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FIBER-OPTIC COMMUNICATIONS

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OPTICAL COMMUNICATIONS GROUP

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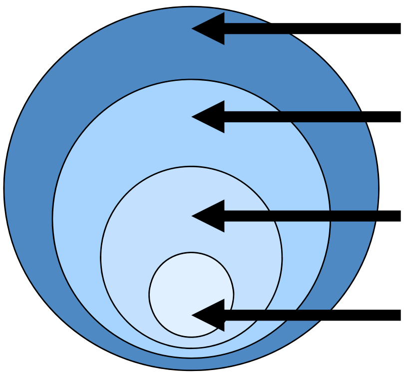
RAY OPTICS

"Light is an electromagnetic wave phenomenon described by the same theoretical principles that govern all forms of EM radiation. Nevertheless, it is possible to describe many phenomena using some approximations"

TEMPORAL EVOLUTION



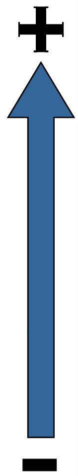
QUANTUM OPTICS (PARTICLES)
Einsten-Planck

ELECTROMAGETIC OPTICS (FOURIER)
Maxwell

WAVE OPTICS (SCALAR)
Newton

RAY OPTICS (GEOMETRIC)
Galileo

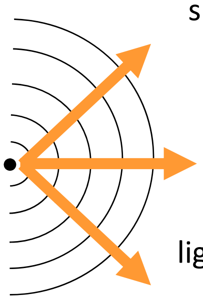
λ<<D



RAY OPTICS: POSTULATES (I)

1. Light travels in the form of **rays**

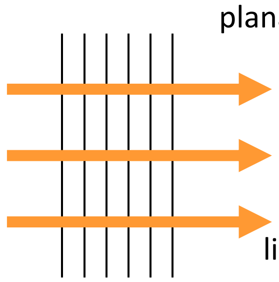
spherical wavefront



light rays

λ<<D →

planar wavefront



light rays

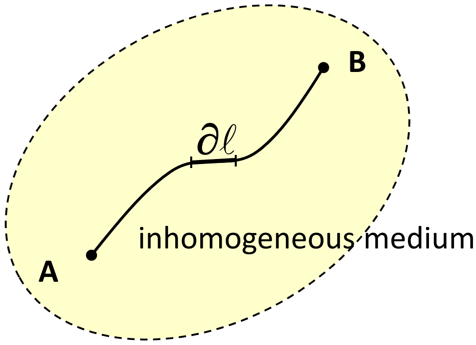
2. An optical medium is characterized by its **refractive index** ($n \geq 1$), which is defined as the ratio of the speed of ligth in free space ($c=3 \cdot 10^8$ m/s) to that in the medium (v)

$$\mathbf{n} \equiv \frac{\mathbf{c}}{\mathbf{v}} \geq 1 \quad \rightarrow \quad \mathbf{t} = \frac{\mathbf{d}}{\mathbf{v}} = \frac{\mathbf{nd}}{\mathbf{c}}$$

nd ≡ optical path length

RAY OPTICS: POSTULATES (II)

3. In an **inhomogeneous medium** the refractive index n(r) is a function of the position r = (x,y,z). The optical path length along a given path between points A and B is therefore:



$$\int_A^B n(r)\partial\ell \quad \rightarrow \quad t = \frac{1}{c}\int_A^B n(r)\partial\ell$$

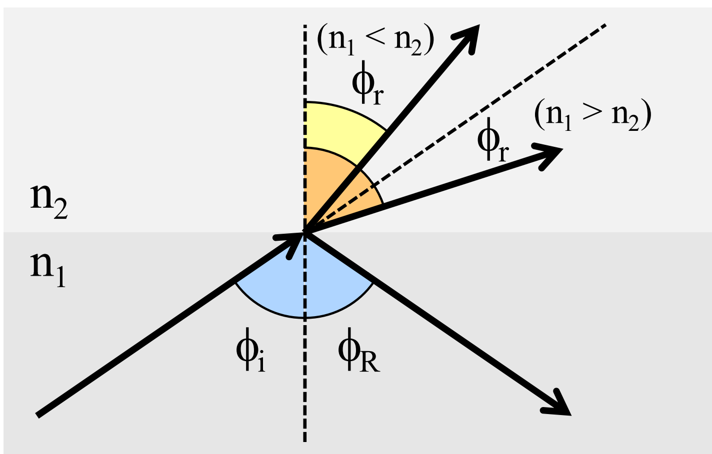
$$\frac{\delta}{\delta r}\left[\int_A^B n(r)\partial\ell\right] = 0$$

4. Light rays travel along the path of least time (**Fermat's Principle**)

REFLECTION & REFRACTION (I)

Reflection Law

$$\phi_R = \phi_i$$



Snell's Law

$$n_1 \sin\phi_i = n_2 \sin\phi_r$$

$$n_1 \cos\theta_i = n_2 \cos\theta_r$$

External Refraction (n_1<n_2) $\Rightarrow$ $\phi_r < \phi_i$

Internal Refraction (n_1>n_2) $\Rightarrow$ $\phi_r > \phi_i$

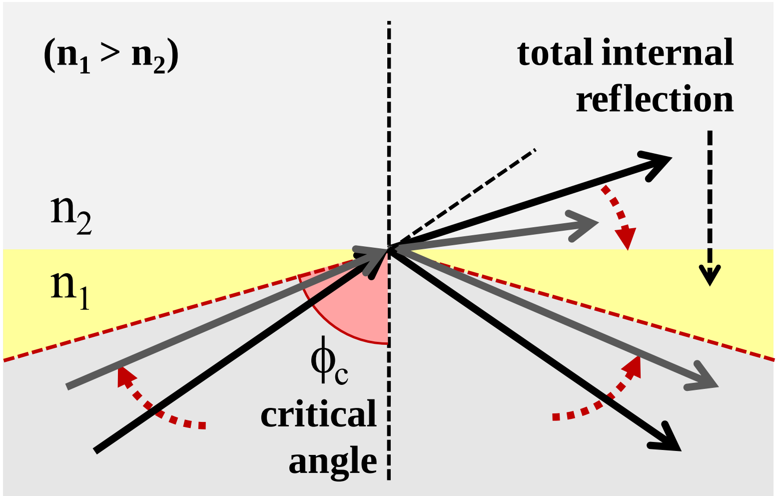
REFLECTION & REFRACTION (II)

Total Internal Reflection

$$n_1 \cdot \sin(\phi_i) = n_2 \cdot \sin(\phi_r)$$

$$\sin(\phi_r) = \boxed{\frac{n_1}{n_2}\sin(\phi_i)}$$

$\Downarrow$ **(n_1>n_2)**



$$\begin{cases} \sin(\phi_i) \in [0, n_2/n_1] & \rightarrow \quad \sin(\phi_r) \le 1 \\ \sin(\phi_i) \in (n_2/n_1, 1] & \rightarrow \quad \cancel{\sin(\phi_r) > 1} \end{cases}$$

$\Rightarrow$ $(P_r = 0)$ **All light remains in medium n_1**

FIBER-OPTIC COMMUNICATIONS

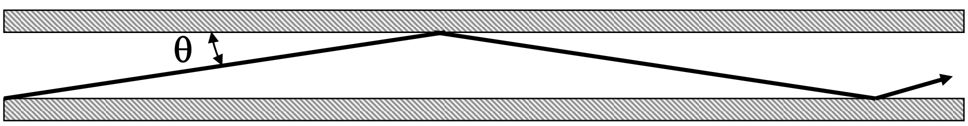
REFLECTION & REFRACTION (II)

Critical Angle $\Rightarrow$ $\sin(\phi_c) = \frac{n_2}{n_1} \rightarrow \boxed{\phi_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)}$

Relative Refractive Index $\Rightarrow$ $\Delta \equiv \frac{n_1 - n_2}{n_1}$

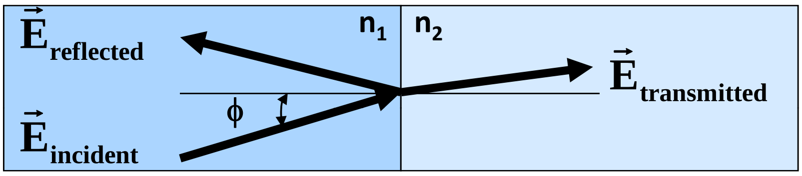
Paraxial Optics

Small propagation angles $\Rightarrow$ $\sin\theta \approx \theta$



REFLECTION & REFRACTION (III)

Energy interchange



Reflectance ⇨ $$\rho = \frac{\vec{E}_{reflected}}{\vec{E}_{incident}}$$

Reflectivity ⇨ $$R = \frac{P_{reflected}}{P_{incident}} = |\rho|^2 \xrightarrow{\text{normal incidence}} \boxed{R = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2}$$

Transmitance ⇨ $$t = \frac{\vec{E}_{transmitted}}{\vec{E}_{incident}}$$

Transmitivity ⇨ $$T = \frac{P_{transmitted}}{P_{incident}} = 1 - R$$

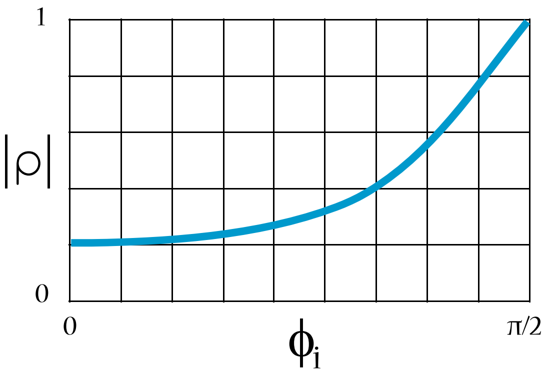
REFLECTION & REFRACTION (III)

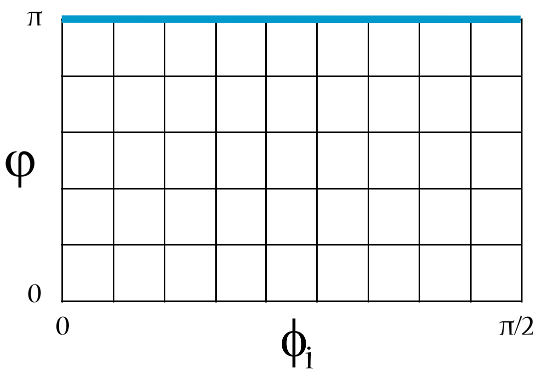
Amplitude Reflectance

$$\boxed{\rho = \frac{\vec{E}_{reflected}}{\vec{E}_{incident}} = \frac{n_1 \cos\phi_i - n_2 \cos\phi_r}{n_1 \cos\phi_i + n_2 \cos\phi_r}} \xrightarrow[\phi_i \approx \phi_r]{} R = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2$$

External Reflection ($n_1<n_2$)

$$n_1 < n_2, \phi_i < \phi_r \rightarrow \rho < 0 \rightarrow \begin{cases} |\rho|^2 \in \left[\left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2, 1\right] \\ \varphi = \pi \end{cases}$$





REFLECTION & REFRACTION (III)

Internal Reflection ($n_1>n_2$)

$$\rho=\frac{n_1\cos\phi_i-n_2\cos\phi_r}{n_1\cos\phi_i+n_2\cos\phi_r}=\frac{n_1\cos\phi_i-jn_2\left(\dfrac{\sin^2\phi_i}{\sin^2\phi_c}-1\right)^{\frac{1}{2}}}{n_1\cos\phi_i+jn_2\left(\dfrac{\sin^2\phi_i}{\sin^2\phi_c}-1\right)^{\frac{1}{2}}}$$

$$\sin\phi_r=\frac{n_1}{n_2}\sin\phi_i\rightarrow\cos\phi_r=\left(1-\left(\frac{n_1}{n_2}\right)^2\sin^2\phi_i\right)^{\frac{1}{2}}=\left(1-\frac{\sin^2\phi_i}{\sin^2\phi_c}\right)^{\frac{1}{2}}=j\left(\frac{\sin^2\phi_i}{\sin^2\phi_c}-1\right)^{\frac{1}{2}}$$

$$\sin\phi_c=\frac{n_2}{n_1}$$

REFLECTION & REFRACTION (III)

Internal Reflection ($n_1>n_2$)

$$\rho=\frac{n_1\cos\phi_i-n_2\cos\phi_r}{n_1\cos\phi_i+n_2\cos\phi_r}=\frac{n_1\cos\phi_i-jn_2\left(\dfrac{\sin^2\phi_i}{\sin^2\phi_c}-1\right)^{\frac{1}{2}}}{n_1\cos\phi_i+jn_2\left(\dfrac{\sin^2\phi_i}{\sin^2\phi_c}-1\right)^{\frac{1}{2}}}$$

$$\phi_i\le\phi_c\rightarrow\begin{cases}|\rho|^2=\left(\dfrac{n_1\cos\phi_i-n_2\left(1-\dfrac{\sin^2\phi_i}{\sin^2\phi_c}\right)^{\frac{1}{2}}}{n_1\cos\phi_i+n_2\left(1-\dfrac{\sin^2\phi_i}{\sin^2\phi_c}\right)^{\frac{1}{2}}}\right)^2\in\left[\left(\dfrac{n_1-n_2}{n_1+n_2}\right)^2,1\right]\\ \arg\{\rho\}=0\end{cases}$$

REFLECTION & REFRACTION (III)

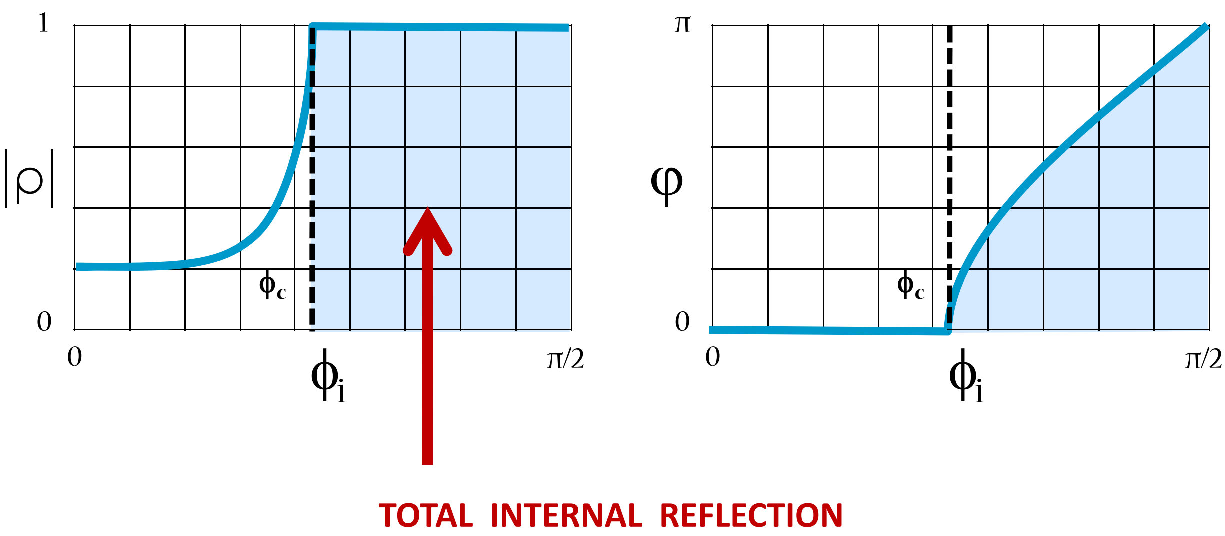
Internal Reflection $(n_1>n_2)$

$$\rho = \frac{n_1\cos\phi_i - n_2\cos\phi_r}{n_1\cos\phi_i + n_2\cos\phi_r} = \frac{n_1\cos\phi_i - jn_2\left(\dfrac{\sin^2\phi_i}{\sin^2\phi_c}-1\right)^{\frac{1}{2}}}{n_1\cos\phi_i + jn_2\left(\dfrac{\sin^2\phi_i}{\sin^2\phi_c}-1\right)^{\frac{1}{2}}}$$

$$\phi_i \geq \phi_c \rightarrow \begin{cases} |\rho|^2 = 1 \\ \arg\{\rho\} = 2\mathrm{tg}^{-1}\left\{\dfrac{n_2\left(\dfrac{\sin^2\phi_i}{\sin^2\phi_c}-1\right)^{\frac{1}{2}}}{n_1\cos\phi_i}\right\} = 2\mathrm{tg}^{-1}\left\{\dfrac{\left(\sin^2\phi_i - \sin^2\phi_c\right)^{\frac{1}{2}}}{\cos\phi_i}\right\} \in [0,\pi] \end{cases}$$

REFLECTION & REFRACTION (III)

Internal Reflection $(n_1>n_2)$



$\Delta\lambda$ - Δf RELATIONSHIP

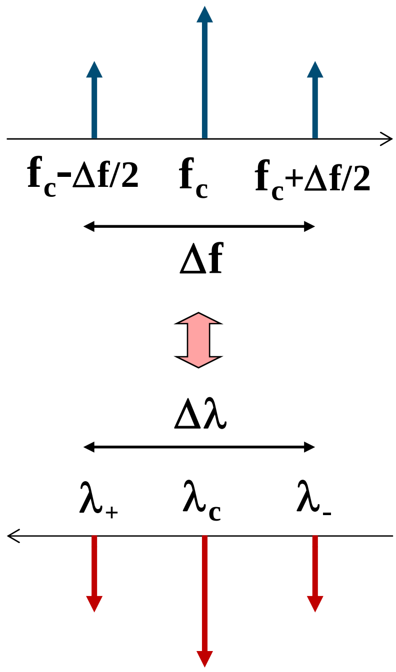
$$\lambda = \frac{c}{f} \rightarrow \partial\lambda = -\frac{c}{f^2}\partial f \rightarrow \int_{\lambda_-}^{\lambda_+}\partial\lambda = -\int_{f_c-\Delta f/2}^{f_c+\Delta f/2}\frac{c}{f^2}\partial f$$

$$\Delta\lambda = c\cdot\frac{1}{f}\Bigg|_{f_c-\Delta f/2}^{f_c+\Delta f/2} = c\left(\frac{1}{f_c+\frac{\Delta f}{2}} - \frac{1}{f_c-\frac{\Delta f}{2}}\right)$$

$$\Delta\lambda = c\cdot\frac{\Delta f}{\left(f_c+\frac{\Delta f}{2}\right)\left(f_c-\frac{\Delta f}{2}\right)} = c\cdot\frac{\Delta f}{f_c^2-\left(\frac{\Delta f}{2}\right)^2}$$

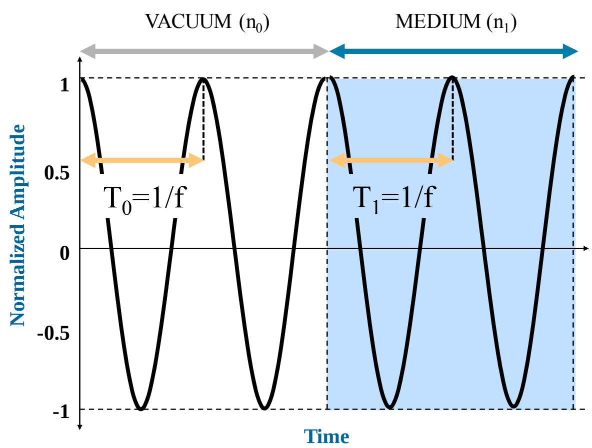
$\Delta f << f_c$ $\Rightarrow$ $\boxed{\Delta\lambda \approx \frac{c}{f_c^2}\Delta f}$

0.8 nm $\approx$ 100 GHz
WDM grid (ITU)



