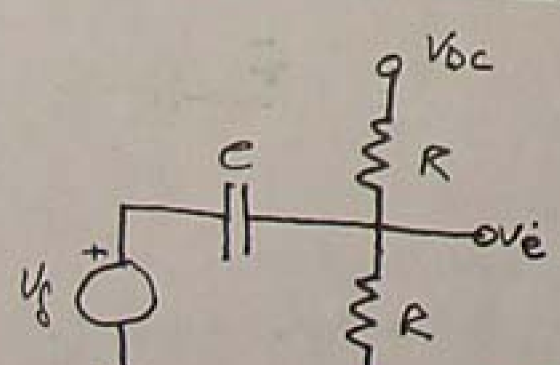
② Solució habitual



a) Sumar $\frac{V_{DC}}{2}$ a v_g $\Rightarrow v_i = \overbrace{\frac{V_{DC}}{2}}^{DC} + \overbrace{v_g}^{AC}$

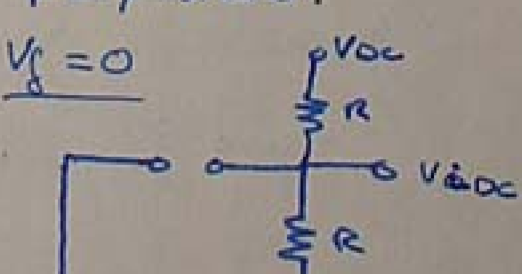
b) Amplificar el component AC

CIRCUIT PER AL CAS a):



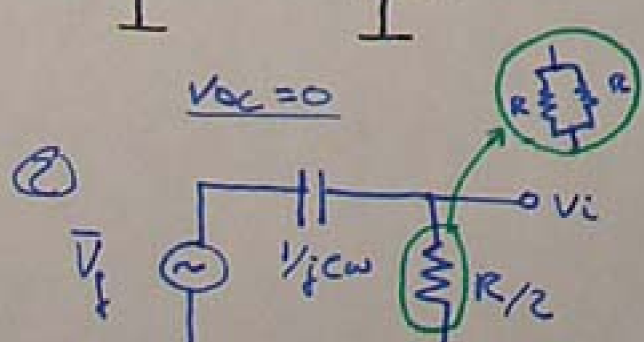
Superposició:

① $v_g = 0$



$v_{iDC} = \frac{V_{DC}}{2}$

② $V_{DC} = 0$

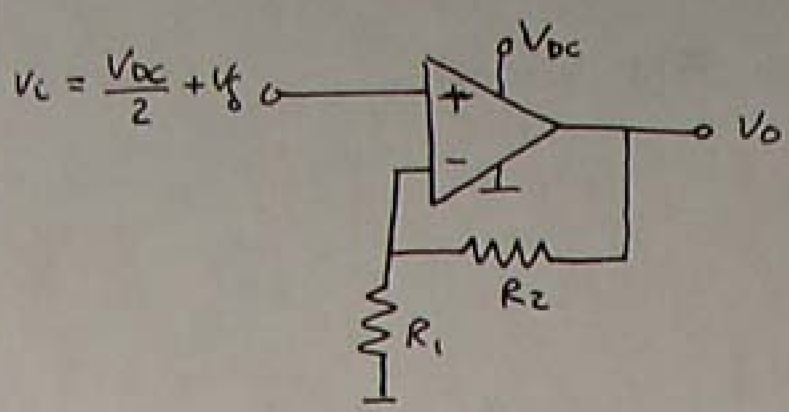


$\bar{V}_{iAC} = \bar{V}_g \dfrac{R/2}{\frac{R}{2} - \frac{j}{C\omega}} \underset{\uparrow}{\simeq} \bar{V}_g \Rightarrow \boxed{v_{iAC}(t) = v_g(t)}$

