

## Practice with the second loopy question

### Introduction

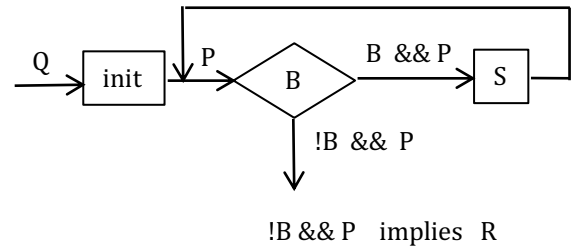
We practice finding a loop condition  $B$  by using the second loopy question: Is  $!B \ \&\& \ P \Rightarrow R$  true? Thus, we look for  $!B$  that makes  $!B \ \&\& \ P \Rightarrow R$  true and complement  $!B$  to get  $B$ .

Here's the invariant  $P$  and postcondition for our first example.

$P$ :  $s$  is the sum of  $m..k-1$  and  $m \leq k \leq n$   
 $R$ :  $s$  is the sum of  $m..n-1$

Knowing that  $P$  is true, and doing some pattern matching with  $P$  and  $R$ , we see that  $R$  will be true if  $k = n$ . Therefore,  $!B$  is  $k = n$ , so the loop condition  $B$  is  $k \neq n$ . Looking at the restriction on  $k$  in invariant  $P$ , we can write the loop condition at  $k < n$  if we want. Thus, we use either

**while** ( $k \neq n$ ) { ... }      or      **while** ( $k < n$ ) { ... }



### A second example

Here are the invariant and postcondition for a loop to calculate the minimum value in array segment  $b[0..n-1]$ :

$P$ :  $v = \text{minimum of } b[0..k-1]$  and  $0 \leq k \leq n$   
 $R$ :  $v = \text{minimum of } b[0..n-1]$

Using reasoning like we did the first example, you can see that we get the same answer for  $B$  as in the previous example.

**while** ( $k \neq n$ ) { ... }      or      **while** ( $k < n$ ) { ... }

### Computing $z = b^c$

Here are the invariant and postcondition for a loop to store  $b^c$  in  $z$ , given  $c \geq 0$ :

$P$ :  $b^c = z * x^y$  and  $y \geq 0$   
 $R$ :  $z = b^c$

Again doing pattern matching, we see that  $R$  will be true when  $P$  is true and  $x^y = 1$ . That last formula,  $x^y = 1$ , is true, when  $y = 0$ . So our loop condition is  $y \neq 0$ :

**while** ( $y \neq 0$ ) { ... }

### Exercises

In the two examples below, find the loop condition. Answers are at the end of the pdf script for this video.

1.  $P$ :  $s$  is the sum of  $k..n-1$  and  $m \leq k \leq n$   
 $R$ :  $s$  is the sum of  $m..n-1$
2.  $P$ :  $v = \text{minimum of } b[k..n]$  and  $0 \leq k \leq n$   
 $R$ :  $v = \text{minimum of } b[0..n]$  and  $0 \leq k \leq n$

### Answers

In the first exercise, doing pattern matching on  $P$  and  $R$ , we see that  $k = m$  is needed. Therefore the loop condition is  $k \neq m$ . This can be written as  $m < k$  if you want, since  $m \leq k \leq n$ :

**while** ( $k \neq m$ ) { ... }      or      **while** ( $m < k$ ) { ... }

In the second exercise, pattern matching on  $P$  and  $R$ , we see that  $k = 0$  is needed. Therefore, the loop condition is  $k \neq 0$ . This can be written as  $0 < k$  if you want, since  $0 \leq k \leq n$ :

**while** ( $k \neq 0$ ) { ... }      or      **while** ( $0 < k$ ) { ... }