## SOLUCIÓN DEL TEST DEL 16-1-01

$$\begin{split} H &= \sum_{i=1}^{L} p_i \cdot log_2 \Bigg( \frac{1}{p_i} \Bigg) = \sum_{i=1}^{L-l} 0.5^i \cdot log_2(2^i) + 0.5^{L-l} \cdot log_2 \Big( 2^{L-l} \Big) = \\ &2 \cdot \Big( l - (L+1) \cdot 0.5^L \Big) + (L-1) \cdot 0.5^{L-l} = 2 - 2 \cdot L \cdot 0.5^L - 2 \cdot 0.5^L + L \cdot 0.5^{L-l} - 0.5^{L-l} = \\ &2 - 2 \cdot L \cdot 0.5^L - 2 \cdot 0.5^L + L \cdot 0.5^L \cdot 0.5^{L-l} - 0.5^L \cdot 0.5^{-l} = 2 - 2 \cdot 0.5^L - 2 \cdot 0.5^L = 2 - 4 \cdot 0.5^L + 1 \cdot 0.5^L +$$

2) a) 
$$111*x(n) = 1 \quad a$$
 
$$1 \quad a$$
 
$$\frac{1}{1} \quad a + 1 \quad a \Rightarrow y_{K}$$

$$-1-1-1*x(n) = -1 -a$$

$$-1 -a$$

$$-1 -a$$

$$-1 -a - 1 - a \Rightarrow y_{\nu}$$

HLSE  $\rightarrow \sum_{i} (y - y_K)^2$  min para que la media = 0

$$y - \text{media} = 0.8, -0.7, -0.2, 0.3$$
 $\parallel$ 
 $0.3$ 

$$(-0.2)^{2} + (-1.7 - a)^{2} + (-1.2 - a)^{2} + (0.3 - a)^{2} = 1.8^{2} + (0.3 + a)^{2} + (0.8 + a)^{2} + (0.3 + a)^{2} \Rightarrow$$

$$= 0.04 + 2.89 + a^{2} + 3.4a + 1.44 + a^{2} + 2.4a + 0.09 + a^{2} - 0.6a = 3.24 + 0.09 + a^{2} + 0.6a + 0.64 + a^{2} + 1.6a + 0.09 + a^{2} + 0.6a \Rightarrow 4.46 + 5.2a = 4.06 + 2.8a \Rightarrow 0.4 = -2.4a \Rightarrow a = -0.16667$$

$$\begin{array}{c}
\overbrace{111...1}^{n} \\
\underbrace{000...0}_{k}
\end{array} \to d_{\min} = n \to r = n-1 \to (*)$$

$$\frac{d_{\min} \ge 2e + 1}{r \ge 2e} \rightarrow e \left\lfloor \frac{d_{\min} - 1}{2} \right\rfloor = n \text{ par, } e = \frac{n - 2}{2}$$

$$n \text{ impar, } e = \frac{n - 1}{2}$$

a) 
$$e = \frac{n-1}{2} \implies (*) e = \frac{r}{2}$$

b) 
$$d = 2e$$

c) 
$$d = 2e \rightarrow {n-2, n \text{ par} \over n-1, n \text{ impar}}$$

4) c)
$$v_t = 18300 \text{ bps}$$
 $v_t = v_m \cdot q_1$ 
 $q_1 = \log_2(A_1)$ 
 $v_1 = 24400 \text{bps}$ 

$$W_{QAM} = \frac{1+\alpha}{T} \rightarrow 4000 = (1+\alpha) \cdot v_m \rightarrow \alpha = 0, v_m = 4000$$

$$18300 = v_m \cdot q \text{ con } 2000 \le v_m \le 4000 \Rightarrow q_1 = 6 \frac{\text{bits}}{\text{simbolo}}$$

$$9.15 \ge q \ge 4.57 \qquad A_1 = 2^6 = 64 \text{ simbolos}$$

$$q_1 = 5, 6, 7, 8, 9 \qquad v_m = 3050 \text{ bauds}$$

$$v_1 = 24400 = 3050 \cdot q$$

$$q' = 8 \rightarrow A' = 2^8 = 256 \text{ simbolos}$$

$$q = 5 \rightarrow v_m = 3660 \rightarrow q' = 6.66$$

$$q = 6 \rightarrow v_m = 3050 \rightarrow q' = 8 \rightarrow 2^8 = 256$$

$$q = 7 \rightarrow v_m = 2614 \rightarrow q' = 9.33$$

$$q = 8 \rightarrow v_m = 2287.5 \rightarrow q' = 10.66$$

$$q = 9 \rightarrow v_m = 2033.3 \rightarrow q' = 12 \rightarrow 2^{12} = 4096$$
5) a)