$\frac{R}{2} \gg \frac{1}{C\omega}$

$$\boxed{v_i = v_{iDC} + v_{iAC} = \frac{V_{DC}}{2} + v_g}$$

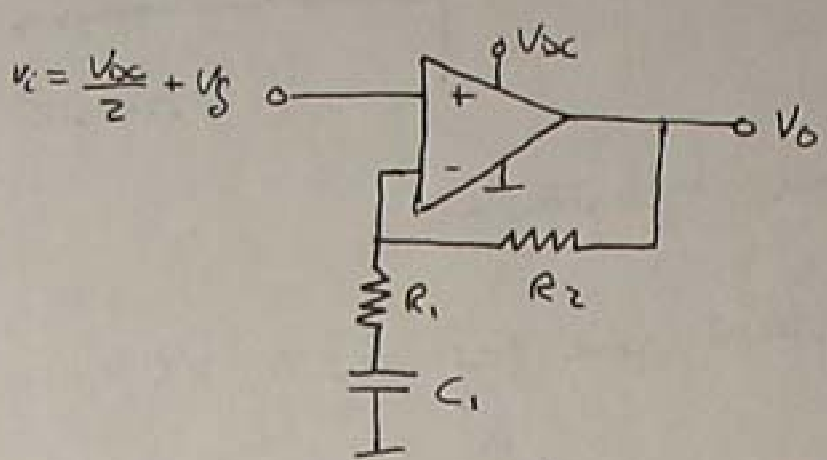
CIRCUIT PER EL CAS b):



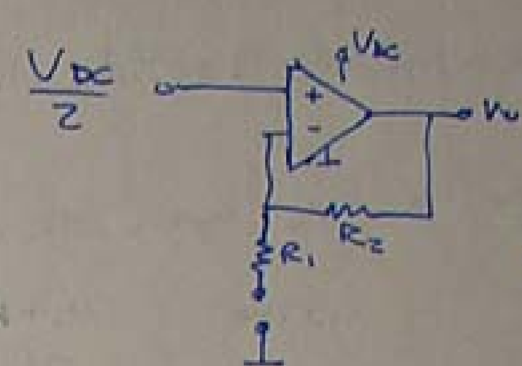
$V_o = K v_i = \underbrace{\frac{15 V_{DC}}{2}}_{} + 15 V_g$

↳ Saturació ⇒ No zona lineal

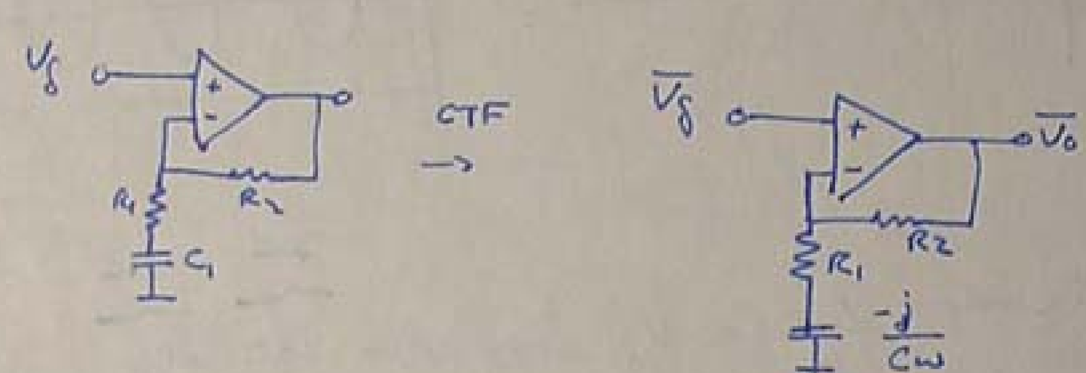
Solució: Afegir un condensador



Superposició:

① 

$V_{o_{DC}} = \left(1 + \frac{R_2}{\infty}\right)\frac{V_{DC}}{2} = \frac{V_{DC}}{2}$

② 

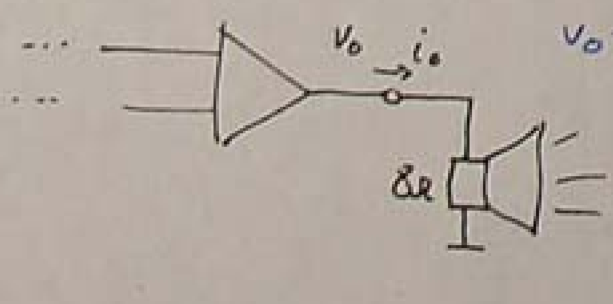
$\bar{V}_o = \left(1 + \frac{R_2}{R_1 - \frac{j}{C_1\omega}}\right)\bar{V}_g$

$\bar{V}_o \underset{\frac{1}{C_1\omega} \ll R_1}{=} \left(1 + \frac{R_2}{R_1}\right)\bar{V}_g = K V_g$

