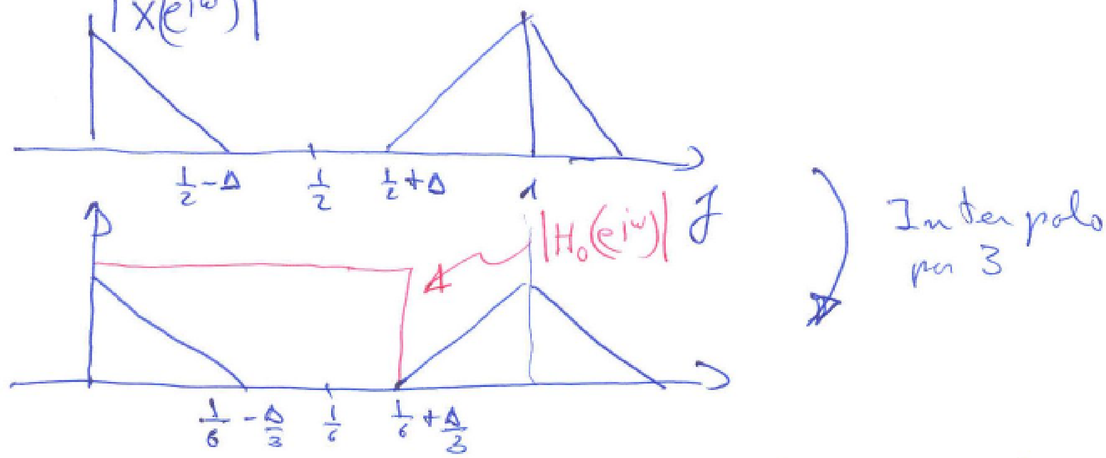
Problema 1

a) La condición que impondremos será que no se produzca aliasing.



La condición a cumplir será: $\frac{1}{6} + \frac{\Delta}{3} = \frac{1}{5}$;

luego $\Delta = 3\left(\frac{1}{5} - \frac{1}{6}\right) \Rightarrow \boxed{\Delta = 1/10}$

finalmente: $f_{max} = \frac{1}{2} - \frac{1}{10} = 2/5$

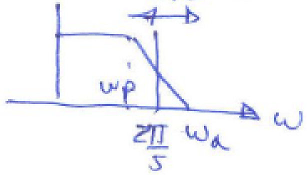
b) $X(e^{j\omega}) \to \boxed{\uparrow 3} \xrightarrow{X(e^{j3\omega})} \boxed{H_0} \xrightarrow{X(e^{j3\omega})H(e^{j\omega})} \boxed{\uparrow 2} \to Y_0(e^{j\omega})$

$$Y_0(e^{j\omega}) = X(e^{j\omega 6})\, H_0(e^{j2\omega})$$

$$X(e^{j\omega}) \to \boxed{\uparrow N} \xrightarrow{X(e^{j\omega N})} \boxed{H_1} \to Y_1(e^{j\omega}) = X(e^{j\omega N})\, H_1(e^{j\omega})$$

c) $N = 6$ y $H_1(e^{j\omega}) = H_0(e^{j\omega 2})$

d)



$\omega_a = \frac{2\pi}{5} + \frac{2\pi}{21}$ y $\omega_p = \frac{2\pi}{5} - \frac{2\pi}{21}$

e) Nuevo ancho de banda: la condición será ahora:

$2\pi\left(\frac{1}{6} + \frac{\Delta}{3}\right) = 2\pi\left(\frac{1}{5} + \frac{2}{21}\right) \Rightarrow \Delta = 3\left(\frac{1}{5} - \frac{1}{6} + \frac{2}{21}\right) \Rightarrow f_{max} = \frac{1}{2} - \frac{27}{70}$

Problema 2

a) $x[n] \longrightarrow \boxed{T_1} \xrightarrow{x[-n]} \boxed{T_2} \longrightarrow x[-n+n_1] = y_1[n]$

$$Y_1(e^{j\omega}) = TF\{y_1[n]\} = X(e^{-j\omega})\,e^{j\omega n_1}; \quad \begin{cases} x[-n] \stackrel{\mathcal{F}\{\cdot\}}{\longleftrightarrow} X(e^{-j\omega}) \\ x[n-n_1] \stackrel{\mathcal{F}\{\cdot\}}{\longleftrightarrow} X(e^{j\omega})\,e^{-j\omega n_1} \end{cases}$$

b) $\mathcal{F}\{y_2[n]\} = \mathcal{F}\{r_{y_1 y_1}[n]\} = \mathcal{F}\{y_1[n] * y_1^*[-n]\} = |Y_1(e^{j\omega})|^2 =$