Hamming  $\binom{n}{7}, 4$ 

$$G = (I_k : P)$$

$$\downarrow I_4$$

$$Y(D) = R(D) + D^r \cdot X(D)$$

$$R(D) = D^r \cdot X(D) \mod g(D) \Rightarrow D^r \cdot X(D) \mid g(D)$$

$$R(D) = c(D)$$
Nos interesa hallar las  $Y(D)$  de las  $X(D)$  de la base.
$$n = k + r \rightarrow 7 = 4 + r \rightarrow r = 3$$

$$*Y(D) = 1000 \quad P^3$$

\*X(D) = 1000 = D<sup>3</sup>

D<sup>3</sup> · X(D) = D<sup>6</sup>

$$\frac{D^3 + D^2 + 1}{D^3 + D^2 + D}$$
Y(D) =  $\frac{D^2 + D}{R(D)} + D^3 \cdot \frac{D^3}{X(D)} = D^6 + D^2 + D$ 

$$\frac{D^5 + D^3}{D^5 + D^4 + D^2}$$

$$\frac{D^5 + D^4 + D^2}{D^4 + D^3 + D}$$

$$\frac{D^4 + D^3 + D}{D^3 + D = R(D)}$$

 $=455 \cdot (10^{-3})^3 = 455 \cdot 10^{-9} = 4.55 \cdot 10^{-7}$ 

$$E = \frac{H}{\overline{L}}$$

$$\begin{cases} H = 2 - 4 \cdot \left(\frac{1}{2}\right)^{L} \Big|_{L=5} = 2 - 4 \cdot \frac{1}{32} = 2 - \frac{1}{8} = \frac{15}{8} = 1.875 \\ H = \sum_{i=1}^{5} P_{i} \cdot \log_{2}\left(\frac{1}{P_{i}}\right) = \frac{1}{2} \cdot \log_{2}(2) + \left(\frac{1}{2}\right)^{2} \cdot \log_{2}(2)^{2} + \left(\frac{1}{2}\right)^{3} \cdot \log_{2}(2)^{3} + \left(\frac{1}{2}\right)^{4} \cdot \log_{2}(2)^{4} = \frac{1}{2} + \frac{1}{4} \cdot 2 + \frac{1}{8} \cdot 3 + \frac{1}{16} \cdot 4 = \frac{1}{2} + \frac{1}{2} + \frac{3}{8} + \frac{8}{16} = \frac{15}{8} = 1.875 \end{cases}$$

$$A^{-\frac{1}{2}}$$

$$\begin{bmatrix} D & \frac{1}{16} \\ E & \frac{1}{16} \end{bmatrix} \stackrel{\frac{1}{8}}{F} \begin{bmatrix} \overline{4} & 0 \\ \end{bmatrix}$$

$$\overline{L} = \frac{1}{2} \cdot 1 + \frac{1}{4} \cdot 2 + \frac{1}{8} \cdot 3 + \frac{1}{16} \cdot 4 = \frac{1}{2} + \frac{1}{2} + \frac{3}{8} + \frac{8}{16} = \frac{15}{8} = 1.875$$

$$E = 1$$

$$H = 0.25 \cdot \log_2\left(\frac{1}{0.25}\right) + 0.5 \cdot \log_2\left(\frac{1}{0.5}\right) + 0.15 \cdot \log_2\left(\frac{1}{0.15}\right) + 0.1 \cdot \log_2\left(\frac{1}{0.1}\right)$$

$$= 0.5 + 0.5 + 0.4105 + 0.3322 = 1.7427 \frac{\text{bits}}{\text{simbolo}}$$

donde 
$$\log_2(x) = \frac{\log(x)}{\log 2}$$

$$d_{\min} \ge 2e + 1$$
  $\Rightarrow$   $4 \ge 2e$   $\Rightarrow$   $e = 2$   $r > 2e$   $\Rightarrow$   $r = 4$ 

e-perfecto 
$$\Rightarrow$$
 (ver debajo)  $\Rightarrow$  n = 5  $\rightarrow$  código  $\begin{pmatrix} 5, 1 \\ n & k \end{pmatrix}$   $\begin{pmatrix} n = k + r \\ 5 = k + 4 \rightarrow 1 \end{pmatrix}$ 

$$P_{error} = P_{\#e \ge 3} \approx P_{\#e = 3} = \binom{5}{3} \cdot p^3 \cdot (1 - p)^2 \approx \binom{5}{3} \cdot p^3 = 10 \cdot (10^{-2})^3 = 10^{-5}$$

