Insight into the invariant of the shortest path algorithm
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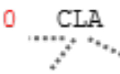
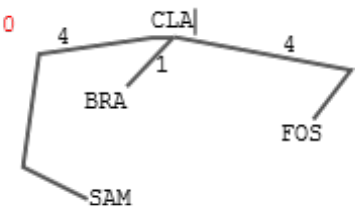
Our boat landed on the southern shore of a long lake. The few people living there called this town CLA. Three roads left the town. We decided to construct a red list of towns and their shortest distances from CLA. To start, the shortest distance from CLA to CLA is 0 miles, that's obvious! We put that on the list.

CLA, 0



We sent scouts to follow the three roads and report back. They came back. They had walked to a town named BRA, 1 mile away, to a town named SAM, 4 miles away, and to FOS, also 4 miles away.

CLA, 0

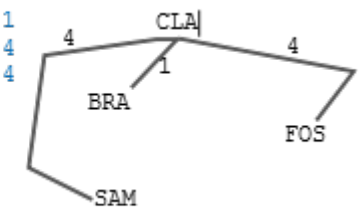


We collected the information about the distances to these three new towns in a blue table. We couldn't put them in the red table because we didn't know whether they were the shortest routes to those towns.

After studying the blue list and looking at the map, it was clear that the shortest road to BRA was 1 mile. But there might be roads to SAM and FOS, perhaps going through BRA.

CLA, 0

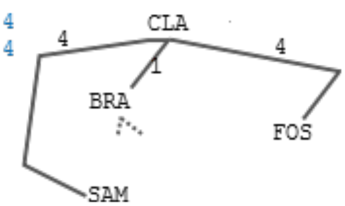
BRA, 1
SAM, 4
FOS, 4



Since we now knew the shortest road to BRA —1 mile— we moved that information to the red table. But now we had to look at roads leaving BRA. We asked the scout who went there how many roads left BRA. Two, he said. We sent scouts to find the towns at the end of those roads.

CLA, 0
BRA, 1

SAM, 4
FOS, 4

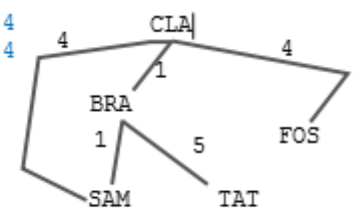


The scouts came back. There was a 1-mile road to SAM and a 5-mile road to a new town, TAT.

That was interesting! We had found a shorter road from CLA to SAM, only 2 miles long!

CLA 0
BRA, 1

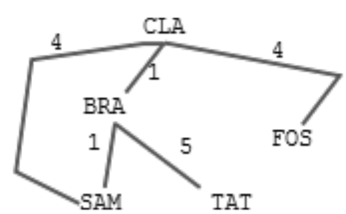
SAM, 4
FOS, 4



So, we changed the shortest known CLA-to- SAM distance to 2 and added the shortest known distance from CLA to TAT, which was 6 miles.

CLA, 0
BRA, 1

SAM, 2
FOS, 4
TAT, 6



Again, from the blue list and the map, we *knew* that the shortest distance from CLA to SAM was 2 miles. There could not be a shorter road. There were no more roads leaving CLA and BRA, and any path from CLA through TAT or FOS would be longer than 4 miles. And we began to see the following:

> Choose a node in the blue set with minimum distance (in this case, SAM). That distance *is* the shortest distance from the start node to that node.

So, we moved `SAM` to the red list and sent scouts out to find towns neighboring `SAM`. There was only one, to a new town called `BIR`. We added that information to our blue table.

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CLA, 0    FOS, 4
BRA, 1    TAT, 6
SAM, 2    BIR, 3
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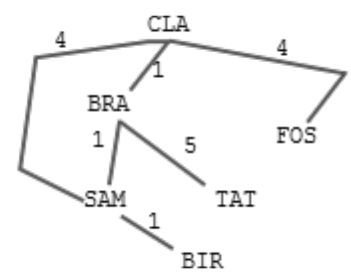

We were tired and stopped for the day.

You can see that the towns are partitioned into three sets:

- A red set, called the *settled* set S. For each town in this set, the shortest distance from the start town is known.
- A blue set, called the *frontier* set F. These towns have been visited at least once. The shortest distance from the start town to each of these towns is known over roads that were traversed, but there *might* be shorter roads that haven't yet been traversed.
- A black set, called the *far-off* set. These towns haven't been visited yet.