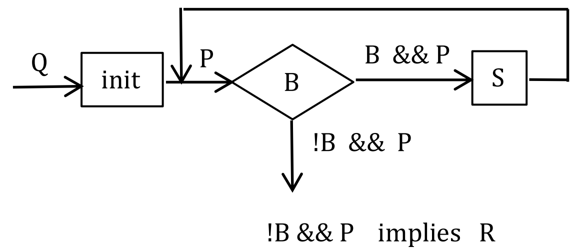
Practice with the third and fourth loopy questions

Introduction

The third and fourth loop questions are:

3. Does the repetend make progress toward termination?
   To see this, we generally give an expression whose value should decrease during execution of the repetend.
4. Does the repetend keep P true: Is {B && P} S {P} true?



Summation

We develop the repetend of a loop that adds the values in range m..n-1. Here is the relevant information:

P: s = sum of m..k-1 and $m \le k \le n$
B: $k < n$
Progress: Decrease the expression n – k

The way to decrease the expression is to add 1 to k: This means that one more value has to be added to s. Since s contains the sum of m..k-1, the next value to add is k. The repetend is:

{B && P} s= s + k; k= k+1; {P}

Here are two more exercises for you to do. The answers can be found at the end of the script for this video. Please stop the video and do them —be careful.

1. P: s = sum of k..n-1 and $0 \le k \le n$
   B: $k > m$
   Progress: decrease k

2. P: v = minimum of b[0..k] and $0 < k \le n$
   B: $k < n$
   Progress: increase k

We developed the repetend in an informal fashion. A later video shows how this repetend can almost be calculated.

Exponentiation

We are working on a loop to calculate b^c (b to the power c) for $c \ge 0$. Here are the invariant, the loop cnodition, and our way of getting closer to termination:

P: b^c = z * x^y and $y \ge 0$
B: $0 < y$
Progress: Decrease y

The simplest y to decrease y is to subtract 1 from it. To see how to maintain the invariant when subtracting 1 from y, we rewrite use this property of exponentiation: x^y = x * x^(y-1). Therefore,

z * x^y = z * x * x^(y-1).

Thus, if we subtract 1 from y, we can maintain the invariant by storing z*x in z, yielding the repetend:

{B && P} z= z*x; y= y-1; {P}

Earlier, we developed the initialization and loop condition, and the final algorithm is shown below. It makes c iterations. In lecture, we will see how this can be reduced greatly.

x= b; y= c; **while** (y != 0) {z= z*x; y= y-1; };

Answer to exercises

1. To make progress toward termination, use k= k-1; To keep invariant P true, a value has to be added to s. We use

s= s + k–1; k= k–1;   or   k= k–1; s= s + k;

2. To make progress toward termination, use k= k+1; For P to be true after that, we need v = minimum of b[0..k]. We can use either

v= Math.min(v, b[k+1]); k= k+1;   or   k= k+1; v= Math.min(v, b[k]);