$$\begin{split} &2^{r}=1+\binom{n}{1}+\binom{n}{2}\text{ ya que }e=2\\ &2^{r}=1+n+\frac{n!}{2!(n-2)!}=1+n+\frac{n\cdot(n-1)}{2}\\ &2^{4}-1=n+\frac{n^{2}-n}{2}\longrightarrow 30=2n+n^{2}-n\longrightarrow n^{2}+n\cdot30=0\\ &n=\frac{-1\pm\sqrt{1+120}}{2}=\frac{-1\pm11}{2}\longrightarrow \frac{5}{-6}\longrightarrow n=5\\ &n=k+r\\ &5=k+4\longrightarrow k=1\longrightarrow codigo(5,1)\\ &P_{E}=P_{\text{Re}23}\approx P_{\text{Re}3}\\ &c)\\ &1\quad A\\ &2\quad B\\ &3\quad C\\ &4\quad D\\ &5\quad AC\\ &6\quad CB\\ &7\quad BC\\ &8\quad CBA\\ &9)\ d)\\ &QAM\cdot 4\qquad PAM\\ &\alpha_{1}=0.3\qquad i\alpha_{2}^{2},A\uparrow\uparrow\\ &(\%_{N})\qquad P_{12}=P_{13}=P_{13}\\ &P_{13}=P_{13}=P_{13}\\ &P_{12}=P_{13}=P_{13}\\ &P_{13}=N_{v}\cdot Q\left[\sqrt{\frac{3(1+\alpha)}{A^{2}}\cdot(\%_{N})}\right]\qquad P_{12}=N_{v}\cdot Q\left[\sqrt{\frac{3(1+\alpha)}{A^{2}-1}\cdot(\%_{N})}\right]\\ &N_{v,2}=4\cdot\left(1-\frac{1}{2}\right)=2\qquad N_{v,2}\cong 2\\ &P_{11}=2\cdot Q\left(\sqrt{\frac{3(1+\alpha)}{A^{2}}\cdot(\%_{N})}\right)\qquad P_{E}=2\cdot Q\left(\sqrt{\frac{3(1+\alpha)}{A^{2}-1}\cdot18000\cdot(\%_{N})}\right)\\ &1.3\cdot\left(\frac{S}{N_{1}}\right)=\frac{3\cdot(1+\alpha)}{A^{2}-1}\cdot18000\cdot\left(\frac{S}{N_{1}}\right)\longrightarrow (A^{2}-1)=\frac{5400}{1.3}\cdot(1+\alpha) \approx A^{2}=41538.46\cdot(1+\alpha)\\ &sicndo\ 0<\alpha<1\rightarrow \alpha=0\rightarrow A^{2}=41538.46\\ &\alpha=1\rightarrow A^{2}=83076.92 \end{split} \label{eq:particles}$$
 un cuadrado perfecto entre ambos

 $65536 = 256^2 \rightarrow \alpha = 0.57 \rightarrow A \in \{203.8, 288.23\} \rightarrow A = 2^q = 256 \rightarrow q = 8 \frac{\text{bits}}{\text{simbolo}}$ 

10) d)  

$$L = 2^{5} - 1 = 31, p^{0}(D) = D^{2} + D^{4}, p^{31}(D) = p^{0}(D)$$
a)  

$$p^{n}(D) = 1 + D + D^{3} + D^{4}$$

$$D \cdot p^{n}(D) = D + D^{2} + D^{4} + D^{5} \quad | D^{5} + D^{2} + 1$$

$$1 + D^{2} \quad + D^{5} \quad 1$$

$$D^{4} + D + 1 = 11001 \rightarrow No$$
b)  

$$p^{n}(D) = 1 + D^{4}$$

$$D \cdot p^{n}(D) = D + D^{5} \quad | D^{5} + D^{2} + 1$$

$$D^{5} + D^{2} + 1 \quad 1$$

$$D^{2} + D + 1 = 110 \rightarrow No$$
c)  

$$p^{n}(D) = D^{3} + D^{4}$$

$$D \cdot p^{n}(D) = D^{4} + D^{5} \quad | D^{5} + D^{2} + 1$$

$$D^{5} + D^{2} + 1 \quad 1$$

$$D^{4} + D^{2} + 1 \neq p^{0}(D) \rightarrow No$$

11) a)