$v_{o_{AC}}(t) = K \cdot V_g(t)$

$$\boxed{V_{oT} = V_{oAC} + V_{oDC} = K \cdot V_g(t) + \frac{V_{DC}}{2}}$$

AMPLIFICADOR D'ÀUDIO ACTUAL:

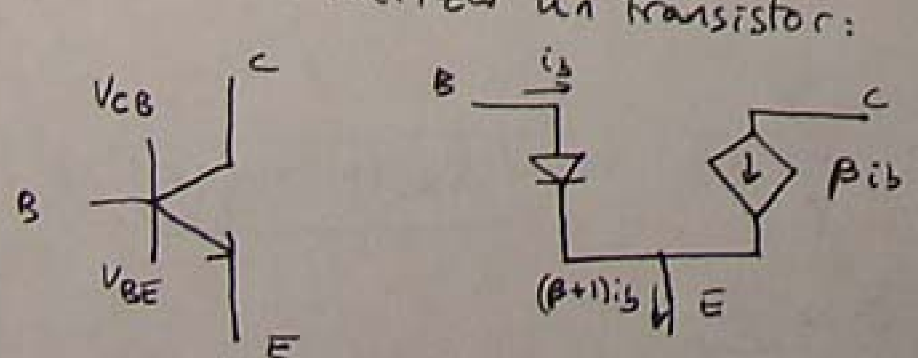


$v_o = \frac{V_{DC}}{2} + K V_g$

$V_{o\,max} \simeq V_{DC} = 8V$

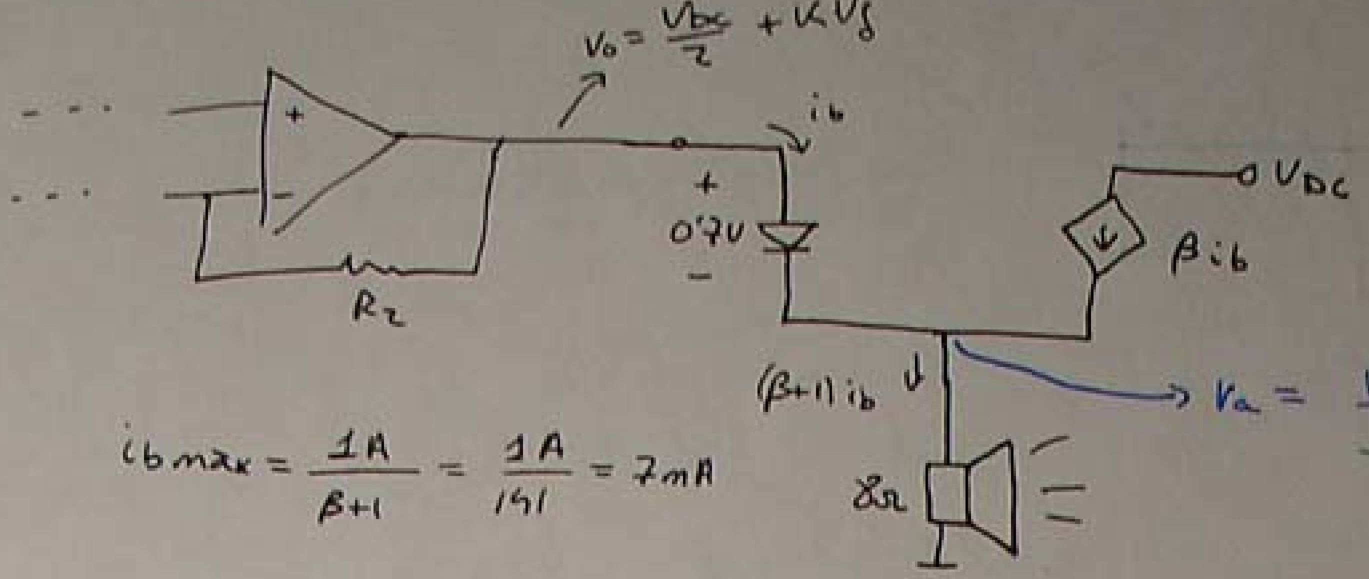
$i_{o\,max} = \frac{8V}{8\Omega} = 1A$ → El amplificador no pot donar tant corrent.

