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The simple proof of this theorem is due to Prof Curran Muhlberger of CS, Cornell.

Proof. Switch the direction of each edge of the graph. Now, every node has outdegree > 0.

Start with any node; follow an edge leaving it; on the new node, follow an edge leaving it; etc. This process can go on forever, since each node has an edge leaving it.

Since the number of nodes is finite, some node must be visited twice. So, stop the process the first time a node `N`, say, is repeated. We have this a cycle `N -> N1 -> ... -> N`.

Reverse the edges of the graph again, getting back the original graph. The cycle `N -> N1 -> ... -> N` in the reversed graph is transformed into this cycle in the original graph: `N <- N1 <- ... <- N`.

Q.E.D. (Quit End Done)