 $y(n) = \{1.5, 0.6, -0.7\}$ 

		( , , ,						
		Уk			$\eta = y - y_k$			$\sigma_{\eta}^{2} = \frac{\sum n^{2}}{3}$
1	1	1	1.5	0.5	0.5	-0.9	-1.2	0.83
1	-1	1	-0.5	-0.5	0.5	1.1	-0.2	0.5
-1	1	1	0.5	0.5	0.5	0.1	-1.2	0.56
-1	-1	-1	-1.5	-0.5	2.5	2.1	-0.2	3.56

12) b)

ECM = 
$$E\{a^2\}\cdot DCM + \underbrace{\frac{\sigma_{\eta}^2}{h^2(0)}}_{0} \approx E\{a^2\}\cdot DCM$$

$$DCM = \frac{\sum_{n \neq 0} h^2(n)}{h^2(0)} \rightarrow ser\'{a} \ el \ que \ tenga \ P_E \ menor \Rightarrow ECM \ menor \Rightarrow DCM \ menor, ya \ que \ no \ hay \ ruido \ \left(\sqrt[8]{N_2}\right)$$

```
GRUPO A:
a)
     0.0165 0.168 0.0255
               0.11
                      1.12
                               0.17
                      0.022
                               0.224 \quad 0.034
h(n) = 0.0165 \ 0.278 \ 1.1675 \ 0.394 \ 0.034 \rightarrow DCM = 0.1716
b)
     -0.03 \quad 0.159 \quad -0.02505
              -0.2
                       1.06
                                 -0.167
                       -0.04 -0.212 -0.0334
h(n) = -0.03 - 0.041 \quad 0.9949 \quad -0.379 \quad -0.0334 \rightarrow DCM = 0.1488
c)
     -0.0465 0.1755 0.0165
                -0.31
                        1.17
                                   0.11
                         -0.062 0.234 0.022
h(n) = 0.0465 -0.1345 1.1245 0.344 0.022 \rightarrow DCM = 0.1099
d)
     -0.0165 0.1425 0.045
                -0.11
                          0.95
                                   0.3
                         -0.022 \quad 0.19 \quad 0.06
                          0.973 \quad 0.49 \quad 0.06 \rightarrow DCM = 0.2588
h(n) = -0.0165 0.325
GRUPO B:
a)
     0.033 0.336 0.051
              0.11
                     1.12
                             0.17
                     0.011 \quad 0.112 \quad 0.017
h(n) = 0.033 \quad 0.446 \quad 1.182 \quad 0.282 \quad 0.017 \rightarrow DCM = 0.2
b)
     -0.06 0.318 -0.0501
              -0.2
                      1.06
                               -0.167
                      -0.02 -0.106 -0.0167
h(n) = -0.06 \ 0.118 \ 0.9898 \ -0.061 \ -0.0167 \rightarrow DCM = 0.0219
c)
```

c)
$$-0.033 \quad 0.285 \quad 0.09$$

$$-0.11 \quad 0.95 \quad 0.3$$

$$-0.011 \quad 0.095 \quad 0.03$$

$$h(n) = -0.033 \quad 0.175 \quad 1.029 \quad 0.395 \quad 0.03 \rightarrow DCM = 0.1781$$

a) DCM = 
$$0.18 = 0.65 \cdot 0.1716 + 0.35 \cdot 0.2$$

b) DCM = 
$$0.1043 = 0.65 \cdot 0.1488 + 0.35 \cdot 0.0219$$

c) DCM = 
$$0.087 = 0.65 \cdot 0.1099 + 0.35 \cdot 0.045$$

d) DCM = 
$$0.1337 = 0.65 \cdot 0.2588 + 0.35 \cdot 0.1781$$

$$\Delta_{\nu}(n+1) = \alpha \cdot \Delta_{\nu}(n) + \frac{\beta}{y^{2}(n)}$$

$$E\{\Delta_{\nu}(n+1)\} = \frac{1}{L_{e} \cdot R_{y}(0)}$$

$$E\{\Delta_{\nu}(n+1)\} \approx E\{\Delta_{\nu}(n)\}$$

$$E\left\{\alpha \cdot \Delta_{\upsilon}(n) + \frac{\beta}{y^{2}(n)}\right\} = \alpha \cdot E\left\{\Delta_{\upsilon}(n)\right\} + \beta \cdot \underbrace{\frac{1}{E\left\{y^{2}(n)\right\}}}_{R_{\chi}(0)}$$

$$(1-\alpha)\cdot \mathrm{E}\{\Delta_{v}(n)\} = \frac{\beta}{\mathrm{R}_{v}(0)}$$

$$E\{\Delta_{\nu}(n)\} = \frac{\beta}{(1-\alpha)\cdot R_{\nu}(0)} = \frac{1}{L_{e}\cdot R_{\nu}(0)} \Rightarrow L_{e} = \frac{1-\alpha}{\beta}$$

