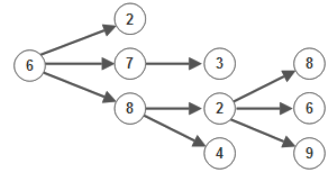


## Introduction to Trees

We often write a *list* of values 6, 7, and 3 like this: (6, 7, 3). The list could also be depicted as shown to the right, with each element having a pointer to its successor in the list.

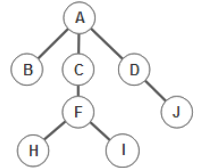


If each element can have 0 or more successors, as shown to the right, we have what we call a *tree*. Here, the first element, 6, has three successors: 2, 7, and 8. The first element's successor 2 has no successors. The first element's successor 7 has one successor.



Thus, a list is a special form of a tree.

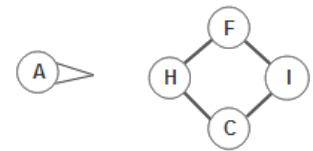
To the right, we draw a tree as computer scientists generally draw trees. No arrow heads are used; it is assumed that all lines point down. And, think of elements as *nodes*, with *edges*—the lines—connecting certain nodes. For now, think of the letters not as values but as names of the nodes. Node A at the top is called the *root* of the tree, and nodes B, H, I, and J, which have no successors, are called *leaves*. Computer scientists generally draw trees upside down!



It's important to realize we are not discussing a *data structure*. Remember: a *data structure* is an organization or format for storing and managing data, usually to make some operations efficient. Here, we're not showing how to implement trees, or how to store them in a data structure. We're just introducing type *tree*—without any operations at the moment.

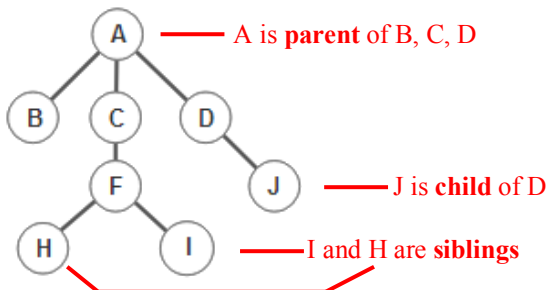
### These are not trees

The first image to the right is not a tree. A tree cannot have a node that has an edge leading from itself back to itself. The second image is also not a tree because node C has two parents, H and I. A node can have at most one parent, and only the root node can have no parents.



### Tree terminology

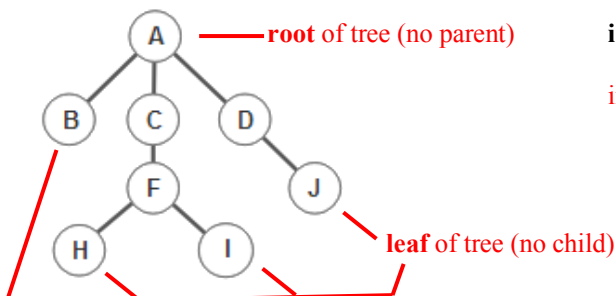
You now know that a tree consists of a bunch of *nodes*, some of which are connected by *edges*. In the tree shown below, for example, nodes A and D are connected by an edge, while nodes A and F are not connected by an edge. The diagram below also describes the notion of a *parent* of a node, a *child* of a node, and a *sibling* of a node. Based on that, the definitions of *ancestor* and *descendent* are obvious from their English meaning.



**ancestor:** parent, parent's parent, parent's parent's parent, etc.  
**descendent** or **descendant:** child, child's child, child's child's child, etc.

J's **ancestors** are D and A.  
 C's **descendants** are F, H, and I

With those definitions, we define the root of a tree, the leaves of a tree, and the internal nodes of a tree:



**internal node** or **inner node:** has a child

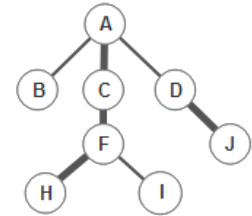
internal (inner) nodes are A, B, C, D, F

The *size* of a tree is the number of nodes in it.

The *degree* of a node is its number of children. Thus, a leaf has degree 0.

## Paths

A (downward) *path* is a sequence of nodes and edges that leads from the first node down to one of its descendants (or itself). We describe a path by giving the sequence of nodes in it. For example, in the tree to the right, we have highlighted two paths using thick edges: the path (D, J) and the path (A, C, F, H).

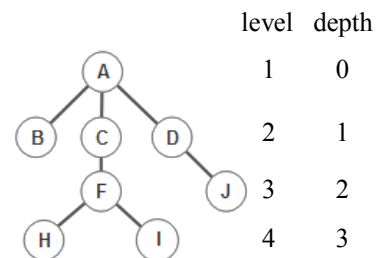


The *length* of the path is the number of edges in it. Note: (B) describes the path of length 0 from B to B; it contains no edges.

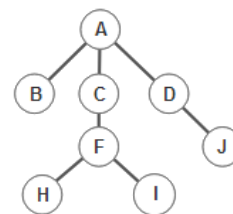
At times, one *does* talk about not only downward paths but paths from any node to any other node. For example, we could consider the path from node F to node B: (F, C, A, B). But for now, we will talk only about downward paths.

## Depth, level, height, and width

The *depth* of a node is the length of the path from the root to the node. The *level* of the node is 1 + (its depth). Sorry, this can be confusing, but that's the terminology. We won't be using the *level*.



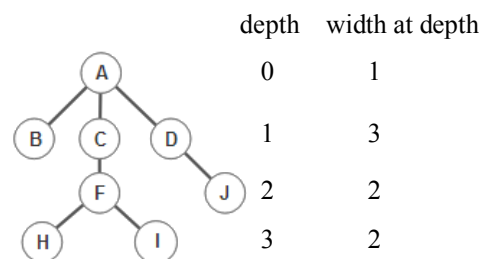
The height of a node is the length of a longest path from that node to a leaf. The height of an empty tree is 0. The height of a tree is the height of its root.



height of B, H, I, J: 0  
height of F, D: 1  
height of C: 2  
height of A: 3  
height of tree: 3

The width of a tree at depth  $d$  is the number of nodes at depth  $d$ . The width of a tree is its maximum width over all depths.

In the tree to the right, the width at depth 1 is 3 —there are three nodes at that depth: B, C, and D. That is the maximum width over all depths, so the width of the tree is 3.



## Forest

A *forest* is just a set of 0 or more disjoint trees. By disjoint we mean that no two trees in the forest have a node in common.

## Subtrees

Look at the first tree to the right. We can think of C simply as a node. It may contain values of some sort, and it has parent A and child F.

But we can also think of *subtree* C: That is, the tree whose root is C, as shown in yellow in the second tree to the right.

So, we have two views of C: it's simply a node, or it's the root of a subtree. Get used to these two ways of thinking of a node of a tree.

