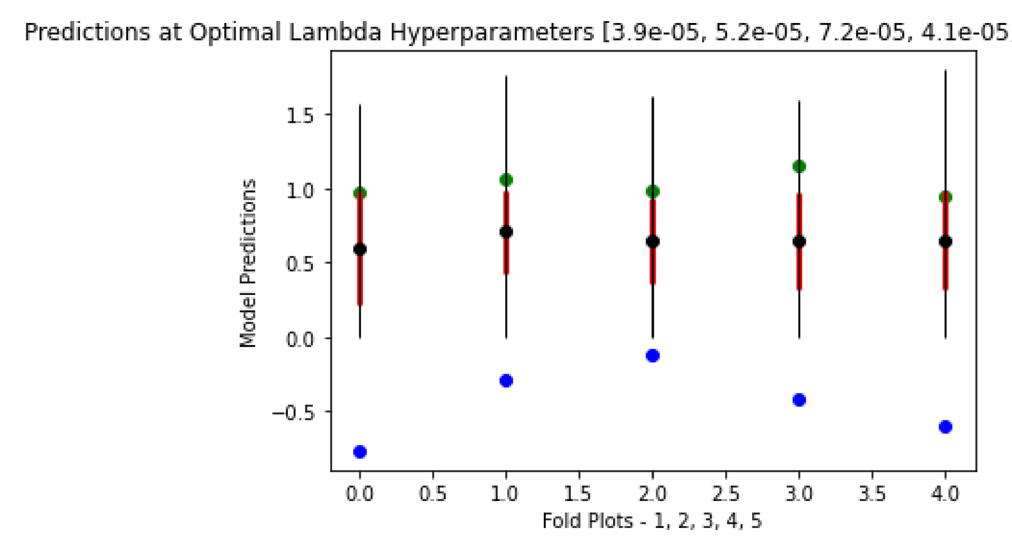
In [1]:
```python
import numpy as np
import scipy.io
from sklearn.model_selection import cross_val_score, KFold, GridSearchCV, train_test_split
from sklearn.ensemble import RandomForestClassifier
from sklearn.metrics import mean_squared_error, accuracy_score
from sklearn.linear_model import ElasticNet



#Problem 1 - Cross Validation


#Problem setup

#read in data
data = scipy.io.loadmat('HW3_1.mat')

X = data['X'] #351 count of both X and y
y = data['y']
lbda = data['lambdas'] #lbda in range [0, 100] (500 count)
lbda = lbda.flatten() #2d to 1d the lambda hyperparameter array

mix_param = 0.95 #alpha hyperparameter

#perform cross-validation (5 fold so 20% of data is used for testing/test error)

# create outer cv
cv_outer = KFold(n_splits=5, shuffle=True, random_state=1)

# perform cv operation in each fold
ideal_model_test = list()
ideal_model_train = list()

test_mse = list()
train_mse = list()

for train_ix, test_ix in cv_outer.split(X):

    # split data to train and test
    X_train, X_test = X[train_ix, :], X[test_ix, :]
    y_train, y_test = y[train_ix], y[test_ix]

    # create inner cv
    cv_inner = KFold(n_splits=5, shuffle=True, random_state=1)

    # define the model
    model = ElasticNet(l1_ratio = mix_param) #l1_ratio parameter sets the alpha hyperparameter

    # define param grid (search space -> looking for optimal lambda)
    parametersGrid = {
                      "alpha": lbda, #alpha parameter sets the lambda hyperparameter
                      }


    # define search for hyperparameter tuning
    tuning = GridSearchCV(model, parametersGrid, cv=cv_inner, refit=True)
    # search
    optimal_fit = tuning.fit(X_train, y_train)

    # get the best performing model and the optimal lambda
    best_model = optimal_fit.best_estimator_


    #To get test errors for graphing
    # evaluate best lambda estimator model on X_test dataset
    y_pred_test = best_model.predict(X_test)
    y_pred_train = best_model.predict(X_train)

    ideal_model_test.append(y_pred_test)
    ideal_model_train.append(y_pred_train)

    #for 1 (c)
    # evaluate the model for test errors (mse)
    test_error = mean_squared_error(y_test, y_pred_test)
    train_error = mean_squared_error(y_train, y_pred_train)



    # store the mse
    test_mse.append(test_error)
    train_mse.append(train_error)
    val_mse = cross_val_score(model, X, y, scoring = 'neg_mean_squared_error', cv = cv_inner) # score of inner

    # store rmse (errors)
    test_rmse = np.sqrt(test_mse)
    train_rmse = np.sqrt(train_mse)
    val_rmse= np.sqrt(np.abs(val_mse))
```