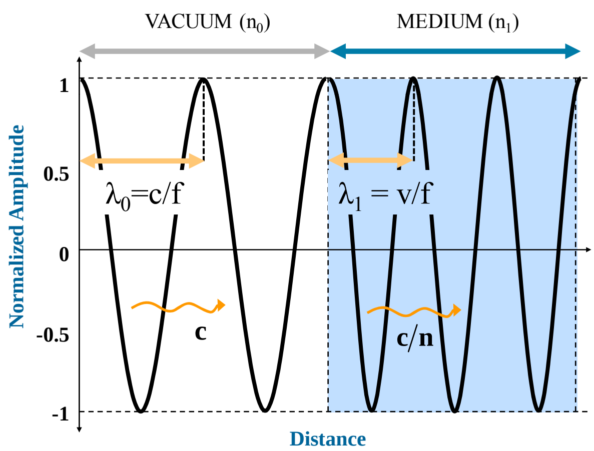
Frequency

Evolution of field amplitude over time in a fixed distance



Wavelength

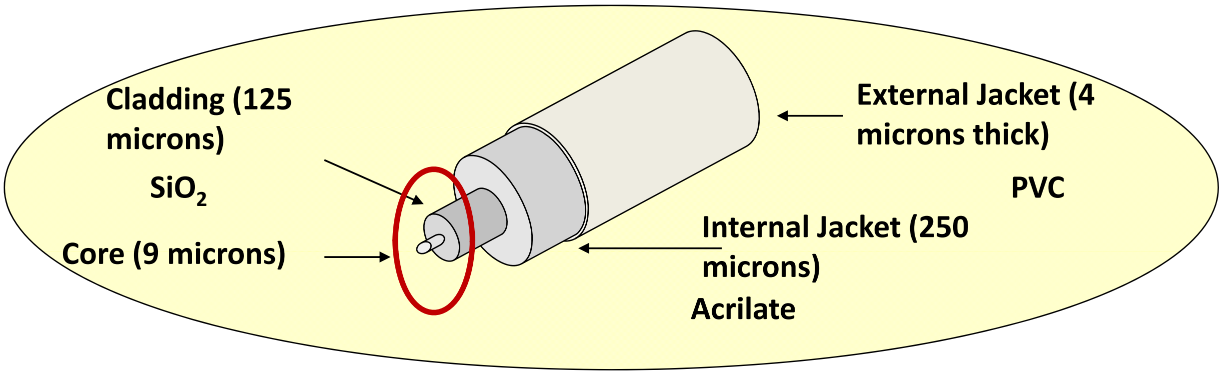
Evolution of field amplitude over distance in a fixed time

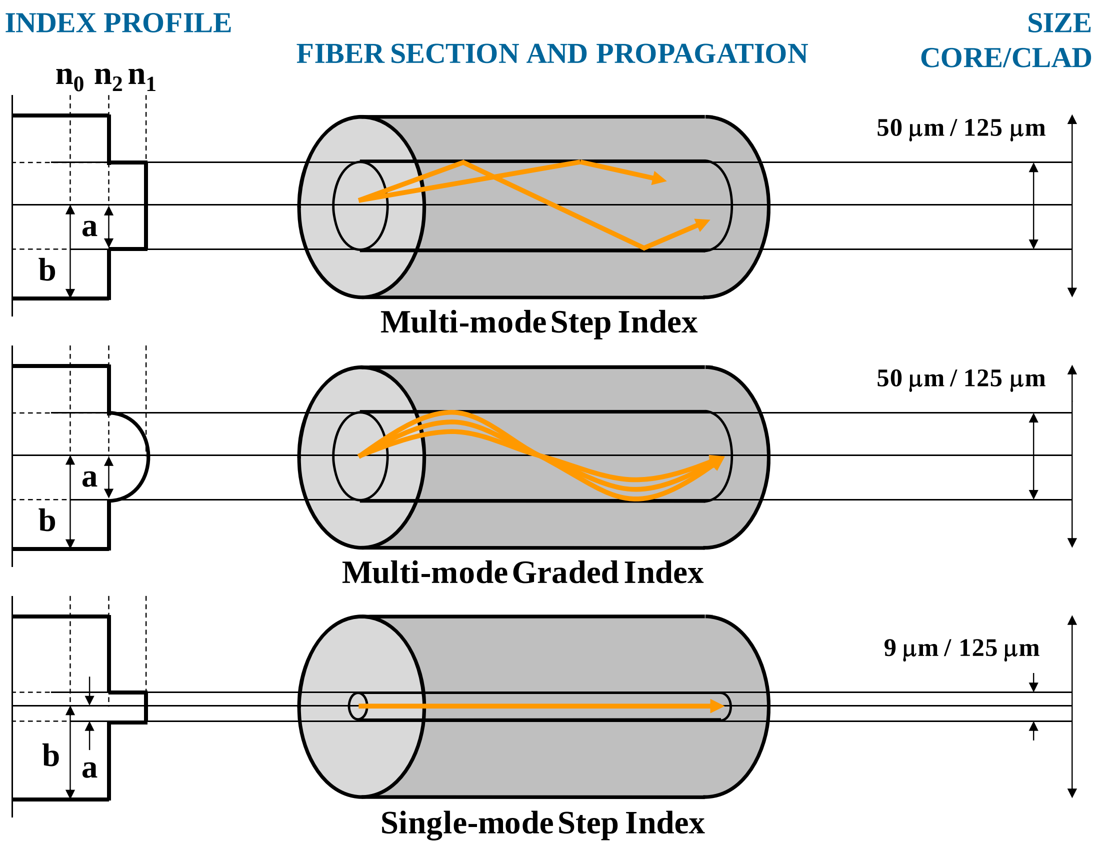


TYPES OF FIBERS

Definintion

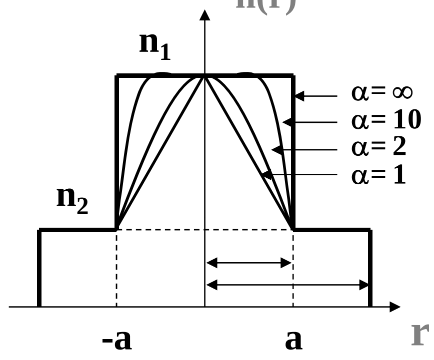
"An Optical Fiber is a dielectric cilindrical waveguide capable of guiding light at certain frequencies with low attenuation and high bandwidth"





GRaded INdex Fibers (I)

$$n(r)=\begin{cases} n_1\left(1-\dfrac{n_1^2-n_2^2}{n_1^2}(r/a)^{\alpha}\right)^{\frac{1}{2}} & r<a \\ n_1\left(1-\dfrac{n_1^2-n_2^2}{n_1^2}\right)^{\frac{1}{2}}=n_2 & r\geq a \end{cases}$$



r: radial distance

α: profile parameter

- $\alpha=\infty$ ⇨ **Step Index**
- $\alpha=1$ ⇨ **Triangular Index**
- $\alpha=2$ ⇨ **Parabolic Index**

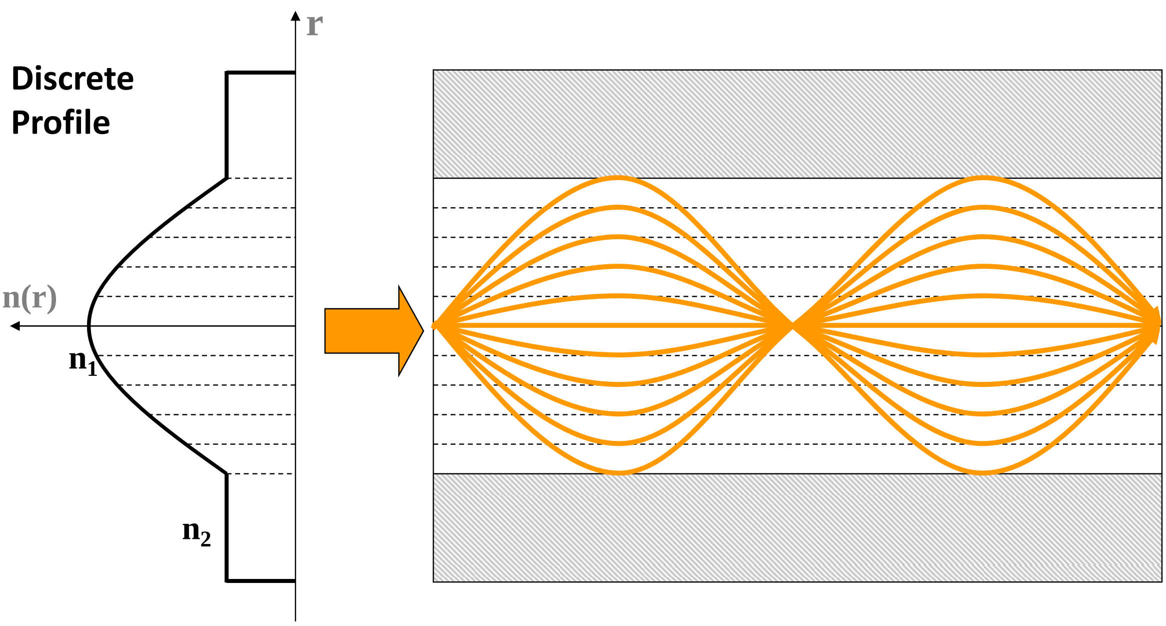
$$\frac{n_1^2-n_2^2}{n_1^2}\approx 2\frac{n_1-n_2}{n_1}\equiv 2\Delta \quad \Rightarrow \quad n(r)\approx\begin{cases} n_1\left(1-2\Delta(r/a)^{\alpha}\right)^{\frac{1}{2}} & r<a \\ n_1(1-2\Delta)^{\frac{1}{2}}=n_2 & r\geq a \end{cases}$$

Paraxial O. ($n_1 \approx n_2$)

Δ: relative refractive index

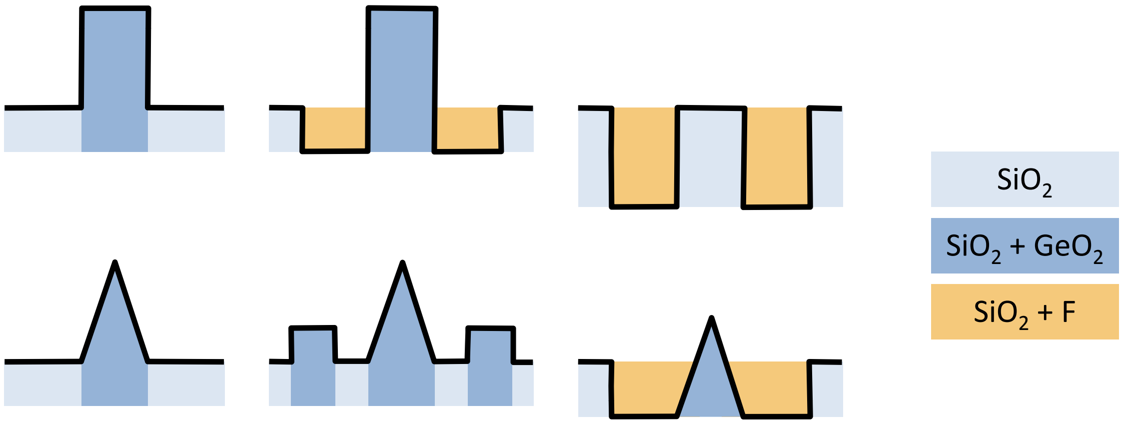
GRaded INdex Fibers (I)

All rays have the same delay no matter the incidence angle (same optical path). The propagation is synchronized.



Advanced Index Profiles

standard fibers



dispersion shifted fibers

CHARACTERISTIC PARAMETERS

Static Parameters — No propagation distance dependence

Geometrical → **Core/Cladding Diameter**: Together with index profile, numerical aperture, and frequency; determines the SM/MM behaviour of the fiber

Optical
- **Index Profile: n(r)**: Transversal evolution of the fiber core's refractive index
- **Numerical Aperture: NA**: Core-cladding refractive indices quadratic difference which determines the acceptance angle of the fiber

Dynamic Parameters — Propagation distance dependence

Attenuation: Optical Power reduction per unit length

Dispersion (Bandwidth): Optical pulses spreading per unit length

ESTANDARDIZATION

Multimode Fibers

MULTIMODE FIBER *62,5/125*	**ISO/IEC 793**
Numerical Aperture	NA = 0,275 (+/- 0,015)
Index Profile	Step index
Relative Refractive Index	1.90 %
Core Diameter	62,5 µm (+/- 3 µm)
Cladding Diameter	125 µm (+/- 1 µm)
Silicon Coating	245 µm (+/- 10 µm)
Operation Wavelenght	850 & 1300 nm
Attenuation @ 850 nm	3 - 3,2 dB/km
Attenuation @ 1300 nm	0,7 - 0,8 dB/km
Bandwidth @ 850 nm	200 - 300 MHz/Km
Bandwidth @ 1300 nm	400 - 600 MHz/Km

MULTIMODE FIBER *50/125*	**ITU-T G.651**
Numerical Aperture	NA= 0,18 a 0,24 (+/- 10%)
Index Profile	Graded index
Average Refractive Index	1,43
Core Diameter	50 µm (+/- 3 µm)
Cladding Diameter	125 µm (+/- 3 µm)
Silicon Coating	245 µm (+/- 10 µm)
Concentricity Error	6%
Core Circularity Error	6%
Cladding Circularity Error	2%
Attenuation @ 850 nm	2,7 - 3 dB/km
Attenuation @ 1300 nm	0,7 - 0,8 dB/km
Bandwidth @ 850 nm	300 - 500 MHz/Km
Bandwidth @ 1300 nm	500 - 1000 MHz/Km

Singlemode Fibers

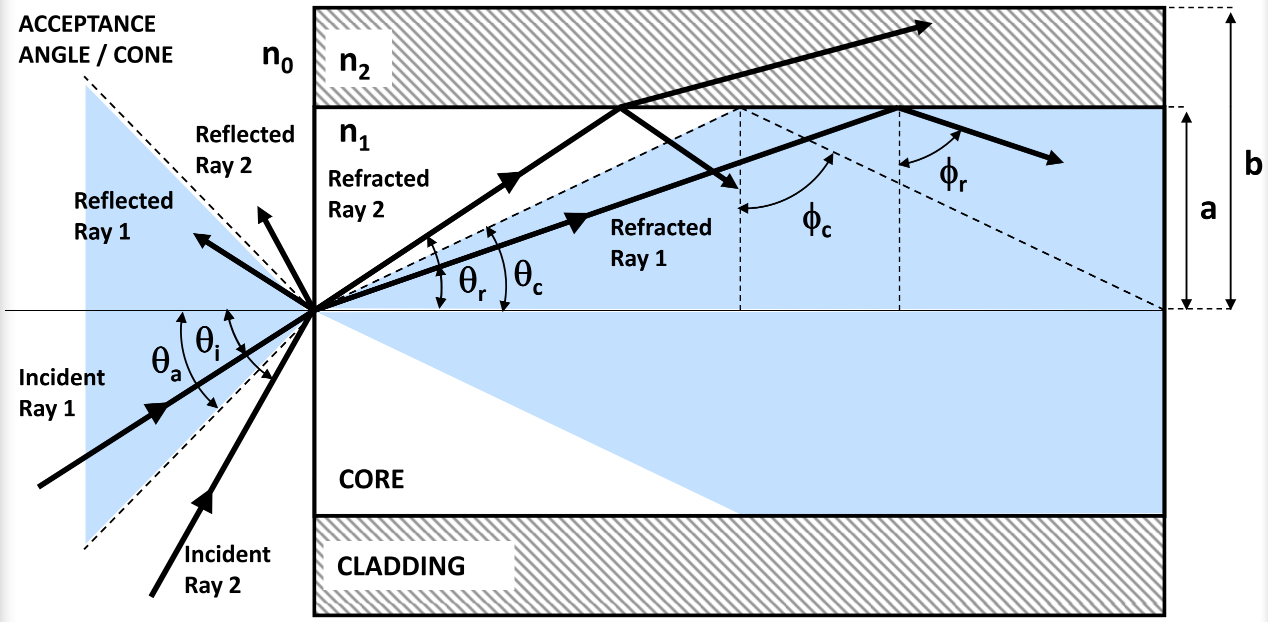
SINGLE MODE FIBER "STAN*DARD"*	**ITU-T G.652**
Cutting Wavelength	1,18 - 1,27 µm
Modal Field diameter	9,3 (8 - 10) µm (+/- 10%)
Cladding Diameter	125 µm (+/- 3 µm)
Silicon Coating	245 µm (+/- 10 µm)
Cladding Circularity Error	2%
Modal Field Concentricity Error	1µm
Attenuation @ 1300 nm	0,4 - 1 dB/km
Attenuation @ 1550 nm	0,25 - 0,5 dB/km
Chromatic Disp. @ 1285-1330 nm	3,5 ps/km/nm
Chromatic Disp. @ 1270-1340 nm	6 ps/km/nm
Chromatic Disp. @ 1550 nm	20 ps/Km/nm

SINGLE MODE FIBER "DISPERSION SHIFTED"	**ITU-T G.653**
Cutoff Wavelength	1,05 - 1,15 µm
Modal Field diameter	8 (7 - 8,3) µm (+/- 10%)
Cladding Diameter	125 µm (+/- 3 µm)
Silicon Coating	245 µm (+/- 10 µm)
Cladding Circularity Error	2%
Modal Field Concentricity Error	1µm
Attenuation @ 1300 nm	< 1 dB/Km
Attenuation @ 1550 nm	0,25 - 0,5 dB/Km
Chromatic Disp. @ 1525-1575 nm	3,5 ps/Km/nm

SINGLE MODE FIBER "MINIMUM ATTENUATION"	**ITU-T G.654**
Modal Field diameter	125 µm (+/- 3 µm)
Cladding Circularity Error	2%
Modal Field Concentricity Error	1µm
Silicon Coating	245 µm (+/- 10 µm)
Attenuation @ 1550 nm	0,15 - 0,25 dB/Km
Chromatic Disp. @ 1550 nm	20 ps/Km/nm

SINGLE MODE FIBER "NON-ZERO DISPERSION SHIFTED"	**ITU-T G.655**
Modal Field diameter	8,4 µm (+/- 0,6 µm)
Cladding Diameter	125 µm (+/- 1 µm)
Cutoff Wavelength	1260 nm
Attenuation @ 1550 nm	0,22 - 0,30 dB/Km
Chromatic Disp. @ 1550 nm	4,6 ps/Km/nm
Non-Zero Dispersion Region	1540 - 1560 nm

PROPAGATION IN O.F.