$$\frac{1 - \frac{1}{2}}{\frac{1}{2}} = 1 = L_e \rightarrow No$$

$$\frac{1 - \frac{2}{3}}{\frac{3}{2}} = \frac{\frac{1}{3}}{\frac{3}{2}} = 0.22 = L_e \rightarrow No$$

$$\frac{1 - \frac{1}{3}}{\frac{1}{2}} = \frac{\frac{2}{3}}{\frac{1}{2}} = 1.33 = L_e \rightarrow No$$

$$\sigma_{\eta} = 0 \rightarrow \text{óptimo} = \text{inversor canal}$$

$$\rho_{\rm x}(0) = 0.86$$

$$\rho_{\rm x}(1) = 0.09$$

$$\rho_{\rm x}(2) = -0.02$$

$$\begin{pmatrix} 0.86 & 0.09 & -0.02 \\ 0.09 & 0.86 & 0.09 \\ -0.02 & 0.09 & 0.86 \end{pmatrix} \cdot \begin{pmatrix} c_{.1} \\ c_{0} \\ c_{.1} \end{pmatrix} = \begin{pmatrix} 0.2 \\ 0.9 \\ -0.1 \end{pmatrix} \rightarrow \begin{pmatrix} c_{.1} \\ c_{0} \\ c_{.1} \end{pmatrix} = \begin{pmatrix} 0.1166 \\ 1.05777 \\ -0.2242 \end{pmatrix}$$

$$h(0) = c_{-1} \cdot x_1 + c_0 \cdot x_0 + c_1 \cdot x_{-1} = 0.989$$

$$\overline{c_0} = \frac{c_0}{h(0)} = 1.060157$$

## 15) a)

$$45 - \alpha = 10.07^{\circ}$$

$$v_t = C = W \cdot log_2(1 + \frac{s}{N}) \rightarrow N = 10^{-3}$$

$$S = 124 \cdot 10^{-3} \cdot K^2$$

$$W = 3000$$

$$5 = \log_2(1 + 124 \cdot K^2)$$
  $C = 15000$ 

$$32 = 1 + 124 \cdot K^2$$

$$K = 0.5$$

$$p = 10^{-3}$$
,  $D = 4 con \frac{6 > 4}{L \ge D}$   $\rightarrow$  Se puede suponer canal sin memoria (entrelazado)

$$P_{e}(bloque) = P_{\#e \ge 2} \approx P_{\#e = 2} = {7 \choose 2} \cdot p^{2} \cdot (1-p)^{5} \approx {7 \choose 2} \cdot p^{2} = 21 \cdot (10^{-3})^{2} = 21 \cdot 10^{-6} = 2.1 \cdot 10^{-5}$$

18) a)
$$R_{y}(0) = \frac{\sum_{i} y^{2}(i)}{7} = 79.23$$

$$\Delta_{\rm v} \approx \frac{1}{L_{\rm e} \cdot R_{\rm v}(0)} = \frac{1}{3 \cdot 79.23} = 0.02945$$

$$P_{E} = N_{v} \cdot Q \left[ \sqrt{\frac{3 \cdot (1+\alpha)}{A-1} \cdot \frac{S}{N}} \right], N_{v} = 4 \cdot \left( 1 - \frac{1}{\sqrt{A}} \right)$$

$$\mathbf{v}_{\mathsf{t}} = \mathbf{v}_{\mathsf{m}} \cdot \log_2(\mathbf{A})$$

$$W_{QAM} = (1 + \alpha) \cdot v_m \rightarrow 21600 = 1.5 \cdot v_m \rightarrow v_m = 14400 \text{ baudios}$$

$$28800 = 14400 \cdot \log_2(A) \rightarrow A = 4$$

$$N_v = 4 \cdot \left(1 - \frac{1}{2}\right) = 2 \rightarrow P_E = 2 \cdot Q\left[\sqrt{\frac{3 \cdot 1.5}{3} \cdot \frac{S}{N}}\right] \le 10^{-7}$$

$$Q(x) = 5 \cdot 10^{-8} = 0.5 \cdot e^{-\frac{x^2}{2}} \rightarrow x = 5.6777$$

$$\sqrt{1.5 \cdot \frac{s}{N}} = 5.6777 \rightarrow \frac{S}{N} = 21.49 \equiv 13.32 \text{ dB}$$

No, al propagar errores, es la otra configuración, la autosincronizante

.....00011000.... ....01001000....

$$1 D^2$$

$$c(D) = 1 + D^2$$