$$= |X(e^{-j\omega})|^2 \quad ; \quad \begin{cases} \text{convolución} \\ \text{giro temporal.} \end{cases}$$

c) $X(e^{j\omega}) = \mathcal{F}\{0, \frac{1}{2}, \frac{1}{2}, 0 \ldots\} = \frac{1}{2} + \frac{e^{-j\omega}}{2} = e^{-j\frac{\omega}{2}}\cos\left(\frac{\omega}{2}\right)$

$$Y_1(e^{j\omega})\Big|_{n_1=2} = X(e^{-j\omega})\,e^{j\omega n_1} = e^{j\frac{\omega}{2}}\cos\left(\frac{\omega}{2}\right) e^{-j2\omega} = e^{-j\frac{3}{2}\omega}\cos\left(\frac{\omega}{2}\right)$$

Luego: $\overline{Y}_1[k] = Y_1(e^{j\omega})\Big|_{\omega=\frac{2\pi k}{3}} = e^{-j\pi k}\cos\left(\frac{\pi}{3}k\right) = \{1, -\cos\left(\frac{\pi}{3}\right), -\cos\left(\frac{\pi}{3}\right)\}$

$\overline{Y}_1[k] = \{1, -\frac{1}{2}, -\frac{1}{2}\}$

d) $y_2[n] = IDFT_3\{[Y_1[k]]^2\} = \{\frac{1}{2}, \frac{1}{4}, \frac{1}{4}\}$ $\quad\begin{cases}\text{Por propiedades;} \\ \text{Observan que hay} \\ \text{aliasing en tiempo!}\end{cases}$

e) $y_2[n] \neq r_{y_1 y_1}[n]$ porque $r_{y_1 y_1}$ tiene duración 5; al ser la idft de duración 3 se ha producido aliasing en tiempo. Una solución sería: $y_1 \to DFT_5\{\cdot\} \to |\cdot|^2 \to IDFT_5\{\cdot\} \to r_{y_1 y_1}$

Problema 3

a) $Y(z) = V_2(z) + a\left[V_1(z) + b\left(V_2(z) + X(z)\right)\right]$

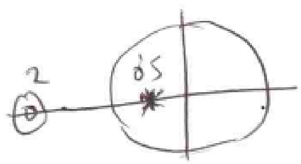
$V_1(z) = z^{-1}\left\{X(z) - a\left[V_1(z) + b\left(V_2(z) + X(z)\right)\right]\right\}$

$V_2(z) = z^{-1}\,V_1(z)$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{ab + az^{-1} + z^{-2}}{1 + az^{-1} + abz^{-2}}$$

b) $H(z) = \dfrac{0'25 + z^{-1} + z^{-2}}{1 + z^{-1} + 0'25z^{-2}}$

ceros : -2 doble
polos : $-1/2$ doble



ROC: $|z| > \frac{1}{2}$

Pasa todo por tener ceros y polos inversos

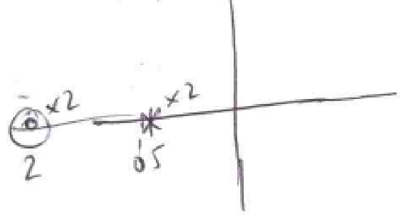
c) $x[n] = \cos\pi n = e^{j\pi n}$

$y[n] = H(-1)\,e^{j\pi n} = e^{j\pi n} = \cos\pi n$

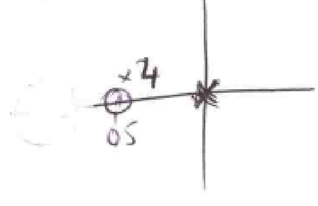
d) $Y(z) = X(z)\,H(z) = \cancel{(1+0'5z^{-1})}\,\dfrac{(0'5 + z^{-1})^2}{(1+0'5z^{-1})^{\cancel{2}}} = \dfrac{0'25 + z^{-1} + z^{-2}}{1 + 0'5z^{-1}}$

$= -2 + 2z^{-1} + \dfrac{9/4}{1 + 0'5z^{-1}} \xrightarrow{z} -2\delta[n] + 2\delta[n-1] + \frac{9}{4}\left(-\frac{1}{2}\right)^n u[n]$

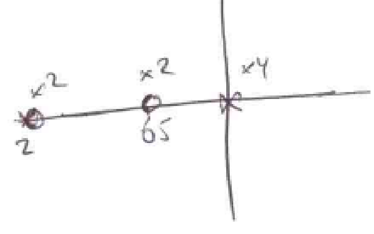
e) H(z)



G(z)



H(z) G(z)



$G(z) = (1 + 0'5z^{-1})^4\,k$

FIR

fase lineal