Merging two adjacent sorted segments

To the left below are two adjacent sorted segments, `b[h..e]` and `b[e+1..k]`. We want an algorithm to merge them in stable fashion into the single sorted segement `b[h..k]` shown to the right.

```
    h         e         k          h                  k
b | 2 4 4 5 7 | 1 2 3 6 7 |     b | 1 2 2 3 4 4 5 6 7 7 |
```

To do this, first copy `b[h..e]` into another array `c[0..e-h]`, as shown below. We have written ? for values in `b[h..e]` not because values aren't there but because we don't care what is in that segment after the copy.

```
    h         e         k          0       e-h
b | ? ? ? ? ? | 1 2 3 6 7 |     c | 2 4 4 5 7 |
```

The goal now is to merge `b[e+1..k]` and `c[0..e-h]` in stable fashion into `b[h..k]`. We show three steps. When an integer is moved, we replace it by ?

Start with this:

```
    h         e         k          0       e-h
b | ? ? ? ? ? | 1 2 3 6 7 |     c | 2 4 4 5 7 |
```

Move smaller of `b[e+1]` and `c[0]`:

```
    h         e         k          0       e-h
b | 1 ? ? ? ? | ? 2 3 6 7 |     c | 2 4 4 5 7 |
```

Move `c[0]`, not `b[e+2]` (stable sort):

```
    h         e         k          0       e-h
b | 1 2 ? ? ? | ? 2 3 6 7 |     c | ? 4 4 5 7 |
```

Move smaller of `b[e+2]` and `c[1]`:

```
    h         e         k          0       e-h
b | 1 2 2 ? ? | ? ? 3 6 7 |     c | ? 4 4 5 7 |
```

That should be enough to give you the idea: At each iteration of a loop, the smallest unmoved (non-?) element in the two segments `b[e+1..k]` and `c[0..e-h]` is moved to the next available position (the first ?) in b[h..].

Inorder to write the loop, we need a loop invariant. We need three variables `i`, `j`, and `m` to indicate three positions in the arrays. We define them below; to the right we show them after the last move shown above.

1. The position `i` in which to place the next merged integer in `b[h..]`.
2. The position `j` of the first unmoved value in `b[e+1..k]`.
3. The position `m` of the first unmoved value in `c[0..e-h]`.

```
    h     i   e     j   k
b | 1 2 2 ? ? | ? ? 3 6 7 |

    0  m   e-h
c | ? 4 4 5 7 |
```

The loop invariant has four parts:

Invariant: `b[h..i-1]` contains the moved values, stably sorted,
`b[j..k]` contains the unmoved values in `b[e+1..k]`,
`c[m..e-h]` contains the unmoved values in `c[0..e-h]`,
`b[h..i-1] ≤ b[j..k]` and `b[h..i-1] ≤ c[m..e-h]`

The algorithm is shown to the right. After truthifying the invariant by initializing `i`, `j`, and `m`, a while-loop moves values as long as both segments `b[j..k]` and `c[m..e-h]` contain a value to move. This makes the code a bit easier to write and to read.

A second loop then moves remaining values in `c[m..e-h]`. There is no need to move remaining values in `b[j..k]` because, if there are any, one can verify that they are already in the correct place at the end of `b[j..k]`.

```
i= h;  j= e+1;  m= 0;

// inv: shown above

// move values as long as b[j..k] and c[m..]
// are not empty
while (j <= k  &&  m <= e-h) {
   if (c[m] <= b[j]) { b[i]= c[m];  m++;  i++; }
   else { b[i]= b[j];  j++;  i++; }
}

while (m <= e-h) {b[i]= b[m];  m++;  i++; }
```

Space and time complexity

The time complexity is O(`k+1-h`). Extra space is used for array `c`, so the space is O(`e+1-h`).