SOLUCIÓ: Utilitzar un transistor:



$\boxed{V_{CB} > 0}$

POSIBLE SOLUCIÓ:

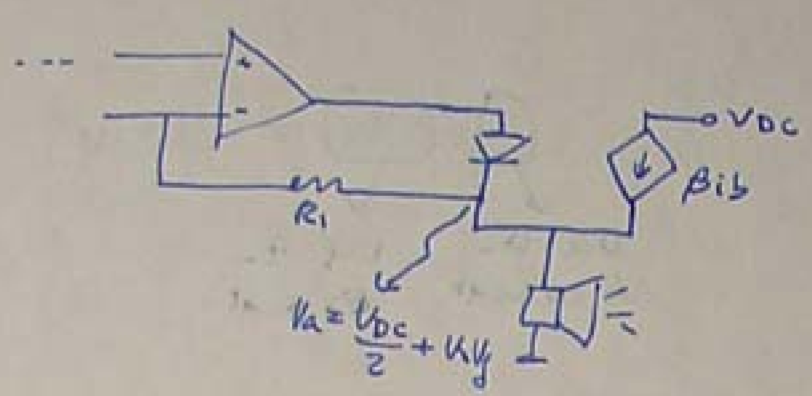


$V_a = \frac{V_{DC}}{2} + KV_g - 0'7$

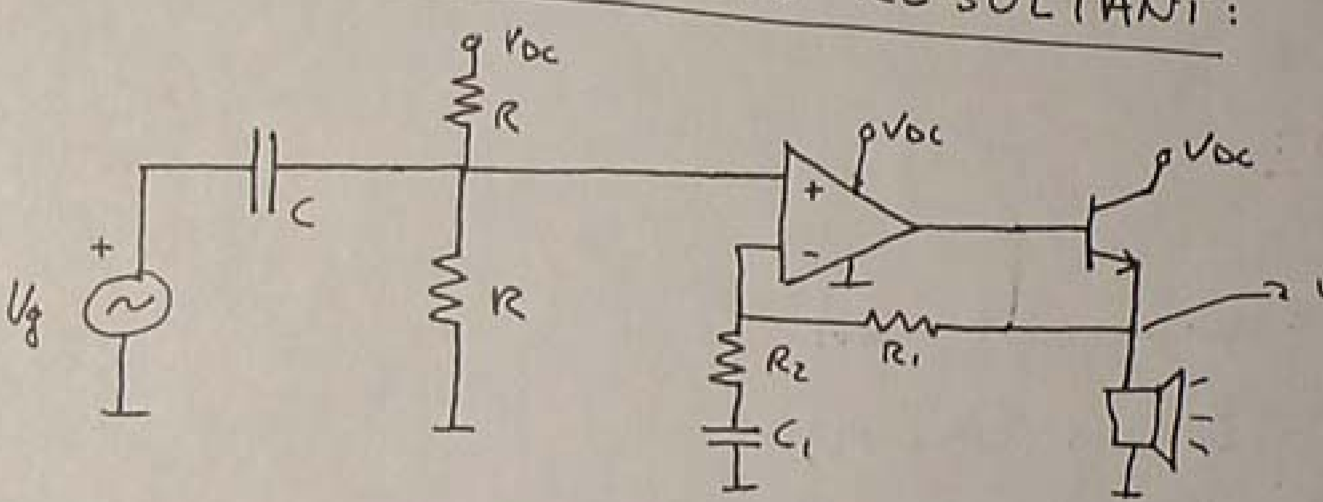
DC AC

$$i_{b\,max} = \frac{1A}{\beta+1} = \frac{1A}{141} = 7mA$$

PETITA MILLORA:



AMPLIFICADOR D'ÀUDIO RESULTANT:



$V_a = \frac{V_{DC}}{2} + KV_g \qquad K = 1 + \frac{R_2}{R_1}$

POTÈNCIA LLIURADA A L'ALTAVEU:

$$P_a(t) = v_a(t) \cdot i_a(t) = \frac{V_a^2(t)}{8\Omega} = \frac{1}{8}\left[\frac{V_{DC}^2}{4} + (KA)^2 \cos^2(\omega t) + V_{DC} \cdot KA \cdot \cos(\omega t)\right] =$$