TOTAL INTERNAL REFLECTION

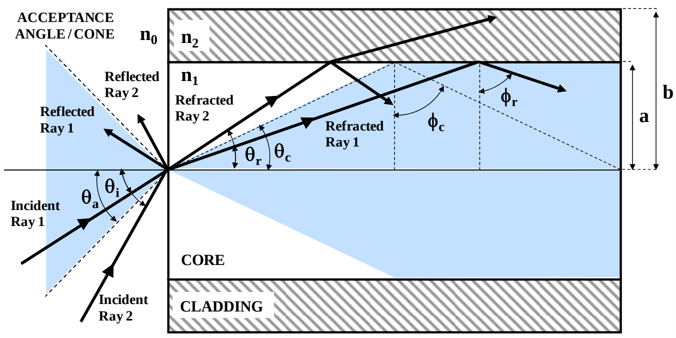
Critical Angle

$$\boxed{\sin\phi_c = \frac{n_2}{n_1}}$$

$$\theta_c = \frac{\pi}{2} - \phi_c$$

$$\sin\theta_c = \cos\phi_c = \left(1 - \left(\frac{n_2}{n_1}\right)^2\right)^{1/2}$$

$$n_0 \sin\theta_a = n_1 \sin\theta_c$$



$$n_0 < n_2 < n_1$$

$$\sin\theta_a = \frac{n_1}{n_0}\sin\theta_c = \frac{n_1}{n_0}\cos\phi_c = \frac{n_1}{n_0}\left(1 - \left(\frac{n_2}{n_1}\right)^2\right)^{1/2} = \boxed{\frac{(n_1^2 - n_2^2)^{1/2}}{n_0}}$$

Acceptance Angle

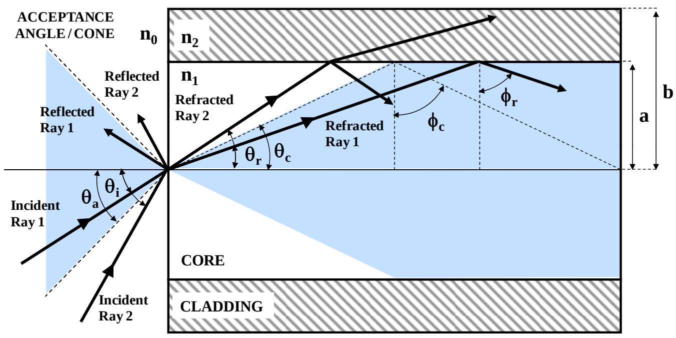
TOTAL INTERNAL REFLECTION

Numerical Aperture

$$NA \equiv (n_1^2 - n_2^2)^{\frac{1}{2}}$$

$$\Delta \equiv \frac{n_1 - n_2}{n_1} \rightarrow NA \approx n_1 (2\Delta)^{1/2}$$

Paraxial Optics

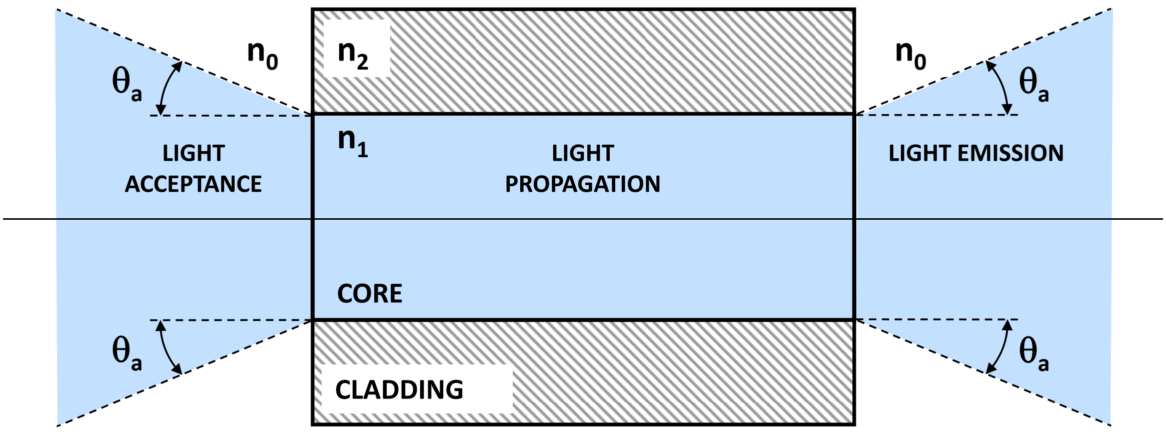


$$n_0 < n_2 < n_1$$

Numerical Aperture quantifies the ability of the optical fiber to accept light coming from outside the core. In this sense, we want NA to be a high value.

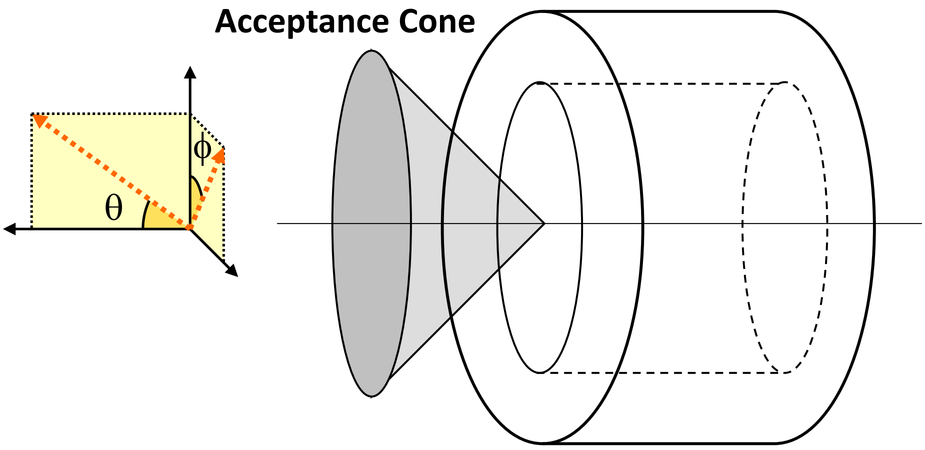
TOTAL INTERNAL REFLECTION

Light Acceptance / Emission



FIBER LIGHT COUPLING

Acceptance Solid Angle



$$\Omega_c = \iint_{\theta_a} \partial\Omega$$

$$\Omega_c = \int_0^{2\pi}\int_0^{\theta_a} \sin\theta\cdot\partial\theta\cdot\partial\phi = 2\pi\left(1-\cos\theta_a\right) = 4\pi\cdot\sin^2\left(\frac{\theta_a}{2}\right) \approx \pi\cdot\theta_a^2$$

$\cos 2\theta = 1 - 2\sin^2\theta$ — Paraxial Optics

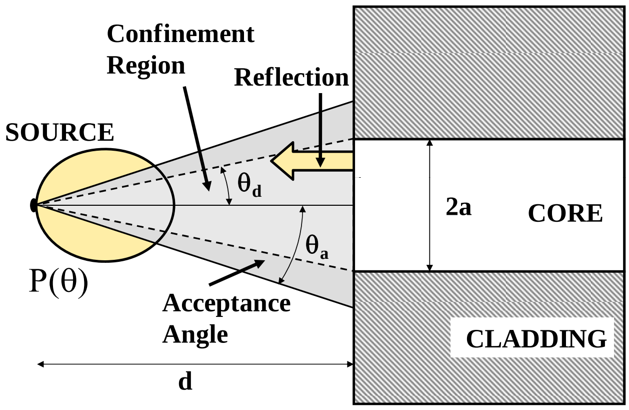
Coupling Efficiency (punctual source)

$$\boxed{\eta_c \equiv \frac{P_{IN}}{P_T}} \longrightarrow L_{dB} \equiv -10\cdot\log\left[\eta_c\right]$$

θ_a → Acceptance Angle

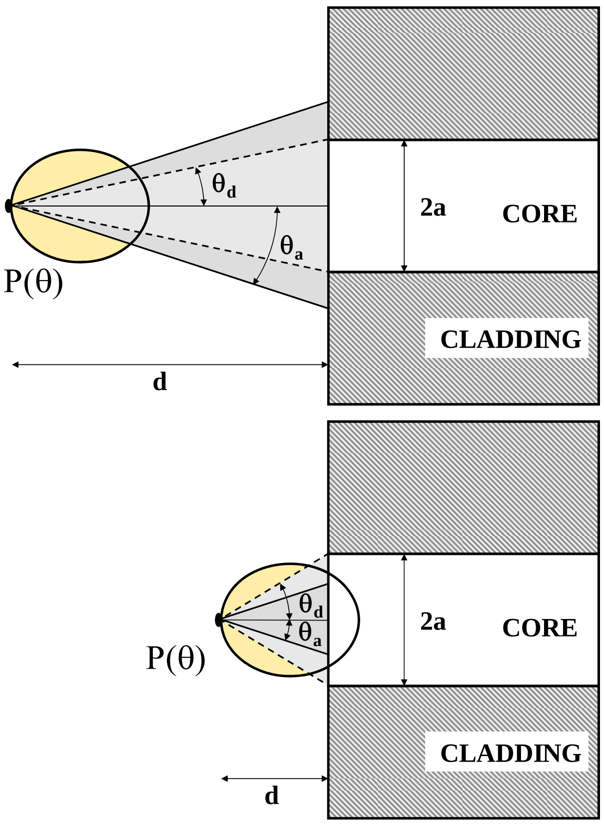
θ_d → Vision Angle

$$\mathrm{tg}\,\theta_d = \frac{a}{d} \qquad R = \left(\frac{n_0 - n_1}{n_0 + n_1}\right)^2$$



$$P_T = \int_0^{2\pi}\int_0^{\pi/2} P(\theta)\sin\theta\cdot\partial\theta\cdot\partial\phi$$

$$P_{IN} = \int_0^{2\pi}\int_0^{\theta}\left(1-R\right)P(\theta)\sin\theta\cdot\partial\theta\cdot\partial\phi$$



FAR SOURCE

distance limit

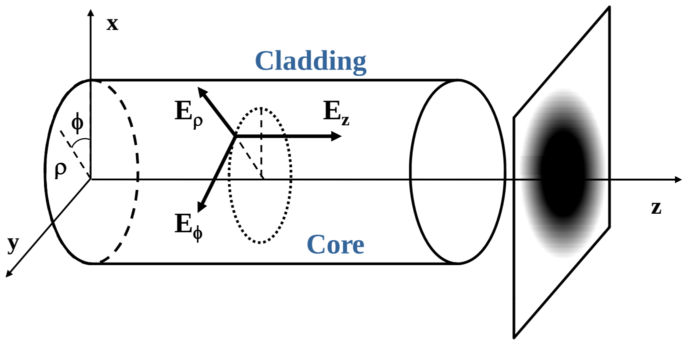
$$\theta_d < \theta_a \rightarrow \theta = \theta_d$$

CLOSE SOURCE

NA limit

$$\theta_a < \theta_d \rightarrow \theta = \theta_a$$

TRANSVERSAL PROPAGATION MODES



❑ A Transversal Propagation Mode of an electromagnetic wave travelling along a waveguide is a particular field distribution measured in a plane perpendicular to the propagation axis.

❑ Each mode belongs to a particular solution of Maxwell's equations inside the waveguide structure given the boundary conditions imposed.

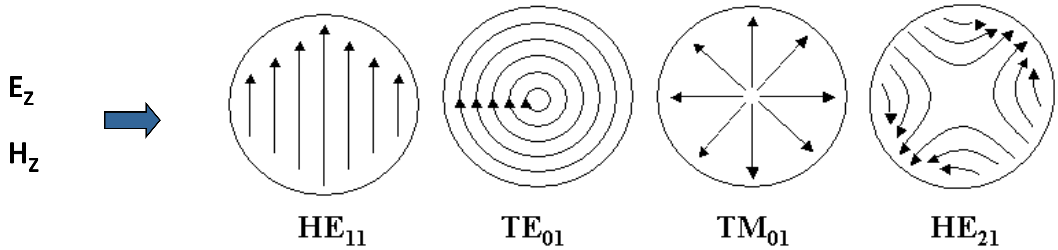
TRANSVERSAL MODES FAMILIES

- **TE (transversal electric)** – The electric field is zero in the propagation direction.
- **TM (transversal magnetic)** – The magnetic field is zero in the propagation direction.
- **TEM (transversal electromagnetic)** – Both the electric and the magnetic field are zero in the propagation direction.
- **HE-EH (hybrids)** – Both the electric and the magnetic field are non-zero in the propagation direction.

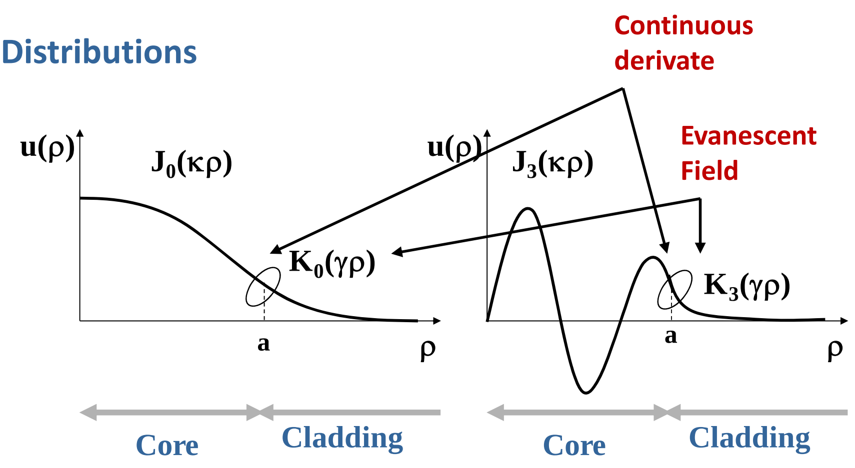
Laser radiation → TEM

OF propagation → HE-EH

Field Distributions

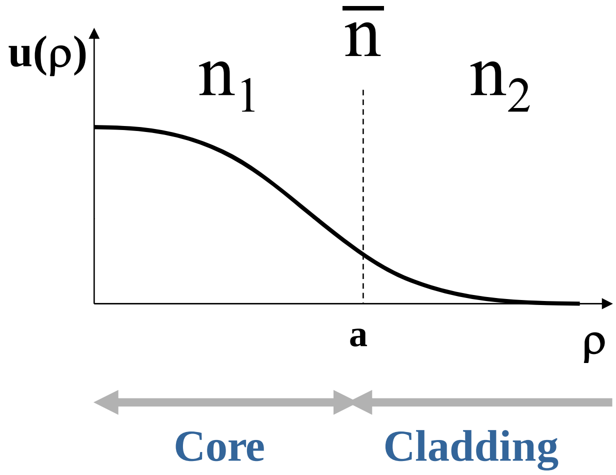


Spacial Distributions



Mode Index (Efective Index)

$$n_2 < \bar{n} < n_1$$



Normalized Propagation Constant

$$b \equiv \frac{\bar{n} - n_2}{n_1 - n_2}$$

Normalized Frequency

$$\boxed{V \equiv 2\pi \frac{a}{\lambda} NA \approx 2\pi \frac{a}{\lambda} n_1 \sqrt{2\Delta}}$$

NA-a trade-off

$n_1 \approx n_2$ paraxial optics

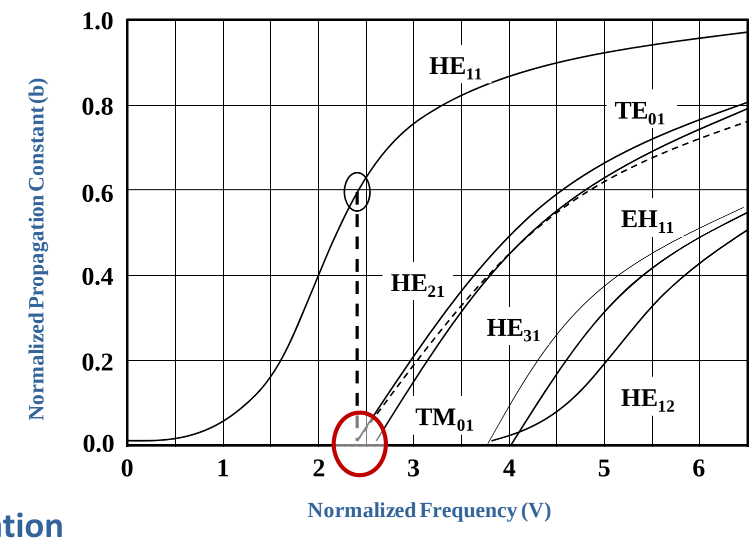
Single Mode condition

$$\boxed{V < 2.405 = V_c}$$

Trade-off

$$V \downarrow \rightarrow \text{single mode} \uparrow$$

$$V \uparrow \rightarrow \% P_{core} / P_{clad} \uparrow$$



Number of Modes approximation

$$M_{SI} \approx \frac{V^2}{2} \quad \leftarrow \text{Step Index Fiber}$$

$$M_{GI} \approx \frac{V^2}{4} \quad \leftarrow \text{GRIN Fiber}$$

Cutoff Wavelength

$$\boxed{\lambda \geq \lambda_c \approx 2\pi \frac{a}{V_c} n_1 \sqrt{2\Delta}}$$

Linear Polarization Modes

$\Delta << 1$

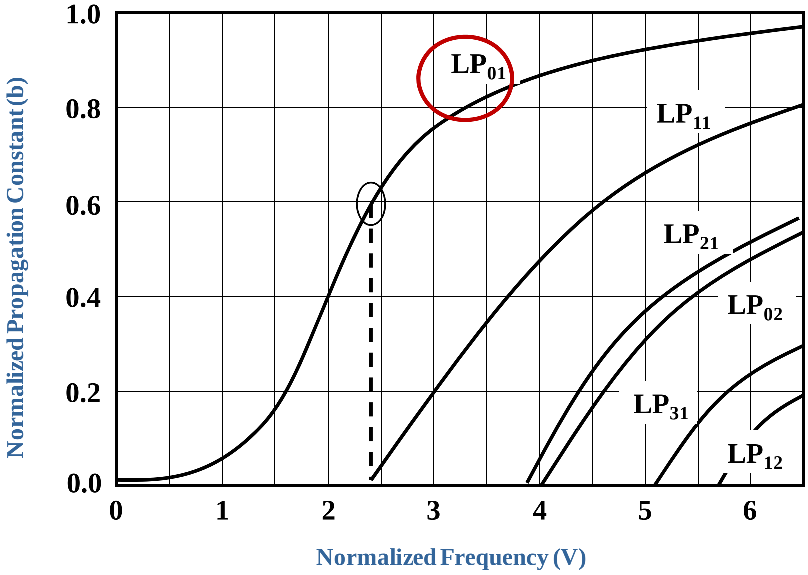
Paraxial Optics

$E_z = H_z = 0$ **TEM**

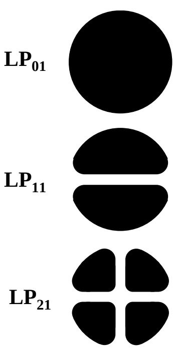
$\mathbf{LP_{ij}}$

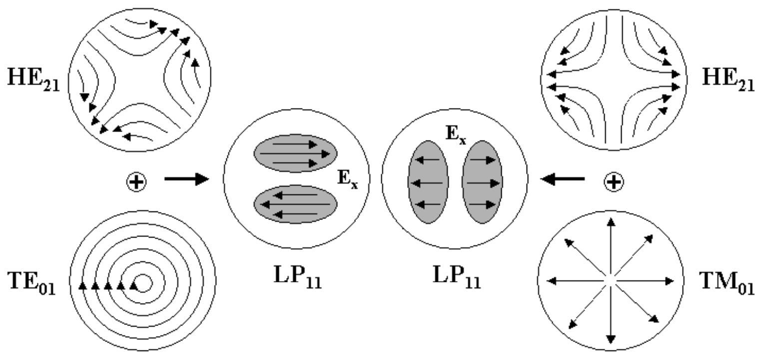
2i revolution maxima

j radial maxima



1. LP_{0n} from HE_{1n}
2. LP_{1n} from TE_{0n}, TM_{0n} & HE_{2n}
3. LP_{mn} ($m \geq 2$) from $HE_{m+1,n}$ & $EH_{m-1,n}$



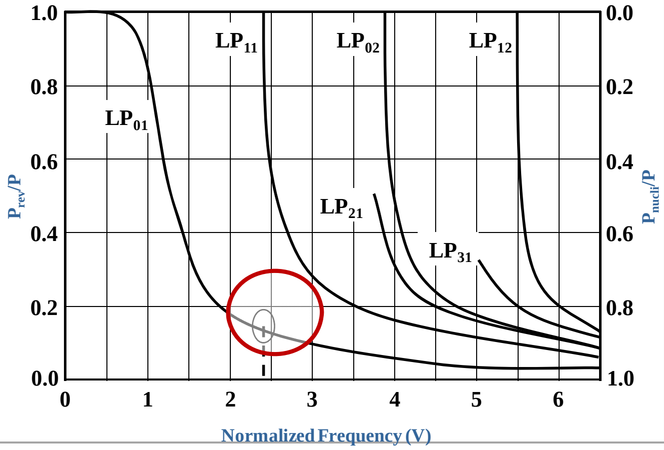


Core/Cladding relative power

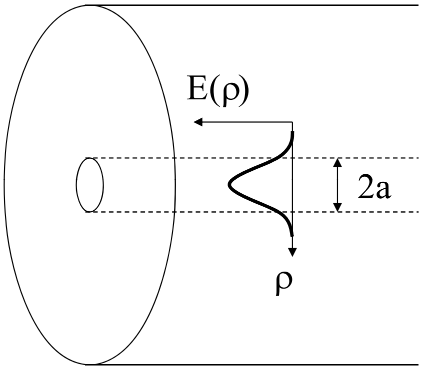
$$\frac{P_{core}}{P} = \left(1 - \frac{\kappa^2}{V^2}\right)\left[1 - \frac{J_m^2(\kappa a)}{J_{m+1}(\kappa a) J_{m-1}(\kappa a)}\right]$$

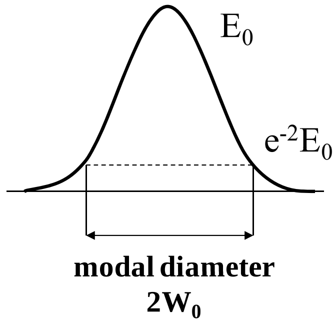
$$\frac{P_{clad}}{P} = 1 - \frac{P_{core}}{P}$$

$$\boxed{\frac{P_{core}}{P} \approx \frac{4}{3\sqrt{M}}}$$



Fundamental Mode (HE_{11} – LP_{01})

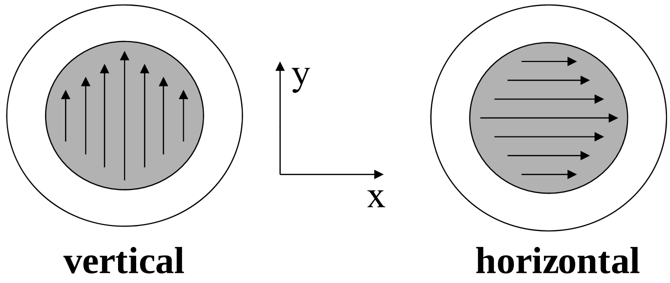




Gaussian Profile

$$E_0 \exp\left[-\frac{1}{2}\left(\frac{\rho}{W_0}\right)^2\right]$$

Polarization Modes ➡

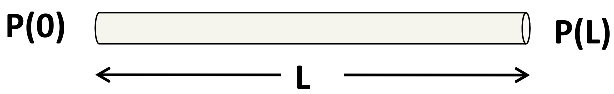


ATTENUATION IN O.F.