In [2]:
```python
#get the needed data for graphing

#min and max mse
minim = list()
maxim = list()
std = list()
mean = list()
opt_lambdas = list()

for i in range(5):
    minim.append(round(ideal_model_test[i][np.argmin(ideal_model_test[i])], 4))
    maxim.append(round(ideal_model_test[i][np.argmax(ideal_model_test[i])], 4))
    std.append(round(np.std(ideal_model_test[i]),4))
    mean.append(round(np.mean(ideal_model_test[i]),4))
    opt_lambdas.append(round(lbda[np.argmax(ideal_model_test[i])], 6))


print('std for each fold {}'.format(std))
print()
print('mean for each fold: {}'.format(mean))
print()
print('minimum: {}'.format(minim))
print()
print('maximum: {}'. format(maxim))
print()

#optimal lbda(better model)
print('optimal lambdas: {}'.format(opt_lambdas))
```

```
std for each fold [0.3843, 0.28, 0.2855, 0.3264, 0.33]

mean for each fold: [0.6017, 0.7082, 0.6453, 0.6494, 0.6511]

minimum: [-0.7691, -0.2947, -0.1185, -0.4245, -0.5953]

maximum: [0.9678, 1.0603, 0.9829, 1.151, 0.9427]

optimal lambdas: [3.9e-05, 5.2e-05, 7.2e-05, 4.1e-05, 1.3e-05]
```

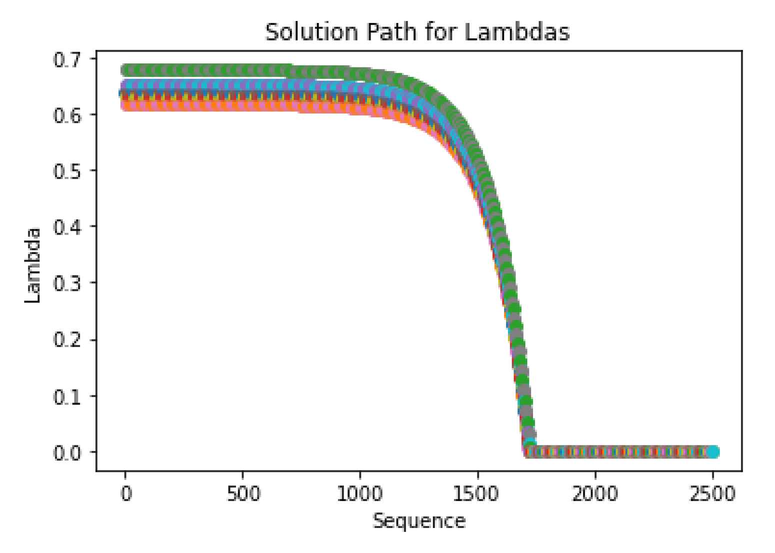
In [3]:
```python
from matplotlib import pyplot as plt

#Probem 1 (b) - model plotting

#plot the mean, std of errors
mins = list(set(mean) - set(minim))
maxs = list(set(maxim) - set(mean))


plt.errorbar(np.arange(5), mean, std, fmt = 'ok', lw = 3, ecolor = 'red')
plt.errorbar(np.arange(5), mean, [mins, maxs], fmt = '.k', ecolor = 'black', lw = 1)

#plot the min and max of errors
x_points = [0.0, 1.0, 2.0, 3.0, 4.0]
max_points = np.full(5, maxim)
min_points = np.full(5, minim)

plt.scatter(x_points, max_points, color = 'green')
plt.scatter(x_points, min_points, color = 'blue')

plt.title('Predictions at Optimal Lambda Hyperparameters {} for each Fold'.format(opt_lambdas))
plt.xlabel('Fold Plots - 1, 2, 3, 4, 5')
plt.ylabel('Model Predictions')

plt.show()
```



