Developing a recursive method: Is a tree a BST?

This development of a recursive method uses the *Mañana Principle* (look it up in JavaHyperText), which says: When, during implementation of a method, you wish you had a certain support method, write your code as if you had it and implement the method later. The support method shows again how useful it is to think of adding a second method, a recursive one, with a new parameter. Finally, this development makes clear how important it is to respect the recursive definition of a binary tree —we'll point that out later what this means.

We deal with a binary tree whose nodes are instances of class `TreeNode`, given to the right.

```
/** An instance is a node of a binary tree. */
public class TreeNode {
   int val; // Value associated with this node.
   TreeNode left; // Left child of this node
                          // ---null if none.
   TreeNode right; // Right child of this node
                          // ---null if none.
}
```

We develop a function `isBST` to determine whether a tree is a Binary Search Tree —a BST.

```
/** = "Tree n is a BST" */
public static boolean isBST(TreeNode n)
```

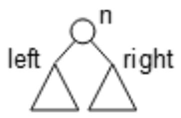
We define BST, written in terms of class `TreeNode`:

A binary tree is a BST iff for each node `n` of the tree, all `val` fields in subtree `n.left` are less than `n.val` and all `val` fields in subtree `n.right` are greater than `n.val`.

By the definition, an empty binary tree is a BST.

Since we are going to write a recursive function, it may help to rewrite the definition of a BST recursively:

A binary tree with root `n` is a BST iff
(1) `n.left` is null or a BST,
(2) all `val` fields in subtree `n.left` are less than `n.val`,
(4) `n.right` is null or a BST,
(3) all `val` fields in subtree `n.right` are greater than `n.val`,



We place a diagram of a binary tree on the right to give guidance in thinking about implementing the function.

Fight a bad tendency

There must be a test `n.left.val` < `n.val`. (if `n.left` != **null**). The tendency is to write that test almost immediately. Fight that tendency! If you don't, you'll be complicating the method by having three tests in the method —testing `n.val`, `n.left.val`, and `n.right.val`.

Look at the above diagram of a binary tree. A binary tree is a node together with a left binary tree and a right binary tree. The diagram doesn't show you anything about the two subtrees. So, when processing node `n`, it may a bad idea to "look inside" the two subtrees at `n.left.val` and `n.right.val`.

Instead, think of having a second function `isBST(n, max)`, write a call `isBST(n.left, n.val)` and have that call verify that `n.val` < `max`. In the same way, we will need a third parameter `min` to verify that all values in subtree `n.right` are greater than `min`.

This leads us to use the *Mañana Principle* and stub in a second function `isBST` with three parameters:

```
/** = "Tree n is a BST and all val fields in it satisfy min < val < max"*/
private static boolean isBST (TreeNode n, int min, int max) {
   return false;
}
```

We can now write function `isBST` with no parameters as shown to the right. It simply calls the function `isBST` with three parameters, and this second function will be recursive. In the earlier part of this JavaHyperText where we discussed the possible need for extra parameters in recursive methods, we also discussed the standard practice of having two methods like this.

```
/** = "Tree n is a BST" */
public static boolean isBST(TreeNode n) {
   return isBST(n, Integer.MIN_VALUE,
                        Integer.MAX_VALUE);
}
```

A small change

Suppose we want to check whether a binary tree with root `n` is a BST. We write the call:

```
isBST(n)
```

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This call immediately results in the call

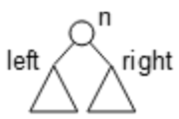
```
isBST(n, Integer.MIN_VALUE, Integer.MAX_VALUE);
```

and because of the requirement that `min` < `val` < `max`, the smallest and largest values of type **int** cannot be in a BST. That's not good, so we change the spec of the two-parameter function to use ≤ instead of <:

```
/** = "Tree n is a BST and all val fields in it satisfy min ≤ val ≤ max"*/
private static boolean isBST (TreeNode n, int min, int max)
```

Implementing the 3-parameter function

We again keep our diagram of a binary tree handy to help guide us in implementing the 3-parameter function, whose spec is above. We proceed in four steps.



1. The empty tree is a binary tree:

   ```
   if (n == null)  return true;
   ```

2. The spec of `isBST` (`n`, `min`, `max`) requires that the `val` fields of all nodes of `n` satisfy `min` ≤ `val` ≤ `max`. Looking at the diagram of a binary tree, node `n` has to be checked, and we do that first:

   ```
   if (n.val < min || max < n.val) return false;
   ```

3. If subtree `n.left` is not empty, then the following properties must be satisfied:
   (1) Subtree `n.left` is a BST (by the first part of the spec and part (1) of the definition of a BST)
   (2) All `val` fields in subtree `n.left` satisfy `min` ≤ `val` ≤ `max` (by the second part of the spec)
   (3) All `val` fields in subtree `n.left` satisfy `val` < `n.val` (for tree `n` to be a BST)

Properties (2) and (3) can be merged into a single property:

   (4) All `val` fields in subtree `n.left` satisfy `min` ≤ `val` ≤ `n.val-1`

Based on properties (1) and (4) a call tells us whether the left subtree has the necessary properties:

```
isBST(n.left, min, n.val-1);
```

4. The right subtree is handled in a similar fashion:

   ```
   isBST(n.right, val+1, max);
   ```

This results in method `isBST` given in the box below.

```
/** = "Tree n is a BST and all val fields in it satisfy min ≤ val ≤ max" */
private static boolean isBST (TreeNode n, int min, int max) {
   if (n == null) return true.
   if (n.val < min || max < n.val) return false;
   return isBST(n.left, min, n.val-1)  &&  isBST(n.right, val+1, max);
}
```

What a beautiful, simple function!