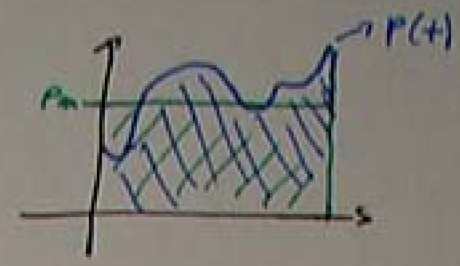
$V_g = A \cos \omega t$

$V_a^2(t) = \frac{V_{DC}^2}{2^2} + K^2 A^2 \cos^2 \omega t + V_{DC} KA \cos \omega t$

$$= \frac{1}{8}\left[\frac{V_{DC}^2}{4} + \underbrace{\frac{(KA)^2}{2} + \frac{(KA)^2}{2} \cos 2\omega t}_{\cos^2 t = \frac{1}{2} + \frac{1}{2}\cos 2t} + V_{DC} \cdot K \cdot A \cdot \cos \omega t\right]$$

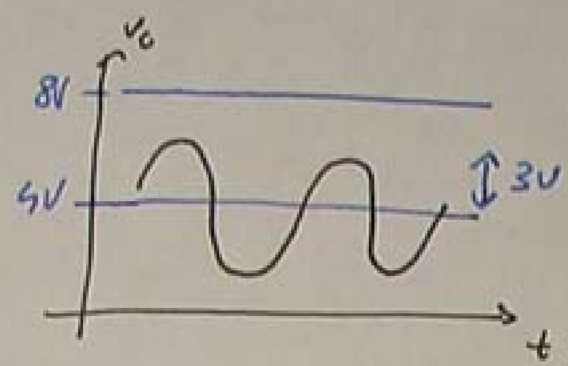
Potència mitja:

$$P_m \triangleq \frac{1}{T}\int_0^T p(\tau)\,d\tau$$



Les àrees són iguals

$$P_m = \frac{1}{8}\left[\frac{V_{DC}^2}{4} + \frac{(KA)^2}{2}\right]$$ → La resta de termes no hi surt pk la seva àrea és nul·la.



$V_{DC} = 8V$
$KA = 3V$

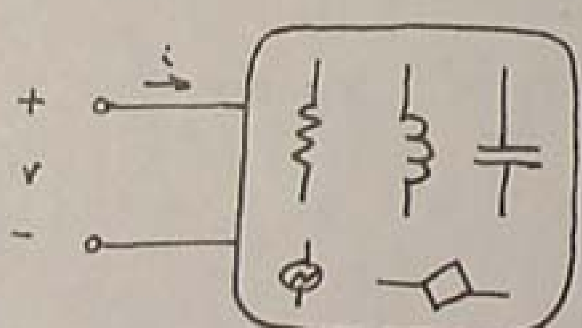
$$P_m = \frac{V_{DC}^2}{32} + \frac{(KA)^2}{16} = 2 + 0'56 = 2'56\,W$$

2: Del DC. No es sent
0'56: Del AC. Es sent

P_m (sonora) ≈ 0'56W

* Podriem controlar el volum fent R_2 variable. VOLUM ≡ + AMPLIFICACIÓ

POTÈNCIA EN R.P.S:



$v(t) = V_p \cdot \cos(\omega t + \phi_v)$

$i(t) = I_p \cdot \cos(\omega t + \phi_i)$

- Potència instantània:

$$P(t) = v(t)\cdot i(t) = V_p \cdot I_p \cdot \cos(\omega t + \phi_v)\cdot\cos(\omega t + \phi_i)$$

$$P(t) = \boxed{\frac{V_p \cdot I_p}{2}\left[\cos(2\omega t + \phi_v + \phi_i) + \cos(\phi_v - \phi_i)\right]}$$

- Potència mitja:

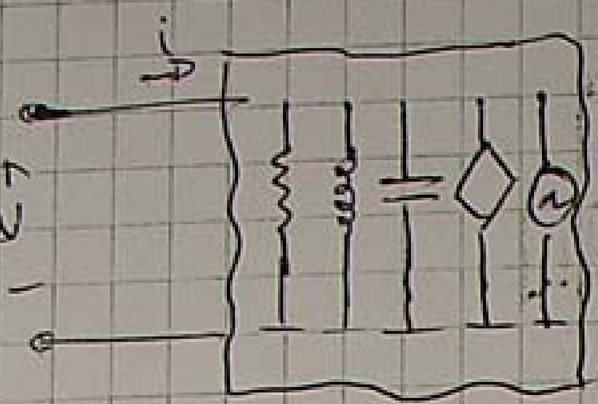
$$\boxed{P_m = \frac{1}{2} V_p \cdot I_p \cdot \cos\phi}$$

$$\boxed{\phi = \phi_v - \phi_i}$$

$\phi = 0 \quad P_m > 0$
$\phi = \frac{\pi}{2} \quad P_m = 0$
$\phi = \pi \quad P_m \leq 0$

FACTOR DE POTÈNCIA: $\cos\phi$

Potència en R.P.S



$V(t) = V_p \cos(\omega t + \Phi_v)$

$I(t) = i_p \cdot \cos(\omega t + \Phi_i)$

Potensia instantanea.

