Homework 1 David Wang

1) Expectations

Model $\rightarrow$ $Y = f(x) + \epsilon$

a) $Err(x) = E[(Y - \hat{f}(x))^2 | X = x]$

$\Rightarrow$ $Y - f(x)$ and $f(x) - \hat{f}(x)$; $= E[((Y - f(x)) + (f(x) - \hat{f}(x)))^2 | X]$

$\Rightarrow$ Distribute square $= E[(Y - f(x))^2 | X] + 2E[Y - f(x) \times (f(x) - \hat{f}(x)) | X] + E[(\hat{f}(x) - \hat{f}(x))^2 | X]$

Notice: $E[Y - f(x) | X] = E[Y|X] - E[Y|X] = 0$

$\Rightarrow$ $= E[((Y - f(x))^2 | X] + E[(f(x) - \hat{f}(x))^2 | X]$

$\Rightarrow$ variance $= \sigma_\epsilon^2 + E[(f(x) - \hat{f}(x))^2 | X]$

$E[(Y - f(x))^2] = E[(f(x) + \epsilon - f(x))^2]$
$= E[(\epsilon)^2] = \sigma_\epsilon^2$

$= \sigma_\epsilon^2 + E[(f(x) - E[\hat{f}(x)] + E[\hat{f}(x)] - \hat{f}(x))^2]$

$\Rightarrow$ $f(x) - E[\hat{f}(x)]$ and $E[\hat{f}(x)] - \hat{f}(x)$

$\Rightarrow$ distribute square $= \sigma_\epsilon^2 + (f(x) - E[\hat{f}(x)])^2 + (E[\hat{f}(x)] - \hat{f}(x))^2 + 2E[(\hat{f}(x) - E[\hat{f}(x)]) \times (E[f(x)] - \hat{f}(x))]$

Not really sure how this step works.

$= \sigma_\epsilon^2 + (F(x) - E[\hat{f}(x)])^2 + E[\hat{f}(x) - E[\hat{f}(x)]]^2$

b) σ_ϵ^2 is the standard error of ϵ

$E[\hat{f}(x)])^2$ is bias squared

$E(\hat{f}(x) - E[\hat{f}(x)])^2$ is variance

2) Covariance

a) Diagonalization $\Sigma = V \Lambda V^T$

$\mathcal{N}(0, I) \rightarrow \mathcal{N}(\mu, \Sigma)$

$\rightarrow \mathcal{N}(\mu, V \Lambda V^T)$

- if $V \Lambda V^T = I$ then $\Lambda = V^{-1} I V = V^{-1} V = I$ ...,

  then $\Sigma = I$

- since distribution is normal (gaussian),

  $\mu = 0$