Homework 2
CS534 Machine Learning, Fall 2021

This homework explores linear classification methods.

Problem 1 - The multivariate normal (10 points)

Consider a dataset containing samples $X \in \mathbb{R}^p$ generated from two multivariate normals with means μ_1, μ_2 and equal covariance Σ. Suppose there are N_1 samples generated from class $\mathcal{N}(\mu_1, \Sigma)$, and N_2 samples generated from class $\mathcal{N}(\mu_2, \Sigma)$. Starting from $P(G = 2|X), P(G = 1|X)$, show that if a sample is more likely to have come from class $g = 2$ then

$$x^T \Sigma^{-1}(\mu_2 - \mu_1) > \frac{1}{2}(\mu_2 + \mu_1)^T \Sigma^{-1}(\mu_2 - \mu_1) - log(N_2/N_1)$$

Problem 2 - Linear discriminant analysis (20 points)

The dataset for this problem contains samples $X \in \mathbb{R}^2$ from two classes

$$X^{(1)} \sim \mathcal{N}(\begin{bmatrix} 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}), \qquad X^{(2)} \sim \mathcal{N}(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix}).$$

Generate these samples using your solution to Homework 1.2.

2.a. Linear discriminant
The linear discriminant function $\delta_k(x)$ is defined as

$$\delta_k(x) = x^T \hat{\Sigma}^{-1} \hat{\mu}_k - \frac{1}{2} \mu_k^T \hat{\Sigma}^{-1} \hat{\mu}_k + \log \pi_k,$$

where π_k is the *prior probability* of class k.

Generate a scatter plot of training samples for $|\{X^{(k)}\}| = 10$, color coding class, and superimpose the following on this plot

- LDA decision boundary $\{x | \delta(x) = 0\}$
- Theoretical Bayes decision boundary: $\{x | f_{X^{(1)}} = f_{X^{(2)}}\}$
- Empirical Bayes decision boundary: $\{x | \hat{f}_{X^{(1)}} = \hat{f}_{X^{(2)}}\}$ (using sample mean, covariance from $|\{X^{(k)}\}| = 10$)

2.b. Linear discriminant - large sample
Repeat 2.a. with $|\{X^{(k)}\}| = 1000$.

Problem 3 - Quadratic discriminant analysis (20 points)

Repeat problem 2 using the quadratic discriminant

$$\delta_k(x) = -\frac{1}{2}\log|\hat{\Sigma}_k| - \frac{1}{2}(x - \hat{\mu}_k)^T \hat{\Sigma}_k^{-1}(x - \hat{\mu}_k) + \log\pi_k.$$