Definition

"Light when propagating down the fiber, it experiences an exponential decay of the optical power over the distance as a consequence of absorption and scattering phenomena"

Attenuation Coefficient



$$\frac{P(L)}{P(0)} = 10^{-\alpha L/10}$$

$$\boxed{\alpha = \frac{1}{L} 10 \log\left(P(0)/P(L)\right)}$$

Units: dB/km

$$e^{-\gamma L} \equiv 10^{-\frac{\alpha L}{10}} \rightarrow \gamma L = \frac{\alpha L}{10} \ln 10 \rightarrow \gamma = \frac{\alpha}{10} \ln 10$$

Units: km^{-1}

Material Absorption

"Any material absorbs energy at certain wavelengths corresponding to the electronic and vibrational resonances of the medium"

Intrinsic Absorption Due to the basic material (SiO_2)

$\lambda < 0.4\mu m$ Electronic resonances (ultraviolet)

$\lambda > 7\mu m$ Vibrational resonances (infrared)

Extrinsic Absorption Due to impurities in the material

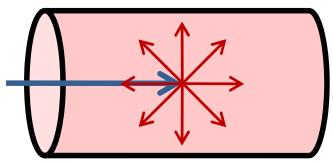
Metals	Fe, Cu, Co, Ni, Mn, Cr ...
Water	OH^+
Dopants	GeO_2, P_2O_5, B_2O_3 ...

Rayleigh Scattering

"Fundamental loss mechanism arising from microscopic density fluctuations which induce small variations of the refractive index of the material"

$$\alpha_R = C/\lambda^4$$

C → 0.7-0.9 (dB/km)μm^4



Waveguide imperfections (Mie Scattering)

"Loss mechanism arising from cilindrical structure imperfections, comparable to the wavelength, of the waveguide"

- ❑ **Irregularities of the core-cladding structure**
- ❑ **Fluctuations of the relative refractive index**
- ❑ **Fluctuations of the core diameter**
- ❑ **Density fluctuations → Stress**
- ❑ **Air bubbles**

The attenuation coefficient depends on the absorption and scattering coefficient of both core and cladding. Given that the cladding penetration depends on the mode so does the attenuation (the bigger the mode order, the bigger the attenuation).

Bending Losses

"When part of the evanescent field is forced to travel at a speed higher than the speed of light due to fiber bending, this energy is radiated outside the propagation mode"

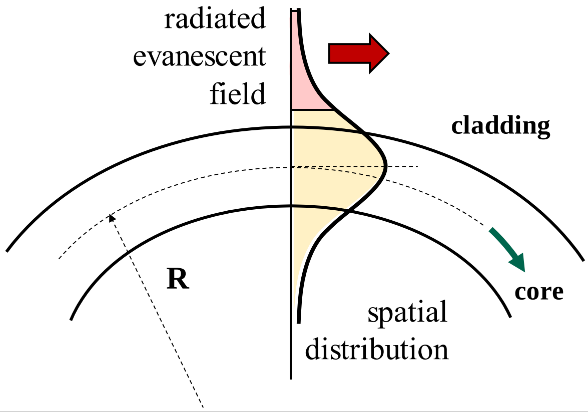
$$\alpha_T = C_1 \exp\left(-C_2 R\right)$$

$$R_c\big|_{MM} \approx \frac{3n_1^2\lambda}{4\pi\left(n_1^2 - n_2^2\right)^{3/2}}$$

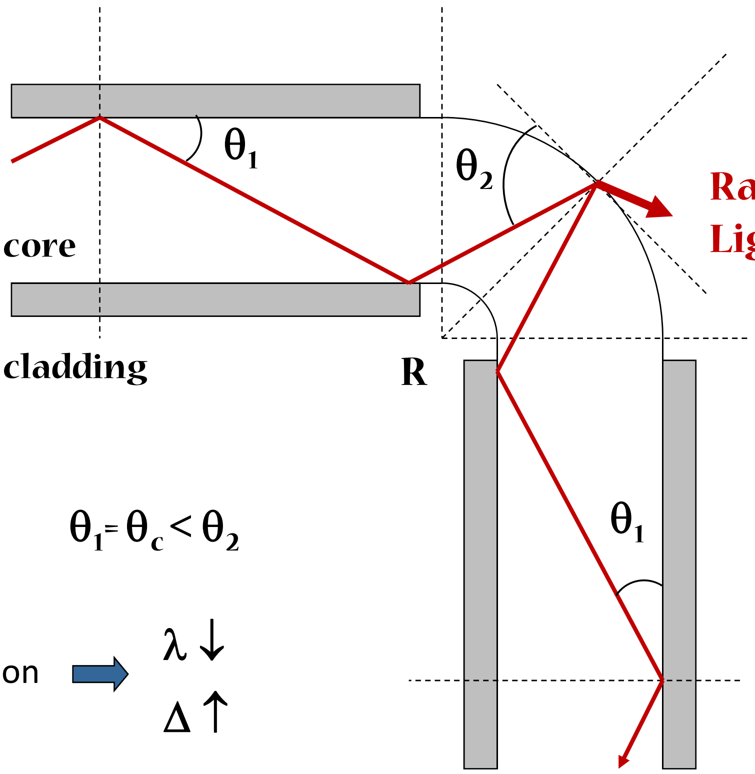
$$R_c\big|_{SM} \approx \frac{20\lambda}{\left(n_1 - n_2\right)^{3/2}}\left(2.748 - 0.996\frac{\lambda}{\lambda_c}\right)^{-3}$$

R: bending radius

R_c: critical bending radius



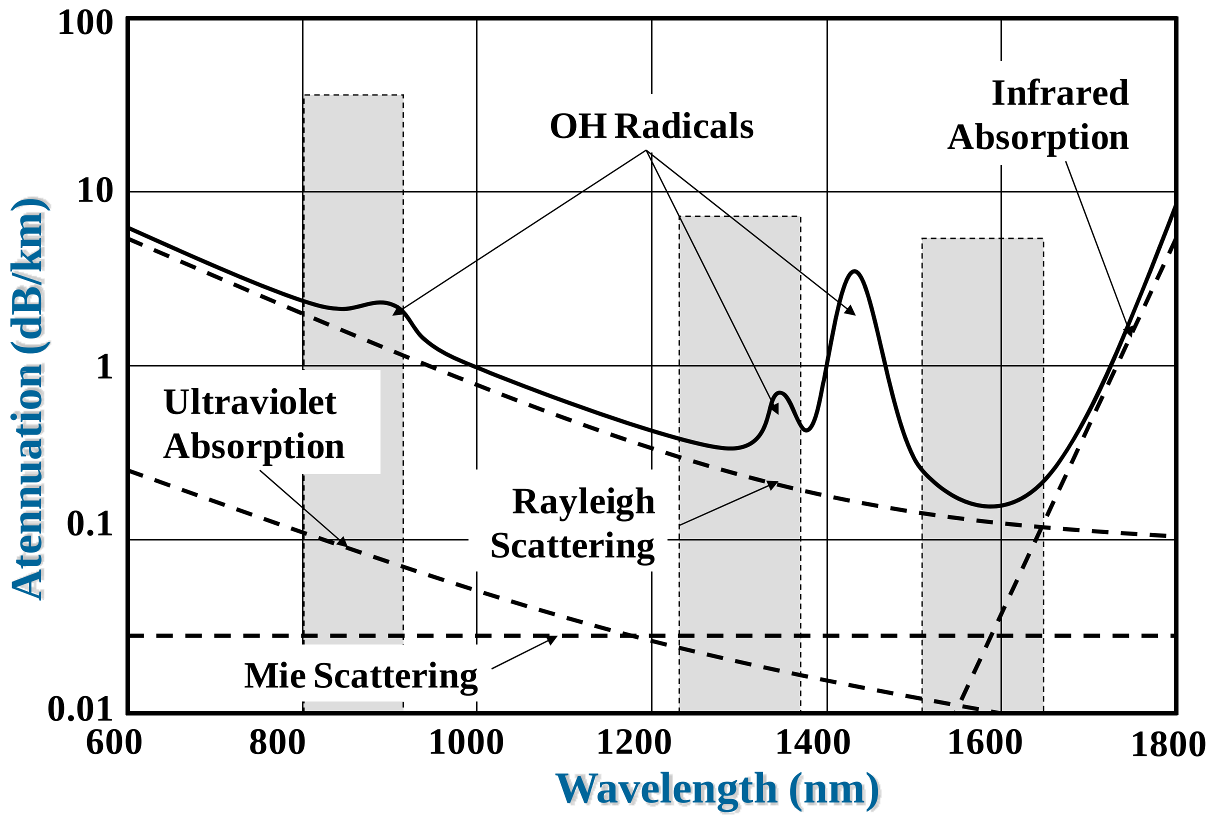
Bending losses intuitive explanation using ray optics



$$\theta_1 = \theta_c < \theta_2$$

solution ➡ $\lambda \downarrow$ $\Delta \uparrow$

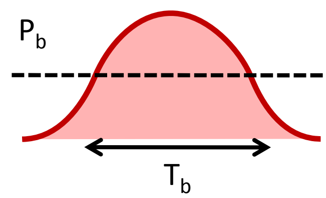
Optical Fiber Attenuation Curve



Distance Limitation due to Attenuation

Bit Energy

$$\boxed{E_b(L) = P_b(L)T_b = \frac{P_b(L)}{R_b}}$$



$$E_b(L) \geq E_{min} \rightarrow \frac{P_b(L)}{R_b} \geq E_{min} \rightarrow \frac{P_b(0)}{R_b} 10^{-\frac{\alpha L}{10}} \geq E_{min} \rightarrow 10^{-\frac{\alpha L}{10}} \geq \frac{E_{min} R_b}{P_b(0)}$$

$$P_b(L) = P_b(0) 10^{-\frac{\alpha L}{10}}$$

$$\rightarrow -\frac{\alpha L}{10} \geq \log\left(\frac{E_{min} R_b}{P_b(0)}\right) \rightarrow \boxed{L \leq \frac{10}{\alpha} \log\left(\frac{P_b(0)}{E_{min} R_b}\right)}$$

Example

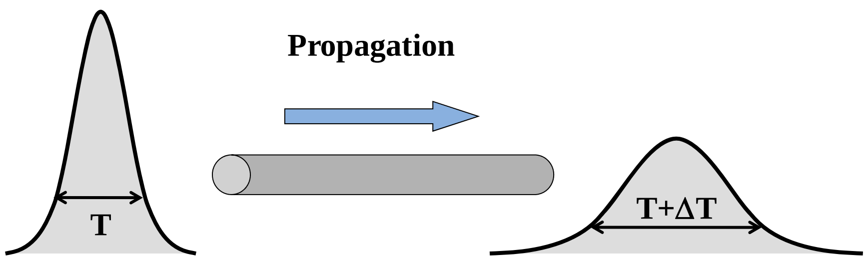
$P_b(0) = 10$ mW
$R_b = 10$ Gb/s
$\alpha = 0.2$ dB/Km
$E_b = 10 \cdot 10^{-18}$ J

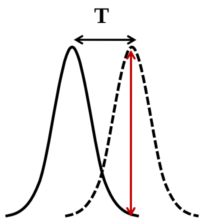
$L_{max} = 250$ Km

DISPERSION IN OPTICAL FIBERS

Definition

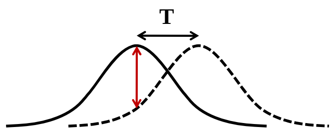
“When an optical pulse propagates through an optical fiber, its energy tends to spread in time producing an increase on the pulse width”





$$\boxed{\Delta T \leq T}$$

Design Criterion



Intermodal (Modal) Dispersion

Each mode propagates at different speed and so a delay among them is induced. Only in Multi-Mode fibers.

Intramodal Dispersion

Group Velocity Dispersion (GVD) or Chromatic Dispersion (CD)

The propagation speed depends on the wavelength and so each spectral component experiences a different delay. Negligible in front of Modal Dispersion.

Material Dispersion

The refractive index of a given material is frequency dependent inducing different propagation speeds for each spectral component.

Waveguide Dispersion

The core-cladding distribution is frequency dependent and so is the effective refractive index.

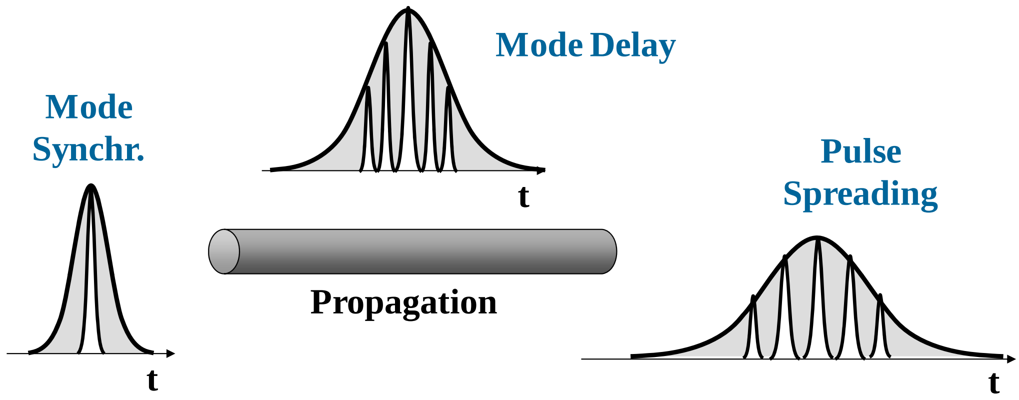
Polarization Mode Dispersion (PMD)

The refractive index is polarization dependent introducing a delay between the x and y component of the light. Negligible in front of Chromatic Dispersion.

DISPERSION IN MULTI-MODE FIBERS

Intermodal (Modal) Dispersion

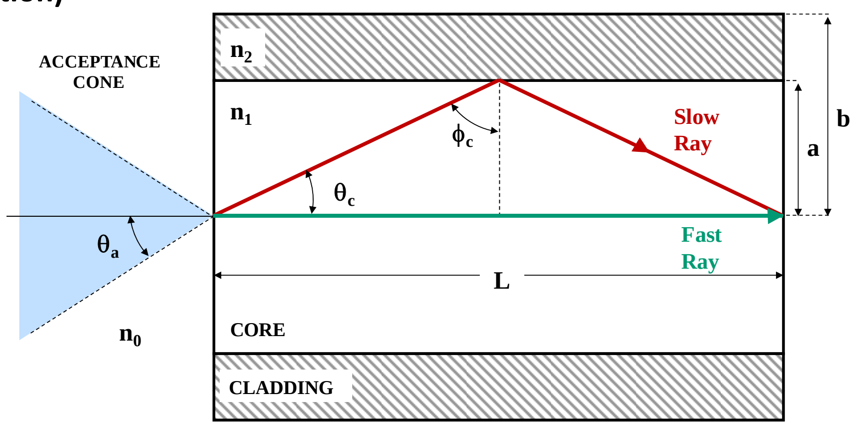
Modal dispersion takes place in multimode fibers as a result of group velocity difference among the propagation modes. The modes are not equaly excited giving a particular profile.



Different path (approximation)

$$t_{min} = \frac{L}{\frac{c}{n_1}} \qquad t_{max} = \frac{\frac{L}{\sin\phi_c}}{\frac{c}{n_1}}$$

$$\sin\phi_c = \frac{n_2}{n_1}$$



$$\tau_{inter} \equiv \frac{t_{max} - t_{min}}{L} = \frac{n_1}{c}\frac{n_1 - n_2}{n_2} = \frac{n_1^2}{n_2 c}\frac{n_1 - n_2}{n_1} = \frac{n_1^2}{n_2 c}\Delta \approx \frac{1}{2n_2 c}NA^2$$

Units: [ns/km]

$$\Delta \equiv \frac{n_1 - n_2}{n_1} \qquad NA \approx n_1 (2\Delta)^{1/2}$$

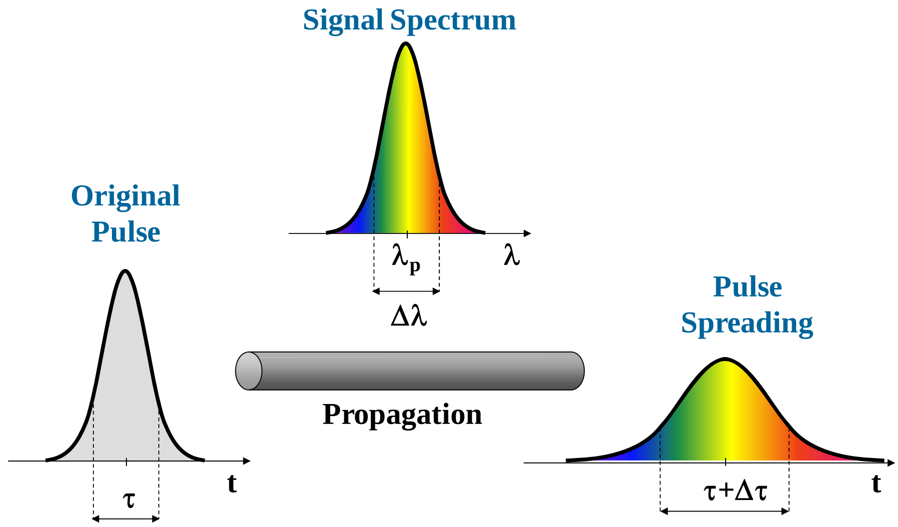
SI Fibers $\boxed{\tau_{mod} = \frac{n_1^2\Delta}{n_2 c} \approx \frac{n_1\Delta}{c}}$

GRIN FIbers
$\alpha_{opt} = 2\cdot(1-\Delta)$ $\boxed{\tau_{mod} \approx \frac{n_1\Delta^2}{8c}}$

DISPERSION IN SINGLE-MODE FIBERS

Group Velocity Dispersion (GVD) – Chromatic Dispersion (CD)

The origin of this kind of dispersion comes from de frequency dependence of the refractive index and so the group velocity.