$p(t) = V(t) \cdot I(t) = \boxed{\frac{1}{2} V_p I_p [\cos(2\omega t + \phi_v + \phi_i) + \cos(\phi_v + \phi_i)}$

Potència mitja

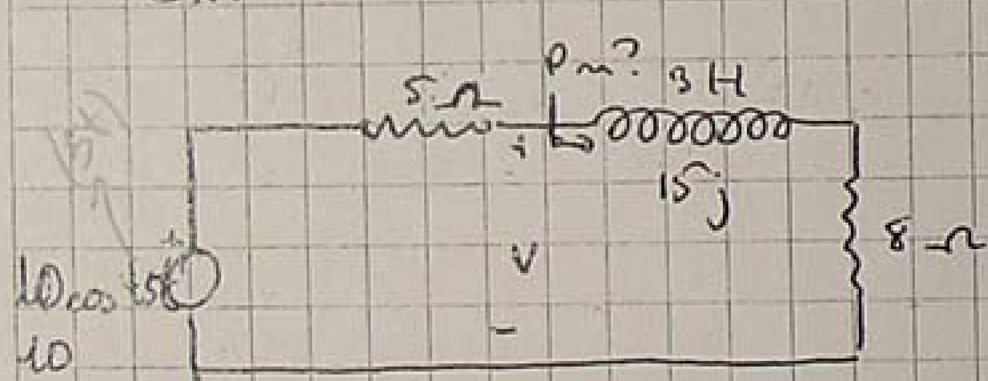
$\boxed{P_m = \frac{1}{2} V_p \cdot I_p \cos \phi}$ $\phi = \phi_v - \phi_i$

$\phi = 0$ $P_m \geq 0$

$\phi = \frac{\pi}{2}$ $P_m = 0$

$\phi = \pi$ $P_m \leq 0$

Ex:



$i = \frac{10}{13 + 15j} = \frac{10 < 0}{19'85 < 49°} = 0'5 < -49°$

$\vec{V} = Z \cdot \bar{I} = 17 < 62° \cdot 0'5 < 49° = 8'5 < 13°$

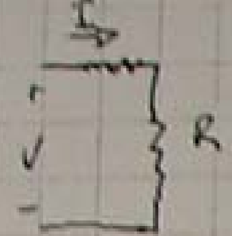
$Z = 8 + 15j = 17 < 62°$

$P_m = \frac{1}{2} V_p I_p \cdot \cos \phi = \frac{1}{2} 8'5 \cdot 0'5 \cdot \cos(13° - (-49°)) = 1 W$

Potència en R.P.S

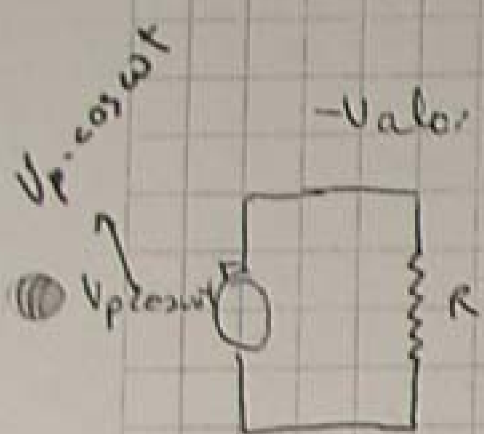
- Potència dissipada per alguns elements

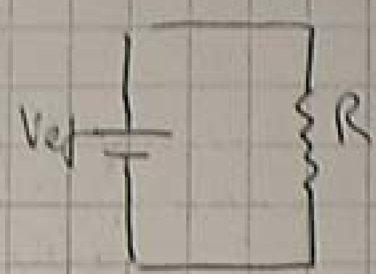
Resistor



$$P_m = \frac{1}{2} V_p \cdot I_p \underbrace{\cos\phi}_{1} = \boxed{\frac{1}{2} V_p \cdot I_p = \frac{1}{2} R I_p^2 = \frac{1}{2} \frac{V_p^2}{R}}$$

