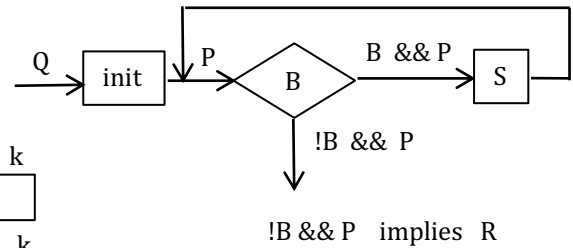
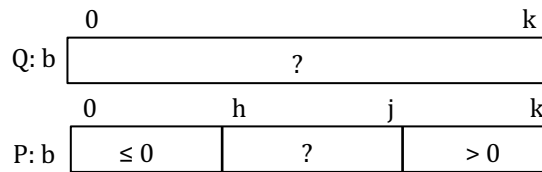


Practice with the first loopy question using array diagrams

We look at two examples of checking the first loopy question.

Consider this precondition and invariant, and remember that we are looking for initialization init to make this Hoare-triple true: $\{Q\} \text{init} \{P\}$.



To make P look like Q , segments $b[0..h-1]$ and $b[j+1..k]$ must be empty. Remembering our formula *Follower – First* for the number of values in a range, to make the first segment empty requires $h - 0 = 0$, so we use the assignment $h = 0$; . Similarly, to make $b[j+1..k]$ empty, we need $k - j = 0$, so we use the assignment $j = k$. Thus, we have:

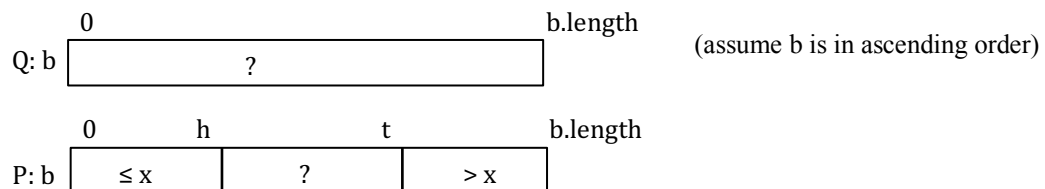
$\{Q\} \ h = 0; j = k; \{P\}$

There is another way to see what to assign to h and j , without using the formula. It's nice to have two different ways of figuring out the initialization, for we can do it both ways and, if we get different results, we know there is an error and we can check our work.

Look at Q : the first unknown is $b[0]$. Look at P : the first unknown in $b[h]$. Therefore, we need $h = 0$ initially. The last unknown in Q is $b[k]$. The last unknown in P is $b[j]$. Evidently, we need $j = k$ initially.

Binary search

This example comes from an algorithm to search a sorted array b for a value x . Precondition Q and invariant P are given below. This example is similar enough to the previous one that you should be able to do it yourself. Please do that. The answer is at the end of the pdf of this script.



Answer to question Segment $b[0..h]$ has to be empty. Using the formula *Follower – First*, we see that $h + 1 - 0 = 0$ has to be true, so we use the assignment $h = -1$; . Similarly, we need $b.length - (t+1) = 0$ initially, so we use the assignment $t = b.length - 1$; . Therefore, the initialization is:

$h = -1; t = b.length - 1;$