Group Delay [s/m]

Lossless Medium

$$\lambda = \frac{c}{f} = -\frac{2\pi c}{\omega}$$

$$\tau_g \equiv \frac{\partial \beta}{\partial \omega} = \frac{n}{c} + \frac{\omega}{c}\frac{\partial n}{\partial \omega} = \frac{\partial \beta}{\partial \lambda}\frac{\partial \lambda}{\partial \omega} = \frac{n}{c} - \frac{\lambda}{c}\frac{\partial n}{\partial \lambda}$$

$$\beta \equiv n\frac{\omega}{c} = \frac{2\pi n}{\lambda} \rightarrow \frac{\partial \beta}{\partial \lambda} = -\frac{2\pi n}{\lambda^2} + \frac{2\pi}{\lambda}\frac{\partial n}{\partial \lambda} \qquad \frac{\partial \lambda}{\partial \omega} = -\frac{2\pi c}{\omega^2} = -\frac{\lambda^2}{2\pi c}$$

β: propagation constant

Group Velocity [m/s]

$$v_g \equiv \frac{1}{\tau_g} = \left(\frac{\partial \beta}{\partial \omega}\right)^{-1} = \frac{c}{n + \omega\frac{\partial n}{\partial \omega}} = \frac{c}{n - \lambda\frac{\partial n}{\partial \lambda}} = \frac{c}{n_g}$$

Group Index

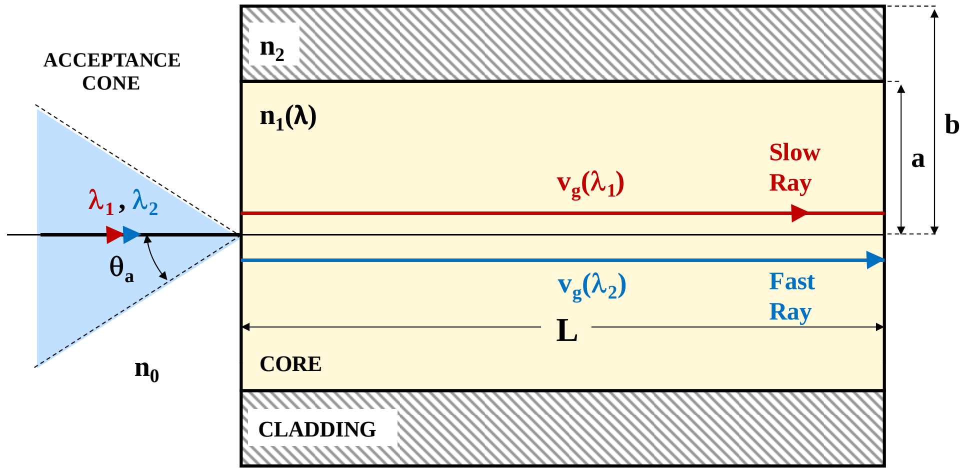
$$n_g \equiv n + \omega\frac{\partial n}{\partial \omega}$$

Phase Velocity [m/s]

$$v_f \equiv \frac{\omega}{\beta} = \frac{c}{n} \xrightarrow{\frac{\partial n}{\partial \lambda}=0} v_f = v_g$$

MATERIAL DISPERSION

Every spectral component experiences a different delay given the different group velocity of each one.



MATERIAL DISPERSION

Pulse Spreading

$$\tau_g \equiv \frac{\partial \beta}{\partial \omega} = \frac{n}{c} - \frac{\lambda}{c}\frac{\partial n}{\partial \lambda}$$

$$\Delta T = \frac{\partial T}{\partial \omega}\Delta\omega = \frac{\partial(\tau_g L)}{\partial \omega}\Delta\omega = L\frac{\partial^2 \beta}{\partial \omega^2}\Delta\omega = L\beta_2\Delta\omega$$

$$\boxed{\beta_2 \equiv \frac{\partial \tau_g}{\partial \omega} = \frac{\partial^2 \beta}{\partial \omega^2}}$$

Dispersion Coefficient

Units: [ps/(GHz·km)]

→ **Higher frequencies travel slower**

$$\Delta T = \frac{\partial T}{\partial \lambda}\Delta\lambda = \frac{\partial(\tau_g L)}{\partial \lambda}\Delta\lambda = LD_M\Delta\lambda$$

Units: [ps/(nm·km)]

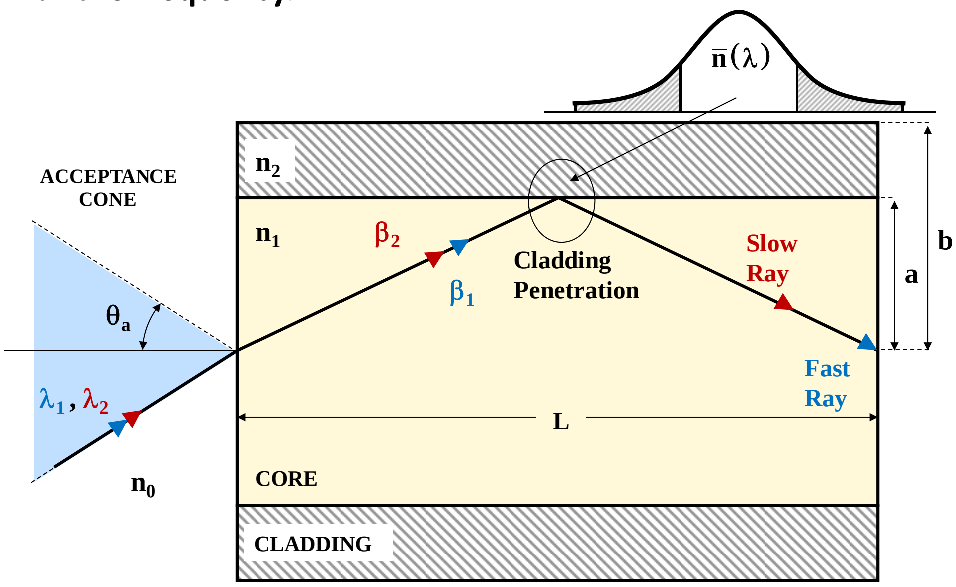
Dispersion Parameter

$$\boxed{D_M \equiv \frac{\partial \tau_g}{\partial \lambda}} = \frac{\partial}{\partial \lambda}\left[\frac{n}{c} - \frac{\lambda}{c}\frac{\partial n}{\partial \lambda}\right] = \frac{1}{c}\left[\cancel{\frac{\partial n}{\partial \lambda} - \frac{\partial n}{\partial \lambda}} - \lambda\frac{\partial n^2}{\partial \lambda^2}\right] = \boxed{-\frac{\lambda}{c}\frac{\partial^2 n}{\partial \lambda^2}}$$

$$D_M = \frac{\partial \tau_g}{\partial \lambda} = \frac{\partial \tau_g}{\partial \omega}\frac{\partial \omega}{\partial \lambda} = -\frac{2\pi c}{\lambda^2}\beta_2$$

WAVEGUIDE DISPERSION

The group velocity of each mode is frequency dependent, even though the material dispersion is zero, because the core-cladding distribution varies with the frequency.



COMBINED MATERIAL & WAVEGUIDE DISPERSION

$$D = -\frac{2\pi c}{\lambda^2}\frac{\partial \tau_g}{\partial \omega} = -\frac{2\pi}{\lambda^2}\left(2\frac{\partial \bar{n}}{\partial \omega} + \omega\frac{\partial^2 \bar{n}}{\partial \omega^2}\right) = D_M + D_W$$

[s/m]

$$b \equiv \frac{\bar{n} - n_2}{n_1 - n_2} \qquad V \equiv 2\pi\frac{a}{\lambda}NA$$

$$\boxed{\tau_{crom} \equiv |D| \cdot \Delta\lambda \geq 0}$$

$$\bar{n} = n_2 + b(n_1 - n_2) \approx n_2(1 + b\Delta)$$

$$D_W = -\frac{2\pi\Delta}{\lambda^2}\left[\frac{n_{2g}^2}{n_2\omega}V\frac{\partial^2(Vb)}{\partial V^2} + \frac{\partial n_{2g}}{\partial \omega}\frac{\partial(Vb)}{\partial V}\right]$$

empirically

$$n_g \equiv n + \omega\frac{\partial n}{\partial \omega}$$

$$D_W \approx -\frac{n_{1g} - n_{2g}}{c\lambda}\frac{1.984}{V^2}$$

$$D_M = -\frac{2\pi}{\lambda^2}\frac{\partial n_{2g}}{\partial \omega} = \frac{1}{c}\frac{\partial n_{2g}}{\partial \lambda}$$

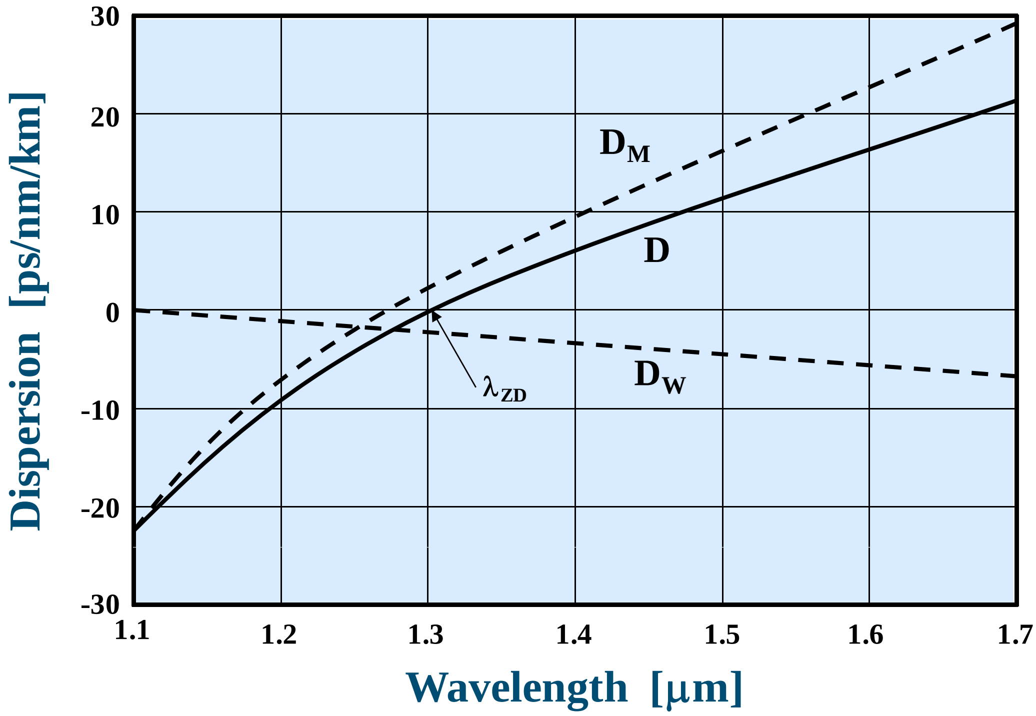
n_{2g}: cladding group index

b: normalized propagation constant

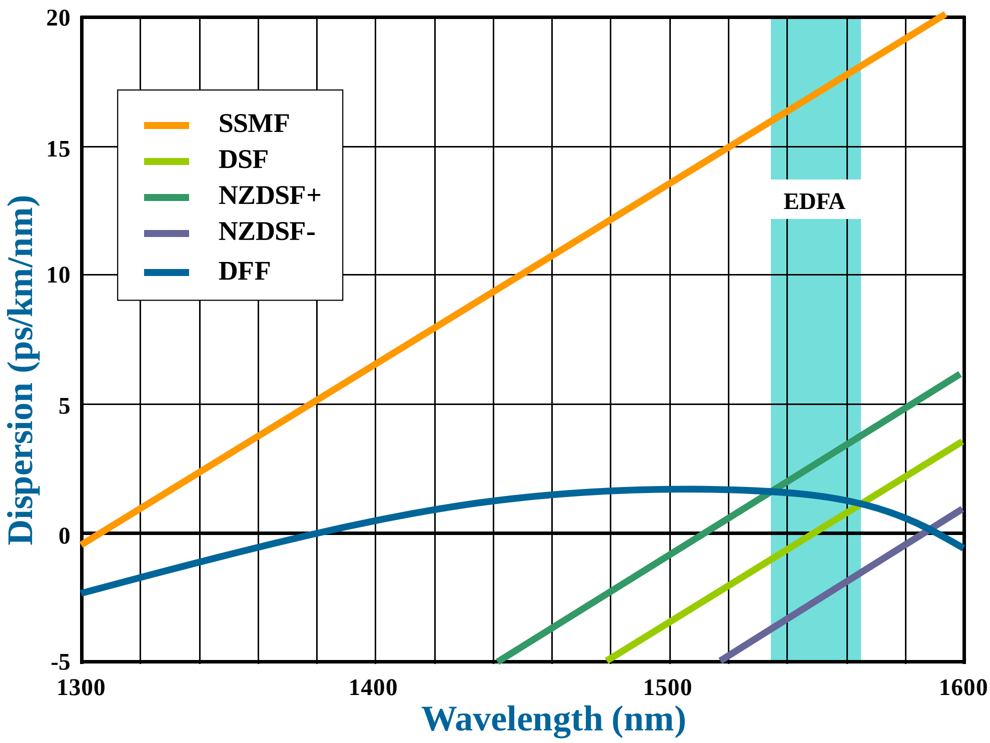
Δ: relative refractive index

V: normalized frequency

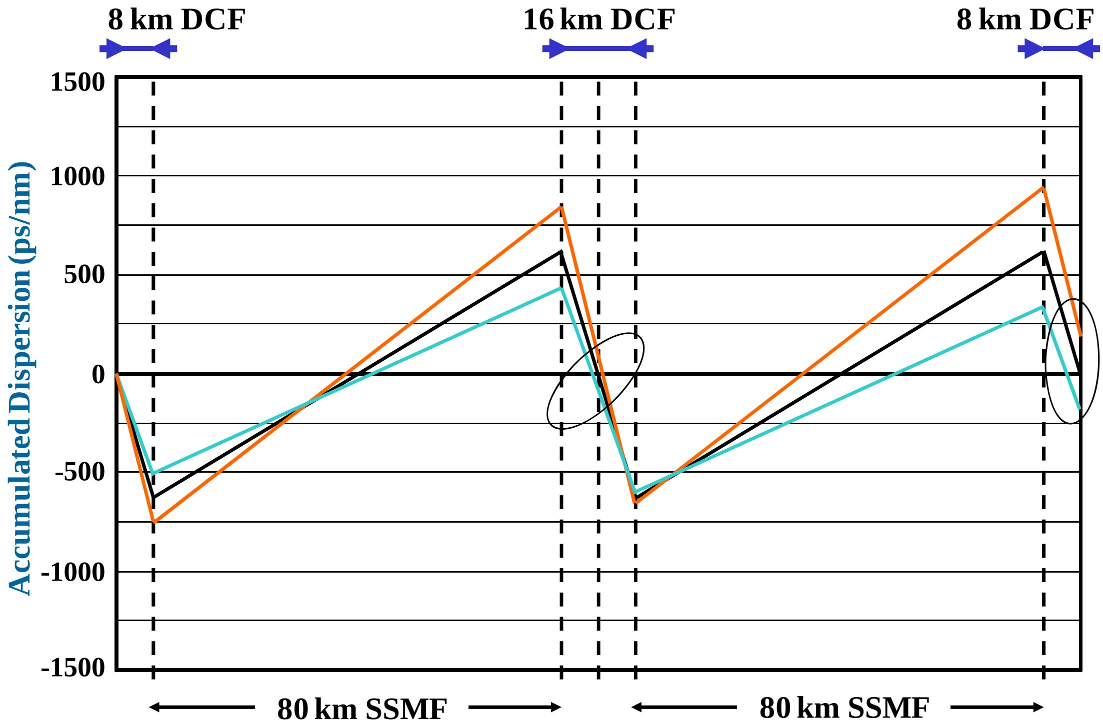
COMBINED MATERIAL & WAVEGUIDE DISPERSION



Dispersion Characteristics of Different Commercial Fibers

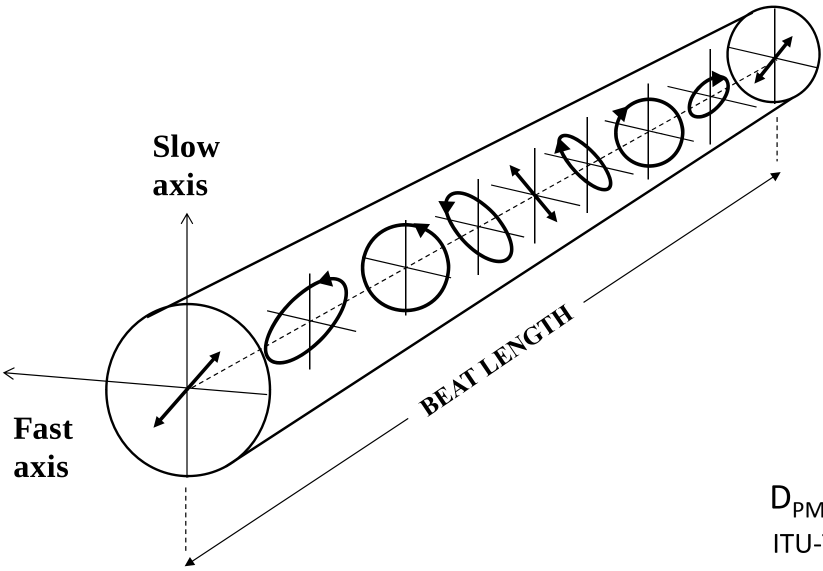


Dispersion Maps using DCF



Polarization Mode Dispersion (PMD)

The origin of this kind of dispersion comes from the polarization dependence of the refractive index. As a consequence, the light experiences different group delays regarding its polarization.



