Merge Sort

```
/** b[h..e] and b[e+1..k] are sorted. Stably
  * swap their values so that b[h..k] is sorted.
  */
public static void merge(
                        int b[], int h, int e, int k)
```

Earlier, we developed method `merge`, who spec is shown appears to the right. It merges two adjacent sorted segments of an array into a single sorted segment. It does so stably, meaning that two equal values remain in the same relative position. The method takes time O(`k+1-h`) and uses space O(`e+1-h`).

```
/** Sort b[h..k]. */
public static void mergeSort(int[] b, int h, int k) {
  if (h >= k)  return;  // if b[h..k] has size 0 or 1

  int e= (h + k) / 2;
  mergeSort(b, h, e);        // Sort b[h..e]
  mergeSort(b, e + 1, k);  // Sort b[e+1..k]
  merge(b, h, e, k);          // Merge the 2 segments
}
```

We use method `merge` to write recursive sorting method `mergeSort`, to the right. It is simple enough that its correctness needs little explanation.

To sort a whole array `c`, use the call

```
mergeSort(c, 0, c.length-1);.
```

Space complexity

Method `merge` requires an extra array of size `e+1-h`. Here, `e` is the average of `h` and `k`, so `mergeSort` requires space O(`(k+1-h)/2`) while the call on `merge` is being executed. That's actually O(`k+1-h`). The recursive calls also require space, but half as much, and not at the same time. So the space requirement is O(`k+1-h`).

When sorting a whole array `c`, as shown above, `mergeSort` requires space O(`c.length`). That is one drawback of `mergeSort`.

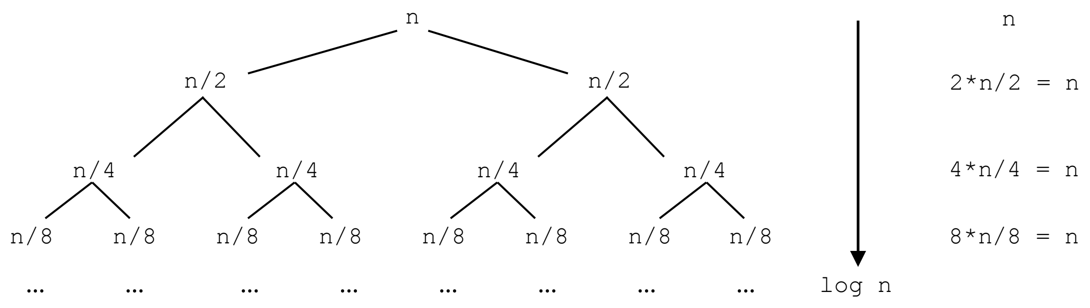
Time complexity

Consider using `mergeSort` to sort an array of size `n`. Two recursive calls are made, each to sort an array of size at most `ceil(n/2)`. Each of these recursive calls makes two further recursive calls on array segments of size at most `ceil(n/4)`, and so on.

The tree shown below depicts these recursive calls. Since at each level the size the segments being sorted is halved, the maximum depth of recursion is $\log_2$ `n`.

How much work is required at each level? At the top level, time O(`n`) is required to merge the two adjacent segements. This is shown in the column on the right. At the next level, O(`n/2`) time is required to merge two segments of size `n/4`; this is done twice, so the time required at the second level is O(`2*n/2`), which is also O(`n`). In the same way, it can be seen that the time required at each level is O(`n`).

Since there are log `n` levels, the total time required by `mergeSort` is `n` log `n`.



See the next page, please.

Remembering merge Sort and quick Sort

Some people have trouble remembering `merge Sort` and `quick Sort`. Here's one way to think about them.

- `merge Sort` relies on method `merge` to merge to adjacent sorted segments
  `merge Sort` recurses and then merges.
- `quick Sort` relies on the partition algorithm to place values on one or the other side of the pivot.
  `quick Sort` partitions and then recurses.

```
/** Sort b[h..k]. */
public static void mergeSort(int[] b, int h, int k) {
   if (h >= k)  return;  // if b[h..k] has size 0 or 1

   int e= (h + k) / 2;
   mergeSort(b, h, e);         // Sort b[h..e]
   mergeSort(b, e + 1, k);   // Sort b[e+1..k]
   merge(b, h, e, k);    // Merge b[h..e] and b[e+1..k]
}
```

```
/** Sort b[h..k]. */
public static void quickSort(int[] b, int h, int k) {
   if (h >= k)  return;  // if b[h..k] has size 0 or 1

   int j= partition(b, h, k);
   // b[h..j-1] ≤ b[j] ≤b[j+1..k]
   quickSort(b, h, j-1);       // Sort b[h..j-1]
   quickSort(b, j+1, k);       // Sort b[j+1..k]
}
```