In [21]:
```python
#Problem 1 (c) - error plotting and discussion

errors_dict = {'Test Errors': test_rmse, 'Train Errors': train_rmse, 'Validation Errors': val_rmse}

fig, ax = plt.subplots()

ax.boxplot(errors_dict.values(), showmeans = True)
ax.set_xticklabels(errors_dict.keys())

plt.ylabel('RMSE')
plt.title('Error Types for Elastic Model')

plt.show()


# Errors Discussion


# First, let us start with the training errors. These errors are the ones we get when we trained the elastic mo
# Hence, these errors tell us how well the model learned from the training data. From our data, our training er
# This is to be expected as we trained the model directly on the training data, hence, good results are expecte

# Secondly, let us talk about the validation errors. These errors tell us how well the parameters of the model
# Hence, in theory, this should make the validation error have a lower RMSE since the model should fit better c
# My results do not reflect that and I think that is mainly due to errors where I did not split the validation

# Finally, let us talk about the test errors. These errors are completely unseen data and the training model is
# Hence, it provides a very good indicator to the model's generalizability.
# My results reflect good results in general where the RMSE is relatively low. However, there is a bit of a hig

# In general, all 3 errors show good results as a RMSE of between 0.200 to 0.500 shows that the Elastic Net mod
# With the exception of validation error, the test and train error results were expected where the model perfo
# Hence, I would say that the models selected by nested k-fold CV proprely optimized the bias-variance trade-of
```

In [5]:
```python
#Problem 2 - Elastic Net Logistic Regression

import warnings
warnings.filterwarnings('ignore')

#IRLS
#solve optimization problem by minimizing cost function iteratively by solving weighted least squares problem

from scipy import linalg

N = 351 #number of values in X

weights = np.linspace(1, 2, N) #34 iteratively reweighted X
Xw = X * np.sqrt(weights)[:, None]
yw = y * np.sqrt(weights)
newX = linalg.lstsq(Xw, yw)[0]
newX = np.reshape(newX, (351, 34))


#run the elastic net model on the reweighted values of X

mix_param = 0.95 #alpha hyperparameter

#perform cross-validation (5 fold so 20% of data is used for testing/test error)

# create outer cv
cv_outer = KFold(n_splits=5, shuffle=True, random_state=1)

# perform cv operation in each fold
ideal_model_test = list()
ideal_model_train = list()

test_mse = list()
train_mse = list()

for train_ix, test_ix in cv_outer.split(X):

    # split data to train and test
    X_train, X_test = newX[train_ix, :], newX[test_ix, :]
    y_train, y_test = y[train_ix], y[test_ix]

    # create inner cv
    cv_inner = KFold(n_splits=5, shuffle=True, random_state=1)

    # define the model
    model = ElasticNet(l1_ratio = mix_param) #l1_ratio parameter sets the alpha hyperparameter

    # define param grid (search space -> looking for optimal lambda)
    parametersGrid = {
                      "alpha": lbda, #alpha parameter sets the lambda hyperparameter
                      }


    # define search for hyperparameter tuning
    tuning = GridSearchCV(model, parametersGrid, cv=cv_inner, refit=True)
    # search
    optimal_fit = tuning.fit(X_train, y_train)

    # get the best performing model and the optimal lambda
    best_model = optimal_fit.best_estimator_


    #To get test errors for graphing
    # evaluate best lambda estimator model on X_test dataset
    y_pred_test = best_model.predict(X_test)
    y_pred_train = best_model.predict(X_train)

    ideal_model_test.append(y_pred_test)
    ideal_model_train.append(y_pred_train)


#Soft-thresholding
#Using thresholding helper module to implement soft thresholding
# https://pywavelets.readthedocs.io/en/latest/ref/thresholding-functions.html

#Soft-threshold for every lambda value

import pywt

threshold_data = list()

for i in range(len(lbda)):
    for j in range(5):
        threshold_data.append(pywt.threshold(ideal_model_test[j], lbda[i], mode='soft'))
```

In [22]:
```python
#plot the result

import statistics

x_plot = list(range(0, len(threshold_data)))

for i in range(len(threshold_data)):
    plt.scatter(x_plot[i], statistics.mean(threshold_data[i]))

plt.title('Solution Path for Lambdas')
plt.xlabel('Sequence')
plt.ylabel('Lambda')
plt.show()
```



In [ ]: