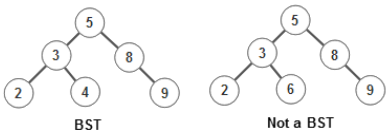
Binary search tree (BST)

A binary search tree, or BST, is a binary tree that satisfies:

1. Each node contains a value.
2. For each node, all the values in its left subtree are less than its value.
3. For each node, all the values in its right subtree are greater than its value.



An example of a BST appears to the right above. It's important to realize that *all* nodes in the left subtree must be less than the node's value. The rightmost binary tree above is not a BST because the value 6 is not less than 5.

Here's one important consequence of properties 2 and 3 of a BST: *A BST cannot contain duplicate values*.

Below, in discussing algorithms that process BSTs, we'll assume that the nodes are of class `Node` and that the class contains the usual fields: **int** `val`, `Node left`, and `Node right`.

Advantages of a BST

A BST has two obvious advantages over a normal binary tree.

(1) When searching for a value `v`, if `v` is less than than the root value, one has to look only in the left subtree. Similarly if `v` is greater than the root value, one has to look only in the right subtree. Therefore, the maximum number of nodes to be tested is 1 + (height of the tree). If the tree is balanced, searching for a value takes time O(log `n`) for a tree with `n` nodes.

To the right above, function `isIn` searches BST `t` for `v`. It uses a loop instead of recursion. It is just as simple as a recursive version, and it is faster because no method calls are necessary.

```
/** = "t is not null and v is in tree t" */
public static boolean isIn(Node t, int v) {
   // inv: if v is in the original tree, v in t
   while (t != null) {
      if (v == t.val) return true;
      t= v < t.val ? t.left : t.right;
   }
   return false;
}
```

(2) We can enumerate the values in a BST in sorted (ascending) order simply by performing an inorder traversal. See method `print` to the right.

```
/** Print t's values, sorted.
  * Precondition: t is not null. */
public static boolean print(Node t) {
   if (t.left != null) print(t.left);
   System.out.println(t.val);
   if (t.right != null) print(t.right);
```

Inserting a value into a BST

Consider inserting 6 into the BST at the top of the page. 6 > 5, so 6 belongs in 5's right subtree. 6 < 8, so 6 belongs in 8's left subtree. Since 8 doesn't have a left subtree, so insert 6 as 8's left subtree.

Consider inserting 10 into the BST. 10 > 5, so 10 belongs in 5's right subtree. 10 > 8, so 10 belongs in 8's right subtree. 10 > 9, so 10 belongs in 9's right subtree. Since 9 doesn't have a right subtree, insert 10 as 9's right subtree.

Evidently, inserting `v` into BST `t` requires searching `t` for `v`; when that search ends without finding `v`, `v` is added.

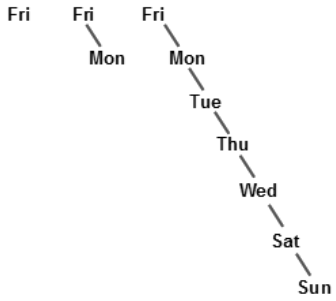
To the right we write `insert`. To write it, we started with the body of `isIn(t, v)` and massaged it to fit the new specification.

The while loop has condition true because the repetend will return when `v` is found or `v` is added to `t`. Note the additional condition in the loop invariant: `t` is not null.

```
/** Insert v into BST t if it is not there.
  * Return true if it was inserted, false otherwise
  * Precondition: t is not null. */
public static boolean insert(Node t, int v) {
   // inv: t != null; if v is in the original tree, v in t
   while (true) {
      if (t.val == v) return false;
      if (v < t.val) {
         if (t.left != null) t= t.left;
         else { t.left= new Node(v); return true; }
      } else { // v > t.val
         if (t.right != null) t= t.right;
         else { t.right= new Node(v); return true; }
      }
   }
}
```

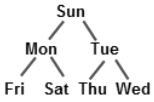
The main change comes when testing which subtree `v` belongs in (left or right). If that subtree exists, assign that subtree to `t`. If that subtree doesn't exist, create a new node with value `v` and null subtrees make it that subtree, and return.

The problem of balancing a BST



Suppose we insert the days of the week into an empty BST (whose values are strings) in alphabetical order. To the right, we show a BST with one node, `Fri` (day), inserted, then a BST with `Mon` inserted into that BST, and finally the BST with all the days of the week inserted. Because we inserted the days in alphabetical order, the BST looks like a linked list.

One of the advantages tauted of a BST is the efficient search for a value. But in such an unbalanced tree, the worst-case search time in a BST with `n` nodes is O(`n`).



On the other hand, if we inserted the values into the BST in a different order, we could produce the BST tree that appears to the left. And in such a complete tree, thus the worst-case search time, as well as the height of the tree, is O(log `n`).

The problem is that inserting nodes into a BST (or even deleting nodes) may make it become very imbalanced, so that its height is nowhere near the logarithm of its size. Further, there is no simple and efficient way to make it balanced again. Because of this, computer scientists in the 1960s, 1970s, and later explored extensions to BSTs that kept them balanced. We explore them briefly in another item under entry *trees* in the JavaHyperText.

The term *balanced* is discussed in another item under entry *trees* in the JavaHyperText. For now, just think of *balanced* as the height of the tree (that is, the longest path from the root to a leaf) is as small as possible or close to as small as possible.

Determining whether a binary tree is a BST

Consider determining whether a binary tree `t` with **int** values is a BST. Let's develop a recursive boolean function `isBST(t, …)`.

Property 3 of a BST —all values in the right subtree of a node are greater than the node's value— requires that a recursive call on the right subtree be given a minimum value for nodes in the subtree. For example, if the root value is 50, the recursive call has to now that all nodes in the right subtree have to be ≥ 51. Therefore, function `isBST` needs a parameter `h` (say) with the requirement that all nodes values be ≥ `h`.

```
/** = "t is a BST with values in the range h..k."
   * Precondition: t is not null.*/
public static boolean isBST(Node t, int h, int k) {
  if (t.val < h  ||  k < t.val) return false;
  if (t.left != null  &&  !isBST(t.left, h, t.val–1))
       return false;
  return t.right == null || isBST(t.right, t.val+1, k);
}
```

In the same way, because of property 2, `isBST` needs a parameter `k` (say) with the requirement that all nodes values be ≤ `k`. Therefore, we envision a method with specification given in the method above.

A call on the `root` of a binary tree will be

```
isBST(root, Integer.MIN_VALUE, Integer.MAX_VALUE)
```

Now think of writing the body of function `isBST`. First, return false if the root is not in range `h..k`.

Second, if the left subtree is not null, (1) it must be a BST and (2) all values in it have to be in the range `h..t.val-1`. The second if-statement checks these conditions and returns false if they are not true.

Third, the last statement, a return statement, checks the same kind of conditions for the right subtree.

How did we think of this function and its specification? By looking at the definition of a BST. The awareness of the need for parameters `h` and `k` came from investigatingthe definition.