

Tarea # 4, tarea grupal

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Calcular las derivadas por definición de las funciones:
Definición de una derivada:

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$f(x) = mx + b$$

$$\begin{aligned} \lim_{h \rightarrow 0} \frac{m(x+h) + b - (mx+b)}{h} &= \lim_{h \rightarrow 0} \frac{m(x+h) + \cancel{b} - mx - \cancel{b}}{h} \\ &= \lim_{h \rightarrow 0} \frac{mx + mh - mx}{h} \\ &= \lim_{h \rightarrow 0} \frac{\cancel{mx} + mh - \cancel{mx}}{h} \\ &= \lim_{h \rightarrow 0} \frac{mh}{h} \\ &= \lim_{h \rightarrow 0} \frac{m\cancel{h}}{\cancel{h}} \\ &= m \end{aligned}$$

$$g(x) = x^3$$

$$\begin{aligned}
 \lim_{h \rightarrow 0} \frac{(x+h)^3 - x^3}{h} &= \lim_{h \rightarrow 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 \cancel{-x^3}}{h} \\
 &= \lim_{h \rightarrow 0} \frac{3x^2h + 3xh^2 + h^3}{h} \\
 &= \lim_{h \rightarrow 0} \frac{h(3x^2 + 3xh + h^2)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\cancel{h}(3x^2 + 3xh + h^2)}{\cancel{h}} \\
 &= \lim_{h \rightarrow 0} 3x^2 + 3xh + h^2 \\
 &= 3x^2 + 3 \cdot 0x + 0^2 \\
 &= 3x^2
 \end{aligned}$$

$$g(x) = c \text{ constante}$$

$$\begin{aligned} \lim_{h \rightarrow 0} \frac{c - c}{h} &= \lim_{h \rightarrow 0} \frac{\cancel{c} - \cancel{c}}{h} \\ &= \lim_{h \rightarrow 0} \frac{0}{h} \\ &= \lim_{h \rightarrow 0} \frac{\cancel{0}}{h} \nearrow 0 \\ &= 0 \end{aligned}$$

$$g(x) = \sqrt{x+1}$$

$$\begin{aligned}
\lim_{h \rightarrow 0} \frac{\sqrt{x+h+1} - \sqrt{x+1}}{h} &= \lim_{h \rightarrow 0} \frac{\sqrt{x+h+1} - \sqrt{x+1}}{h} \cdot \frac{\sqrt{x+h+1} + \sqrt{x+1}}{\sqrt{x+h+1} + \sqrt{x+1}} \\
&= \lim_{h \rightarrow 0} \frac{x+h+1 - (x+1)}{h(\sqrt{x+h+1} + \sqrt{x+1})} \\
&= \lim_{h \rightarrow 0} \frac{\cancel{x} + \cancel{h} + 1 - \cancel{x} - 1}{h(\sqrt{x+h+1} + \sqrt{x+1})} \\
&= \lim_{h \rightarrow 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{x+h+1} + \sqrt{x+1})} \\
&= \lim_{h \rightarrow 0} \frac{1}{\sqrt{x+h+1} + \sqrt{x+1}} \\
&= \frac{1}{\sqrt{x+0+1} + \sqrt{x+1}} \\
&= \frac{1}{\sqrt{x+1} + \sqrt{x+1}} \\
&= \frac{1}{2\sqrt{x+1}}
\end{aligned}$$

$$g(x) = \cos x$$

$$\begin{aligned}
& \lim_{h \rightarrow 0} \frac{\cos(x+h) - \cos(x)}{h} \\
&= \lim_{h \rightarrow 0} \frac{\cos(x+h) - \cos(x)}{h} \\
&= \lim_{h \rightarrow 0} \frac{\cos(x) \cdot \cos(h) - \sin(x) \cdot \sin(h) - \cos(x)}{h} \\
&= \lim_{h \rightarrow 0} \frac{[\cos(x) \cdot \cos(h) - \cos(x)] - \sin(x) \cdot \sin(h)}{h} \\
&= \lim_{h \rightarrow 0} \frac{\cos(x)[\cos(h) - 1] - \sin(x) \cdot \sin(h)}{h} \\
&= \lim_{h \rightarrow 0} \frac{\cos(x)[\cos(h) - 1]}{h} + \lim_{h \rightarrow 0} \frac{-\sin(x) \cdot \sin(h)}{h} \\
&= \cos(x) \lim_{h \rightarrow 0} \left(\frac{[\cos(h) - 1]}{h} \right) - \sin(x) \lim_{h \rightarrow 0} \frac{\sin(h)}{h} \\
&= -\cos(x) \lim_{h \rightarrow 0} \left(\frac{[1 - \cos(h)]}{h} \right) - \sin(x) \lim_{h \rightarrow 0} \frac{\sin(h)}{h} \\
&= -\cos(x) \lim_{h \rightarrow 0} \left(\frac{[1 - \cos(h)]}{h} \right) - \sin(x) \lim_{h \rightarrow 0} \frac{\sin(h)}{h} \\
&= -\cos(x) \cdot 0 - \sin(x) \cdot 1 \\
&= -\sin(x)
\end{aligned}$$

Encuentre la recta tangente a la curva $y = x^3$ en el punto $(2, 8)$

$$\begin{aligned}f(x) &= x^3 \\f'(x) &= 3x^2\end{aligned}$$

Una vez tenemos la recta tangente, evaluamos en f' , con esto obtendremos la pendiente de la recta tangente.

$$f'(2) = 3 \cdot 2^2$$

$$m = 12$$

Ahora con la ecuación punto-pendiente obtenemos la ecuación de la recta:

$$y - y_0 = m(x - x_0)$$

sustituimos valores y luego despejamos y:

$$y - 8 = 12(x - 2)$$

la ecuación de la recta tangente a x^3 en el punto $(2, 8)$ es:

$$y = 12x - 16$$