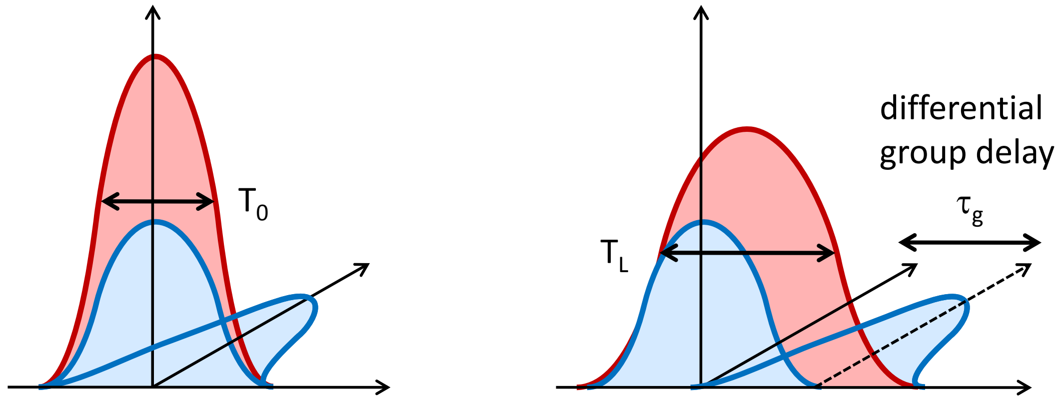
Polarization is a random process

$$\Delta T = D_{PMD}\sqrt{L}$$

D_{PMD} [ps/km$^{1/2}$]: PMD Parameter

ITU-T G.652.b : D_{PMD}=0.2-0.5 ps/km$^{1/2}$

Distance Limitation due to PMD



D_{PMD}=1 ps/km$^{1/2}$

ISI CRITERION

$$\Delta T \le T_b$$

$$\Delta T = D_{PMD}\sqrt{L}$$

$$\boxed{L \le \frac{1}{D_{PMD}^2 R_b^2}}$$

10 G L_{max} = 10000 Km

100 G L_{max} = 100 Km

MULTI-MODE FIBERS BANDWIDTH

Multi-Mode Fiber Transfer Function in Linear Regime

$$H(\omega) = \sum_{q=-\infty}^{\infty} c_q \cdot e^{-j\beta_q(\omega)L}$$

β_q: q-mode propagation constant

c_q: mode amplitude $\sum_{q=-\infty}^{\infty} c_q < \infty$

Taylor

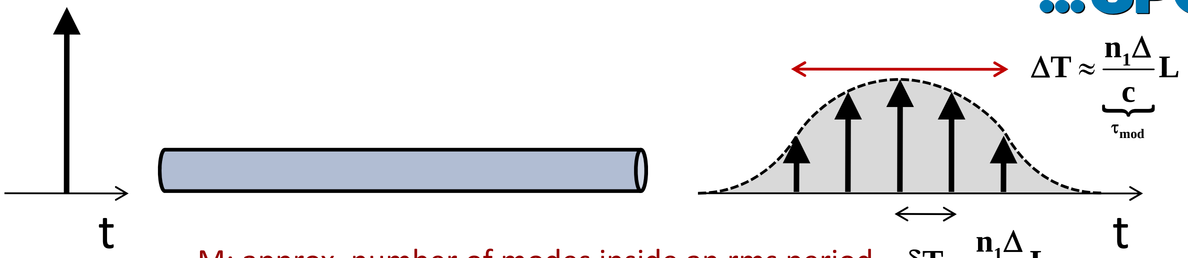
$$\beta_q(\omega) \approx \beta_{0,q} + \overbrace{\beta_{1,q}(\omega-\omega_c)}^{\text{delay}} + \overbrace{\cancel{\frac{1}{2}\beta_{2,q}(\omega-\omega_c)^2}}^{\text{dispersion}} + \overbrace{\cancel{\frac{1}{6}\beta_{3,q}(\omega-\omega_c)^3}}^{\text{dispersion slope}}$$

$$\beta_{n,q} \equiv \left.\frac{\partial \beta_q^n}{\partial \omega^n}\right|_{\omega=\omega_c}$$

$\beta_{1,q}$: q-mode group delay

Pure Delay Fiber

$$\boxed{H(\omega) = \sum_{q=-\infty}^{\infty} c_q \cdot e^{-j\beta_{1,q}L(\omega-\omega_c)}} \qquad \beta_{1,q} \equiv \left.\frac{\partial \beta_q}{\partial \omega}\right|_{\omega=\omega_c}$$



$$\Delta T \approx \underbrace{\frac{n_1 \Delta}{c}}_{\tau_{mod}} L$$

M: approx. number of modes inside an rms period

$$\delta T \approx \frac{n_1 \Delta}{c M} L$$

$$H(\omega) = \sum_{q=-\infty}^{\infty} c_q \cdot e^{-j\beta_{1,q} L(\omega-\omega_c)} \Rightarrow h(t) = \sum_{q=-\infty}^{\infty} c_q \cdot \partial\left(t - \underbrace{\{\beta_{1,0} L + q \cdot \delta T\}}_{\beta_{1,q} L}\right)$$

$$h(t) = \partial(t - \beta_{1,0} L) * \underbrace{\sum_{q=-\infty}^{\infty} c_q \cdot \partial(t - q \cdot \delta T)}$$

$$c_q \equiv a(q \cdot \delta T)$$

$$a(t) \cdot \sum_{q=-\infty}^{\infty} \partial(t - q \cdot \delta T)$$

$$\boxed{H(f) = A(f) * \frac{1}{\delta T} \sum_{q=-\infty}^{\infty} \partial\left(f - \frac{q}{\delta T}\right) \cdot e^{-j\beta_{1,0} L(f - f_c)}}$$

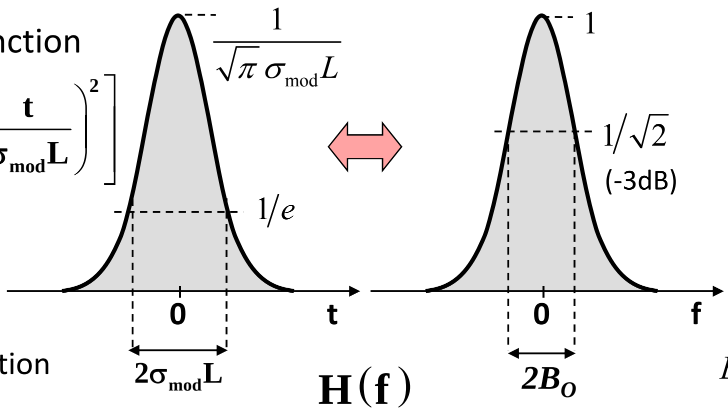
$a(t)$ **Time Domain** **Frequency Domain** $A(f)$

Gaussian Function

$$\frac{1}{\sqrt{\pi}\sigma_{mod} L} \exp\left[-\left(\frac{t}{\sigma_{mod} L}\right)^2\right]$$

$$\sigma_{mod} \approx \frac{n_1 \Delta}{2c}$$

$\sigma_{mod} L$: Std. Deviation



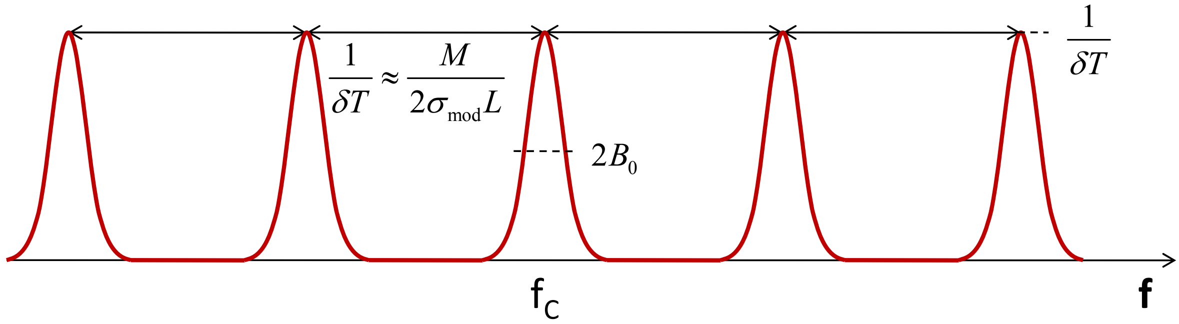
Gaussian Function

$$\exp\left[-(\pi f \cdot \sigma_{mod} L)^2\right]$$

$$B_O = \frac{\sqrt{2 \ln 2}}{2\pi \sigma_{mod} L}$$

B_0: 3-dB bandwidth

$H(f)$



SINGLE-MODE FIBERS BANDWIDTH

Single-Mode Fiber Transfer Function in Linear Regime

$$H(\omega) = e^{-j\beta(\omega)L}$$

Taylor

$$\beta(\omega) \approx \beta_0 + \underbrace{\cancel{\beta_1(\omega-\omega_c)}}_{\text{delay}} + \underbrace{\frac{1}{2}\beta_2(\omega-\omega_c)^2}_{\text{dispersion}} + \underbrace{\cancel{\frac{1}{6}\beta_3(\omega-\omega_c)^3}}_{\text{dispersion slope}}$$

$$\beta_n \equiv \left.\frac{\partial\beta^n}{\partial\omega^n}\right|_{\omega=\omega_c}$$

$$h(t) = \sqrt{\frac{1}{2\pi\beta_2 L}}\, e^{j\left(\frac{t^2}{2\beta_2 L} - \frac{\pi}{4}\right)}$$

freq. dependant delay

constant amplitude !!

Pure Dispersive Fiber

$$\boxed{H(\omega) = e^{-j\frac{1}{2}\beta_2 L(\omega-\omega_c)^2}} \qquad \beta_2 = -\frac{\lambda^2}{2\pi c} D$$

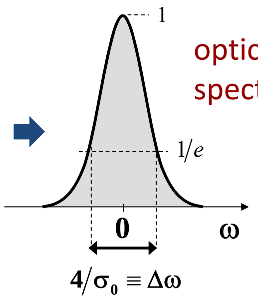
β_2: dispersion coefficient

D: dispersion parameter

GAUSSIAN PULSES PROPAGATION

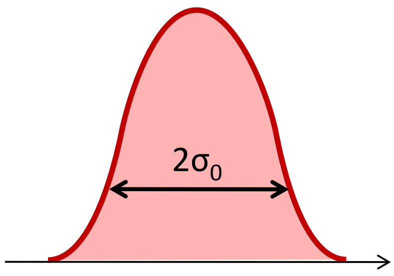
$$\underbrace{A_0 e^{-\left(\frac{t}{\sigma_0}\right)^2}}_{a_0(t)} * \underbrace{\frac{1}{\sqrt{\pi\sigma_{crom}L}} e^{-\left(\frac{t}{\sigma_{crom}L}\right)^2}}_{h_{crom}(t)}$$

$$\boxed{a_0(t) = A_0 e^{-\left(\frac{t}{\sigma_0}\right)^2}}$$

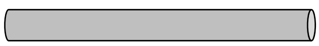


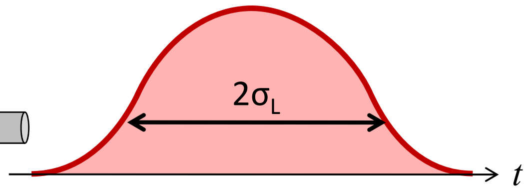
optical spectrum

$$\boxed{|a_L(t)| = A_L e^{-\left(\frac{t}{\sigma_L}\right)^2}}$$



$$h(t)$$





$$a_L(t) = \underbrace{\overbrace{A_0\left(\frac{\sigma_0}{\sigma_L}\right)}^{A_L} e^{-\frac{t^2}{\sigma_L^2}}}_{\text{gaussian amplitude}} \underbrace{e^{j\frac{2\beta_2 L}{\sigma_0^4+(2\beta_2 L)^2}t^2}}_{\text{chirp}} \underbrace{e^{-j\frac{1}{2}\mathrm{atg}\left(\frac{2\beta_2 L}{\sigma_0^2}\right)}}_{\text{phase}}$$

$$\sigma_L^2 = \sigma_0^2 + \left(\underbrace{2\beta_2 L/\sigma_0}_{\sigma_{crom}L}\right)^2$$

$$\sigma_{crom} \equiv 2\beta_2/\sigma_0 = \beta_2\,\Delta\omega/2$$

Broadening Factor

$$\frac{\sigma_L}{\sigma_0} = \left(1 + \left(2\beta_2 L/\sigma_0^2\right)^2\right)^{\frac{1}{2}}$$

MULTI-MODE FIBERS BANDWIDTH

Multi-Mode Fiber Transfer Function in Linear Regime

$$H(\omega)=\sum_{q=-\infty}^{\infty} c_q \cdot e^{-j\beta_q(\omega)L}$$

β_q: q-mode propagation constant

c_q: mode amplitude $\quad \sum_{q=-\infty}^{\infty} c_q < \infty$

Taylor

$$\beta_q(\omega)\approx\beta_{0,q}+\underbrace{\beta_{1,q}(\omega-\omega_c)}_{\text{delay}}+\underbrace{\frac{1}{2}\beta_{2,q}(\omega-\omega_c)^2}_{\text{dispersion}}+\underbrace{\cancel{\frac{1}{6}\beta_{3,q}(\omega-\omega_c)^3}}_{\text{dispersion slope}}$$

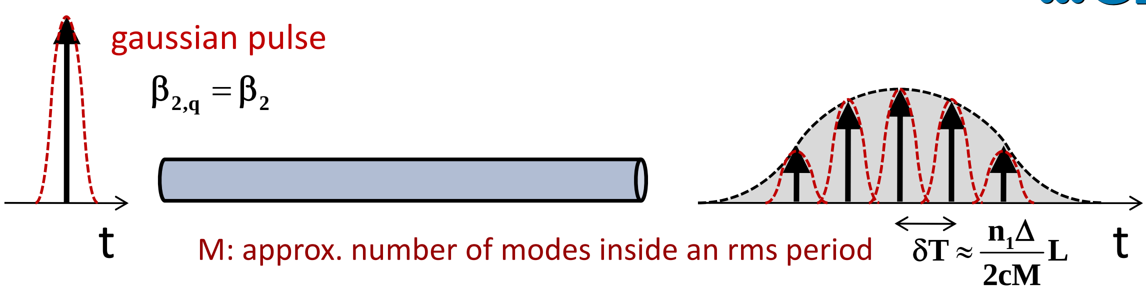
$$\beta_{n,q}\equiv\left.\frac{\partial\beta_q^n}{\partial\omega^n}\right|_{\omega=\omega_c}$$

$\beta_{1,q}$: q-mode group delay

$\beta_{2,q}$: q-mode disp. coefficient

Pure Delay-Dispersive Fiber ⇒

$$\boxed{H(\omega)=\sum_{q=-\infty}^{\infty} c_q \cdot e^{-j\left\{\beta_{1,q}(\omega-\omega_c)+\frac{1}{2}\beta_2(\omega-\omega_c)^2\right\}L}}$$



M: approx. number of modes inside an rms period $\quad \delta T\approx\frac{n_1\Delta}{2cM}L$

$$H(\omega)=\sum_{q=-\infty}^{\infty} c_q \cdot e^{-j\left\{\beta_{1,q}(\omega-\omega_c)+\frac{1}{2}\beta_2(\omega-\omega_c)^2\right\}L}=\underbrace{e^{-j\frac{1}{2}\beta_2(\omega-\omega_c)^2L}}_{H_{CD}(\omega)}\cdot\sum_{q=-\infty}^{\infty} c_q \cdot e^{-j\beta_{1,q}(\omega-\omega_c)L}$$

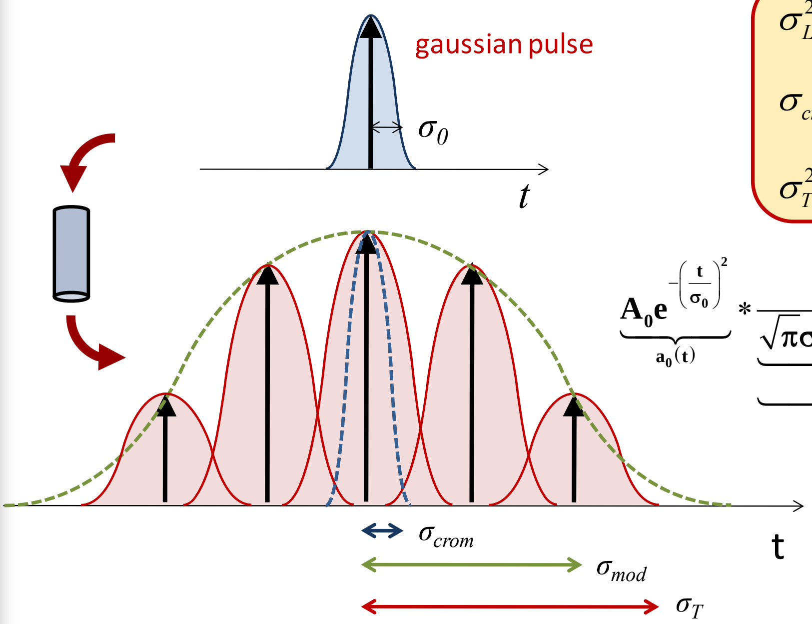
$$c_q\equiv a(q\cdot\delta T)$$

$$h(t)=\underbrace{FT^{-1}\left\{e^{-j\frac{1}{2}\beta_2(\omega-\omega_c)^2L}\right\}}_{h_{CD}(t)}*\partial(t-\beta_{1,0}L)*\left\{a(t)\cdot\sum_{q=-\infty}^{\infty}\partial(t-q\cdot\delta T)\right\}$$

$$\boxed{H(f)=e^{-j\frac{1}{2}\beta_2(\omega-\omega_c)^2L}\left\{A(f)*\frac{1}{\delta T}\sum_{q=-\infty}^{\infty}\partial\left(f-\frac{q}{\delta T}\right)\right\}\cdot e^{-j\beta_{1,0}L(f-f_c)}}$$

Same module than in pure modal dispersion

Combined Chromatic and Modal Dispersion



$$\sigma_L^2 \approx \underbrace{\sigma_0^2 + (\sigma_{crom}L)^2 + (\sigma_{mod}L)^2}_{\text{broadening}}$$

$$\sigma_{crom} = \frac{2\beta_2}{\sigma_0} \quad , \quad \sigma_{mod} \approx \frac{n_1\Delta}{2c}$$

$$\sigma_T^2 \equiv \sigma_{crom}^2 + \sigma_{mod}^2$$

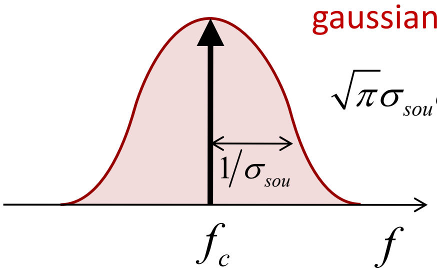
$$\underbrace{\mathbf{A_0 e}^{-\left(\frac{t}{\sigma_0}\right)^2}}_{\mathbf{a_0(t)}} * \underbrace{\underbrace{\frac{\mathbf{1}}{\sqrt{\pi}\sigma_{\mathbf{crom}}\mathbf{L}}\mathbf{e}^{-\left(\frac{t}{\sigma_{crom}L}\right)^2}}_{\mathbf{h_{crom}(t)}} * \underbrace{\frac{\mathbf{1}}{\sqrt{\pi}\sigma_{\mathbf{mod}}\mathbf{L}}\mathbf{e}^{-\left(\frac{t}{\sigma_{mod}L}\right)^2}}_{\mathbf{h_{mod}(t)}}}_{\mathbf{h(t)}}$$

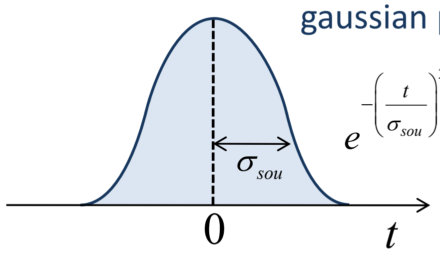
$$h(t) = \frac{1}{\sqrt{\pi}\sigma_T L} e^{-\left(\frac{t}{\sigma_T L}\right)^2}$$

Equivalent Fiber Transfer Function

Non-ideal Sources

Equivalent to an ideal source modulated by a Gaussian pulse

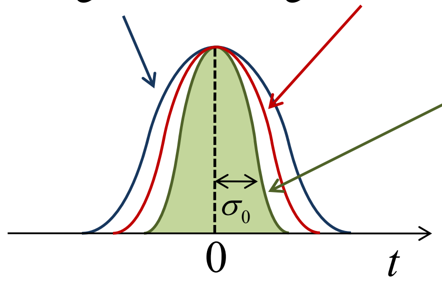




Non-ideal Sources Modulation

ideal source $\sigma_{sou} = \infty$

$$e^{-\left(\frac{t}{\sigma_{mod}}\right)^2} e^{-\left(\frac{t}{\sigma_{sou}}\right)^2} = e^{-\left(\frac{t}{\sigma_0}\right)^2}$$



$$\sigma_0^2 = \frac{\sigma_{mod}^2 \sigma_{sou}^2}{\sigma_{mod}^2 + \sigma_{sou}^2}$$

