Introduction to Recursion

A definition of a thing is *recursive* if its meaning includes the thing. In other words, *recursion* occurs when a thing is defined in terms of itself.

For example, the non-negative powers of 2 can be defined recursively as follows:

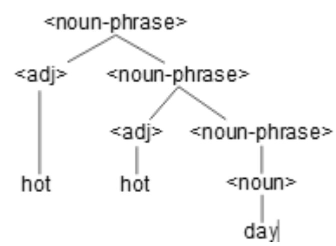
$2^0 = 1$
$2^k = 2*2^{k-1.}$ for $k > 0$

To the right, we show how this definition can be used to calculate 2^2. The calculation shows two uses of the definition in the case k > 0 and one use of the definition of 2^0.

2^2
= <by definition of 2^k with k = 2>
$2*2^1$
= <by definition of 2^k with k = 1>
$2*(2*2^0)$
= <by definition of 2^k with k = 0>
2*2*1
= <arithmetic>
4

Below we give a *grammar* for noun phrases, which could be part of a grammar that defines the syntax of English. Line (1) says that `dog` and `day` are nouns. The symbol "::=" is used simply to separate the term being defined from its definitions(s). In the same way, line (2) defines three adjectives. Line (3) defines a noun phrase to be either a noun or an adjective followed by a noun phrase. Aha! A recursive definition.

(1) `<noun> ::= dog | day`
(2) `<adj>  ::= hot | nice | sunny`
(3) `<noun-phrase> ::= <noun> | <adj> <noun-phrase>`



To the right, we give a "tree" that uses this grammar for noun phrases to show that `hot hot day` is a noun phrase. At the top, you see the use of the recursive definition "An adjective followed by a noun phrase is a noun phrase". You can see that definition used a second time. You see one use of the definition "a noun is a noun phrase", one use of the definition "`day` is a noun", and two uses of the definition "`hot` is an adjective".

Each time the recursive definition in line (3) is used, another adjective is added. Thus, a noun phrase can have 0 or more adjectives, and the same adjective can appear over and over in a noun phrase. A grammar defines syntax, not semantics.

Here's one more recursive definition, of the set of ancestors of a person `p`:

`p`'s ancestors consist of (1) `p`'s parents and (2) the ancestors of `p`'s parents

Writing recursive functions

Above, we gave a definition of the nonnegative powers of 2. Such a mathematical definition can be transformed easily, almost automatically, into a Java function, as shown to the right. You can write almost any recursive mathematical definition into a Java function in this fashion.

```
/** = 2^k.
  * Precondition k >= 0. */
public static int pow(int k) {
   if (k == 0) return 1;
   return 2 * pow(k-1);
}
```

To show you what recursive functions may look like, we present on the right below a function that returns the number of digits in the decimal representation of a number `n`.

If `n` < 10, the answer is 1 (even if `n` is 0).

The comments in the function tell you that for `n` ≥ 10, the answer is 1 plus the number of digits in `n` /10. That's the value that the return statement returns. Ae can calculate numDigits(352):

```
/** = number of digits in the decimal
  *   representation of n.
  *   e.g. numDigits(0) = 1,
  *        numDigits(35) = 2,
  *        numDigits(1356) = 4.
  *  Precondition: n >= 0. */
public static int numDigits(int n) {
   if (n < 10) return 1;
   // n = (n/10)*10  + n%10
   // So, #digits in n is #(n/10) + 1
   return numDigits(n/10) + 1;
`
```

numDigits(352)
= <with n = 352, the value of numDigits(n/10) + 1 is returned.>
numDigits(35) + 1
= <with n = 35, the value of numDigits(n/10) + 1 is returned.>
numDigits(3) + 1 + 1
= <with n = 3, the value 1 is returned>
1 + 1 + 1
= <arithmetic>
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