- Valor eficaç





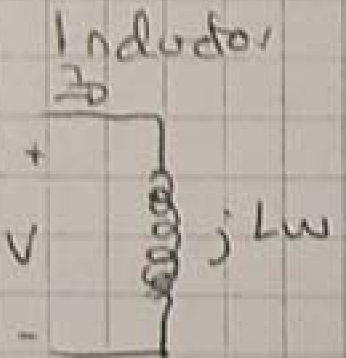
$$P_m = \frac{1}{2} \frac{V_p^2}{R} \Rightarrow P_m = \frac{V_{ef}^2}{R}$$

$$\downarrow$$

$$V(ef) = \frac{V_p}{\sqrt{2}}$$

$$I(ef) = \frac{I_p}{\sqrt{2}}$$

Inductor



$$P_m = \frac{1}{2} V_p I_p \cos\phi$$

$$\bar{V} = jL\omega \bar{I} \longrightarrow V_p = L\omega I_p$$

$$\hookrightarrow \phi_V = \frac{\pi}{2} + \phi_I \qquad \phi = \phi_V - \phi_I = \frac{\pi}{2}$$

$$P_m = \frac{1}{2} V_p I_p \overset{0}{\cos(\pi/2)} = 0$$

Condensador

Idem $\phi = \pi/2$ $P_m = 0$

Potencia en valor eficaç

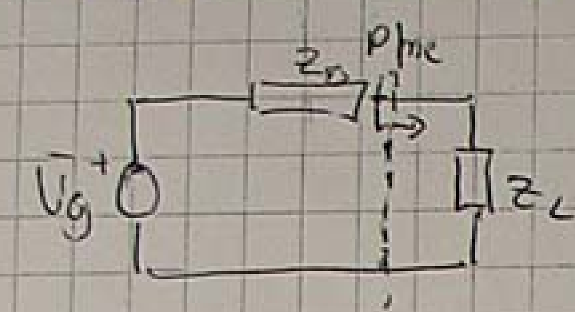
$P_m = \underbrace{V_{ef} \cdot I_{ef}}_{P_\sim} \cos\phi \; (\omega)$

$\Rightarrow \quad P_\sim = V_{ef} \cdot I_{ef}$

$\uparrow$

$\cos\phi \rightarrow$ Factor de potència $\quad$ P aparent

MÀXIMA TRANSFERENCIA DE POTÈNCIA



Quin valor de Z_L maximiza P_{me}

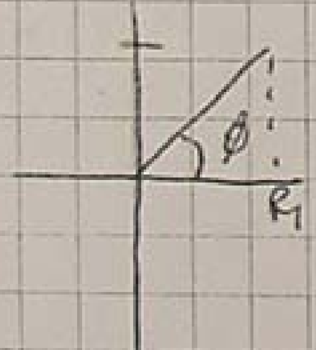
$Z_g = R_g + j\,x_g$

Generador $\quad$ Càrrega $\quad Z_L = R_L + j\,x_L$

$\bar{V} = \bar{V}_g \cdot \frac{R_L + j\,\omega_L}{R_g + R_L + j(x_g + x_L)} \Rightarrow V_p = |\bar{V}| = V_{gp} \frac{\sqrt{R_L^2 + X_L^2}}{\sqrt{(R_g + R_L)^2 + (X_g + X_L)^2}}$

$\bar{I} = \bar{V}_g \cdot \frac{1}{R_g + R_L + j(X_g + X_L)} \rightarrow I_p = |\bar{I}| = V_{gp} \frac{1}{\sqrt{(R_g + R_L)^2 + (X_g + X_L)^2}}$

$j\,X_L - Z_L$



$\cos\phi = \frac{R_L}{\sqrt{R_L^2 + X_L^2}}$

$P_m = \frac{1}{2} V_p I_p \cdot \cos\phi \Rightarrow \frac{1}{2} V_{gp}^2 \cdot \frac{\sqrt{R_L^2 + X_L^2}}{(R_g + R_L)^2 + (X_g + X_L)^2} \quad \frac{R_L}{\sqrt{R_L^2 + X_L^2}}$

$P_m = \frac{1}{2} V_{gp}^2 \frac{R_L}{(R_g + R_L)^2 + (X_g + X_L)^2}$

Maximitzar el $P_m \Leftrightarrow X_L = -X_g \Rightarrow P_m = Z = \frac{1}{2} V_{gp}^2 \frac{R_L}{(R_g + R_L)}$