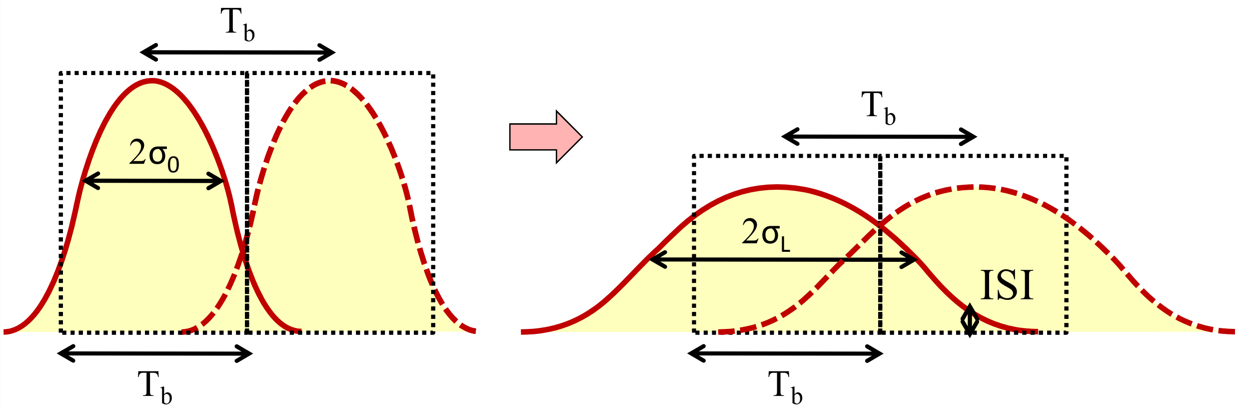
$$\sigma_{mod} \gg \sigma_{sou} \rightarrow \sigma_{sou}$$

$$\sigma_{mod} \ll \sigma_{sou} \rightarrow \sigma_{mod}$$

Maximum Distance due to Dispersion

$$p_0(t) = A_0 e^{-\left(\frac{t}{\sigma_0}\right)^2}$$

$$\left|p_L(t)\right| = A_L e^{-\left(\frac{t}{\sigma_L}\right)^2}$$



ISI CRITERION

$$\boxed{2\sigma_L \le \sqrt{2}T_b} \Rightarrow \mathrm{ISI} = e^{-\left(\frac{t}{\sigma_L}\right)^2}\Bigg|_{\substack{t=T_b \\ \sigma_L = T_b/\sqrt{2}}} = e^{-\left(\sqrt{2}\frac{\cancel{T_b}}{\cancel{T_b}}\right)^2} = \frac{1}{e^2} \approx 13.5\%$$

Maximum Distance due to Dispersion

$$\sigma_L^2 = \sigma_0^2 + \underbrace{\left(\frac{2\beta_2 L}{\sigma_0}\right)^2}_{\sigma_{crom}^2} + \underbrace{\left(\frac{n_1\Delta}{2c}L\right)^2}_{\sigma_{mod}^2} \le \frac{T_b^2}{2} \longrightarrow \boxed{L \le \left(\frac{\frac{T_b^2}{2} - \sigma_0^2}{\left(\frac{2\beta_2}{\sigma_0}\right)^2 + \left(\frac{n_1\Delta}{2c}\right)^2}\right)^{\frac{1}{2}}}$$

NRZ $\xrightarrow{2\sigma_0 = T_b}$

$$L_{max,NRZ} \le \left(\frac{\frac{T_b^2}{2} - \frac{T_b^2}{4}}{\left(\frac{4\beta_2}{T_b}\right)^2 + \left(\frac{n_1\Delta}{2c}\right)^2}\right)^{\frac{1}{2}} = \frac{1}{2R_b\sqrt{\left(4R_b\beta_2\right)^2 + \left(\frac{n_1\Delta}{2c}\right)^2}} = \frac{1}{2R_b\sqrt{\left(4R_b\frac{|D|\lambda_p^2}{2\pi c}\right)^2 + \left(\frac{n_1\Delta}{2c}\right)^2}}$$

RZ $\xrightarrow{2\sigma_0 = T_b/2}$

$$L_{max,RZ} \le \left(\frac{\frac{T_b^2}{2} - \frac{T_b^2}{16}}{\left(\frac{8\beta_2}{T_b}\right)^2 + \left(\frac{n_1\Delta}{2c}\right)^2}\right)^{\frac{1}{2}} \approx \frac{2}{3R_b\sqrt{\left(8R_b\beta_2\right)^2 + \left(\frac{n_1\Delta}{2c}\right)^2}} = \frac{2}{3R_b\sqrt{\left(8R_b\frac{|D|\lambda_p^2}{2\pi c}\right)^2 + \left(\frac{n_1\Delta}{2c}\right)^2}}$$

Maximum Distance due to Modal Dispersion

$$L \le \left(\frac{\frac{T_b^2}{2} - \sigma_0^2}{\left(\cancel{\frac{2\beta_2}{\sigma_0}}\right)^2 + \left(\frac{n_1\Delta}{2c}\right)^2} \right)^{\frac{1}{2}} \longrightarrow \boxed{L \le \frac{2c}{n_1\Delta}\left(\frac{T_b^2}{2} - \sigma_0^2\right)^{\frac{1}{2}}}$$

NRZ $\xrightarrow{2\sigma_0 = T_b}$ $L_{max,NRZ} \le \frac{c}{R_b n_1 \Delta}$

RZ $\xrightarrow{2\sigma_0 = T_b/2}$ $L_{max,RZ} \le \frac{4c}{3R_b n_1 \Delta} \approx \frac{4}{3} L_{max,NRZ}$

$\propto \frac{1}{R_b}$

Example

R_b = 100 Mb/s
n_1 = 1.5
Δ = 0.02

$L_{max,NRZ} \approx$ 100 m
$L_{max,RZ} \approx$ 133 m

Maximum Distance due to Chromatic Dispersion

$$L \le \left(\frac{\frac{T_b^2}{2} - \sigma_0^2}{\left(\frac{2\beta_2}{\sigma_0}\right)^2 + \left(\cancel{\frac{n_1\Delta}{2c}}\right)^2} \right)^{\frac{1}{2}} \longrightarrow \boxed{L \le \frac{\sigma_0}{2\beta_2}\left(\frac{T_b^2}{2} - \sigma_0^2\right)^{\frac{1}{2}}}$$

NRZ $\xrightarrow{2\sigma_0 = T_b}$ $L_{max,NRZ} \le \frac{\pi c}{4R_b^2 |D| \lambda_p^2}$

RZ $\xrightarrow{2\sigma_0 = T_b/2}$ $L_{max,RZ} \le \frac{\sqrt{7}\,\pi c}{16R_b^2 |D| \lambda_p^2} \approx \frac{2}{3} L_{max,NRZ}$

$\propto \frac{1}{R_b^2}$

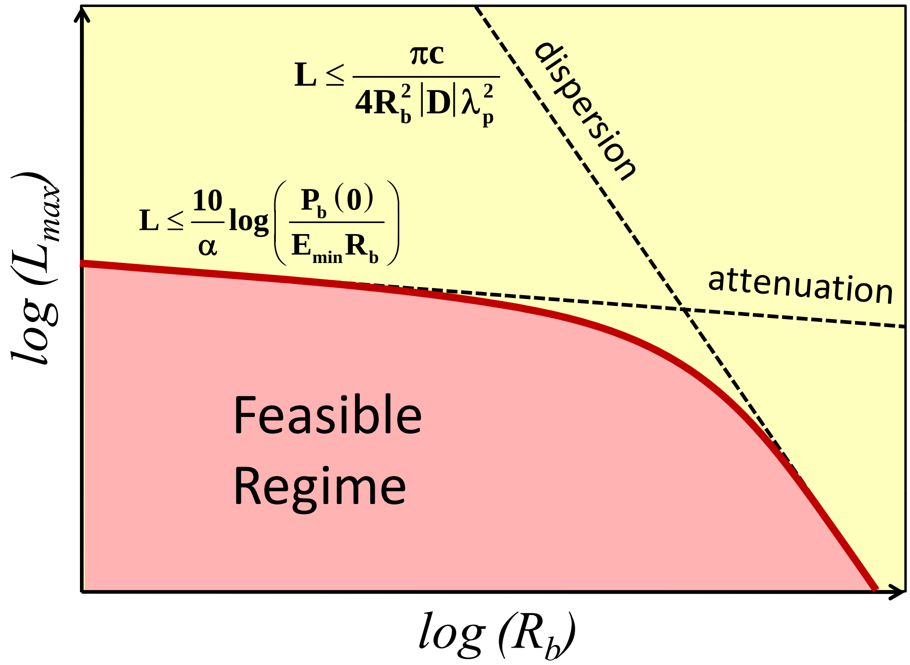
Example

R_b = 10 Gb/s
$|D|$ = 17 ps/nm/Km
λ_p = 1550 nm

$L_{max,NRZ} \approx$ 60 Km
$L_{max,RZ} \approx$ 40 Km

Attenuation / Dispersion influence on the tx. distance

Maximum Distance vs. Bit Rate



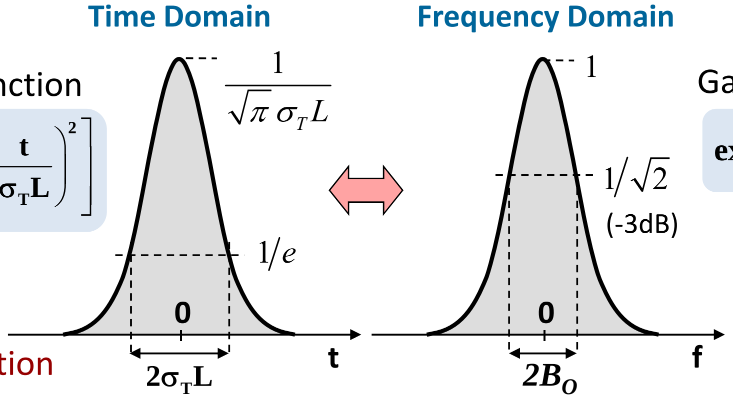
Equivalent Fiber Transfer Function: Optical Bandwidth

$\mathbf{h(t)}$ — Gaussian Function

$$\frac{1}{\sqrt{\pi}\sigma_T L}\exp\left[-\left(\frac{t}{\sigma_T L}\right)^2\right]$$

$$\sigma_T^2 \equiv \sigma_{crom}^2 + \sigma_{mod}^2$$

$\sigma_T L$: Std. Deviation



$\mathbf{H(f)}$ — Gaussian Function

$$\exp\left[-(\pi f \cdot \sigma_T L)^2\right]$$

$$B_O = \frac{\sqrt{2\ln 2}}{2\pi\,\sigma_T L}$$

B_0: 3-dB BW

Equivalent Fiber Bandwidth per unit Length

$$B_O = \frac{\sqrt{2\ln 2}}{2\pi\sigma_T L} = \frac{\sqrt{2\ln 2}}{\pi\tau_T L} \Rightarrow \boxed{f_O \equiv B_O L = \frac{\sqrt{2\ln 2}}{\pi\tau_T}} \text{ [Hz·m]} \qquad \tau_T \equiv 2\sigma_T \text{ [s]}$$

$$\boxed{f_0\sigma_T = \frac{\sqrt{2\ln 2}}{2\pi}}$$

	SI units	Prop. units	
f_0	[Hz·m],	[GHz·km]:	Bandwidth per unit length
τ, σ	[s/m],	[ns/km]:	Dispersion

Electrical vs. Optical Bandwidth

TX Electrical → Source Optical → (fiber) → Photo-Detector → RX Electrical

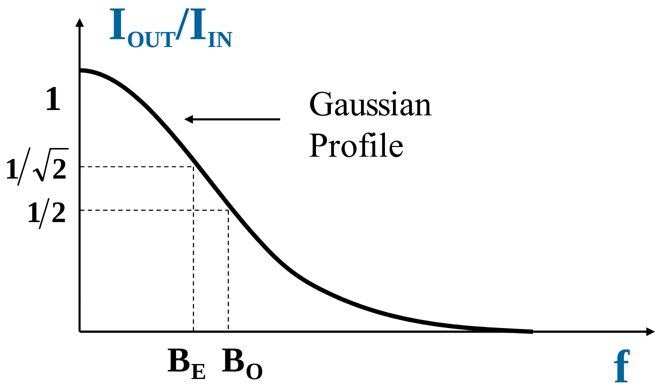
I_{IN} P_{IN} P_{OUT} I_{OUT}

$$|H_E|^2 \equiv \frac{P_{E-IN}}{P_{E-OUT}} = \frac{I_{OUT}^2}{I_{IN}^2}$$

$$B_E \equiv f\Big|_{|H_E|^2=\frac{1}{2}} \rightarrow \boxed{\frac{I_{OUT}}{I_{IN}} = \frac{1}{\sqrt{2}}}$$

$$|H_O|^2 \equiv \frac{P_{O-IN}}{P_{O-OUT}} = \frac{P_{OUT}}{P_{IN}} = \frac{I_{OUT}}{I_{IN}}$$

$$B_O \equiv f\Big|_{|H_O|^2=\frac{1}{2}} \rightarrow \boxed{\frac{I_{OUT}}{I_{IN}} = \frac{1}{2}}$$



$$\boxed{B_O > B_E}$$

$$|H|^2 = \exp\left[-\alpha f^2\right]$$ Gaussian Profile

$$B_O \rightarrow \exp\left[-\alpha B_O^2\right] = \frac{1}{2} \rightarrow B_O = \sqrt{\frac{\ln 2}{\alpha}}$$

$$B_E \rightarrow \exp\left[-\alpha B_E^2\right] = \frac{1}{\sqrt{2}} \rightarrow B_E = \sqrt{\frac{\ln 2}{2\alpha}}$$

$$\boxed{B_O = \sqrt{2} B_E}$$

$$|H|^2 = \exp\left[-\ln 2\left(\frac{f}{B_0}\right)^2\right] = \exp\left[-\frac{\ln 2}{2}\left(\frac{f}{B_E}\right)^2\right]$$

$$\boxed{R_B \leq 2B_E}$$

Nyquist

Effective Channel's Bandwidth

"What limits the transmission speed is the electrical bandwidth"

$$\sigma_T f_0 = 0.1874 \Rightarrow f_E = \frac{f_O}{\sqrt{2}} \rightarrow \boxed{\sigma_T f_E = 0.1325} \Rightarrow \boxed{B_{E,C} = \frac{f_E}{L}}$$

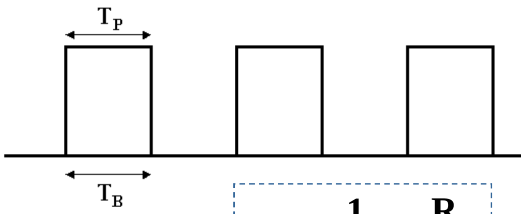
$$2\sigma_T = \tau_T$$

$$\sigma_T^2 \equiv \sigma_{crom}^2 + \sigma_{mod}^2$$

f_E [Hz·m], [GHz·km]
τ, σ [s/m], [ns/km]

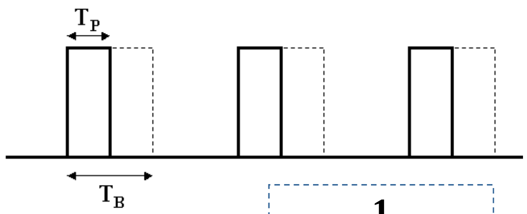
Signal's Bandwidth: $R_B - B_E$ Relationship

NRZ modulation

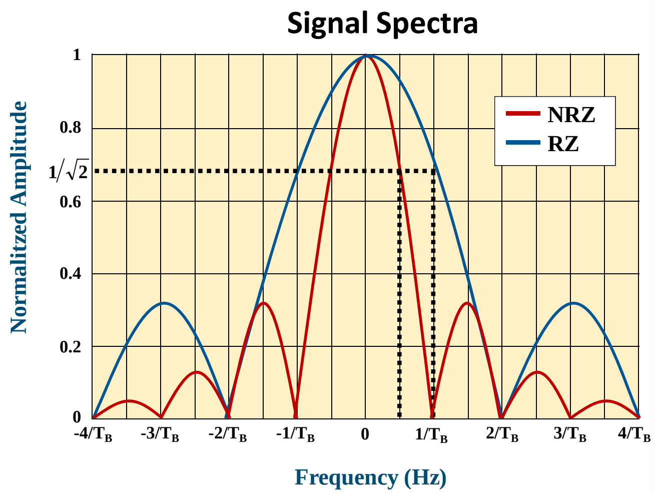


$$B_E = \frac{1}{2T_B} = \frac{R_B}{2}$$

RZ modulation



$$B_E = \frac{1}{T_B} = R_B$$



Maximum Distance due to Dispersion (Bandwidth Criterion)

NRZ Modulation

$$B_{E,S} = \frac{R_B}{2} \le B_{E,C} = \frac{f_E}{L} = \frac{\sqrt{\ln 2/\pi^2}}{2\sigma_T L} \longrightarrow \boxed{R_B \le \frac{\sqrt{\ln 2/\pi^2}}{\sigma_T L}}$$

RZ Modulation

$$B_{E,S} = R_B \le B_{E,C} = \frac{f_E}{L} = \frac{\sqrt{\ln 2/\pi^2}}{2\sigma_T L} \longrightarrow \boxed{R_B \le \frac{\sqrt{\ln 2/\pi^2}}{2\sigma_T L}}$$

$$\boxed{\begin{aligned} R_{B,max,RZ} &= \frac{1}{2} R_{B,max,NRZ} \\ L_{max,RZ} &= \frac{1}{2} L_{max,NRZ} \end{aligned}}$$

Example: Chromatic Dispersion

$R_b = 10\ Gb/s$
$|D| = 17\ ps/nm/Km$
$\lambda_c = 1550\ nm$
$\Delta f = R_b$

$\sigma_{crom} = \frac{1}{2}\beta_2 \Delta\omega$

$L_{max,NRZ} \approx 40\ Km$
$L_{max,RZ} \approx 20\ Km$

APPENDIX

PROPAGATION IN O.F.

