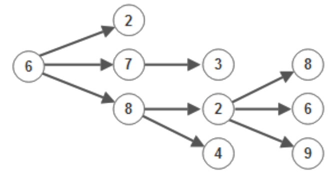
Introduction to Trees

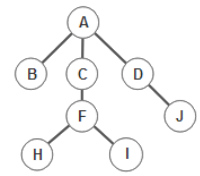
We often write a *list* of values 6, 7, and 3 like this: (6, 7, 3). The list could also be depicted as shown to the right, with each element having a pointer to its successor in the list.

If each element can have 0 or more successors, as shown to the right, we have what we call a *tree*. Here, the first element, 6, has three successors: 2, 7, and 8. The first element's successor 2 has no successors. The first element's successor 7 has one successor.



Thus, a list is a special form of a tree.

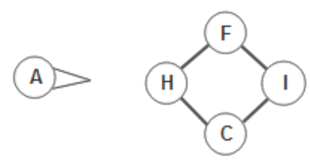
To the right, we draw a tree as computer scientists generally draw trees. No arrow heads are used; it is assumed that all lines point down. And, think of elements as *nodes*, with *edges* —the lines— connecting certain nodes. For now, think of the letters not as values but as names of the nodes. Node `A` at the top is called the *root* of the tree, and nodes `B`, `H`, `I`, and `J`, which have no successors, are called *leaves*. Computer scientists generally draw trees upside down!



It's important to realize we are not discussing a *data structure*. Remember: a *data structure* is an organization or format for storing and managing data, usually to make some operations efficient. Here, we're not showing how to implement trees, or how to store them in a data structure. We're just introducing type *tree* —without any operations at the moment.

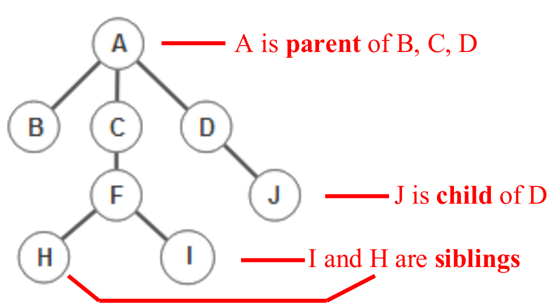
These are not trees

The first image to the right is not a tree. A tree cannot have a node that has an edge leading from itself back to itself. The second image is also not a tree because node `C` has two parents, `H` and `I`. A node can have at most one parent, and only the root node can have no parents.



Tree terminology

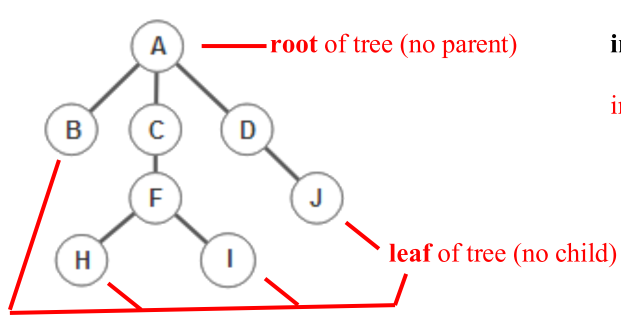
You now know that a tree consists of a bunch of *nodes*, some of which are connected by *edges*. In the tree shown below, for example, nodes `A` and `D` are connected by an edge, while nodes `A` and `F` are not connected by an edge. The diagram below also describes the notion of a *parent* of a node, a *child* of a node, and a *sibling* of a node. Based on that, the definitions of *ancestor* and *descendent* are obvious from their English meaning.



ancestor: parent, parent's parent, parent's parent's parent, etc.
descendent or **descendant**: child, child's child, child's child's child, etc.

J's **ancestors** are D and A.
C's **descendents** are F, H, and I

With those definitions, we define the root of a tree, the leaves of a tree, and the internal nodes of a tree:



internal node or **inner node**: has a child

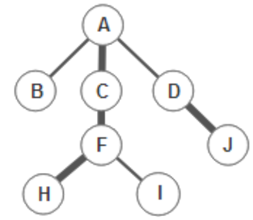
internal (inner) nodes are A, B, C, D, F

The *size* of a tree is the number of nodes in it.

The *degree* of a node is its number of children. Thus, a leaf has degree 0.

Paths

A (downward) *path* is a sequence of nodes and edges that leads from the first node down to one of its descendents (or itself). We describe a path by giving the sequence of nodes in it. For example, in the tree to the right, we have highlighted two paths using thick edges: the path (`D`, `J`) and the path (`A`, `C`, `F`, `H`).

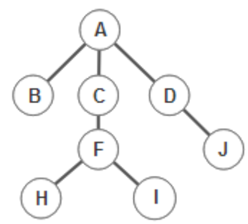


The *length* of the path is the number of edges in it. Note: (`B`) describes the path of length 0 from `B` to `B`; it contains no edges.

At times, one *does* talk about not only downward paths but paths from any node to any other node. For example, we could consider the path from node `F` to node `B`: (`F`, `C`, `A`, `B`). But for now, we will talk only about downward paths.

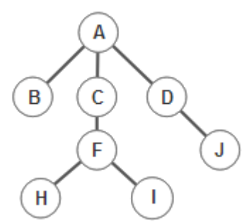
Depth, level, height, and width

The *depth* of a node is the length of the path from the root to the node. The *level* of the node is 1 + (its depth). Sorry, this can be confusing, but that's the terminology. We won't be using the *level*.



level	depth
1	0
2	1
3	2
4	3

The height of a node is the length of a longest path from that node to a leaf. By definition, an empty tree has height -1. The height of a tree is the height of its root.



height of `B`, `H`, `I`, `J`: 0

height of `F`, `D`: 1

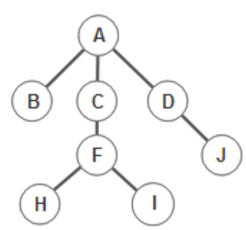
height of `C`: 2

height of `A`: 3

height of tree: 3

The width of a tree at depth `d` is the number of nodes at depth `d`. The width of a tree is its maximum width over all depths.

In the tree to the right, the width at depth 1 is 3 —there are three nodes at that depth: `B`, `C`, and `D`. That is the maximum width over all depths, so the width of the tree is 3.



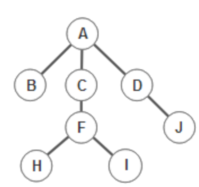
depth	width at depth
0	1
1	3
2	2
3	2

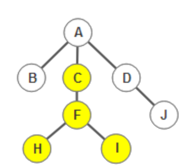
Forest

A *forest* is just a set of 0 or more disjoint trees. By disjoint we mean that no two trees in the forest have a node in common.

Subtrees

Look at the first tree to the right. We can think of `C` simply as a node. It may contain values of some sort, and it has parent `A` and child `F`.





But we can also think of *subtree* `C`: That is, the tree whose root is `C`, as shown in yellow in the second tree to the right.

So, we have two views of `C`: it's simply a node, or it's the root of a subtree. Get used to these two ways of thinking of a node of a tree.