WAVE EQUATION

Maxwell's Equations

$$\left|\begin{array}{l} \nabla\times\vec{E} = -\dfrac{\partial\vec{B}}{\partial t} \\ \nabla\times\vec{H} = \vec{J}_f + \dfrac{\partial\vec{D}}{\partial t} \\ \nabla\cdot\vec{D} = \rho_f \\ \nabla\cdot\vec{B} = 0 \end{array}\right.$$

$$\vec{D} = \varepsilon_o\vec{E} + \vec{P}$$
$$\vec{B} = \mu_0\vec{H} + \vec{M}$$

$$c = \left(\frac{1}{\mu_0\varepsilon_0}\right)^{\frac{1}{2}}$$

no conductor medium without free charges

OPTICAL FIBER

no magnetic medium

$$\left|\begin{array}{l} \nabla\times\vec{E} = -\dfrac{\partial\vec{B}}{\partial t} \\ \nabla\times\vec{H} = \dfrac{\partial\vec{D}}{\partial t} \\ \nabla\cdot\vec{D} = 0 \\ \nabla\cdot\vec{B} = 0 \end{array}\right.$$

$$\vec{D} = \varepsilon_o\vec{E} + \vec{P}$$
$$\vec{B} = \mu_0\vec{H}$$

E, H: electric & magnetic field vectors

D, B: electric & magnetic flux density

P, M: polarization & magnetization density

ε_0, μ_0: free-space electrical permittivity & magnetic permeability

J_f: electric current density

ρ_f : free-charges concentration

c: free-space speed of light

Free-Space Wave Equation

$$\vec{P}=0$$
$$\vec{M}=0$$
$$\vec{J}_f=0$$
$$\rho_f=0$$

$$\nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}$$
$$\nabla\times\vec{H}=\cancel{\vec{J}_f}+\frac{\partial\vec{D}}{\partial t}$$
$$\nabla\cdot\vec{D}=\cancel{\rho_f}$$
$$\nabla\cdot\vec{B}=0$$

$$\vec{D}=\varepsilon_0\vec{E}+\cancel{\vec{P}}$$
$$\vec{B}=\mu_0\vec{H}+\cancel{\vec{M}}$$

$$\nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}=-\mu_0\frac{\partial\vec{H}}{\partial t}\longrightarrow\nabla\times\nabla\times\vec{E}=-\mu_0\frac{\partial(\nabla\times\vec{H})}{\partial t}=-\mu_0\varepsilon_0\frac{\partial^2\vec{E}}{\partial t^2}$$
$$\nabla\times\vec{H}=\frac{\partial\vec{P}}{\partial t}=\varepsilon_0\frac{\partial\vec{E}}{\partial t}$$

$$\nabla\cdot\vec{D}=\varepsilon_0\nabla\cdot\vec{E}=0\longrightarrow\nabla\times\nabla\times\vec{E}=\nabla\left(\cancel{\nabla\cdot\vec{E}}\right)-\nabla^2\vec{E}=-\nabla^2\vec{E}$$
$$\nabla\cdot\vec{M}=\mu_0\nabla\cdot\vec{H}=0$$

$$\nabla\times\nabla\times\vec{E}=-\nabla^2\vec{E}=-\frac{1}{c}\frac{\partial^2\vec{E}}{\partial t}$$

$$c=\left(\frac{1}{\mu_0\varepsilon_0}\right)^{\frac{1}{2}}$$

$$\boxed{\nabla^2\vec{E}-\frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2}=0}$$

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Wave Equation in linear, non-dispersive, homogeneous, and isotropic media

$$\vec{P}=\varepsilon_o\chi\vec{E}$$
$$\vec{D}=\varepsilon_o\vec{E}+\vec{P}=\varepsilon_o(1+\chi)\vec{E}=\varepsilon\vec{E}$$
$$\varepsilon\equiv\varepsilon_o(1+\chi)$$

$$\boxed{\nabla^2\vec{E}-\frac{1}{v^2}\frac{\partial^2\vec{E}}{\partial t^2}=0}$$

$$v=\left(\frac{1}{\mu_0\varepsilon}\right)^{\frac{1}{2}}$$
$$v=\frac{c}{n}\rightarrow n=\frac{c}{v}=\left(\frac{\varepsilon}{\varepsilon_o}\right)^{\frac{1}{2}}=(1+\chi)^{\frac{1}{2}}$$

χ: electrical susceptibility (scalar)
ε: medium's electrical permittivity
n: medium's refractive index

Inhomogeneous Medium

$$\vec{P}=\varepsilon_o\chi(r)\vec{E}$$

In an inhomogeneous medium both electrical susceptibility and permittivity are position dependent (and so is the refractive index)

Anisotropic medium

$$\mathbf{P}_i = \sum_j \varepsilon_0 \chi_{ij} \mathbf{E}_j$$

χ_{ij}: susceptibility tensor

In an anisotropic medium, the relationship between electric field and polarization density vectors depends on the direction of field vector and they are generally not parallel

Dispersive medium

$$\vec{\mathbf{P}}(\mathbf{t}) = \varepsilon_0 \int_{\infty}^{\infty} \chi(\mathbf{t}-\mathbf{t}')\cdot\vec{\mathbf{E}}(\mathbf{t}')\mathbf{dt}'$$

$\varepsilon_0\chi(t)$: impulsive response

In a dispersive medium, the relationship between electric field and polarization density vectors is dynamic (with memory) rather than instanta-neous

Non-linear medium

$$\vec{\mathbf{P}} = \varepsilon_0\left(\chi^{(1)}\vec{\mathbf{E}} + \chi^{(2)}\vec{\mathbf{E}}^2 + \chi^{(3)}\vec{\mathbf{E}}^3 + \ldots\right)$$

$\chi^{(i)}$: i-order non-linear coefficient

In a non-linear medium, the relationship between electric field and polarization density vectors is generally not linear

Wave Equation in Optical Fibers

$$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\nabla\times\vec{H} = \cancel{\vec{J}_f} + \frac{\partial\vec{D}}{\partial t}$$

$$\nabla\cdot\vec{D} = \cancel{\rho_f}$$

$$\nabla\cdot\vec{B} = 0$$

$$\vec{D} = \varepsilon_0\vec{E} + \vec{P}$$

$$\vec{B} = \mu_0\vec{H} + \cancel{\vec{M}}$$

$$\nabla\times\nabla\times\vec{E} = -\frac{\partial(\nabla\times\vec{B})}{\partial t} = -\mu_0\frac{\partial(\nabla\times\vec{H})}{\partial t}$$

$$= -\mu_0\frac{\partial^2\vec{D}}{\partial t^2} = -\varepsilon_0\mu_0\frac{\partial^2\vec{E}}{\partial t^2} - \mu_0\frac{\partial^2\vec{P}}{\partial t^2}$$

Dielectric medium

Non-magnetic medium

$$\boxed{\nabla^2\vec{E} = -\mu_0\varepsilon_0\frac{\partial^2\vec{E}}{\partial t^2} - \mu_0\frac{\partial^2\vec{P}}{\partial t^2}}$$

Isotropic, linear medium
(far from resonances)

$$\vec{P}(r,t) = \varepsilon_0\int_{\infty}^{\infty}\chi(r,t-t')\cdot\vec{E}(r,t')dt'$$

Fourier Transform

$$\tilde{\vec{\mathbf{E}}}(\mathbf{r},\omega) \equiv \int_{-\infty}^{\infty}\vec{\mathbf{E}}(\mathbf{r},\mathbf{t})\mathbf{e}^{-j\omega t}\mathbf{dt}$$

Monochromatic light

$$\tilde{\vec{\mathbf{P}}}(\mathbf{r},\omega) = \varepsilon_0\tilde{\chi}(\mathbf{r},\omega)\tilde{\vec{\mathbf{E}}}(\mathbf{r},\omega)$$

$$\boxed{\nabla^2\tilde{\mathbf{E}} = -\omega^2\mu_0\varepsilon_0\tilde{\mathbf{E}} - \omega^2\mu_0\tilde{\mathbf{P}} = -\omega^2\mu_0\varepsilon_0\left(1+\tilde{\chi}\right)\tilde{\mathbf{E}} = -\frac{\omega^2}{\mathbf{c}^2}\left(1+\tilde{\chi}\right)\tilde{\mathbf{E}} = -\mathbf{k}^2\tilde{\mathbf{E}}}$$

wave number $\mathbf{k} \equiv \frac{\omega}{\mathbf{c}}(1+\tilde{\chi})^{\frac{1}{2}}$

$$\boxed{\nabla^2\tilde{E}+k^2\tilde{E}=0}$$

Helmholtz Equation

Each component of Electric and Magnetic fields satisfy this wave equation

Planar Wave with z propagation

$$U = A\exp(-jkz) = A\exp(-j\beta z)\exp\left(-\frac{1}{2}\alpha z\right)$$

β: propagation constant
α: absorption coeff.

$$k \equiv \frac{\omega}{c}(1+\tilde{\chi})^{\frac{1}{2}} = \beta - j\frac{1}{2}\alpha \longleftarrow \tilde{\chi}=\chi'+j\chi''$$

Lossy medium

$$\chi'' << \chi'$$

$$\beta = \frac{\omega}{c}(1+\chi')^{\frac{1}{2}}\left\{\frac{1}{2}\left[\sqrt{1+\left(\frac{\chi''}{1+\chi'}\right)^2}+1\right]\right\}^{\frac{1}{2}} \approx \frac{\omega}{c}(1+\chi')^{\frac{1}{2}}\left[1+\frac{1}{8}\left(\frac{\chi''}{1+\chi'}\right)^2\right] \approx \frac{\omega}{c}(1+\chi')^{\frac{1}{2}}$$

$$\alpha = \frac{\omega}{c}(1+\chi')^{\frac{1}{2}}\left\{2\left[\sqrt{1+\left(\frac{\chi''}{1+\chi'}\right)^2}-1\right]\right\}^{\frac{1}{2}} \approx \frac{\omega}{c}\frac{\chi''}{(1+\chi')^{\frac{1}{2}}}$$

refractive index

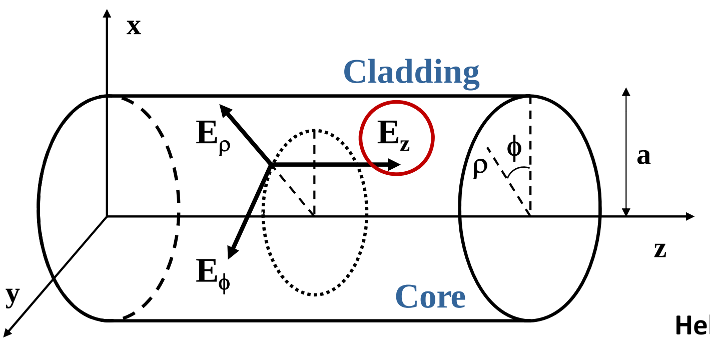
Phase velocity

$$v \equiv \frac{\omega}{\beta} \rightarrow \beta = \frac{\omega}{v} = \frac{\omega}{c}n \rightarrow n = \frac{c}{\omega}\beta \approx (1+\chi')^{\frac{1}{2}}$$

$$\boxed{n \approx (1+\chi')^{\frac{1}{2}}}$$

Transversal Propagation Modes

"An Optical Transversal Mode refers to a particular solution of wave equation that satisfies the boundary conditions imposed by the waveguide and its transversal field distribution remains constant with propagation"



Laplacian Operator in Cartesian and Cylindrical coordinates

$$\nabla^2\Psi = \frac{\partial^2\Psi}{\partial x^2}+\frac{\partial^2\Psi}{\partial y^2}+\frac{\partial^2\Psi}{\partial z^2}$$

$$\nabla^2\Psi = \frac{\partial^2\Psi}{\partial \rho^2}+\frac{1}{\rho}\frac{\partial\Psi}{\partial\rho}+\frac{1}{\rho^2}\frac{\partial^2\Psi}{\partial\phi^2}+\frac{\partial^2\Psi}{\partial z^2}$$

Helmholtz Equation (lossless)

$$\nabla^2\vec{E}+k_0^2n^2\vec{E}=0$$

Variable separation

Propagation z

Periodicity ϕ

$$\boxed{E_z(\rho,\phi,z) = F(\rho)\Phi(\phi)Z(z) = F(\rho)e^{-jm\phi}e^{-j\beta z}}$$

Propagation Modes in Step-Index Fibers

$$n = \begin{cases} n_1 & \rho < a \\ n_2 & \rho \geq a \end{cases}$$

$$\kappa^2 \equiv k_0^2 n_1^2 - \beta^2$$

$$\gamma^2 \equiv \beta^2 - k_0^2 n_2^2$$

β: Propagation constant

$$\frac{\partial^2 F}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial F}{\partial \rho} + \left(\kappa^2 - \frac{m^2}{\rho^2}\right) F = 0 \qquad \rho < a \quad \text{Core}$$

$$\frac{\partial^2 F}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial F}{\partial \rho} + \left(\gamma^2 + \frac{m^2}{\rho^2}\right) F = 0 \qquad \rho \geq a \quad \text{Cladding}$$

We exclude solutions that tend to ∞ for $r \to 0$ or that are not zero when $r \to \infty$

$$E_Z(\rho) = \begin{cases} A \cdot J_m(\kappa\rho) e^{-jm\phi} e^{-j\beta z} & \rho \leq a \\ C \cdot K_m(\gamma\rho) e^{-jm\phi} e^{-j\beta z} & \rho > a \end{cases}$$

$$H_Z(\rho) = \begin{cases} B \cdot J_m(\kappa\rho) e^{-jm\phi} e^{-j\beta z} & \rho \leq a \\ D \cdot K_m(\gamma\rho) e^{-jm\phi} e^{-j\beta z} & \rho > a \end{cases}$$

$J_m(\cdot)$: Bessel functions of 1st kind and order m

$K_m(\cdot)$: Bessel functions of 2nd modified kind and order m

$$E_\rho = -\frac{j}{\kappa^2}\left[\beta \frac{\partial E_Z}{\partial \rho} + \mu_0 \frac{\omega}{\rho}\frac{\partial H_Z}{\partial \phi}\right]$$

$$E_\phi = -\frac{j}{\kappa^2}\left[\frac{\beta}{\rho}\frac{\partial E_Z}{\partial \phi} - \mu_0 \omega \frac{\partial H_Z}{\partial \rho}\right]$$

$$H_\rho = -\frac{j}{\kappa^2}\left[\beta \frac{\partial H_Z}{\partial \rho} - \varepsilon_0 n^2 \frac{\omega}{\rho}\frac{\partial E_Z}{\partial \phi}\right]$$

$$H_\phi = -\frac{j}{\kappa^2}\left[\frac{\beta}{\rho}\frac{\partial H_Z}{\partial \phi} + \varepsilon_0 n^2 \omega \frac{\partial E_Z}{\partial \rho}\right]$$

Boundary Conditions

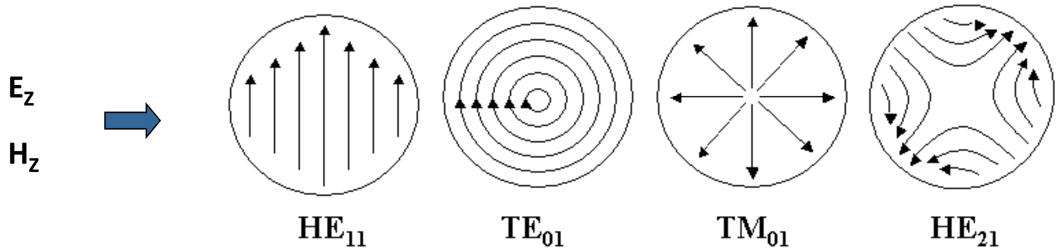
Boundary conditions force that the tangential components (ϕ i z) of E and H fields must coincide in the core-cladding separation area. We have 4 equations (**Eρ, Eϕ, Hρ, Hϕ**) and 4 unknowns (**A,B,C,D**). A non-trivial solution will be available when the coefficient matrix's determinant is zero.

Eigenvalue Equation

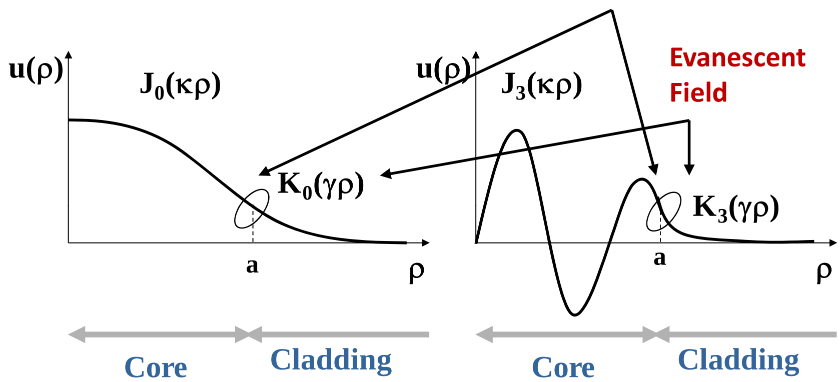
$$\left[\frac{J_m'(\kappa a)}{\kappa J_m(\kappa a)} + \frac{K_m'(\gamma a)}{\gamma K_m(\gamma a)}\right]\left[\frac{J_m'(\kappa a)}{\kappa J_m(\kappa a)} + \frac{n_2^2}{n_1^2}\frac{K_m'(\gamma a)}{\gamma K_m(\gamma a)}\right] = \left[\frac{2m\beta(n_1 - n_2)}{a\kappa^2\gamma^2}\right]^2$$

For each value of **m** an equation system is defined with multiple solutions from which we can extract the β_{mn} value that determines the propagation condition (**propagation mode**).

Field Distributions



Spacial Distributions



Propagation Condition

$$\kappa^2 \equiv k_0^2 n_1^2 - \beta^2$$
$$\gamma^2 \equiv \beta^2 - k_0^2 n_2^2$$

$$K_m(\gamma\rho) \xrightarrow{\gamma\rho\to\infty} e^{-\gamma\rho} \to \gamma > 0 \to \beta \geq k_0 n_2$$
$$F \text{ real} \to \kappa \text{ real} \to \beta \leq k_0 n_1$$

$$n_2 k_0 < \beta < n_1 k_0$$
$$n_2 < \frac{\beta}{k_0} \equiv \bar{n} < n_1$$

$\bar{n}$: mode index (effective index)

cutoff

$$\beta = n_2 k_0 \rightarrow \gamma = 0$$
$$\bar{n} = n_2 \to \kappa = k_0 \left(n_1^2 - n_2^2\right)^{\frac{1}{2}} \leq 0$$

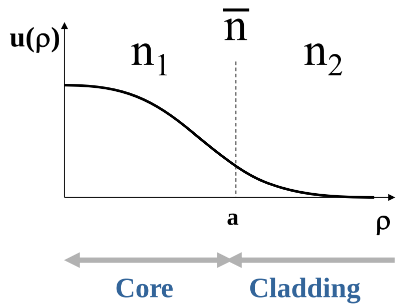
non-real

$$\beta = n_1 k_0 \rightarrow \kappa = 0$$
$$\bar{n} = n_1 \to \gamma = k_0 \left(n_1^2 - n_2^2\right)^{\frac{1}{2}} \leq 0$$

Normalized Frequency

$$\kappa^2 + \gamma^2 = \left(n_1^2 - n_2^2\right) k_0^2 = NA^2 k_0^2$$
$$X \equiv \kappa a \quad Y \equiv \gamma a$$
$$X^2 + Y^2 = \left(2\pi \frac{a}{\lambda} NA\right)^2 \equiv V^2$$

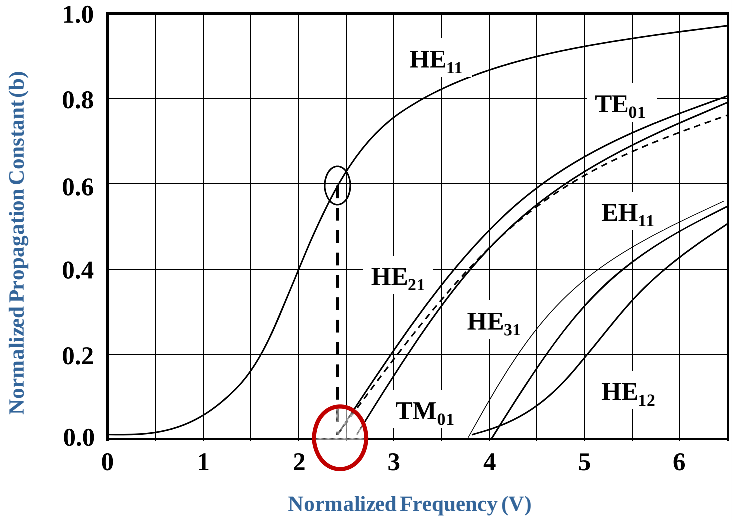
$$\boxed{V \equiv 2\pi \frac{a}{\lambda} NA \underset{n_1 \approx n_2}{\approx} 2\pi \frac{a}{\lambda} n_1 \sqrt{2\Delta}}$$



NA-a trade-off

Normalized Propagation Constant

$$b \equiv \frac{\bar{n} - n_2}{n_1 - n_2} = \frac{\beta/k_0 - n_2}{n_1 - n_2}$$



Single-mode Condition

$$\left.\begin{matrix} m = 0 \\ \gamma = 0 \end{matrix} \rightarrow J_0(\kappa a)\right|_{\gamma=0} = J_0(V) = 0$$

$$\boxed{V < 2.405 = V_c}$$

Trade-off

$$V \downarrow \rightarrow \text{single mode} \uparrow$$

$$V \uparrow \rightarrow \% P_{core}/P_{clad} \uparrow$$

Number of modes

$$M_{SI} \approx \frac{V^2}{2}$$ ← Step Index Fiber

$$M_{GI} \approx \frac{V^2}{4}$$ ← Graded Index Fiber

Cut-off Wavelength

$$\boxed{\lambda_c \approx 2\pi \frac{a}{V_c} n_1 \sqrt{2\Delta}}$$